

Article

Thermodynamic Comparison of Two- and Three-Stage Cascade Refrigeration Systems: A Dynamic Energy Assessment in Hospital Environments

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Abstract

The design of ultra-low-temperature refrigeration systems has gained increasing importance in recent years, driven by the growing demand for very-low-temperature storage cabinets for many applications, such as the preservation of vaccines and biological materials. In the first part of this study, the thermal design of both two-stage and three-stage cascade refrigeration systems was developed using the Engineering Equation Solver (EES). The systems were evaluated under evaporator temperatures of $-85\text{ }^{\circ}\text{C}$, $-80\text{ }^{\circ}\text{C}$, and $-75\text{ }^{\circ}\text{C}$ and ambient temperatures ranging from $-5\text{ }^{\circ}\text{C}$ to $40\text{ }^{\circ}\text{C}$. For the two-stage configuration, the R170/R161 refrigerant pair was assessed as an alternative to the conventional R170/R290 combination. In the three-stage configuration, the performance of the R1150/R170/R290 and R1150/R170/R161 refrigerant combinations was analyzed. The results indicate that under identical operating conditions, the three-stage system demonstrated lower compressor power consumption and reduced exergy destruction compared to the two-stage configuration, while achieving higher coefficients of performance (COPs) and exergy efficiency. In the second part of the study, long-term dynamic simulations of energy consumption for both two-stage and three-stage systems were carried out using TRNSYS 18 under varying ambient temperature conditions. The simulations were performed for hospital installation rooms located in three different cities: Muğla (Turkey), Milan (Italy), and Warsaw (Poland). The results of the dynamic simulations indicate that the use of R161 leads to significant energy savings compared to R290. Specifically, when comparing the three-stage cascade system using R1150/R170/R161 with the conventional two-stage R170/R290 system, energy consumption reductions of 20.5% in Warsaw, 23.6% in Milan, and 26.6% in Muğla were achieved.



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1. Introduction

According to the definition established by the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE), ultralow-temperature (ULT) refrigeration systems are classified as units operating within the temperature range of $-50\text{ }^{\circ}\text{C}$ to $-100\text{ }^{\circ}\text{C}$ [1]. The excessively high compression ratios associated with ultralow-temperature refrigeration render single-stage vapor-compression cycles ineffective and unsuitable for practical implementation, thereby necessitating advanced refrigeration configurations,

such as cascade refrigeration systems, which employ thermally coupled circuits operating with refrigerants specifically selected for their respective temperature levels [2]. Cascade refrigeration systems are widely used across a broad range of sectors, including hypothermia-related medical treatments, cryogenic processes such as the cryopreservation of equipment and liquefied gas production, food storage and distribution systems, supermarkets, small-scale refrigeration units, and air-conditioning applications [3]. The demand for ultralow-temperature freezers operating at $-80\text{ }^{\circ}\text{C}$ has increased substantially, becoming particularly critical for vaccine manufacturing, cold-chain preservation, logistics, and end-use delivery [4]. Over the forecast period from 2020 to 2030, the ultra-low-temperature refrigeration market is expected to grow at a compound annual growth rate of 4.4% [5].

According to the most recent report of the International Institute of Refrigeration [6], the refrigeration and air-conditioning sector currently represents approximately 20% of global energy consumption, and electricity demand for space cooling is projected to increase to nearly three times its current level by 2050 [7]. In this context, it is of great importance to develop solutions to optimize ultra-low-temperature refrigeration systems by reducing power consumption.

Various studies on cascade refrigeration systems are available in the literature. Ye et al. [8] proposed a multi-objective optimization framework for a cascade refrigeration system using R290 as the high-temperature refrigerant and R170 in the low-temperature stage, integrating the Response Surface Methodology with a desirability function. They developed and validated quadratic models with high accuracy ($R^2 = 0.9995$ for COP, 0.9977 for total exergy destruction, and 0.9990 for exergy efficiency) and identified optimal operating conditions at an evaporator temperature of $-97.87\text{ }^{\circ}\text{C}$ and a condenser temperature of $30.39\text{ }^{\circ}\text{C}$, resulting in a COP of 0.60, total exergy destruction of 112.45 W, and an exergy efficiency of 40.73%. The deviations between predicted and actual values remained below 3%, confirming the robustness of the proposed multi-objective optimization approach for enhancing the thermodynamic performance of cascade refrigeration systems. Pektezel [9] performed a thermal design and performance assessment of an ultra-low-temperature three-stage cascade refrigeration system using six different refrigerant combinations. They found that the R1150/R170/R717 configuration delivered the best performance, achieving a COP of 0.65 and an exergy efficiency of 0.35 at an evaporator temperature of $-85\text{ }^{\circ}\text{C}$. In a subsequent phase, the author applied machine learning using a Multilayer Perceptron (MLP) model, yielding very low test-set prediction errors: Mean Absolute Error (MAE) values of 0.0006 for COP, 0.0193 for total compressor power, 0.0003 for exergy efficiency, and 0.0194 for total exergy destruction. Ji et al. [10] investigated the comprehensive performance of an ultra-low-temperature cascade refrigeration system intended for vaccine cold-chain applications below $-80\text{ }^{\circ}\text{C}$ by developing an integrated energy–exergy–environmental–exergoeconomic analysis framework. They compared eight environmentally friendly refrigerant pairs and identified propane–ethane (R290–R170) as the optimal option, outperforming the conventional R404A–R508B cascade system. Quantitatively, the R290–R170 configuration improved the COP by 5.94%, reduced power consumption by 5.68%, lowered CO_2 emissions by 29.67%, and decreased the exergoeconomic cost by approximately 5%. Roy and Mandal [11] carried out a numerical comparison of a cascade refrigeration system operating with two refrigerant combinations, namely R41–R404A and R170–R161, under identical operating conditions. They reported that replacing R41–R404A with R170–R161 enhanced the COP by approximately 6.7–8.9% over the examined evaporator temperature range, while reducing system exergy losses by about 16.2–13.4% as the evaporator temperature increased from $-60\text{ }^{\circ}\text{C}$ to $-30\text{ }^{\circ}\text{C}$. Moreover, the R170–R161 configuration achieved an exergetic efficiency of nearly 59% at an evaporator temperature of $-30\text{ }^{\circ}\text{C}$, indicating superior thermodynamic performance. Kayes et al. [12] conducted an integrated

thermodynamic, economic, and environmental analysis of a triple-cascade refrigeration system using hydrocarbon refrigerants (1-butene/heptane/m-xylene), followed by multi-objective optimization using parametric analysis, Box–Behnken modeling, and a genetic algorithm combined with the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP) methods. For a system capacity of 10 kW, the optimal operating conditions were identified at an evaporator temperature of $-101.02\text{ }^{\circ}\text{C}$, a condenser temperature of $36.55\text{ }^{\circ}\text{C}$, a low-temperature cycle condenser temperature of $-69.05\text{ }^{\circ}\text{C}$, and a medium-temperature cycle condenser temperature of $-34.65\text{ }^{\circ}\text{C}$, resulting in a COP of 0.71, an exergy efficiency of 0.51, and an annual total plant cost of USD 38,262. The exergy analysis revealed that the high-temperature cycle compressor (19.3%) and the throttling valve (15.5%) accounted for the majority of total exergy destruction. Ye et al. [13] proposed a thermodynamic improvement framework for an ultra-low-temperature cascade refrigeration system employing R290 and R170 in the high- and low-temperature circuits, respectively, and combined parametric analysis with artificial neural network modeling. They reported that the system achieved a maximum COP of about 0.593 and an exergy efficiency of 38.38%, while the minimum total compressor power input and total exergy destruction were 185.6 W and 114.5 W, respectively, depending on operating conditions. The ANN model, trained and tested with 80 datasets, exhibited strong predictive capability, yielding mean absolute errors of 0.0027 for COP, 0.9090 for total compressor power, 1.0314 for total exergy destruction, 0.1691 for exergy efficiency, and 1.1438 and 1.0230 for the discharge temperatures of the high- and low-temperature compressors, respectively, with correlation coefficients between 0.9886 and 0.9994. Pektezel and Ozdemir [7] conducted a comprehensive investigation on the thermodynamic optimization of a conventional three-stage cascade refrigeration system. They developed a detailed model in EES and applied Response Surface Methodology, identifying optimal operating conditions of $-90.04\text{ }^{\circ}\text{C}$ for the evaporator and $-56.41\text{ }^{\circ}\text{C}$, $-10.94\text{ }^{\circ}\text{C}$, and $26.08\text{ }^{\circ}\text{C}$ for the condenser at the low-, medium-, and high-temperature stages, respectively. Under these conditions, the system achieved a maximum COP of 0.697 and an exergy efficiency of 41.4%, while total exergy destruction and compressor power consumption were reduced to 5.545 kW and 11.394 kW, respectively. Walid Faruque et al. [14] conducted a detailed thermodynamic assessment of a triple cascade vapor refrigeration system charged with hydrocarbon refrigerants, aiming to identify the most suitable refrigerant configuration for ultra-low-temperature operation through mathematical modeling. By performing energy and exergy analyses under varying evaporator and condensation temperatures in the LTC and MTC, they reported that system performance strongly depended on operating conditions, with the maximum COP and exergy efficiency reaching 0.5931 and 54.45%, respectively, at an evaporator temperature of $-100\text{ }^{\circ}\text{C}$. They also reported that the 1-butene/heptane/m-xylene configuration delivered the highest thermodynamic performance at evaporator temperatures of $-120\text{ }^{\circ}\text{C}$ and $-110\text{ }^{\circ}\text{C}$, whereas the 1-butene/toluene/m-xylene combination became the most favorable option at $-100\text{ }^{\circ}\text{C}$ and $-90\text{ }^{\circ}\text{C}$, indicating a strong dependence of optimal refrigerant selection on operating temperature.

Table 1 presents the properties of the refrigerants used in the study [15]. In this study, the performance of both two-stage and three-stage cascade refrigeration systems was investigated. For the two-stage system, R170 was used as the low-temperature cycle (LTC) refrigerant, while R290 and R161 were employed as high-temperature cycle (HTC) refrigerants. In the three-stage system, R1150 was selected as the LTC refrigerant, R170 for the medium-temperature cycle (MTC), and R290 and R161 as the HTC refrigerants. The refrigerants used in this study are classified as A3, indicating high flammability. It should be emphasized that according to IEC 60335-2-89 [16], the allowable refrigerant charge limit for

A3 refrigerants can reach up to 0.5 kg for commercial refrigeration systems [17]. In practice, however, commercially available ultra-low-temperature freezers operating with R170/R290 typically use significantly lower refrigerant charges. This indicates that refrigerant charge levels used in practical applications are well below the safety limits set by international standards. Therefore, the use of A3 refrigerants in such applications is already feasible and widely implemented. For hospitals, in addition to the refrigerant charge limits specified in IEC 60335-2-89, practical installations require an appropriate risk assessment, adequate room ventilation, leak detection where required, and compliance with relevant national and international safety standards. In practice, ultra-low-temperature freezers charged with hydrocarbons are commonly installed in dedicated laboratory rooms rather than patient-care areas, where these safety requirements can be readily implemented. Therefore, although the present study focuses on thermodynamic performance and long-term energy consumption, the practical application of the investigated A3 refrigerants should always be considered in accordance with applicable safety regulations. It should also be noted that R161 is classified as an A3 refrigerant because of its high flammability and low toxicity. In addition, R161 has a Global Warming Potential (GWP) of approximately 4 (100-year time horizon), which is substantially lower than that of many conventional refrigerants and is one of the reasons it has attracted increasing attention as a low-GWP alternative. In fact, as is well known, low-GWP refrigerants are attracting strong interest because they address climate impact, regulation, and system performance all at once [18–21].

