RESEARCH-ARTICLE

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Abstract

The use of machine and deep learning techniques to predict outcomes in legal proceedings is a highly debated topic among legal scholars and policymakers. These technologies have the potential for supporting judicial decision-making, assist litigants, and analyze biases within the legal process. However, challenges remain, notably in the reluctance of judges to adopt such tools due to concerns over judicial independence, the normative correctness, accuracy, and robustness of algorithmic decisions, and the transparency of AI systems. It is claimed that methods are needed to validate AI-based judicial predication mechanisms. This paper contributes to addressing these challenges by developing a rich computational normative framework for judicial case-based reasoning grounded in Defeasible Deontic Logic. We explore legal CBR, focusing on inconsistencies and incomplete knowledge within case bases, and emphasize the importance of normative explanations to ensure transparency and justification in legal decision-making. By reconstructing CBR and deontic explanations, we provide a formal mechanism for validating AI-based judicial predictions where cases are represented using a fine-grained deontic language.

CCS Concepts

• **Theory of computation** → **Proof theory**; **Automated reasoning**; • **Applied computing** → **Law**.

Keywords

Deontic Logic, Defeasible Logic, Legal Explanation, Case-based Reasoning

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1 Introduction

The growing interest in leveraging *machine learning* and *deep learning* to predict legal outcomes is evident in both academic literature and discussions among policymakers [see, e.g., 6, 9, 21, 43, 52, 55, 57].

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These advanced AI techniques are seen as potential tools for various applications in the legal field. For instance, they can serve as algorithmic decision predictors that:

(1) **Support Judges:** These tools can assist judges by providing data-driven insights and predictions for individual cases, potentially improving the consistency and efficiency of judicial decision-making.
(2) **Assist Litigants:** Litigants can use these AI tools to estimate their likelihood of winning a case, helping them make informed decisions about whether to pursue legal action or seek settlements.
(3) **Examine Biases:** AI can analyze and identify biases in legal decision-making processes, shedding light on disparities related to ethnicity, gender, socioeconomic status, and other factors [9].

Despite the potential benefits, the integration of AI in judicial contexts faces significant challenges. Judges, for example, show reluctance towards adopting AI tools due to several reasons:

(1) **Judicial Independence:** There is a concern that AI could undermine the independent exercise of judicial power, potentially influencing decisions unduly.
(2) **Normative Issues and Accuracy:** Judges and legal scholars question whether the outcomes generated by AI predictors are normatively correct, accurate, and robust. The alignment of AI decisions with legal and ethical standards is not always clear.
(3) **Transparency and Explainability:** As is well-known, AI algorithms often lack transparency. This “black box" nature makes it difficult for users to understand how predictions are made, which is problematic in a field that requires explainable and accountable decision-making.

Given these challenges, it is crucial to develop robust methods and normative benchmarks for validating AI-based judicial prediction mechanisms. Validation ensures that these AI systems are reliable, accurate, and ethical in their predictions [37].

In response to this need, our paper proposes a general framework for *judicial deontic case-based reasoning* (CBR). CBR has played an important role in AI & law research. While different models have been adopted, factor-based representations have been most popular, where *factors are features of cases that support possible judicial outcomes* (i.e., *plaintiff wins*, *defendant wins*). This line of research started with HYPO [5, 53], in whose framework a factor-based representation enables various patterns of analogical reasoning: citing a precedent, distinguishing it, and reasoning *a fortiori*. The importance of legal CBR for machine learning has been advanced in [55].

John Horty has studied the logical properties of legal CBR, developing a method for determining whether a certain decision is consistent or inconsistent with a case base [32]. Applying the idea of *a fortiori* reasoning, he has argued that any decision in a new case would be inconsistent with a precedent if that decision is different from the precedent's, even though the new case has a more (or equally) inclusive set of facts favoring the precedent's conclusion and a less (or equally) inclusive set of factors against that conclusion. Further developments include the so-called *reason model*, wherein the reason distinguished in a case is the specified set of elements that the judge considers to provide sufficient support to the case decision, outweighing the factors to the contrary.

Recent research, such as in [12, 16, 17, 56], has argued that some of the usual assumptions in legal CBR can be relaxed or weakened in order to deal with more realistic scenarios. In particular, in the context of the reason model:

- **Inconsistent case bases -** The concept of consistency assumes that the initial background case base is consistent, which is not a realistic assumption. Horty [33] outlined a broader interpretation of the reason model's constraint concept, extending it to also encompassing inconsistent case bases; recent investigations have been accordingly developed (such as [12, 48]);
- **Incomplete knowledge -** The idea of complete knowledge in the context of legal CBR means that any case base is such that all situations have been decided in favor of the defendant or the plaintiff and, for example, that all features in the base have a direction (i.e., they are pro or con factors, which means that they support the plaintiff or the defendant). This assumption can also be unrealistic [16, 46].

Our framework work with incomplete and potentially inconsistent case bases leveraging **Defeasible Deontic Logic** (DDL), which is a logic that accommodates in an efficient way deontic reasoning with incomplete and potentially conflicting normative information [30].

By integrating this component, we aim to develop AI systems that are not only predictive but also transparent and normatively aligned with judicial standards. This paper somehow complements the research effort of [17], where it was proposed an inference mechanism with incomplete knowledge for the identification of factors, among the features that are present in a case base, and for the determination of their direction [7], i.e., if they are in favor of the plaintiff or the defendant. That approach was based on the logical framework described in [39, 40] and adopted the view that a legal case-based reasoner is nothing but a binary classifier [16, 41].

The limits of the above-mentioned lines of research are that they flatten the reasoning mechanism by simply connecting features and outcomes—a case is a rule linking the facts of the case and its outcome, being it a decision in favor either of the plaintiff or of the defendant—and they do not offer a deontic description of cases. In [19] a preliminary work was developed using propositional Defeasible Logic, which was used to spot arguments supporting the decision but without any deontic analysis. Our contribution makes a significant step forward by using DDL and by concatenating different types of factual and deontic rules supporting the different judicial decisions and, above all, the relevant deontic outcomes behind those decisions. This way, we contribute with the following results:

Result 1: As highlighted in recent works such as [18], an account of judicial cases may require that deontic qualifications and other normative positions of the parties are formally described. We achieve this in the context of CBR by extending the model with no computational cost, thanks to DDL.

Result 2: The use of DDL rules allows us to capture the idea of *ratio decidendi*, which can be defined as the underlying justification structure in a judicial decision. This structure consists of a sequence of reasoning steps determining the precedential outcome of a decision and encompasses a set of potentially conflicting arguments [11, 42, 51]. We consider it here as a possibly complex but minimal set of deontic reasoning steps grounded in the specific facts of the case, which consider conflicts and justify the judicial decision; the ratio decidendi can also be used to predict possible undecided cases.

Result 3: Once we have devised a reasoning mechanism to capture the procedures internal to each judicial case and its ratio decidendi, we introduce a second reasoning layer on top of the first one. This second layer intertwines with the first layer and is meant to reconstruct how different judicial precedents interact, enabling us to resolve undecided cases and explain them.

The layout of the article is as follows. Section 2 provides a variant of DDL used in the remainder of the paper and that allows for the account of directed deontic statements in judicial cases. Section 3 reconstructs the reason-based standard model of CBR developed by Horty. Section 4 explains how to use DDL as a tools for first layer to model the reasoning mechanism internal to judicial cases. Section 5 proposes proof theory for the second layer and for inferring judicial decisions for undecided cases; the proof theory relaxes consistency and completeness assumptions of case bases. Section 6 uses the previous framework to argue in what sense a decided case can be explained in a case base. Some conclusions end the paper.