Table 1. Thermophysical characteristics of the refrigerants considered in this study [15].

Working Fluid	ODP *	GWP _{100-yr} (CO ₂ -eq *)	ASHRAE Safety Class	NBP * (°C)	Molecular Weight (kg mol ^{−1})	Critical Temperature (°C)	Critical Pressure (MPa)
R1150	0	4	A3	−103.77	28.05	9.2	5.04
R170	0	5.5	A3	−88.58	30.07	32.17	4.87
R290	0	<1	A3	−42.11	44.10	96.74	4.25
R161	0	4	A3	−37.55	48.06	102.1	5.01

* Ozone Depletion Potential, Carbon Dioxide Equivalent, Normal Boiling Point.

A review of the literature indicates that previous studies on cascade refrigeration systems have primarily focused on thermodynamic energy and exergy analyses, artificial intelligence-based performance prediction, and optimization using various techniques. Although several studies have investigated dynamic building energy simulations or refrigeration systems separately, comprehensive studies integrating ultra-low-temperature cascade refrigeration systems with long-term dynamic energy consumption simulations under varying ambient conditions remain very limited. In particular, no previous study has simultaneously investigated the long-term dynamic energy consumption of ultra-low-temperature cascade refrigeration systems installed in hospital equipment rooms, compared alternative refrigerant combinations, and evaluated the influence of equipment-room ventilation strategies under different climatic conditions. Therefore, the novelty of the present study lies in integrating these complementary aspects within a single modelling framework, combining detailed thermodynamic analysis with long-term dynamic energy-consumption simulations.

2. Models of Two- and Three-Stage Cascade Refrigeration Systems

Figure 1 illustrates the components and flow diagram of the two-stage cascade refrigeration system. The LTC compressor increases the temperature and pressure of the working fluid and directs it to the cascade heat exchanger (CHX). The CHX functions as a condenser for the LTC cycle and as an evaporator for the HTC cycle. In the CHX, heat is transferred from the lower-temperature cycle to the higher-temperature cycle. After

condensation in the LTC condenser, the working fluid, whose temperature has decreased, is directed to the LTC expansion valve, where both its temperature and pressure are reduced. The fluid then enters the evaporator, which extracts heat from the refrigerated chamber in which it is located by means of the ultra-low-temperature refrigerant flowing through it, thereby providing ultra-low-temperature storage conditions. In the upper cycle, the refrigerant is compressed by the HTC compressor, increasing its temperature and pressure. The fluid is subsequently directed to the condenser, where it rejects heat to the ambient environment. After releasing heat to the surroundings and experiencing a temperature drop, the refrigerant passes through the HTC expansion valve, where its temperature and pressure decrease. The refrigerant then enters the CHX, which operates as the evaporator of the HTC cycle, absorbs the heat transferred from the LTC cycle, and thus completes the cascade refrigeration cycle.

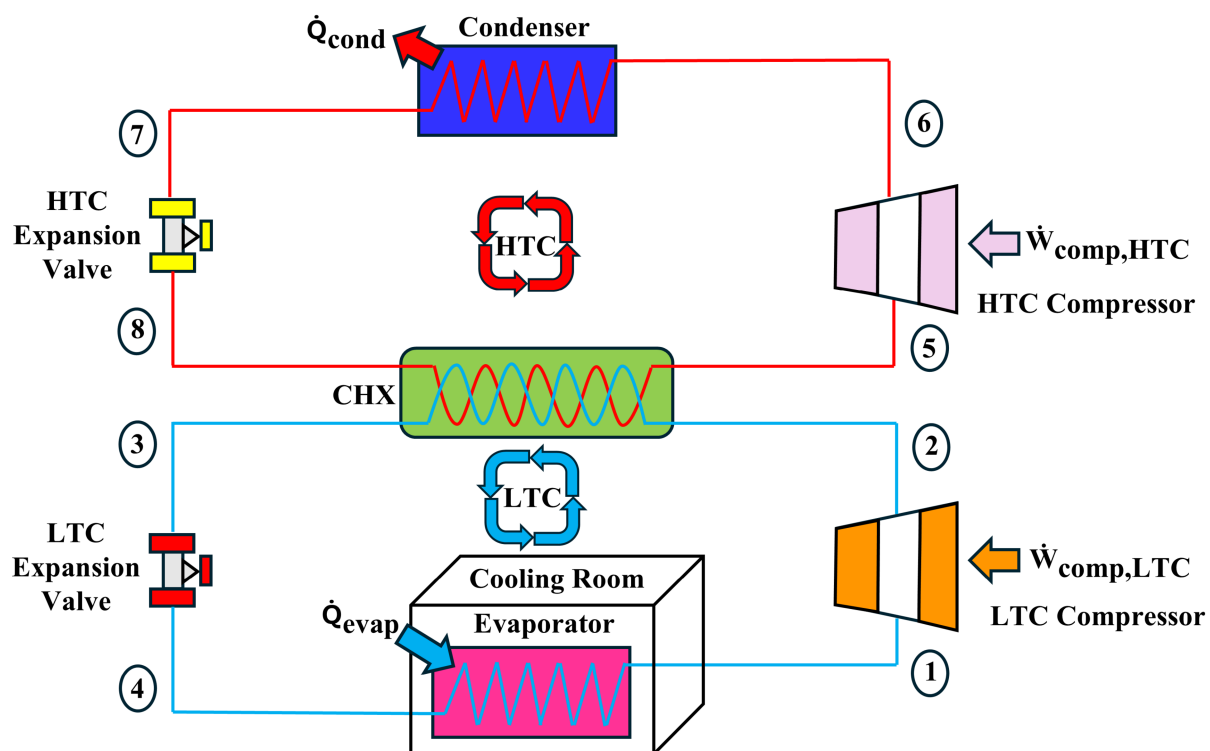


Figure 1. Schematic diagram of the two-stage cascade refrigeration system.

Figure 2 illustrates the piping diagram of the three-stage cascade refrigeration system. The system consists of three interconnected sub-cycles, namely the low-temperature cycle (LTC), the medium-temperature cycle (MTC), and the high-temperature cycle (HTC). In the LTC, the ultra-low-temperature refrigerant flowing through the evaporator is the primary factor enabling the desired ultra-low-temperature storage conditions. The refrigerant is compressed by the LTC compressor, increasing its temperature and pressure, and then directed to the first cascade heat exchanger (CHX-I). While CHX-I acts as a condenser for the LTC, it serves as an evaporator for the MTC. In CHX-I, heat is transferred from the LTC to the MTC. Following heat transfer, the refrigerant, at reduced temperature, passes through the LTC expansion valve, where its temperature and pressure decrease, and subsequently enters the evaporator, extracting heat from the refrigerated chamber and maintaining the required low-temperature conditions. In the MTC, the compressor raises the refrigerant's temperature and pressure, which are then directed to the second cascade heat exchanger (CHX-II). CHX-II functions as a condenser for the MTC and as an evaporator for the HTC. In this heat exchanger, heat is transferred from the MTC to the

HTC, thereby reducing the refrigerant temperature. The refrigerant then passes through the MTC expansion valve, experiencing a drop in temperature and pressure, and is redirected to CHX-I. Since CHX-I operates as the evaporator of the MTC, it absorbs the heat transferred from the LTC, thereby completing the MTC. In the HTC, the refrigerant is compressed by the compressor, increasing its temperature and pressure, and then flows to the condenser, where heat is rejected to the ambient environment. After releasing heat to the surroundings and experiencing a temperature decrease, the refrigerant passes through the HTC expansion valve, resulting in a reduction in temperature and pressure. The refrigerant then enters CHX-II, which operates as the HTC evaporator, absorbs the heat transferred from the MTC, and thus completes the HTC and the overall three-stage cascade refrigeration cycle.

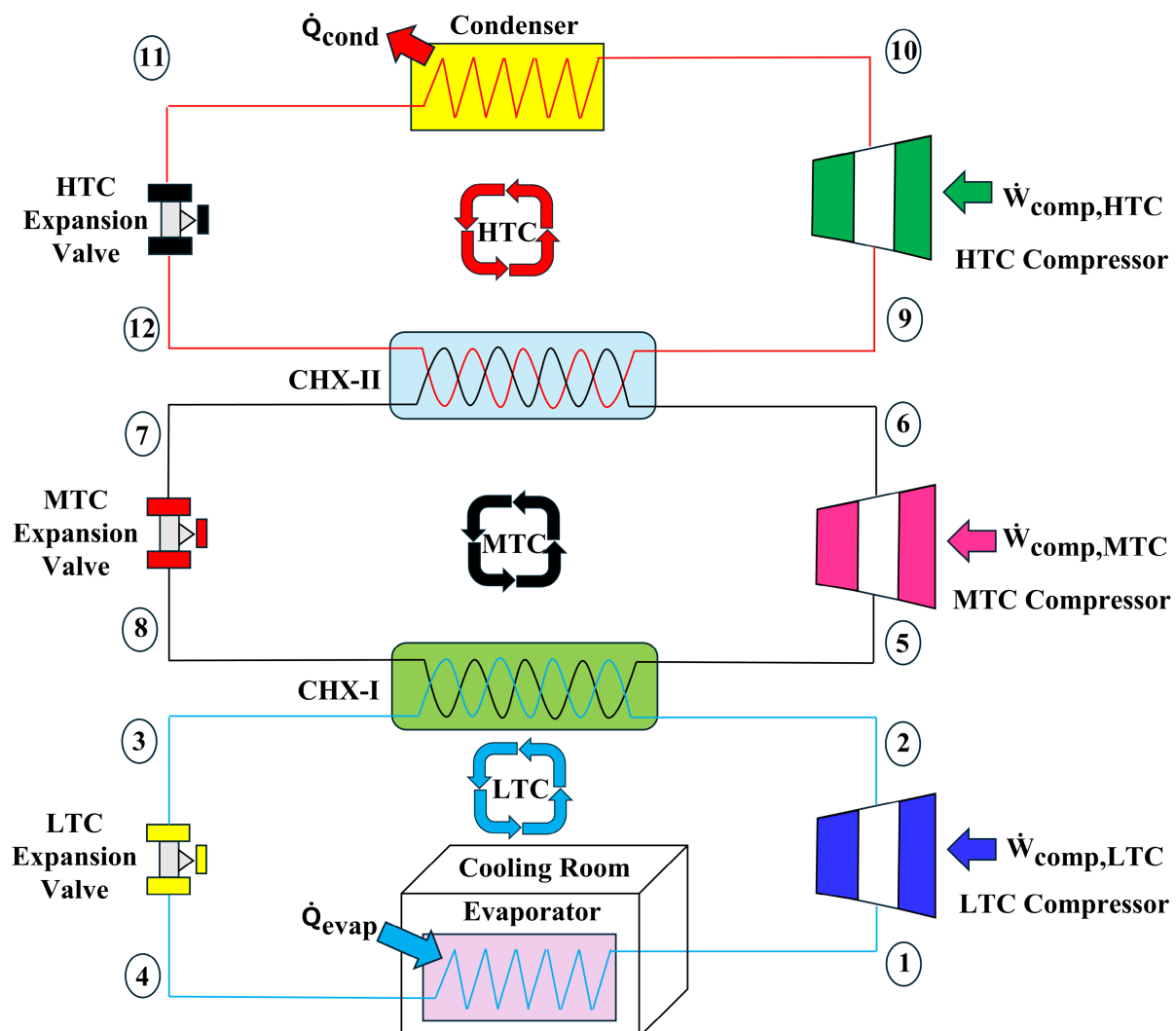


Figure 2. Piping and instrumentation diagram of the three-stage cascade refrigeration system.

2.1. Energy and Exergy Analysis of Cascade Refrigeration Models

The thermal design of the refrigeration cycle was performed using the Engineering Equation Solver (EES) [22]. The modeling assumptions include steady-state operation of all system components, with pressure drops in the piping neglected, as well as heat gains and losses. Changes in kinetic and potential energy are assumed to be insignificant, and the electrical power consumption of the evaporator and condenser fans is excluded from the analysis. Fan power consumption is typically small compared to the compressor power, especially in ultra-low-temperature systems where compression work dominates the total energy consumption. Therefore, neglecting fan power does not significantly affect the

comparative evaluation between different system configurations and refrigerant pairs. The compression and throttling processes are treated as adiabatic. A 5 °C temperature difference is assumed between the evaporator and the ultra-low-temperature cooling chamber. The refrigeration cycles are modeled assuming saturated vapor at the evaporator outlet and saturated liquid at the condenser outlet. Accordingly, superheating and subcooling effects are not included in the analysis. The adopted assumptions are commonly used in thermodynamic analyses of cascade refrigeration systems. Nevertheless, including pressure drops, heat gains/losses, fan power consumption, compressor superheating, and condenser subcooling may influence the absolute performance values. Pressure drops and heat transfer losses would generally increase compressor pressure ratios, leading to higher compressor power consumption, higher discharge temperatures, lower COP, and increased exergy destruction. Likewise, including the condenser and evaporator fan power would reduce the overall system COP due to the additional electrical power consumption. The inclusion of subcooling at the condenser and cascade heat exchanger exits would increase the refrigerating effect by lowering the enthalpy before expansion, thereby improving the overall COP. Conversely, while superheating is practically essential to prevent liquid slugging in the compressors, it would generally increase compressor discharge temperature and compressor work. These assumptions are applied consistently to all investigated operating conditions, refrigerant configurations, and both two-stage and three-stage systems. Therefore, the comparative assessment and the observed performance trends presented in this study are not expected to change significantly.

In the present study, three temperature levels are considered: ambient air temperature, refrigerant condensation temperature, and heat sink temperature used in the exergy formulation. The ambient temperature is treated as a variable operating condition, while the condensation temperature is assumed to be higher than the ambient temperature by a fixed temperature difference ($\Delta T = 10\text{ °C}$), representing the practical approach temperature in the condenser. For the exergy analysis, the heat sink temperature (T_H) is set equal to the reference dead-state temperature ($T_0 = 25\text{ °C}$) [23], which is used solely as the thermodynamic reference state and remains constant throughout the analysis. Following the conventional exergy analysis approach presented by Cengel and Boles [24], the dead-state temperature was fixed at 25 °C and used as the common thermodynamic reference environment for all investigated operating conditions. It is important to note that T_0 is not assumed to be equal to the refrigerant condensation temperature. Under this definition, the exergy associated with heat transfer in the condenser becomes negligible, and the corresponding irreversibilities are included in the condenser exergy destruction term. Although the ambient temperature varies between -5 °C and 40 °C , maintaining a constant dead-state temperature does not affect the comparative evaluation presented in this study, as all operating conditions and refrigerant configurations are assessed relative to the same thermodynamic reference environment. It should be noted that different choices of the dead-state temperature may slightly influence the absolute values of the calculated exergy efficiencies. However, because an identical reference state is consistently applied to all investigated cases, the relative performance comparison and the observed trends remain unchanged.

Air is assumed to serve as the heat-transfer medium in both the evaporator and the condenser. While the ambient temperature ranged from -5 °C to 40 °C , the ambient pressure was assumed to be 101.325 kPa. In the comparative energy and exergy analyses of the refrigerants, a cooling capacity of 1.5 kW was assumed. For the long-term dynamic simulation, a cooling capacity of 0.3 kW was applied when the cabinet door remained closed, while 1.5 kW was used from the time of door opening until steady-state conditions

were re-established. Additionally, evaporator temperatures of $-85\text{ }^{\circ}\text{C}$, $-80\text{ }^{\circ}\text{C}$, and $-75\text{ }^{\circ}\text{C}$ were considered in the energy and exergy comparisons.

To ensure a consistent, physically meaningful comparison between the two-stage and three-stage cascade refrigeration systems, the intermediate temperature levels were carefully selected based on the overall temperature lift between the evaporator and condenser. In this study, the condenser temperature is defined as $10\text{ }^{\circ}\text{C}$ higher than the ambient temperature. For a representative case with an ambient temperature of $25\text{ }^{\circ}\text{C}$, the condenser temperature becomes $35\text{ }^{\circ}\text{C}$. The evaporator temperature is taken as $-85\text{ }^{\circ}\text{C}$, resulting in a total temperature lift of approximately $120\text{ }^{\circ}\text{C}$ across the system. For the two-stage system, the intermediate cascade temperature is set to $-25\text{ }^{\circ}\text{C}$, corresponding to an equal division of the total temperature lift between the two stages. For the three-stage system, the intermediate temperatures are defined as $-45\text{ }^{\circ}\text{C}$ and $-15\text{ }^{\circ}\text{C}$. These values divide the overall temperature lift into three segments, providing a reasonable distribution of temperature differences across the stages. Although an equal division of the total temperature lift could be considered, the intermediate temperatures in the three-stage system are selected to achieve a more realistic, balanced distribution of compression work across the stages. Due to the severe operating conditions of the low-temperature stage, a non-uniform distribution of temperature lift provides a more practical representation of cascade system operation. This selection of intermediate temperatures ensures that both systems operate under comparable thermodynamic conditions, enabling a fair and consistent performance comparison.

To verify that the comparison between the two-stage and three-stage cascade refrigeration systems is not biased by the selected intermediate temperatures, a robustness analysis was conducted at the reference operating condition of an evaporator temperature of $-85\text{ }^{\circ}\text{C}$, an ambient temperature of $25\text{ }^{\circ}\text{C}$, and a cooling capacity of 1.5 kW . As shown in Table 2, the intermediate temperature of the two-stage R170/R290 system was varied between $-35\text{ }^{\circ}\text{C}$ and $-15\text{ }^{\circ}\text{C}$, while the first and second intermediate temperatures of the three-stage R1150/R170/R161 system were varied between $-40\text{ }^{\circ}\text{C}$ and $-50\text{ }^{\circ}\text{C}$ and between $-10\text{ }^{\circ}\text{C}$ and $-20\text{ }^{\circ}\text{C}$, respectively. The results indicate that moderate variations in intermediate temperatures have a slight effect on the absolute values of COP and total compressor power consumption. However, the overall performance trends remain unchanged. For all investigated intermediate-temperature combinations, the three-stage R1150/R170/R161 system consistently achieves higher COP and lower total compressor power consumption than the conventional two-stage R170/R290 system. These findings confirm that the selected intermediate temperatures ($-25\text{ }^{\circ}\text{C}$ for the two-stage system and $-45\text{ }^{\circ}\text{C}/-15\text{ }^{\circ}\text{C}$ for the three-stage system) are representative engineering design values and that the principal conclusions of this study are not dependent on the specific intermediate-temperature selection.

Table 2. Robustness analysis of the effect of intermediate temperatures on the performance of the two-stage and three-stage cascade refrigeration systems.

R170/R290, $\dot{Q}_{\text{evap}} = 1.5\text{ kW}$ $T_{\text{evap}} = -85\text{ }^{\circ}\text{C}$, $T_{\text{env}} = 25\text{ }^{\circ}\text{C}$		
T_3	COP	$\dot{W}_{\text{comp,total}}$
-35	0.7173	2.09
-30	0.7315	2.05
-25	0.7313	2.05
-20	0.7179	2.09
-15	0.6925	2.17

Table 2. Cont.

R1150/R170/R161, $\dot{Q}_{\text{evap}} = 1.5 \text{ kW}$ $T_{\text{evap}} = -85 \text{ }^{\circ}\text{C}$, $T_{\text{env}} = 25 \text{ }^{\circ}\text{C}$			
T_3	T_7	COP	$\dot{W}_{\text{comp,total}}$
−40	−10	0.8151	1.84
−40	−15	0.8109	1.85
−40	−20	0.7984	1.88
−45	−10	0.8242	1.82
−45	−15	0.8232	1.82
−45	−20	0.8132	1.84
−50	−10	0.8251	1.82
−50	−15	0.8277	1.81
−50	−20	0.8208	1.83

The COPs of both the two- and three-stage cascade refrigeration systems are determined using Equation (1):

$$\text{COP} = \frac{\dot{Q}_{\text{evap}}}{\dot{W}_{\text{comp,total}}}, \quad (1)$$

where the total compressor power consumption of the two-stage system is calculated with Equation (2) and that of the three-stage system with Equation (3):

$$\dot{W}_{\text{comp,total}} = \dot{W}_{\text{comp,LTC}} + \dot{W}_{\text{comp,HTC}} \quad (2)$$

$$\dot{W}_{\text{comp,total}} = \dot{W}_{\text{comp,LTC}} + \dot{W}_{\text{comp,MTC}} + \dot{W}_{\text{comp,HTC}} \quad (3)$$

The exergy at each state point (i) of both two- and three-stage systems is evaluated using Equation (4):

$$e_i = (h_i - h_0) - T_0(s_i - s_0) \quad (4)$$

The flow exergy at the two- and three-stage system state points is obtained by multiplying the specific exergy by the corresponding mass flow rate, as given in Equation (5):

$$\dot{E}_i = \dot{m}_i e_i \quad (5)$$

The total exergy destruction of the two-stage system is calculated using Equation (6), while that of the three-stage system is determined using Equation (7):

$$\dot{E}_{\text{dest,total}} = \dot{E}_{\text{dest,evap}} + \dot{E}_{\text{dest,comp,LTC}} + \dot{E}_{\text{dest,CHX}} + \dot{E}_{\text{dest,valve,LTC}} + \dot{E}_{\text{dest,comp,HTC}} + \dot{E}_{\text{dest,cond}} + \dot{E}_{\text{dest,valve,HTC}} \quad (6)$$

$$\dot{E}_{\text{dest,total}} = \dot{E}_{\text{dest,evap}} + \dot{E}_{\text{dest,comp,LTC}} + \dot{E}_{\text{dest,CHX,I}} + \dot{E}_{\text{dest,valve,LTC}} + \dot{E}_{\text{dest,comp,MTC}} + \dot{E}_{\text{dest,CHX,II}} + \dot{E}_{\text{dest,valve,MTC}} + \dot{E}_{\text{dest,comp,HTC}} + \dot{E}_{\text{dest,cond}} + \dot{E}_{\text{dest,valve,HTC}} \quad (7)$$