2 Background 1: DDL

Defeasible Logic (DL) [4, 45] is a simple, flexible, and efficient rule-based non-monotonic formalism, whose strength lies in its constructive proof theory that allows drawing meaningful conclusions from a (potentially) conflicting and incomplete knowledge base. In non-monotonic systems, more accurate conclusions can be obtained when more pieces of information become available. Many variants of DL have been proposed for the logical modelling of different application areas, especially for legal reasoning (for an overview of the literature, see [30]).

In this research, we focus on the Defeasible Deontic Logic's framework advanced in [24], that allows us to model a large variety of normative concepts, as well as to determine what prescriptive behaviours are in force in a given situation. The logic is integrated with directed deontic statements, following the approach in [28].

We now present the main elements of the logic, and start by defining the language of a defeasible deontic theory. Let PROP = $\{a, b, c, \dots\}$ be a set of propositional atoms, Lab = $\{r, s, w, \} \dots$ be a set of arbitrary labels (the names of the rules), and Agents = $\{\alpha, \delta, \pi, \dots\}$ a finite set of agents including the plaintiff π and the defendant δ. Lower-case Roman letters denote literals. Accordingly, PLit = PROP ∪ $\{\neg l \mid l \in \text{PROP}\}$ is the set of *plain literals*, the set of *deontic literals* is ModLit = $\{\Box l, \neg\Box l, \Diamond l, \neg\Diamond l \mid l \in \text{PLit} \land \Box \in \{\mathsf{O}, \mathsf{O}^{\alpha}_{\beta} | \alpha, \beta \in \text{Agents}\}, \Diamond \in \{\mathsf{P}, \mathsf{P}^{\pi}_{\delta} | \pi, \delta \in \text{Agents}\}\}$, and finally, the set of *literals* is Lit = PLit ∪ ModLit.

Example 2.1. Consider for example the modal literals $\mathsf{O}^{\alpha}_{\beta} l$ and $\mathsf{P}^{\pi}_{\delta} l$: the former one means that l is obligatory such that α is the right holder and β is the duty bearer of such an obligation, the latter one means that l is permitted such that π is the right holder and δ is the addressee of such a permission.

The *complement* of a literal l is denoted by $\sim l$. We have three cases: (1) l is a plain literal such that, if l is a positive literal p then $\sim l$ is $\neg p$, and if l is a negative literal $\neg p$ then $\sim l$ is p; (2) l is a modal literal $\Box l$ where $\Box \in \{\mathsf{O}, \mathsf{O}^{\alpha}_{\beta}\}$ such that $\sim\Box l = \{\neg\mathsf{O}l, \mathsf{O}{\sim}l, \neg\mathsf{P}l, \mathsf{P}{\sim}l, \neg\mathsf{O}^{\alpha}_{\beta} l, \mathsf{O}^{\alpha}_{\beta}{\sim}l, \neg\mathsf{P}^{\alpha}_{\beta} l, \mathsf{P}^{\alpha}_{\beta}{\sim}l\}$; (3) l is a modal literal $\Diamond l$ where $\Diamond \in \{\mathsf{P}, \mathsf{P}^{\alpha}_{\beta}\}$ such that $\sim\Diamond l = \{\mathsf{O}{\sim}l, \neg\mathsf{P}l, \mathsf{O}^{\alpha}_{\beta}{\sim}l, \neg\mathsf{P}^{\alpha}_{\beta} l\}$.

Definition 2.2 (Defeasible Deontic Theory). A *defeasible deontic theory* D is a tuple

$$(F, R, >)$$

where

- F is the finite set of facts,
- R is the finite set of rules, and
- $>$ is a binary relation over R (called *superiority relation*).

The set of facts $F \subseteq$ Lit denotes simple pieces of information that are considered to be always true. A theory is meant to represent our normative knowledge, where the rules encode the norms at stake, and the set of facts corresponds to "*factual information*", or "*given deontic positions*" (as, for instance, indisputable obligations or deontic positions imported from other higher-ranked normative systems). The rules are used to conclude the evidential facts, obligations and permissions that hold given the set of facts.

The set R is finite and contains two *types* of rules[1]: *defeasible rules* and *defeaters*. Rules are also of two *kinds*:

- *Evidence rules* (non-deontic) in R^{E} model evidence or factual statements;
- *Deontic rules* model prescriptive behaviours, which are either *obligation rules* in $R^{\Box}$ where $\Box \in \{\mathsf{O}, \mathsf{O}^{\alpha}_{\beta}\}$ determining when and which obligations are in force, or *permission rules* in $R^{\Diamond}$ representing *strong* (or *explicit*) permissions where $\Diamond \in \{\mathsf{P}, \mathsf{P}^{\alpha}_{\beta}\}$.

Lastly, the *superiority* relation, $> \subseteq R \times R$, solves conflicts among rules' conclusions. Formally:

Definition 2.3 (Rule). A *rule* is an expression of the form $r \colon A(r) \hookrightarrow_X C(r)$, where

(1) $r \in$ Lab is the unique name of the rule;
(2) $A(r) \subseteq$ Lit is the set of antecedents;
(3) An arrow $\hookrightarrow \in \{\Rightarrow, \rightsquigarrow\}$ denotes, respectively, defeasible rules and defeaters;
(4) $X \in \{\mathsf{E}, \mathsf{O}, \mathsf{P}, \mathsf{O}^{\pi}_{\delta}, \mathsf{P}^{\pi}_{\delta}\}$;
(5) $C(r)$ is the consequent, which is a single plain literal $l \in$ PLit.

[1]DDL usually also adopts strict rules, which handle the monotonic part of theories. For the sake of simplicity, we omit them here.

If $X = \mathsf{E}$ then the rule is used to derive non-deontic literals (evidence statements), whilst if X is O, $\mathsf{O}^{\pi}_{\delta}$, P, or $\mathsf{P}^{\pi}_{\delta}$ then the rule is used to derive deontic conclusions (prescriptive statements). The conclusion $C(r)$ is a single literal.

We use some standard abbreviations on rule sets. The set of defeasible rules is $R_{\Rightarrow}$, the set of defeaters is $R_{\rightsquigarrow}$. $R^{\Box}[l]$ is the set of rules with conclusion l and modality $\Box$.

Example 2.4. Consider the following rules:

$$r : \Rightarrow_{\mathsf{O}} good\_faith$$
$$s : contract\_void, \mathsf{O}good\_faith, good\_faith \Rightarrow_{\mathsf{P}^{\delta}_{\pi}} \neg pay$$
$$w : \neg contract\_void \Rightarrow_{\mathsf{O}^{\pi}_{\delta}} pay.$$

Rule r says that it is obligatory by default to act in good faith. Rule s states that, if the contract is void, it is obligatory to act in good faith and we act accordingly, then the defendant δ is permitted of not paying towards the plaintiff π. Rule s states that, if the contract is not void, the defendant δ has the obligation of paying in favor of the plaintiff π. As usual in DDL, the logical intuition is that if rules are applicable, the consequent is derived with the appropriate modality of the rule. For example, if *contract_void* and *good_faith* are in the set F of facts, then we can intuitively draw the following derivation

1.	$\mathsf{O}good\_faith$	(Rule r)
2.	$\mathsf{P}^{\delta}_{\pi}\neg pay$	(1., facts, Rule s)

Formally, DDL proof theory is based on proof tags.

Definition 2.5 (Tagged modal formula). Let $\Box \in$ ModLit. A *tagged modal formula* is an expression of the form $\pm\partial_{\Box} l$, with the following meanings

- $+\partial_{\Box} l$: l is *defeasibly provable* (short, *provable*) with mode $\Box$,
- $-\partial_{\Box} l$: l is *defeasibly refuted* (short, *refuted*) with mode $\Box$.

Accordingly, the meaning of $+\partial_{\mathsf{O}} p$ is that p is provable as an obligation, and $-\partial_{\mathsf{P}^{\delta}_{\pi}} \neg p$ is that we have a refutation for the permission of $\neg p$ in favour of δ towards π. Similarly, for the other modalities.

Definition 2.6 (Proof). Given a defeasible deontic theory D, a *proof P of length m in D* is a finite sequence $P(1), P(2), \ldots, P(m)$ of tagged modal formulas, where the proof conditions defined in the rest of this paper hold. $P(1..n)$ denotes the first n steps of P.

The notational convention '$D \vdash \pm\partial_{\Box} l$' means that there is a proof P for $\pm\partial_{\Box} l$ in D.

Core notions in DL are that of *applicability/discardability* of a rule. This paper uses the one developed in [25, 27]. As knowledge in a defeasible theory is circumstantial, given a defeasible rule like '$r \colon a, b \Rightarrow_{\Box} c$', there are four possible scenarios: the theory defeasibly proves both a and b, the theory proves neither, the theory proves one but not the other. Naturally, only in the first case, where both a and b are proved, we can use r to *support/try to conclude* $\Box c$.

Definition 2.7 (Applicability). Assume a defeasible deontic theory $D = (F, R, >)$.

(1) Rule $r \in R$ is *applicable* at $P(n+1)$, iff for all $a \in A(r)$
   (a) if $a \in$ PLit, then $+\partial_{\mathsf{E}} a \in P(1..n)$,

(b) if $a = \Box q$, then $+\partial_\Box q \in P(1..n)$, with $\Box \in \{\mathsf{O}, \mathsf{P}, \mathsf{O}^\alpha_\beta, \mathsf{P}^\alpha_\beta\}$,
(c) if $a = \neg\Box q$, then $-\partial_\Box q \in P(1..n)$, with $\Box \in \{\mathsf{O}, \mathsf{P}, \mathsf{O}^\alpha_\beta, \mathsf{P}^\alpha_\beta\}$.

(2) Rule $r \in R^{\mathsf{E}}$ is *applicable* at $P(n+1)$ for $+\partial_\Box l$ where $\Box \in \{\mathsf{O}, \mathsf{P}, \mathsf{O}^\alpha_\beta, \mathsf{P}^\alpha_\beta\}$, iff
(d) $r \in R^{\mathsf{E}}[l]$,
(e) for all $a \in A(r)$, $a \in$ PLit, and $A(r) \neq \emptyset$,
(f) $+\partial_\Box a \in P(1..n)$.

Note that *discardability* of a rule is obtained by applying the principle of *strong negation* to the definition of applicability [26], and thus are omitted.

Condition (1) establishes that (1a, 1b) every positive literal has been proved, and (1c) every deontic negative literal has been rejected at a previous derivation steps.

Condition 2 formalises *rule conversion*, which is a way to derive a conclusion with a certain modality by using rules for another modality [27]. In our case, evidence rules can be used to derive obligations and permissions. This is formalised by Condition 2, which reads very easily: there must be an evidence rule in which none of the antecedents is an obligation or permission (hence all of them are plain literals), and, if all such antecedents are proved with the same deontic mode—as obligations or permissions—then the rule becomes applicable in supporting its conclusion as an obligation (resp. permission). Therefore, if we change rule r of above as '$r\colon a, \mathsf{O}b \Rightarrow_{\mathsf{E}} c$', then this new r cannot be used through conversation and may only be used to support 'an evidence c'.

For space reasons, we provide conditions for $+\partial_{\mathsf{O}}$, $+\partial_{\mathsf{P}}$, $+\partial_{\mathsf{O}^\alpha_\beta}$, $+\partial_{\mathsf{P}^\alpha_\beta}$ only, since (i) $-\partial_{\mathsf{O}}$, $-\partial_{\mathsf{P}}$, $-\partial_{\mathsf{O}^\alpha_\beta}$, $-\partial_{\mathsf{P}^\alpha_\beta}$ can be obtained by applying the strong negation principle to the positive counterparts [3], and (ii) the proof conditions for evidential statements are the standard ones for propositional Defeasible Logic [4].

Definition 2.8 (Obligation Proof Conditions). Let $\Box \in \{\mathsf{O}, \mathsf{O}^\alpha_\beta | \alpha, \beta \in \text{Agents}\}$, $\Diamond \in \{\mathsf{P}, \mathsf{P}^\pi_\delta | \pi, \delta \in \text{Agents}\}$.

$+\partial_\Box l$: If $P(n+1) = +\partial_\Box l$ then
(1) $\Box l \in F$, or
(2) $\Box{\sim}l, \neg\Box l, \Diamond{\sim}l, \neg\Diamond l \notin F$ and
(3) $\exists s \in R^\Box_\Rightarrow[l] \cup R^{\mathsf{E}}_\Rightarrow[l]$ s.t.
(3.1) s is applicable if $s \in R^\Box_\Rightarrow[l]$, or
s is applicable for $+\partial_\Box l$ if $s \in R^{\mathsf{E}}_\Rightarrow[l]$, and
(3.2) $\forall w \in R^\Box[{\sim}l] \cup R^\Diamond[{\sim}l] \cup R^{\mathsf{E}}[{\sim}l]$ either
(3.2.1) w is discarded if $w \in R^\Box[{\sim}l] \cup R^\Diamond[{\sim}l]$, or
w is discarded for $+\partial_\Box l$ if $w \in R^{\mathsf{E}}_\Rightarrow[l]$, or
(3.2.2) $\exists z \in R^\Box[l] \cup R^{\mathsf{E}}_\Rightarrow[l]$ s.t.
(3.2.2.1) z is applicable if $z \in R^\Box_\Rightarrow[l]$ or
z is applicable for $+\partial_\Box l$ if $z \in R^{\mathsf{E}}_\Rightarrow[l]$, and
(3.2.2.2) $z > w$.

Definition 2.9 (Permission Proof Conditions). Let $\Box \in \{\mathsf{O}, \mathsf{O}^\alpha_\beta | \alpha, \beta \in \text{Agents}\}$, $\Diamond \in \{\mathsf{P}, \mathsf{P}^\pi_\delta | \pi, \delta \in \text{Agents}\}$.

$+\partial_\Diamond l$: If $P(n+1) = +\partial_\Diamond l$ then
(1) $\Diamond l \in F$, or
(2) $-\partial_\Box{\sim}l \in P(1..n)$, or
(3) $\Box{\sim}l, \neg\Diamond l \notin F$ and
(4) $\exists s \in R^\Diamond_\Rightarrow[l] \cup R^{\mathsf{E}}_\Rightarrow[l]$ s.t.
(4.1) s is applicable if $s \in R^\Diamond_\Rightarrow[l]$, or
s is applicable for $+\partial_\Diamond l$ if $s \in R^{\mathsf{E}}_\Rightarrow[l]$, and
(4.2) $\forall w \in R^\Box[{\sim}l] \cup R^{\mathsf{E}}[{\sim}l]$ either
(4.2.1) w is discarded if $w \in R^\Box[{\sim}l]$, or
w is discarded for $+\partial_\Diamond l$ if $w \in R^{\mathsf{E}}_\Rightarrow[l]$, or
(4.2.2) $\exists z \in R^\Diamond[l] \cup R^{\mathsf{E}}_\Rightarrow[l]$ s.t.
(4.2.2.1) z is applicable if $z \in R^\Diamond_\Rightarrow[l]$ or
z is applicable for $+\partial_\Diamond l$ if $z \in R^{\mathsf{E}}_\Rightarrow[l]$, and
(4.2.2.2) $z > w$.