Exergy efficiency of both two- and three-stage systems is expressed by Equation (8). The exergy efficiency adopted in the present study is consistent with the conventional definition for refrigeration systems, in which the useful exergy product is obtained from the difference between the compressor work input and the total exergy destruction.

$$\eta_{\text{ex}} = 1 - \left(\frac{\dot{E}_{\text{dest,total}}}{\dot{W}_{\text{comp,total}}} \right) \quad (8)$$

Heat balance equations in cascade heat exchangers of a three-stage system are given by Equations (9) and (10), respectively:

$$\dot{m}_{LTC}(h_2 - h_3) = \dot{m}_{MTC}(h_5 - h_8) \quad (9)$$

$$\dot{m}_{MTC}(h_6 - h_7) = \dot{m}_{HTC}(h_9 - h_{12}) \quad (10)$$

Exergy destructions in cascade heat exchangers of a three-stage system are given by Equations (11) and (12), respectively:

$$\dot{E}_{\text{dest,CHX,I}} = \dot{E}_2 + \dot{E}_8 - \dot{E}_3 - \dot{E}_5 \quad (11)$$

$$\dot{E}_{\text{dest,CHX,II}} = \dot{E}_6 + \dot{E}_{12} - \dot{E}_7 - \dot{E}_9 \quad (12)$$

Exergy destruction of the condenser and exergy transferred with heat in the condenser in the three-stage system are given by Equations (13) and (14), respectively:

$$\dot{E}_{\text{dest,cond}} = \dot{E}_{10} - \dot{E}_{11} - \dot{E}_{Q,\text{cond}} \quad (13)$$

$$\dot{E}_{Q,\text{cond}} = \left(1 - \frac{T_0}{T_H}\right) \dot{Q}_{\text{cond}} \quad (14)$$

The exergy destruction in the evaporator and the exergy transferred with heat in the evaporator in the three-stage system are given by Equations (15) and (16), respectively. All temperatures appearing in the exergy equations were converted to Kelvin before the exergy calculations were performed.

$$\dot{E}_{\text{dest,evap}} = \dot{E}_4 - \dot{E}_1 - \dot{E}_{Q,\text{evap}} \quad (15)$$

$$\dot{E}_{Q,\text{evap}} = \left(\frac{T_0}{T_L} - 1\right) \dot{Q}_{\text{evap}} \quad (16)$$

Equation (17) defines T_L . Here, $\Delta T_{\text{evap}} = 5^\circ\text{C}$ was assumed.

$$T_L = T_1 + \Delta T_{\text{evap}} \quad (17)$$

The assumptions given by Equations (18) and (19) were made for the temperature difference of cascade heat exchangers in a three-stage system. Here, $\Delta T_{\text{CHX,I}} = \Delta T_{\text{CHX,II}} = 5^\circ\text{C}$ was assumed for both cascade heat exchangers.

$$T_8 = T_3 - \Delta T_{\text{CHX,I}} \quad (18)$$

$$T_{12} = T_7 - \Delta T_{\text{CHX,II}} \quad (19)$$

In the present study, the compressor isentropic efficiency is not assumed to be constant. Instead, it is calculated for each operating condition using the pressure-ratio-dependent empirical correlation, namely [13,25],

$$\eta_{\text{comp}} = 0.874 - 0.0135 \left(\frac{P_{\text{discharge}}}{P_{\text{suction}}} \right) \quad (20)$$

2.2. Model Validation of the Developed Cascade Refrigeration Systems

Figure 3 presents the comparison and validation of the developed models against results reported in the literature. Figure 3a shows the validation results for the two-stage system, while Figure 3b shows those for the three-stage system. The COP data at an evaporator temperature of -50°C presented in Figure 3a provide a comparative evaluation between the COP value obtained from the experimental study conducted by Dopazo and Fernandez-Seara [25] using a carbon dioxide–ammonia pair and the COP value predicted by the present model after integrating the operating parameters adopted in that study. Sim-

ilarly, the COP data at an evaporator temperature of $-30\text{ }^{\circ}\text{C}$, shown in Figure 3a, compares the experimental COP reported by Sanz-Kock et al. [26] for a carbon dioxide–R134a system with the COP obtained from the present model under the same operating conditions. The COP results presented in Figure 3b compare the COP values reported by Sun et al. [27] for a three-stage cascade refrigeration system employing the R1150/R170/R161 refrigerant group with those predicted by the present model by incorporating the assumptions used in that study. It should be noted that the availability of detailed experimental data for ultra-low-temperature cascade refrigeration systems is very limited, particularly for three-stage configurations. Moreover, published experimental studies generally do not report all thermodynamic state properties, mass flow rates, and component specifications required for an exact reproduction of the investigated systems. Therefore, validation was performed by adopting the operating conditions reported in the literature and comparing the predicted COP values with the published results. Although the refrigerant combinations and operating conditions are not identical to those investigated in the present study, this approach is commonly used to validate thermodynamic refrigeration models when comprehensive experimental datasets are unavailable. The close agreement observed in Figure 3 confirms that the developed models accurately predict the thermodynamic performance of cascade refrigeration systems over a broad operating range.

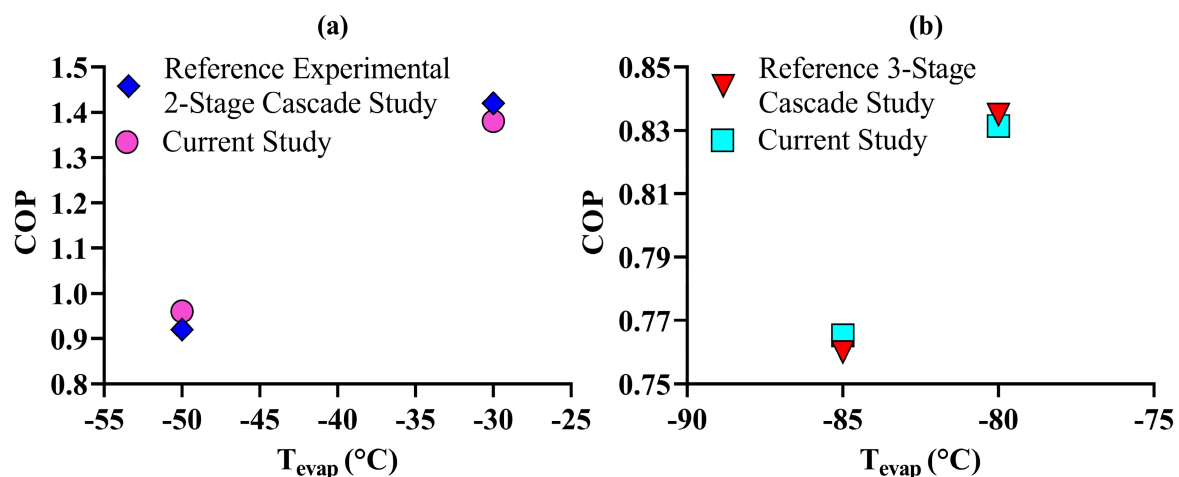


Figure 3. Model validation for the (a) two- and (b) three-stage cascade refrigeration systems.

Table 3 presents the statistical results of the literature-based validation shown in Figure 3. The absolute and relative deviations between the literature-reported COP values and those predicted by the present models are provided to quantitatively assess model accuracy. For all investigated cases, the relative deviation is below 4.5%, demonstrating good agreement with the published data and confirming the reliability of the developed thermodynamic models.

Table 3. Absolute and relative deviations between the present models and the literature data used for model validation.

System	T_{evap} ($^{\circ}\text{C}$)	Absolute Deviation	Relative Deviation (%)
Two-stage	-30	0.04	2.82
Two-stage	-50	0.04	4.35
Three-stage	-85	0.0053	0.70
Three-stage	-80	0.0036	0.43

2.3. TRNSYS Models Developed for Two- and Three-Stage Cascade Refrigeration Systems

After identifying the best possible pair, the R1150/R170/R161 pair that yields the best COP values, we aim to compare it to the “commercial standard” R170/R290, i.e., the most widely used, using a dynamic simulation. The software package TRNSYS v. 18 [28,29] is employed, and reference is made to three locations: Mugla, Turkey; Milan, Italy; and Warsaw, Poland. The three locations differ in climate conditions.

TRNSYS is a well-known software for dynamic simulations and is validated by the ANSI/ASHRAE Standard 140. Currently, TRNSYS is part of the validation project SimQuality.

The hospital’s 3D modelling is carried out using Google SketchUp [30]. The 3D-modelled building is then imported into TRNSYS (type 56) [28] using the TRNSYS plugin for Google SketchUp [31]. All non-geometry information, such as layer material properties, ventilation and infiltration profiles, heat gains, etc., was added in TRNBuild, the type 56 software. The defined characteristics were saved in text files, which were then read by Type 56 during simulations in Simulation Studio. The multizone building model Type 56 calculates the energy balance as the convective and conductive heat flux to the air node defined by the following equation:

$$\dot{Q}_i = \dot{Q}_{\text{sur } f,i} + \dot{Q}_{\text{in } f,i} + \dot{Q}_{\text{vent},i} + \dot{Q}_{g,c,i} + \dot{Q}_{\text{cplg},i}, \quad (21)$$

where $\dot{Q}_i$ is the conductive and convective heat flux, $\dot{Q}_{\text{sur } f,i}$ are the convective gains from surfaces, $\dot{Q}_{\text{in } f,i}$ are the infiltration gains (air flow from outside only), $\dot{Q}_{\text{vent},i}$ are the ventilation gains, $\dot{Q}_{g,c,i}$ are the internal convective gains (by people, equipment, illumination, radiators, etc.) and $\dot{Q}_{\text{cplg},i}$ are the gains related to convective air flow coming from adjacent thermal zones, while radiative heat flows to the walls and windows are defined as

$$\dot{Q}_{r,wi} = \dot{Q}_{g,r,i,wi} + \dot{Q}_{\text{sol},wi} + \dot{Q}_{\text{long},wi} + \dot{Q}_{\text{wall-gain}}, \quad (22)$$

where $\dot{Q}_{r,wi}$ are the radiative gains for the wall surface temperature node, $\dot{Q}_{g,r,i,wi}$ are the radiative air node internal gains received by walls, $\dot{Q}_{\text{sol},wi}$ represents the solar gains through windows received by walls, $\dot{Q}_{\text{long},wi}$ is the long-wave radiation exchange between a wall and all the other walls and windows, and $\dot{Q}_{\text{wall-gain}}$ is the user-specified heat flow to the wall or window surface. The unit of measure of the terms in Equations (21) and (22) is kJh^{-1} .

For the three cases considered, we assume that five cascade refrigerators are collocated in a hospital-dedicated room, as shown in Figure 4.