The set of positive and negative conclusions of a theory is called *extension*. The extension of a theory is computed based on the literals that appear in it; more precisely, the literals in the Herbrand Base of the theory $HB(D) = \{l, {\sim}l \in \text{PLit} | \, l \text{ appears in } D\}$.

Definition 2.10 (Extension). The *extension* $E(D)$ of a defeasible deontic theory D is

$$E(D) = (+\partial_{\mathsf{E}}, -\partial_{\mathsf{E}}, +\partial_{\mathsf{O}}, -\partial_{\mathsf{O}}, \{+\partial_{\mathsf{O}^y_x}\}^{y\in\text{Agents}}_{y\in\text{Agents}}, \{-\partial_{\mathsf{O}^y_x}\}^{y\in\text{Agents}}_{y\in\text{Agents}}, +\partial_{\mathsf{P}}, -\partial_{\mathsf{P}}, \{+\partial_{\mathsf{P}^y_x}\}^{y\in\text{Agents}}_{y\in\text{Agents}}, \{-\partial_{\mathsf{P}^y_x}\}^{y\in\text{Agents}}_{y\in\text{Agents}}),$$

where $\pm\partial_\Box = \{l \in HB(D) | \, D \vdash \pm\partial_\Box l\}$, with $\Box \in \{\mathsf{E}, \mathsf{O}, \mathsf{O}^y_x, \mathsf{P}, \mathsf{P}^y_x\}$.

THEOREM 2.11. *[See [24, 27]] Given a defeasible theory D, its extension $E(D)$ can be computed in time polynomial to the size of the theory.*

PROOF. The result directly follows from observing that Definitions 2.8 and 2.9 are simple instances of Definition 9 and 11 in [24]. So the algorithms of [24] fits for our case as well. □

3 Background 2: Judicial Cases

One of the most influential frameworks of judicial reasoning is the case-based reasoning / precedential constraint proposed by John Horty [31–33, 36]. Horty has reconstructed factor-based representations of CBR [5, 53] and developed such factor-based models into a theory of precedential constraints, i.e., of how a new case must be decided, in order to preserve consistency in the case law.

Horty takes into account the fact that judges may provide explicit reasons for their choice of a certain outcome. This leads to the distinction between the result and the reason model of precedents. In the first model, the message conveyed by the case is only that all factors supporting the case-outcome (pro-factors) outweigh all factors against that outcome (con-factors). In the second, the message is that the factors for the case outcome indicated by the judge outweigh all factors against that outcome. We say *result model* for "the factor-based result model of precedential constraint" and *reason model* for "the factor-based reason model of precedential constraint".

In standard Horty's model the language is based on the set of factors $Atm_0 = Plt \cup Dfd$ where Plt and Dfd are disjoint sets of factors favoring the plaintiff and defendant respectively. In addition, let $Val = \{1, 0, ?\}$ where elements stand for *plaintiff wins*, *defendant wins* and *indeterminacy* respectively. Let $Dec = \{\mathsf{t}(x) : x \in Val\}$ and read $\mathsf{t}(x)$ as "the actual decision/outcome (of the judge/classifier) takes value x". An outcome $\mathsf{t}(1)$ or $\mathsf{t}(0)$ means that, the judge is predicted to decide for the plaintiff or for the defendant. The outcome $\mathsf{t}(?)$ means either outcome would be consistent: the judge may develop the law in one direction or the other. The set Atm denotes $Atm_0 \cup Dec$.

A set $s \subseteq Atm_0$ is called a *fact situation.* A set of atoms X is called a *reason* for an outcome (decision) x if it a set of factors all favoring the same outcome: $X \subseteq Plt$ is a reason for 1 and $X \subseteq Dfd$ is a reason for 0. We can think of reasons X favoring an outcome x as a rule $X \Rightarrow x$. For readability, we make a convention that, for $x \in \{0, 1\}$, let $\overline{x} = 1 - x$ and $\overline{\overline{x}} = x$. Moreover, let $Atm_0^x = Plt$ if $x = 1$, and $Atm_0^x = Dfd$ if $x = 0$.

In the reason model, a *precedent case* (precedent) is a triple $c = (s, X, x)$, where $s \subseteq Atm_0$, $X \subseteq s \subseteq Atm_0^x, x \in \{0, 1\}$. In plain words, $s \cap Atm_0^x$ contains all *pro-factors* in s for x, while $s \cap Atm_0^{\overline{x}}$ all *con-factors* in s for x. X is the *reason of the case*, namely a subset of the pro-factors which the judge considers sufficient to support that outcome, relative to all con-factors in the case.

A *case base CB* (for reason model) is a set of precedential cases. When the reason contains all pro-factors within the situation (i.e., when $c = (s, s \cap Atm_0^x, x)$) all such factors are considered equally decisive. If a case base only contains cases of this type, we obtain what Horty calls "the result model", and note such a case base CB^{res}. The class of all *CBs* and $CB^{\mathrm{res}}s$ are noted $\mathbf{CB}$ and $\mathbf{CB}^{\mathrm{res}}$ respectively.

A case base can be inconsistent when two precedents map the same fact situation to different outcomes. Another scenario is that a consistent case base becomes inconsistent after *update*, namely after expanding it with some new case. Hence maintaining consistency is the crucial concern of legal case-based reasoning.

The following definitions are based on [41] (which elaborate on those in [33, 49]).

Definition 3.1 (Preference relation derived from a case). Let $c = (s, X, x)$ be a case. Then the preference relation $\prec_c$ derived from c is s.t. for any two reasons Y, Y' favoring x and $\overline{x}$ respectively, $Y' \prec_c Y$ iff $Y' \subseteq s \cap Atm_0^{\overline{x}}$ and $X \subseteq Y$.

Definition 3.2 (Preference relation derived from a case base). Let CB be a case base. Then the preference relation $\prec_{CB}$ derived from CB is s.t. for any two reasons Y, Y' favoring x and $\overline{x}$ respectively, $Y' \prec_{CB} Y$ iff $\exists c \in CB$ s.t. $Y' \prec_c Y$.

Definition 3.3 ((In)consistency). A case base CB is inconsistent, if there are two reasons Y, Y' s.t. $Y' \prec_{CB} Y$ and $Y \prec_{CB} Y'$. CB is consistent if it is not inconsistent.

Definition 3.4 (Precedential constraint). Let CB be a consistent case base, X is a reason for x in CB and applicable in a new fact situation s', i.e. $X \subseteq s'$. Updating CB with the new case (s', X, x) meets the precedential constraint, iff $CB \cup \{(s', X, x)\}$ is still consistent.

There is more than one way to satisfy the precedential constraint, depending on how the precedents in CB interacts with the new case. The requirement of consistency dictates the outcome when the "a fortiori" constraint applies: if reason X for x outweighs (i.e., is stronger than) reason $s \cap Atm_0^{\bar{x}}$, *a fortiori* any superset of X outweighs any subset of $s \cap Atm_0^{\bar{x}}$, so that only by deciding for x rather than for $\overline{x}$ consistency is maintained.