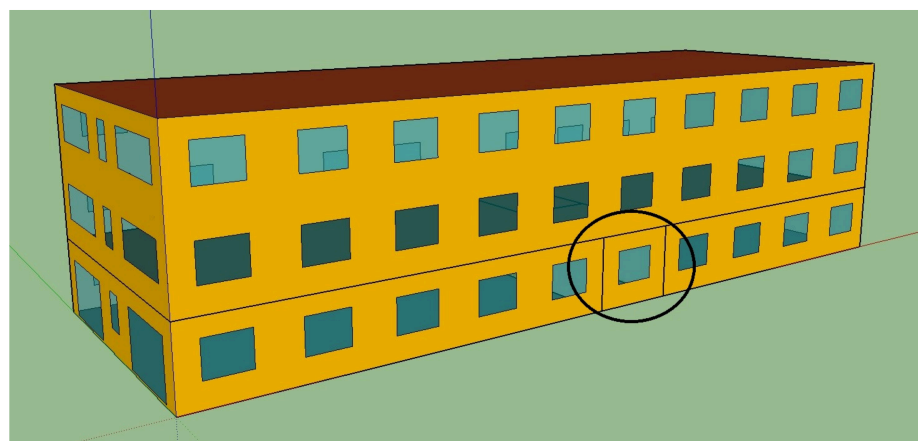


Figure 4. Sketch of the building and the room where five cascade refrigerators are placed.

The climatic data (such as external temperature, solar radiation, etc.) are hourly extracted from the Meteonorm database, as are the solar gains calculated by TRNSYS from the Meteonorm input file. Figure 5 shows the daily average external temperature for the entire year for the three different cities.

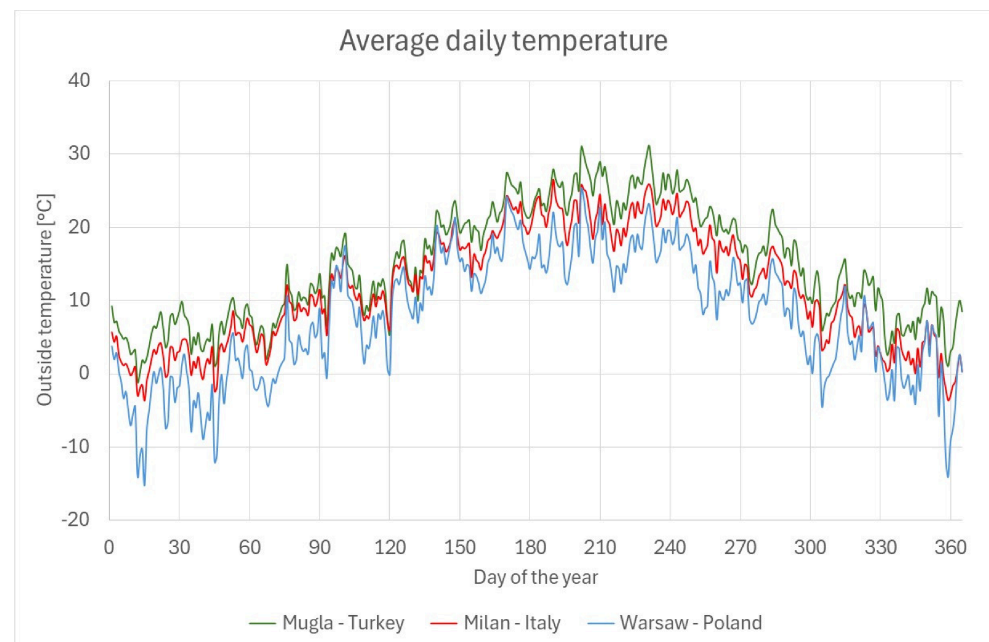


Figure 5. Average daily external temperature for the three considered locations.

The size of the room where the cascade refrigerators are placed is located on the ground-floor level of a hospital building and has a total volume of 57.5 m³ (5.7 m long by 3.6 m wide by 2.8 m ceiling high), with an external wall facing south and a window of 3.2 m² (2.0 m wide by 1.6 m high). The external wall thermal transmittance is 0.5 W/m²K, whilst the window one is 2.8 W/m²K. For the calculation, it is assumed that no heating or cooling system is provided in the room, and an infiltration rate of 0.1 volume per hour of external air is considered. Moreover, the adjacent rooms are maintained at 21 °C in winter and 26 °C in summer.

We assume a small power consumption for each cascade refrigerator; approximately 0.3 kW is the evaporator power in normal conditions. Ten times a day, each door is kept open for approximately 1 min. During this period, consumption increases, and it takes approximately 15 min to reach steady state; in this phase, the evaporator provides 1.5 kW of power. The power dissipated by the condensers of the five refrigerators was implemented in Type 56 using the Internal Gains function. The total heat release was divided into 80% convective and 20% radiative components.

2.4. From EES Data to TRNSYS

The performance data calculated using EES were implemented in TRNSYS through an interpolation procedure. The interpolated values were determined as a function of room temperature, which, in turn, defines the condenser temperature. Accordingly, the performance characteristics provided by EES were interpolated to obtain the corresponding operating conditions during the TRNSYS simulations.

To improve the reliability of the interpolation procedure, the concept of second-law efficiency was adopted. Based on the performance data obtained from EES, namely the compressor power consumption and the evaporator cooling capacity, the Carnot coefficient

of performance (COP_{max}) was first evaluated, allowing the corresponding second-law efficiency to be calculated.

$$COP_{max} = 1/(T_{cond}/T_{evap} - 1) \quad (23)$$

$$\eta_{II} = COP/COP_{max} \quad (24)$$

The second-law efficiency was then correlated to the evaporator and condenser temperature using second-order polynomial curve fits. During the TRNSYS simulations, the interpolated second-law efficiency was multiplied by the COP_{max} ; both COP_{max} and η_{II} were calculated directly within TRNSYS based on the evaporating and condensing temperatures. This approach enabled the actual COP to be evaluated as a function of operating conditions while ensuring a more robust, physically consistent interpolation of the EES performance data.

3. Results

3.1. Energy and Exergy Results

All data presented in the graphs shown in Figures 6–13 were generated assuming a constant cooling capacity of 1.5 kW. Figures 6–9 present the performance results of the two-stage cascade refrigeration system, whereas Figures 10–13 illustrate the performance results of the three-stage cascade refrigeration system.

Figure 6 shows the effects of various environments and evaporator temperatures on the total compressor power consumption of the two-stage cascade refrigeration system for both R170/R290 and R170/R161 refrigerant pairs. An examination of Figure 6 indicates that increasing evaporator temperature reduces compressor power consumption, consistent with previous studies [32]. For instance, at an ambient temperature of 40 °C, increasing the evaporator temperature from −85 °C to −75 °C reduced the compressor power consumption by 17.4% when R170/R290 was used. Figure 6 also reveals that for a fixed evaporator temperature, an increase in ambient temperature results in higher total compressor power consumption. For example, for the R170/R161 refrigerant pair at an evaporator temperature of −80 °C, increasing the ambient temperature from −5 °C to 40 °C increased compressor power consumption from 1.188 kW to 2.291 kW. A comparison of the two refrigerant pairs shows that R170/R161 yields lower compressor power consumption than R170/R290.

Figure 7 illustrates the effects of variations in evaporator and ambient temperatures on the COP of the two-stage cascade refrigeration system for both R170/R290 and R170/R161 refrigerant pairs. An examination of Figure 7 indicates that at a given ambient temperature, increasing the evaporator temperature improves the COP, consistent with findings reported in the literature [13]. For example, for the R170/R290 refrigerant pair at an ambient temperature of 10 °C, increasing the evaporator temperature from −85 °C to −75 °C increased the COP from 0.91 to 1.15. The observed increase in COP with increasing evaporator temperature can be explained by the reduction in the temperature lift between the evaporator and the condenser. A lower temperature lift results in a lower compression ratio, reducing compressor work and improving system efficiency. Another notable observation from Figure 7 is that at a fixed evaporator temperature, an increase in ambient temperature has a detrimental effect on COP. For instance, for the R170/R161 refrigerant pair at an evaporator temperature of −85 °C, increasing the ambient temperature from −5 °C to 40 °C resulted in a 46.3% reduction in COP. An increase in ambient temperature leads to a higher condenser temperature, thereby increasing the compression ratio and compressor power consumption. This situation causes a decrease in COP. When the two refrigerant pairs are compared by COP, R170/R161 outperforms R170/R290 under all operating conditions.

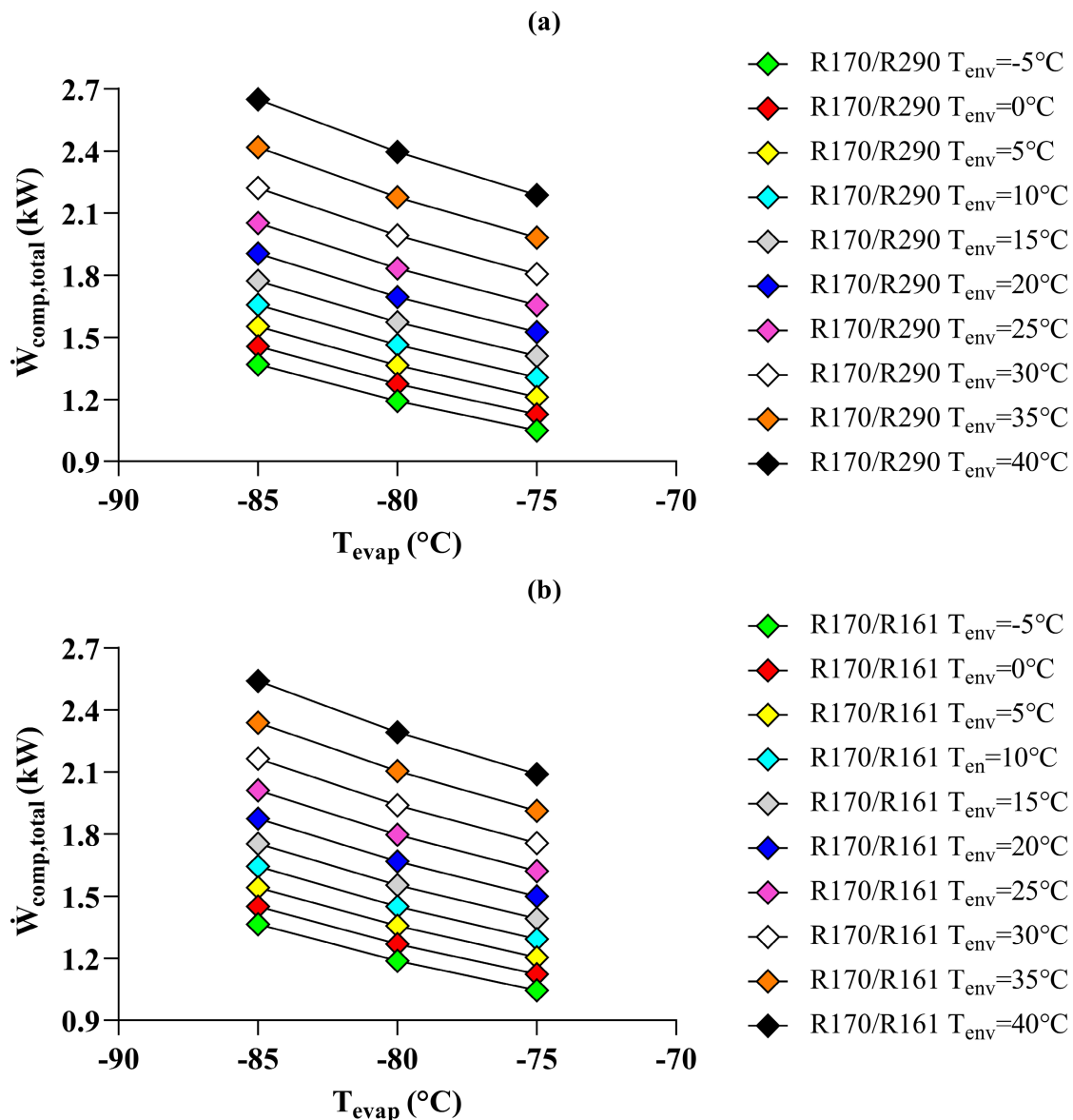


Figure 6. Effect of T_{env} and T_{evap} on $\dot{W}_{comp,total}$ for the two-stage system (a) R170/R290; (b) R170/R161.

Figure 8 illustrates the effects of variations in evaporator and ambient temperatures on the total exergy destruction of the two-stage cascade refrigeration system for the R170/R290 and R170/R161 refrigerant pairs. The results in Figure 8 indicate that at a given ambient temperature, increasing the evaporator temperature reduces total exergy destruction, consistent with previous studies [33]. For example, for the R170/R290 refrigerant pair at an ambient temperature of 25 °C, increasing the evaporator temperature from −85 °C to −75 °C resulted in a 19.9% decrease in total exergy destruction. This behavior can be explained by the reduction in temperature lift between the evaporator and condenser. A lower temperature lift leads to a lower compression ratio, which decreases compressor work and associated irreversibilities. Figure 8 also shows that at a fixed evaporator temperature, total exergy destruction increases with increasing ambient temperature. For instance, for the R170/R161 refrigerant pair at a constant evaporator temperature of −75 °C, increasing the ambient temperature from −5 °C to 40 °C increased the system's total exergy destruction from 0.565 kW to 1.609 kW. This is mainly due to the rise in condenser temperature, which increases the temperature lift and compression ratio. As a result, compressor work and

irreversibilities increase. A general comparison of the refrigerant pairs indicates that the R170/R290 pair results in higher exergy destruction than R170/R161 across all operating conditions considered.

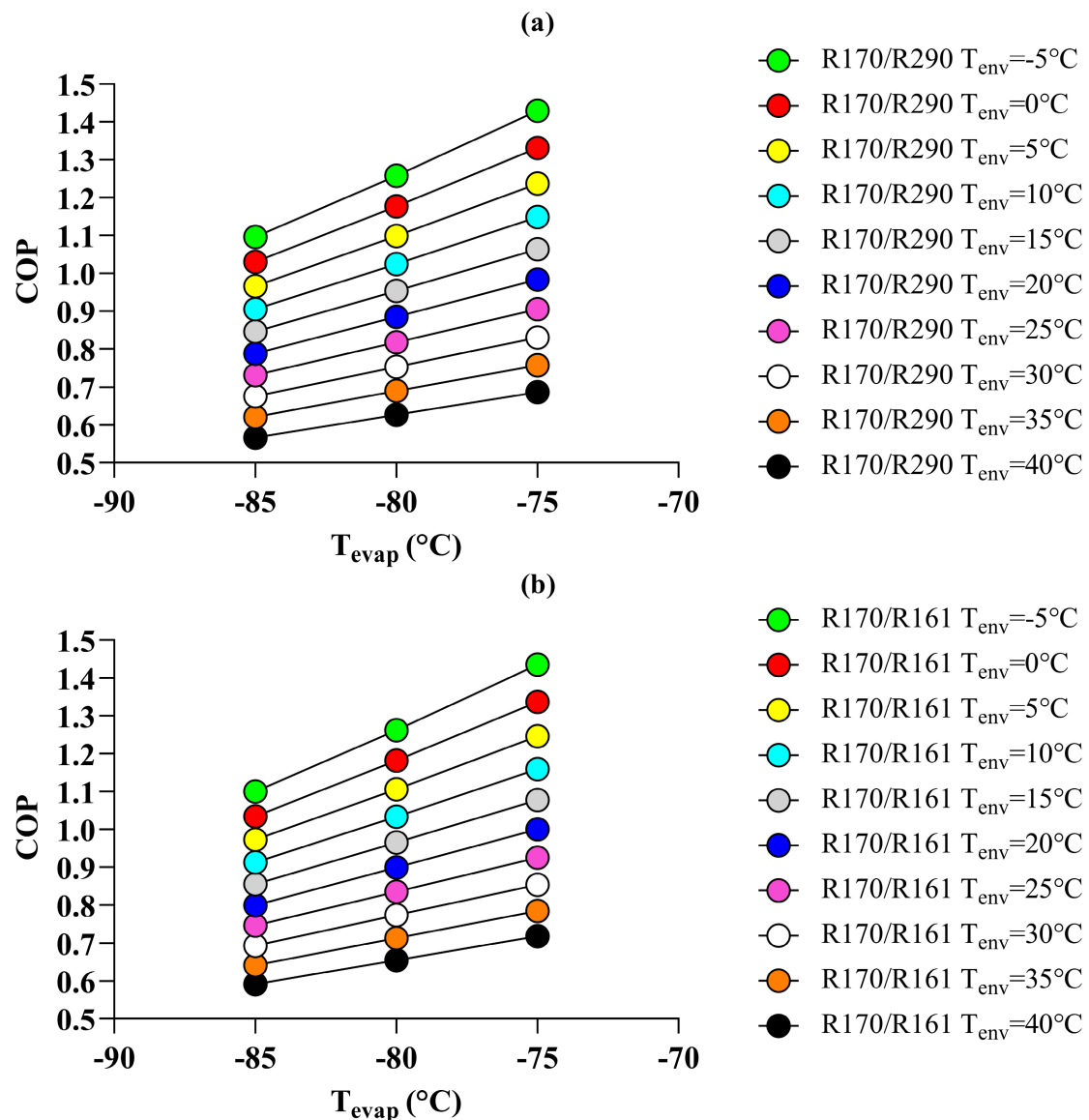


Figure 7. Influence of T_{env} and T_{evap} on COP of the two-stage system (a) R170/R290; (b) R170/R161.

Figure 9 illustrates the effects of variations in ambient and evaporator temperatures on the exergy efficiency of the two-stage cascade refrigeration system for the R170/R290 and R170/R161 refrigerant pairs. An examination of Figure 9 shows that at a given ambient temperature, increasing the evaporator temperature results in higher exergy efficiency, consistent with observations reported in previous studies [34]. For example, for the R170/R290 refrigerant pair at an ambient temperature of 0 $^{\circ}\text{C}$, increasing the evaporator temperature from -85 $^{\circ}\text{C}$ to -75 $^{\circ}\text{C}$ increased the exergy efficiency from 0.40 to 0.43. Another important finding from Figure 9 is that at a fixed evaporator temperature, decreasing the ambient temperature increases the exergy efficiency. For instance, for the R170/R161 refrigerant pair at a constant evaporator temperature of -80 $^{\circ}\text{C}$, reducing the ambient temperature from 40 $^{\circ}\text{C}$ to -5 $^{\circ}\text{C}$ increased exergy efficiency by 92.8%. A comparison of the two refrigerant pairs indicates that R170/R161 provides higher exergy efficiency than R170/R290 under all operating conditions considered. At high ambient

temperatures, the exergy efficiency exhibits a nearly horizontal trend with increasing evaporator temperature. In contrast, at low ambient temperatures, the reduced condenser irreversibilities allow the benefits of higher evaporator temperatures to be directly reflected in the exergy efficiency. The sensitivity of exergy efficiency to the evaporator temperature decreases significantly at high ambient conditions due to the dominance of condenser-related irreversibilities.

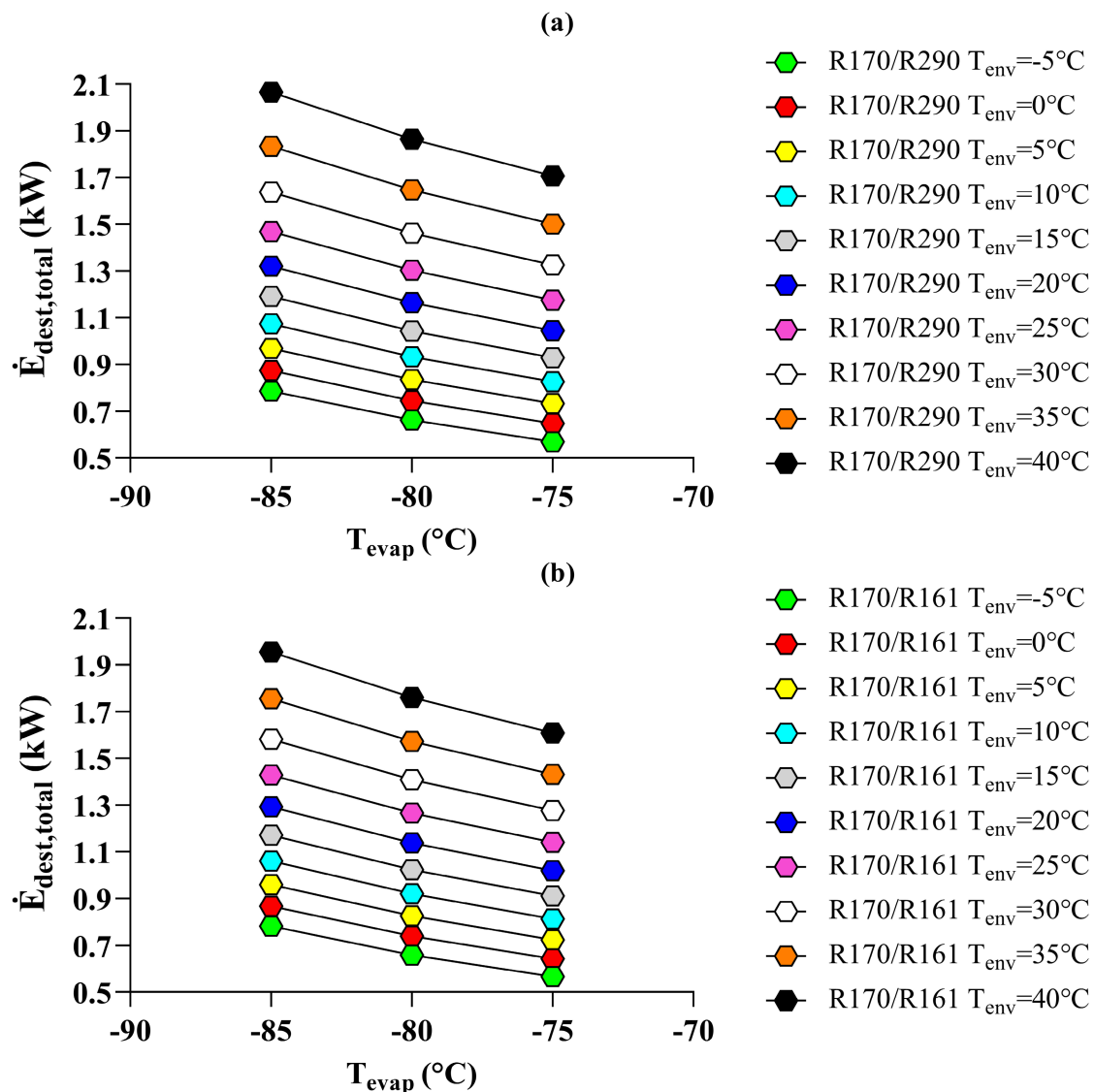


Figure 8. Impact of T_{env} and T_{evap} on $\dot{E}_{\text{dest,total}}$ of the two-stage system (a) R170/R290; (b) R170/R161.