4 Layer 1: Deontic Logic of Cases

4.1 The Basic Variant

Following [19], the reconstruction of Horty's intuition in DDL requires some changes. A major difference with respect to reason-based standard model is that it is not meaningful in DDL to a priori distinguish between factors which favor the plaintiff and factors which favor the defendant: DDL is precisely meant to be an inferential mechanism to establish the direction of factors, so $Atm_0 \subseteq \mathrm{Lit}$ is the set of *factors* of the case language and $Dec \subset \mathrm{Lit}$.

A second major difference is that we can exploit deontic outcomes to have a more fine-grained analysis of cases:

Remark 1. *We may observe the following:*

(1) **Plaintiff's claims explicit** - *in the DDL framework we can make explicit plaintiff's complaint, which consists in set of claims of the plaintiff behind its action, i.e., a set* $\mathtt{Claim}_{\mathsf{E}}$ *of factual statements* $\{f_1, f_2 \dots\}$ *and a set* $\mathtt{Claim}_{\Box}$ *of deontic statements* $\{\Box p, \Diamond q, \dots\}$*;*

(2) **Obligatoriness of judicial outcome** - *for saying that a certain outcome* $x \in \{0, 1, ?\}$ *is obtained in a case c, we have to prove that* $t(x)$ *is a DDL-conclusion in c and* $t(x)$ *must be obligatory, i.e., that* $+\partial \mathrm{O} t(x)$ *holds in the case;*

(3) **Necessity of plaintiff's claims** - *for saying that* $+\partial \mathrm{O} t(1)$ *holds in the case c, we must show that all elements in* $\mathtt{Claim}_{\mathsf{E}}$ *and* $\mathtt{Claim}_{\Box}$ *are proved in c and that they are necessary for* $+\partial \mathrm{O} t(1)$*;*

(4) **Deontic outcomes and burden of proof** - *unless we shift in the burden of proof, we should assume that*
- *if the plaintiff wins in c, then we prove either* $+\partial \mathrm{O} t(1)$ *or* $+\partial_{\mathrm{O}^{\pi}_{\delta}} t(1)$*;*
- *if the defendant wins in c, then we prove either* $+\partial \mathrm{O} t(0)$*,* $+\partial_{\mathrm{O}^{\delta}_{\pi}} t(0)$*,* $-\partial \mathrm{O} t(1)$ *or* $-\partial_{\mathrm{O}^{\pi}_{\delta}} t(1)$*.*

Perhaps, the most challenging aspects are those under points (3) and (4): the former requires to check if the conclusions in plaintiff's claim are necessary to derive $+\partial \mathrm{O} t(1)$, the latter leads to identifying further cases, in comparison to Definition 2.8, to prove the obligatoriness of the outcome in favour of the defendant.

For the first problem, we can exploit the definition of literal contraction as elaborated in [10, 29], which reconstruct AGM contraction in DL. We have however to distinguish different contractions for the various types of conclusions.

Definition 4.1 (Contraction in DDL). Let $\mathtt{Claim}_{\mathsf{E}} = \{f_1, \dots, f_n\}$ and $E(D)$ be the extension of a Defeasible Deontic Theory $D = (F, R, >)$.

$$D^{-}_{\mathtt{Claim}_{\mathsf{E}}} = \begin{cases} D & \text{if } f_1, \dots, f_n \notin \partial^{+}_{\mathsf{E}}(D) \\ (F, R', >') & \text{otherwise} \end{cases}$$

where

$$R' = R \cup \{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_n \rightsquigarrow_{\mathsf{E}} {\sim} f_i \mid i \in \{1, \dots, n\}\}$$
$$>' = > - \{s > r \mid r \in R' - R\}. \quad (1)$$

Similarly, let $\texttt{Claim}_{\Box} = \{\Box p_1, \ldots, \Box p_n, \Diamond q_1, \ldots, \Diamond q_m\}$, $\Box \in \{\mathrm{O}, \mathrm{O}^{\alpha}_{\beta} | \alpha, \beta \in \text{Agents}\}$, $\Diamond \in \{\mathrm{P}, \mathrm{P}^{\pi}_{\delta} | \pi, \delta \in \text{Agents}\}$ and $\# \in \{\Box, \Diamond\}$.

$$D^-_{\{\Box p_1,\ldots,\Box p_n\}} = \begin{cases} D & \text{if } p_1, \ldots, p_n \notin \partial^+_{\Box}(D) \\ (F, R', >') & \text{otherwise} \end{cases}$$
where
$$R' = R \cup \{p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n \rightsquigarrow_{\#} \sim p_i \mid i \in \{1, \ldots, n\}\}$$
$$>' = > - \{s > r \mid r \in R' - R\} \tag{2}$$

and

$$D^-_{\{\Diamond q_1,\ldots,\Diamond q_m\}} = \begin{cases} D & \text{if } q_1, \ldots, q_n \notin \partial^+_{\Diamond}(D) \\ (F, R', >') & \text{otherwise} \end{cases}$$
where
$$R' = R \cup \{q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_n \rightsquigarrow_{\Diamond} \sim q_i \mid i \in \{1, \ldots, n\}\}$$
$$>' = > - \{s > r \mid r \in R' - R\} \tag{3}$$

Definition 4.2 (Necessity of Plaintiff's Claims). Let $D = (F, R, >)$ be a Defeasible Deontic Theory, and $\texttt{Claim}_{\mathsf{E}} = \{f_1, \ldots, f_n\}$ and $\texttt{Claim}_{\Box} = \{\Box p_1, \ldots, \Box p_n, \Diamond q_1, \ldots, \Diamond q_m\}$ be the claims of the plaintiff. $\texttt{Claim}_{\mathsf{E}}$ and $\texttt{Claim}_{\Box}$ are *necessary for the outcome* $+\partial_{\Box} t(1)$, where $\Box \in \{\mathrm{O}, \mathrm{O}^{y}_{x}\}$ iff

- $D \vdash +\partial_{\Box} t(1)$;
- $D \vdash +\partial_{\Box} p_1, \ldots, D \vdash +\partial_{\Box} p_n$ and $D \vdash +\partial_{\Diamond} q_1, \ldots, D \vdash +\partial_{\Diamond} q_m$;
- $D' \nvdash +\partial_{\Box} t(1)$ if $D' = D^-_{\texttt{Claim}_{\mathsf{E}} \cup \texttt{Claim}_{\Box}}$.

Similar considerations apply for defining if the claims are necessary (can be used) for the purposes of the defendant.

Since DDL is the logical engine for case-based reasoning, if we stick to the idea that $Val \in \{1, 0\}$, then the reason model based on DDL simply defines precedents as follows:

Definition 4.3 (Case, basic model). A *case* is a 5-tuple $c = (D, X, \texttt{Claim}_{\mathsf{E}}, \texttt{Claim}_{\Box}, x)$ where

(1) D is a Defeasible Deontic Theory $(F, R, >)$ where $F \subseteq Atm_0$ is the *fact situation* of c;
(2) $x \in \{0, 1\}$, $\Box \in \{\mathrm{O}, \mathrm{O}^{\pi}_{\delta}\}$, and $X \subseteq F$ such that, for any $D' = (X', R, >)$ where $X' \subset X$, either
  (a) if $x = 1$, then $D \vdash +\partial \Box t(1)$ and $D' \vdash -\partial \Box t(1)$, or
  (b) if $x = 0$, then $D \vdash +\partial \Box t(0)$ and $D' \vdash -\partial \Box t(0)$, or $D \vdash -\partial \Box t(1)$ and $D' \vdash +\partial \Box t(0)$; and
(3) $\texttt{Claim}_{\mathsf{E}}$ and $\texttt{Claim}_{\Box}$ are *necessary for the outcome* $+\partial_{\Box} t(x)$.

Notice that the set X of reasons must be minimal [20]. Likewise, the properties of contraction in Definition 4.1 guarantees minimality [10, 29].

Definition 4.3 allows for providing a deontic reconstruction of some well-known intuitions from Horty reason-based model.

However, as we alluded to, since we use DDL as an inferential engine, we do not a priori partition factors into those which favor the plaintiff and those which favor the defendant.

Definition 4.4 (Reason). Let $F \subseteq Atm_0$ be a *fact situation* of the case $c = (D = (F, R, >), X, \texttt{Claim}_{\mathsf{E}}, \texttt{Claim}_{\Box}, x)$. We say that that the set X is a *reason* for the outcome x.

Definitions 3.1-3.4 can be simply adapted on the basis of Definitions 4.3-4.4.

4.2 The General Case

We can go beyond the basic model. Following [12, 16, 56], we argued in [17] that some of the usual assumptions in legal CBR can be relaxed in order to deal with more realistic scenarios. In particular, in the context of the reason model we can abandon the requirements that case bases be consistent and complete.

A first immediate consequence in our framework of the choice of abandoning consistency and completeness is a revision of Definition 4.3, thus admitting that, in any case $c = (D, X, \texttt{Claim}_{\mathsf{E}}, \texttt{Claim}_{\Box}, x)$, $x \in \{0, 1, ?\}$ (in other words, we may explicitly consider undecided cases in *CB*). Another consequence is that we need to prioritize cases when we have to deal with inconsistencies in the case base: conceptually, the order can be built using, e.g., the ranking of courts involved in the previous cases (e.g., supreme courts prevail over courts of appeal, which prevail over courts of first instance) [15] or the importance of values promoted [8].

Since we have to extend the machinery of DDL and embed CBR in it, we adjust our notation by stating that, for $x \in \{0, 1, ?\}$, let $\overline{x} = y$ such that $y \in \{0, 1, ?\} - \{x\}$. In other words, $\forall y \in \{0, 1, ?\}$ we have that $\sim y = \{0, 1, ?\} - \{y\}$. We also state that $\sim(s, x) = (s, \overline{x})$.

Definition 4.5 (Case, general model). A *case* is a 5-tuple $c = (D, X, \texttt{Claim}_{\mathsf{E}}, \texttt{Claim}_{\Box}, x)$ where

(1) D is a Defeasible Deontic Theory $(F, R, >)$ where $F \subseteq Atm_0$ is the *fact situation* of c;
(2) $x \in \{0, 1, ?\}$, $\Box \in \{\mathrm{O}, \mathrm{O}^{\pi}_{\delta}\}$, and $X \subseteq F$ such that, for any $D' = (X', R, >)$ where $X' \subset X$, either
  (a) if $x = 1$, then $D \vdash +\partial \Box t(1)$ and $D' \vdash -\partial \Box t(1)$, or
  (b) if $x = 0$, then (i) $D \vdash +\partial \Box t(0)$ and $D' \vdash -\partial \Box t(0)$, or (ii) $D \vdash -\partial \Box t(1)$ and $D' \vdash +\partial \Box t(0)$;
  (c) if $x =?$, then (i) $D \vdash +\partial_{\Box} t(?)$ and $D' \vdash +\partial_{\Box} t(y)$ where $y \in \{1, 0\}$, or (ii) $D \vdash -\partial_{\Box} t(1)$ and $D \vdash -\partial_{\Box} t(0)$; and
(3) $\texttt{Claim}_{\mathsf{E}}$ and $\texttt{Claim}_{\Box}$ are *necessary for the outcome* $+\partial_{\Box} t(x)$.

We say that D is the theory of a case c if $c = (D, X, \texttt{Claim}_{\mathsf{E}}, \texttt{Claim}_{\Box}, x)$.

Minimality of reasons is ensured also in this case. The main difference with respect to Definition 4.3 is that we handle here also the deontic case where the outcome is ?. This amounts to spotting to sub-cases: the information available in the case positively shows that either outcome in $\{1, 0\}$ would be deontically consistent (2c-(i)), or that the information is not sufficient to support any of them (2c-(ii)).

We can now identify a proof-theoretic notion of *ratio decidendi* of a case c in our framework:

Definition 4.6 (Ratio decidendi). Let $c = (D, X, \texttt{Claim}_{\mathsf{E}}, \texttt{Claim}_{\Box}, x)$ be a case and $\mathcal{P}_c$ be the set of proofs based on D for $+\partial_{\Box} t(x)$. The *ratio decidendi* of c is a minimal subset of $\mathcal{P}_c$.

5 Layer 2: Deontic Logic of Case Bases

Once we have a mechanism for fully representing deontic reasoning within the single cases, we have to devise an appropriate reasoning method for settling undecided cases by considering how cases in our case base *CB* interact. In our proposal, this second layer thus amounts to a new defeasible system to be added on top on the first layer: indeed, we relax the assumption that our case base *CB* is consistent and so defeasibility is needed.

As mentioned above, following [15, 17] we introduce a priority relation among cases in *CB*. Here, for the sake of simplicity we assume that the priority is given, but nothing prevents to adopt Horty's approach and reframe in DDL his procedure for defining a preference order.

Definition 5.1 (Priority among cases). Let *CB* the case base. For any two cases $c, c' \in CB$ we write $c \ll c'$ to denote that *c prevails over* c'. The relation $\ll$ is the priority relation for *CB*.

As usual in legal CBR, analogical reasoning means that precedents can be cited to support the decision of new cases on the basis of the similarity between fact situations. We thus need first of all to establish the similarity between sets of factors[2].

Definition 5.2 (Similarity between sets of factors). Let $D = (\emptyset, R, >)$ be a Defeasible Deontic Theory and $s, s' \subset Atm_0$. The *degree of similarity between s and s' relative to D*, noted $sim_D(s,s')$, is defined as follows:

$$sim_D(s,s') = sim_{D_{\mathsf{E}}}(s,s') + sim_{D_{\Box}}(s,s')$$

such that

$$sim_{D_{\mathsf{E}}}(s,s') = |\{\phi \in Atm_0 : (s, R, >) \vdash +\partial_{\mathsf{E}}\phi \text{ iff } (s', R, >) \vdash +\partial_{\mathsf{E}}\phi\}| \tag{4}$$

$$sim_{D_{\Box}}(s,s') = |\{\phi \in Atm_0 : (s, R, >) \vdash +\partial_{\Box}\phi \text{ iff } (s', R, >) \vdash +\partial_{\Box}\phi\}| \tag{5}$$

$$sim_{D_{\Box}}(s,s') > 0. \tag{6}$$

We state that $sim_D(s,s') > 0$, otherwise $\neg sim_D(s,s')$.

Notice that Definition significantly elaborates in DDL the idea proposed in [17, 41]. We should observe that we distinguish two dimensions of similarity: the factual one ("how many factors do both cases proof-theoretically share?") and the deontic one ("how many deontic conclusions do both cases proof-theoretically share?"). In addition, we assume that deontic similarity cannot be zero, because it would mean that cases diverge in regard to their *rationes decidendi* (see Definition 4.6).

Another key notion of our model is the one of *decidable fact situation for an outcome x*, which consists in developing a deontic defeasible reasoning mechanism working on many cases and exploiting the inferential power of DDL to predict the decision of an undecided fact situation.