Figure 10 illustrates how variations in ambient and evaporator temperatures influence the total compressor power demand of the three-stage cascade refrigeration system using the R1150/R170/R290 and R1150/R170/R161 refrigerant combinations. As shown in Figure 10, increasing the evaporator temperature reduces the compressor power requirement, consistent with trends reported in the literature [9]. For example, at an ambient temperature of 20 °C, increasing the evaporator temperature from −85 °C to −75 °C results in a 15.8% reduction in compressor power consumption for the R1150/R170/R290 combination. In addition, Figure 10 shows that at a constant evaporator temperature, higher ambient temperatures increase total compressor power consumption. Specifically, for the R1150/R170/R161 refrigerant combination operating at an evaporator temperature of −75 °C, raising the ambient temperature from −5 °C to 40 °C increases the compressor

power from 1.046 kW to 1.911 kW. A comparative assessment further indicates that the R1150/R170/R161 refrigerant pair consistently requires less compressor power than the R1150/R170/R290 pair under identical operating conditions.

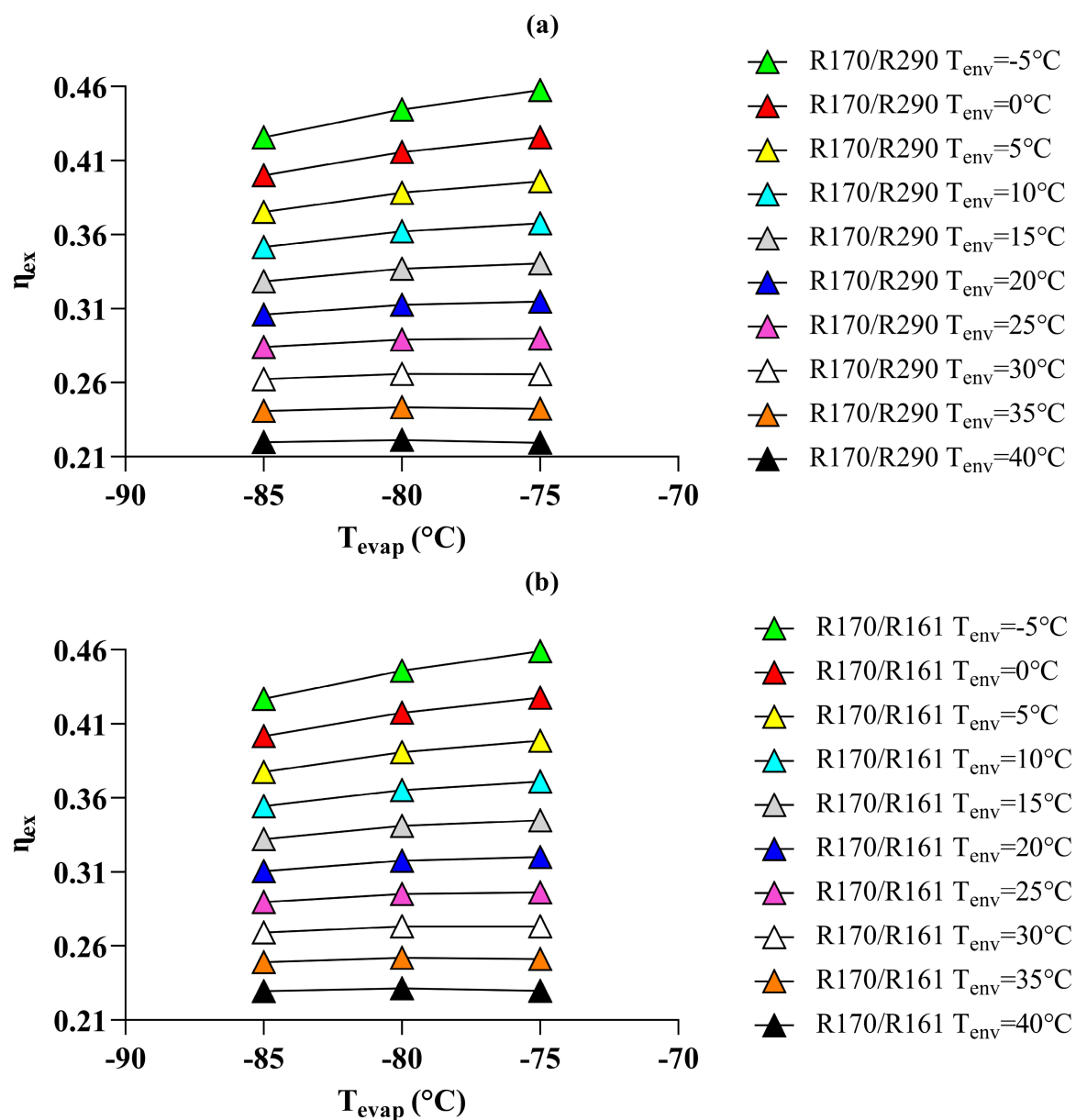


Figure 9. Role of T_{env} and T_{evap} on η_{ex} of the two-stage system (a) R170/R290; (b) R170/R161.

Figure 11 presents the influence of evaporator and ambient temperature variations on the COP of the three-stage cascade refrigeration system employing the R1150/R170/R290 and R1150/R170/R161 refrigerant combinations. As shown in Figure 11, at a constant ambient temperature, increasing the evaporator temperature increases the COP, in agreement with the widely reported literature [35]. For instance, at an ambient temperature of 30 °C, raising the evaporator temperature from -85 °C to -75 °C increases the COP from 0.75 to 0.89 for the R1150/R170/R290 refrigerant combination. Furthermore, Figure 11 shows that at a fixed evaporator temperature, higher ambient temperatures reduce the COP. As an example, for the R1150/R170/R161 refrigerant combination operating at an evaporator temperature of -80 °C, increasing the ambient temperature from -5 °C to 40 °C results in a 43.8% decrease in COP. A comparison of the two refrigerant groups in terms of COP

performance indicates that the R1150/R170/R161 configuration consistently outperforms R1150/R170/R290 across the entire operating range.

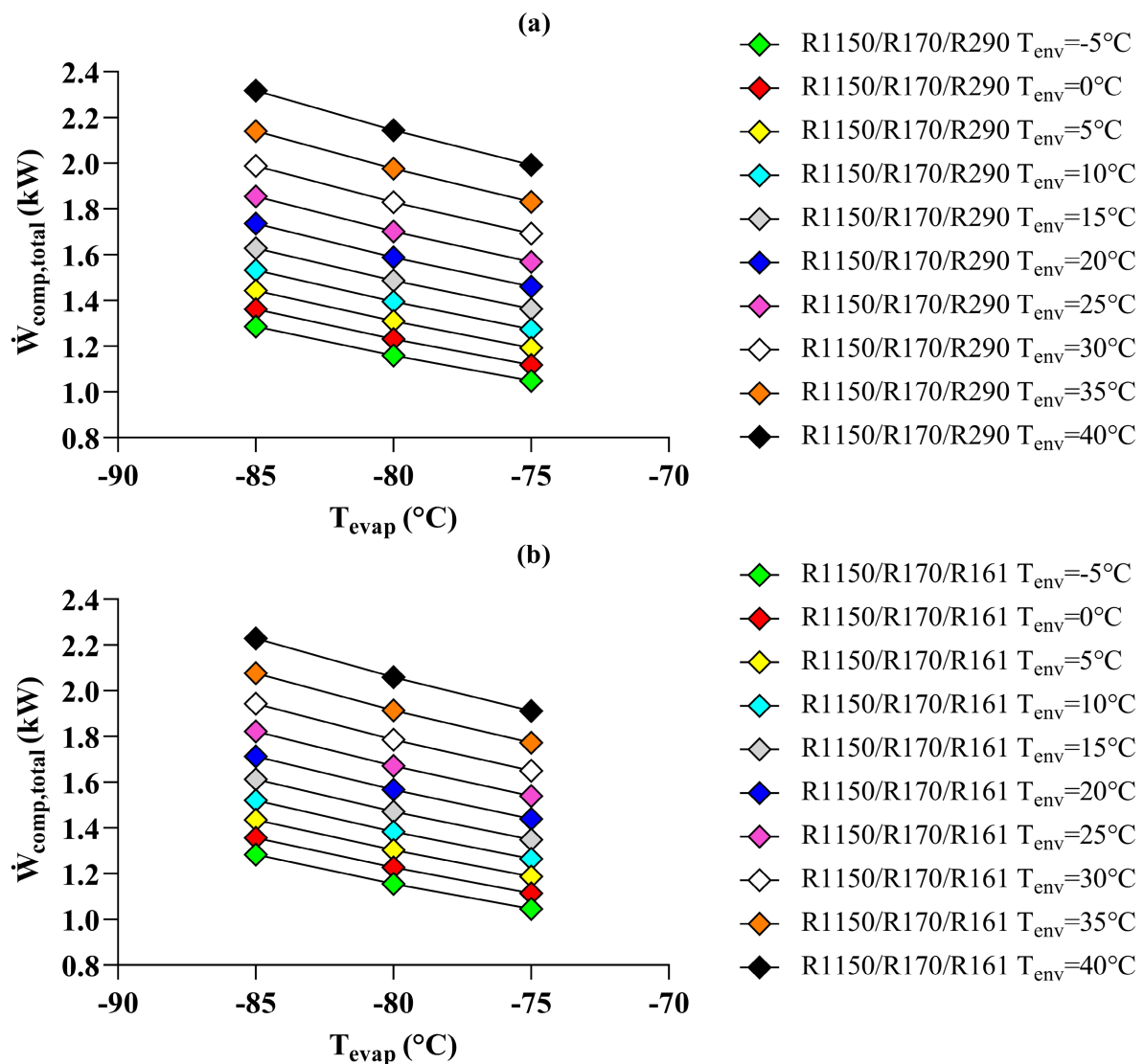


Figure 10. Significance of T_{env} and T_{evap} on $\dot{W}_{comp,total}$ for the three-stage configuration (a) R1150/R170/R290; (b) R1150/R170/R161.