Definition 5.3 (Undecided Fact Situations). Let *s* be a fact situation. We say that *s* is *undecided* in *CB* iff $\nexists c = (D = (s, R, >), X, \mathtt{Claim}_{\mathsf{E}}, \mathtt{Claim}_{\Box}, x) \in CB$ such that $x \in \{0, 1\}$.

[2] The dual notion of distance can be defined. In accordance with [14], it is nothing but the Hamming distance, which counts the cardinality difference of features' values. [16] has shown that, in this context, building similarity between cases via Hamming distance coincides with building it via subset inclusion relation (i.e., via shared properties as done in HYPO). Of course, more refined notions of similarity can be developed, which e.g. contextualize this measure of similarity with respect to specific logical conclusions.

If a fact situation is undecided, it can still be decidable on the basis of *CB*. The idea is to identify a case c' supporting $+\partial_{\Box}t(x)$ which maximizes in *CB* similarity of fact situations such that, if there is a possible attack (i.e., another case c'' with maximal similarity but supporting the opposite judicial outcome $+\partial_{\Box}t(\overline{x})$), then we must check that c', or another case c''' maximally similar and supporting $t(x)$, has priority over c''. It is not hard to see that the following proof conditions correspond to the standard ones for propositional DL, but they use cases to draw conclusions and not not rules. Notice that such proof conditions inherit from DL its skeptical nature, which means that unsolved conflicts between cases prevent to make decidable undecided fact situations.

Definition 5.4 (Proof Conditions for Decidable Fact Situations). We define that a fact situation *s* is decidable in *CB* in favor of *x*, i.e., $+\Delta_{CB}(s, x)$, as follows:

$+\Delta_{CB}(s, x)$: If $+\Delta_{CB}(s, x)$ then either
- (1) $\exists c' = ((s', R', >'), X', \mathtt{Claim}'_{\mathsf{E}}, \mathtt{Claim}'_{\Box}, x) \in CB$ s.t. $sim_{(\emptyset,R',>')}(s,s')$ is maximal in *CB* and
- (2) $\forall c'' = ((s'', R'', >''), X'', \mathtt{Claim}''_{\mathsf{E}}, \mathtt{Claim}''_{\Box}, \overline{x})$ either
  - (2.1) $sim_{(\emptyset,R'',>'')}(s,s'')$ is not maximal in *CB*, or
  - (2.2) $\exists c''' = ((s''', R''', >'''), X''', \mathtt{Claim}'''_{\mathsf{E}}, \mathtt{Claim}'''_{\Box}, x) \in$ $\in CB$ s.t. $sim_{(\emptyset,R''',>''')}(s,s''')$ is maximal in *CB* and $c''' \ll c''$.

$-\Delta_{CB}(s, x)$: If $-\Delta_{CB}(s, x)$ then
- (1) $\nexists c' = ((s', R', >'), X', x) \in CB$ s.t. $sim_{(\emptyset,R',>')}(s,s')$ is maximal in *CB* or
- (2) $\exists c'' = ((s'', R'', >''), X'', \overline{x})$ s.t.
  - (2.1) $sim_{(\emptyset,R'',>'')}(s,s'')$ is maximal in *CB*, and
  - (2.2) $\forall c''' = ((s''', R''', >'''), X''', x) \in CB$ either $sim_{(\emptyset,R''',>''')}(s,s''')$ is not maximal in *CB* or $c''' \not\ll c''$.

As a notational convention, we write $CB \vdash +\Delta(s, x)$ and $CB \vdash -\Delta(s, x)$ to denote, respectively, $+\Delta_{CB}(s, x)$ and $-\Delta_{CB}(s, x)$.

Accordingly, the notion of judicial proof is straightforwardly adapted from the intuition behind Definition 2.6. The main difference is that, for this second layer, we can simply define a proof as a relation linking a set of cases with a judicial outcome for an undecided fact situation.

Definition 5.5 (Decision Proof). A proof for a decision $\pm\Delta_{CB}(s, x)$ is a relation $\Gamma \vdash \pm\Delta_{CB}(s, x)$ where Γ is the set of cases satisfying Definition 5.4.

One may think of the inference of a decidable factual situation $+\Delta_{CB}(s, x)$ as a way to show that there is a set of defeasible deontic theories, including $(s, R, >)$, supporting the conclusion of $t(x)$.

The following is a trivial result showing that, if a fact situation cannot be decided, then we cannot prove any appropriate case supporting $t(1)$ or $t(0)$ and the other way around.

Proposition 5.6 (Coherence in *CB* and Undecidable Fact Situations). *Let CB be a case base.*

$+\Delta_{CB}(s, ?)$: *If* $+\Delta_{CB}(s, ?)$ *then* $-\Delta_{CB}(s, 1)$ *and* $-\Delta_{CB}(s, 0)$.

$-\Delta_{CB}(s, ?)$: *If* $-\Delta_{CB}(s, ?)$ *then either* $+\Delta_{CB}(s, 1)$ *or* $+\Delta_{CB}(s, 0)$.

The following result can also be easily proved.

Proposition 5.7. *Let CB be a case base. If* $+\Delta_{CB}(s,x)$ *and* $+\Delta_{CB}(s,\overline{x})$*, then* $\exists c = ((s,R,>),X,\mathtt{Claim}_\mathsf{E},\mathtt{Claim}_\Box,x)$ *and* $\exists c' = ((s,R',>'),X',\mathtt{Claim}'_\mathsf{E},\mathtt{Claim}'_\Box,\overline{x}) \in CB$ *where both* $c \not\ll c'$ *and* $c' \not\ll c$*. Otherwise, the fact situation s is undecided in CB and, if* $+\Delta_{CB}(s,x)$*, then* $-\Delta_{CB}(s,\overline{x})$.

Proposition 5.7 states that we prove an inconsistency if *CB* contains two cases supporting opposite outcomes but based on theories having the same fact situation.

Definition 5.8 (Case Base Extension). Given a case base *CB*, we define the set of positive and negative conclusions of *CB* as its *case base extension*:

$$E(CB) = (+\Delta, -\Delta),$$

where $\pm\Delta = \{(s,x) | \, CB \vdash \pm\Delta_{CB}(s,x)\}$.

6 Judicial Explanations

Definitions 4.3 and 4.5 identify the reasons for judicial outcomes as the *minimal set of factors* (i.e., facts in DDL original terminology) needed for supporting the inference of x via plaintiff's claims (with $t(1)$) or despite such claims (with $t(0)$) . This idea is imported also in Definition 5.4, which simply uses different defeasible deontic theories as rules with respect to those that are usually employed in the standard proof theory of DDL. This idea corresponds to a basic idea of *normative explanation* of a decision [22, 23, 41, 54].

As recalled in [54], a normative explanation is an explanation where norms are crucial: in legal decision-making, this means to explain why a legal conclusion ought to be the case on the basis of certain norms and facts [1, 23, 38, 41, 47, 50]. In this context, we have two ways for approaching this idea: by focusing on the facts that are used to draw certain normative conclusions, and by considering the arguments used for conclusions, which also use information constituting the ratio decidendi. We adopt the second route and extend [54]'s definition. The idea of judicial explanation can be defined accordingly.

Definition 6.1 (Judicial explanation). Let *CB* a case base. The set of cases Γ is a *judicial explanation* $\mathtt{Expl}((s,x),CB)$ in *CB* for (s,x) iff

- $\Gamma \vdash \pm\Delta_{CB}(s,x)$;
- Γ is a minimal set in *CB*.

Example 6.2. Consider the following very simple case base *CB*:

$$c_1 = (D_1 = (F_1,R_1,>_1),X_1,\mathtt{Claim}_{\mathsf{E}1},\mathtt{Claim}_{\Box 1},0)$$
$$c_2 = (D_2 = (F_2,R_2,>_2),X_2,\mathtt{Claim}_{\mathsf{E}2},\mathtt{Claim}_{\Box 2},1)$$
$$c_3 = (D_3 = (F_3,R_3,>_3),X_3,\mathtt{Claim}_{\mathsf{E}3},\mathtt{Claim}_{\Box 3},0)$$

where

$$\mathtt{Claim}_{\mathsf{E}1} = \{p\}$$
$$\mathtt{Claim}_{\Box 1} = \{\mathsf{O}b\}$$

$$\mathtt{Claim}_{\mathsf{E}2} = \emptyset$$
$$\mathtt{Claim}_{\Box 2} = \{\mathsf{P}q\}$$

$$\mathtt{Claim}_{\mathsf{E}3} = \{a\}$$
$$\mathtt{Claim}_{\Box 3} = \{\mathsf{P}z\}$$

Assume D_1 is as follows:

$$F_1 = \{i,l\},$$
$$R_1 = \{s_1\colon \Rightarrow_\mathsf{O} t(0),\ s_2\colon l \Rightarrow_{\mathsf{P}^\pi_\delta} t(1),\ s_3\colon \mathsf{O}b,p \Rightarrow_\mathsf{O} t(1),$$
$$s_4\colon d \Rightarrow_{\mathsf{O}^\delta_\pi} t(0),\ s_5\colon i \Rightarrow_\mathsf{E} d,\ s_6\colon \Rightarrow_\mathsf{E} p,\ s_7\colon \Rightarrow_\mathsf{O} b\}$$
$$>_1 = \{\langle s_2, > s_1\rangle, \langle s_3 > s_1\rangle, \langle s_4 > s_3\rangle, \langle s_4 > s_2\rangle\}.$$

Assume D_2 is as follows:

$$F_2 = \{i\},$$
$$R_2 = \{s_1\colon \Rightarrow_\mathsf{O} t(0),\ r_2\colon i,\mathsf{P}q \Rightarrow_{\mathsf{O}^\pi_\delta} t(1),\ r_3\colon \Rightarrow_\mathsf{P} q\}$$
$$>_2 = \{\langle r_2 > s_1\rangle\}.$$

Assume D_3 is as follows:

$$F_3 = \{l\},$$
$$R_3 = \{w_3\colon \mathsf{P}z,a \Rightarrow_\mathsf{O} t(0),\ w_4\colon \Rightarrow_\mathsf{P} z,\ w_5\colon l \Rightarrow_\mathsf{E} a\}$$
$$>_3 = \emptyset.$$

Let us consider the undecided factual situation $F_4 = \{i,l,o\}$ and assume $c_1 \ll c_2$ and $c_2 \ll c_3$.

It is easy to see that

$$\begin{aligned} E(D_1) = (&+\partial_\mathsf{E} = \{i,l,d,p\},\\ &-\partial_\mathsf{E} = \emptyset,\\ &+\partial_\mathsf{O} = \{t(0),b\},\\ &-\partial_\mathsf{O} = \{t(1)\},\\ &+\partial_{\mathsf{O}^\delta_\pi} = \{t(0)\},\\ &+\partial_\mathsf{P} = \{t(0),b\},\\ &-\partial_\mathsf{P} = \{t(1),{\sim}b\}\\ &+\partial_{\mathsf{P}^\pi_\delta} = \emptyset) \end{aligned}$$

$$\begin{aligned} E(D_2) = (&+\partial_\mathsf{E} = \{i\},\\ &-\partial_\mathsf{E} = \emptyset,\\ &+\partial_\mathsf{O} = \emptyset,\\ &-\partial_\mathsf{O} = \{t(0)\},\\ &+\partial_{\mathsf{O}^\pi_\delta} = \{t(1)\},\\ &+\partial_\mathsf{P} = \{q\},\\ &-\partial_\mathsf{P} = \emptyset\\ &+\partial_{\mathsf{P}^\pi_\delta} = \{t(1)\}) \end{aligned}$$

$$\begin{aligned} E(D_3) = (\ & +\partial_{\mathsf{E}} = \{l, a\}, \\ & -\partial_{\mathsf{E}} = \emptyset, \\ & +\partial_{\mathsf{O}} = \{t(0)\}, \\ & -\partial_{\mathsf{O}} = \{\sim z\}, \\ & +\partial_{\mathsf{P}} = \{z\}, \\ & -\partial_{\mathsf{P}} = \{t(1)\} \end{aligned}$$

Accordingly, it is easy to check that

$$CB \vdash +\Delta(F_4, 0)$$

since, if $D' = (\emptyset, R_1, >_1)$, then $sim_{D'}(s,s')$ is maximal in CB, and so $\mathsf{Expl}((F_4, 0), CB) = \{c_1\}$.

7 Conclusion

Following the extensive AI & Law literature springing from the study of HYPO and CATO, in the last decade a significant effort has been put in investigating axioms as well as formal properties of factor-based case-based reasoning, and in providing the logical foundations for such a type of reasoning (see, among others, [2, 7, 12, 13, 33–35, 41, 49, 50]). Also due to development of explainable AI [6, 44], the quest for logical foundations of factor-based CBR has been recently focused, e.g., on formal models of argumentative explanation [50] or on logics for classifier systems [16, 17, 41].

[16, 17] proposed a novel approach to address this issue, starting from the intuition, introduced in [41], that a case base can be represented through a binary classifier. One limit of that approach was to flatten knowledge bases by simply connecting fact situations and judicial outcomes.

In this paper, we solved the problem and developed a novel framework for judicial case-based reasoning grounded in Defeasible Deontic Logic. The motivation behind this research stems from the increasing interest and debate around the use of AI techniques, such as machine learning and deep learning, in predicting judicial outcomes. Despite the potential benefits of these technologies in supporting judicial decision-making, assisting litigants, and analyzing biases within the legal process, there remain considerable challenges. These challenges include concerns over judicial independence, the normative correctness of algorithmic decisions, transparency, and robustness.

Our framework addresses these challenges by formalizing judicial CBR to handle factual and deontic inconsistencies as well as incomplete knowledge within case bases using Defeasible Deontic Logic. The framework enriches the traditional view by incorporating a method to generate and validate explanations of judicial decisions through logical and argumentation-based structures. This approach provides a structured way to ensure that AI-based judicial predictions are not only accurate but also transparent and aligned with normative standards.

Key contributions include:

- A general model for judicial case-based reasoning that can deal with inconsistent and incomplete case bases, extending traditional models to more realistic judicial scenarios.
- The formal definition and construction of frameworks derived from case bases, integrating the logical strength of Defeasible Deontic Logic with the argumentative structure essential for legal reasoning.
- A mechanism for determining decidable fact situations in new cases, enhancing the ability of the system to predict outcomes by leveraging established precedents.
- The introduction of normative explanations within this framework, providing a clear, structured way to validate and understand AI-based predictions in judicial contexts.

By reconstructing judicial reasoning within this logical and deontic framework, we have taken a step towards building more reliable and explainable AI systems for judicial decision-making. Future work will focus on empirical validation of the framework using real-world legal datasets and exploring how different prioritization strategies among cases affect the robustness of judicial explanations.

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