Figure 12 depicts the impact of changes in evaporator and ambient temperatures on the total exergy destruction of the three-stage cascade refrigeration system operating with the R1150/R170/R290 and R1150/R170/R161 refrigerant combinations. As shown in Figure 12, at a constant ambient temperature, increasing the evaporator temperature decreases total exergy destruction, consistent with observations from earlier studies [27]. For example, at an ambient temperature of 15°C , raising the evaporator temperature from -85°C to -75°C reduces the total exergy destruction by 18.7% for the R1150/R170/R290 refrigerant group. Moreover, Figure 12 shows that at a fixed evaporator temperature, total exergy destruction increases with increasing ambient temperature. Specifically, for the R1150/R170/R161 refrigerant combination operating at an evaporator temperature of -85°C , increasing the ambient temperature from -5°C to 40°C leads to an increase in total exergy destruction from 0.467 kW to 1.413 kW. An overall comparison further reveals that the R1150/R170/R290 refrigerant group consistently exhibits higher exergy

destruction than the R1150/R170/R161 combination over the entire range of operating conditions examined.

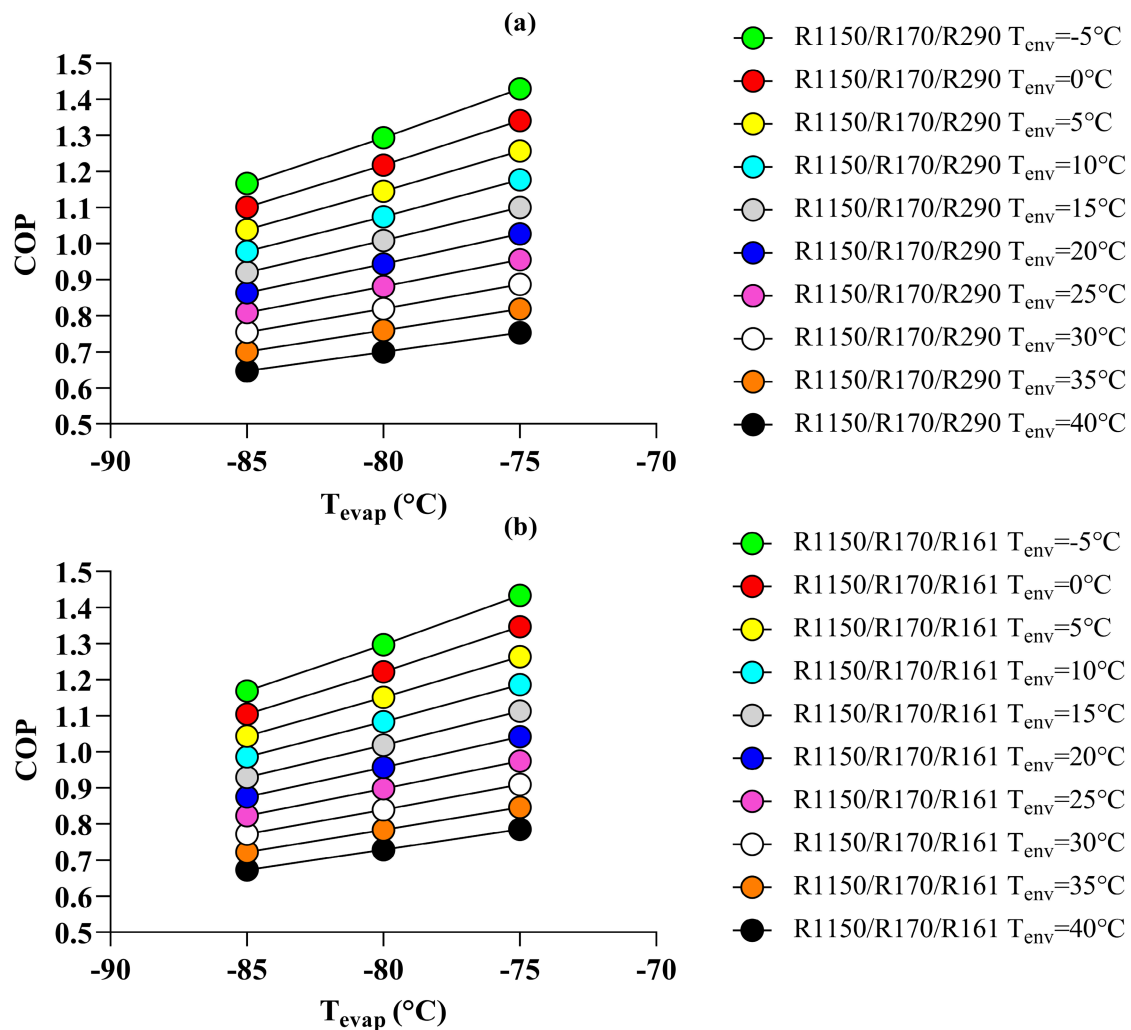


Figure 11. Contribution of T_{env} and T_{evap} to COP of the three-stage configuration (a) R1150/R170/R290; (b) R1150/R170/R161.

Figure 13 demonstrates the influence of ambient and evaporator temperature variations on the exergy efficiency of the three-stage cascade refrigeration system employing the R1150/R170/R290 and R1150/R170/R161 refrigerant combinations. As shown in Figure 13, at a constant ambient temperature, increasing the evaporator temperature enhances the exergy efficiency, in good agreement with previous studies [36]. For example, at an ambient temperature of -5 °C, raising the evaporator temperature from -85 °C to -75 °C increases the exergy efficiency from 0.63 to 0.67 for the R1150/R170/R290 refrigerant group. In addition, Figure 13 shows that at a fixed evaporator temperature, lower ambient temperatures improve exergy efficiency. Specifically, for the R1150/R170/R161 refrigerant combination operating at an evaporator temperature of -75 °C, decreasing the ambient temperature from 40 °C to -5 °C results in an 82.7% improvement in exergy efficiency. A comparative evaluation further indicates that the R1150/R170/R161 refrigerant group consistently achieves higher exergy efficiency than the R1150/R170/R290 combination across all operating conditions investigated. At elevated ambient temperatures, the exergy efficiency shows a nearly flat response to increasing evaporator temperature. By contrast, under low ambient temperatures, the lower irreversibilities in the condenser enable the positive impact of higher evaporator temperatures to be more clearly reflected

in the exergy efficiency. Consequently, at high ambient temperatures, the sensitivity of exergy efficiency to changes in evaporator temperature is markedly reduced due to the predominance of irreversibilities associated with the condenser.

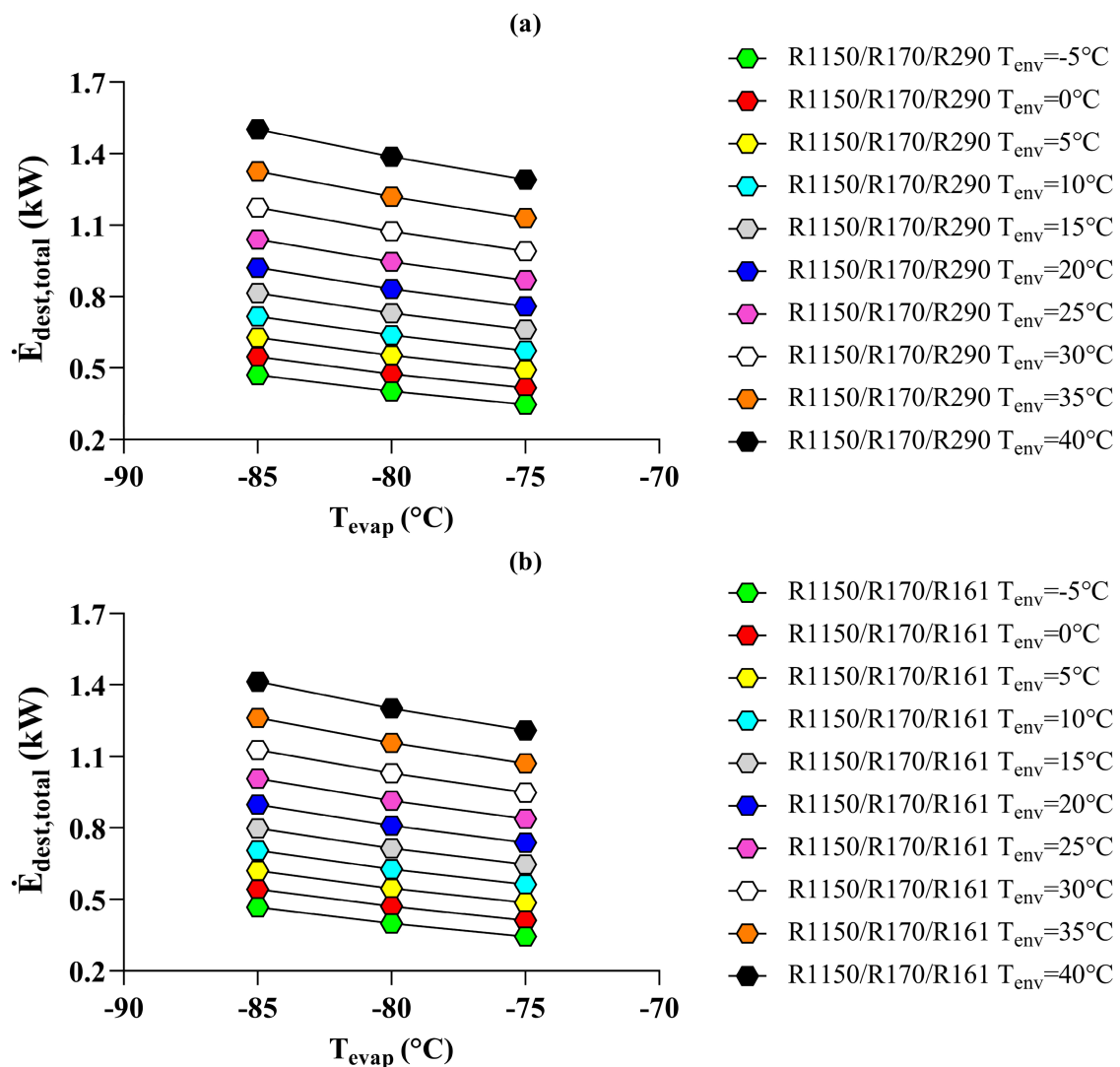


Figure 12. Effect of T_{env} and T_{evap} on $\dot{E}_{\text{dest,total}}$ of the three-stage configuration (a) R1150/R170/R290; (b) R1150/R170/R161.

Table 4 presents the comparison results of two-stage and three-stage cascade refrigeration systems. For instance, consider the R170/R290 refrigerant pair for the two-stage system and the R1150/R170/R290 refrigerant group for the three-stage system at an evaporator temperature of -85 °C and an ambient temperature of 25 °C. Under these conditions, the COP, exergy efficiency, and total compressor power consumption of the two-stage system are 0.73, 0.28, and 2.05 kW, respectively, whereas those of the three-stage system are 0.81, 0.44, and 1.86 kW, respectively. In particular, the three-stage system's reduction in compressor power consumption results in overall performance improvements compared to the two-stage system. A similar comparison was performed for the R170/R161 refrigerant pair in the two-stage system and the R1150/R170/R161 refrigerant group in the three-stage system at an evaporator temperature of -80 °C and an ambient temperature of 10 °C. Under these operating conditions, the two-stage system exhibits a COP of 1.03, an exergy efficiency of 0.37, and a total compressor power consumption of 1.45 kW, while the three-stage system yields a COP of 1.08, an exergy efficiency of 0.55, and a total compressor power

consumption of 1.38 kW. Once again, the three-stage system demonstrates a noticeable advantage over the two-stage system in terms of reduced compressor power consumption.

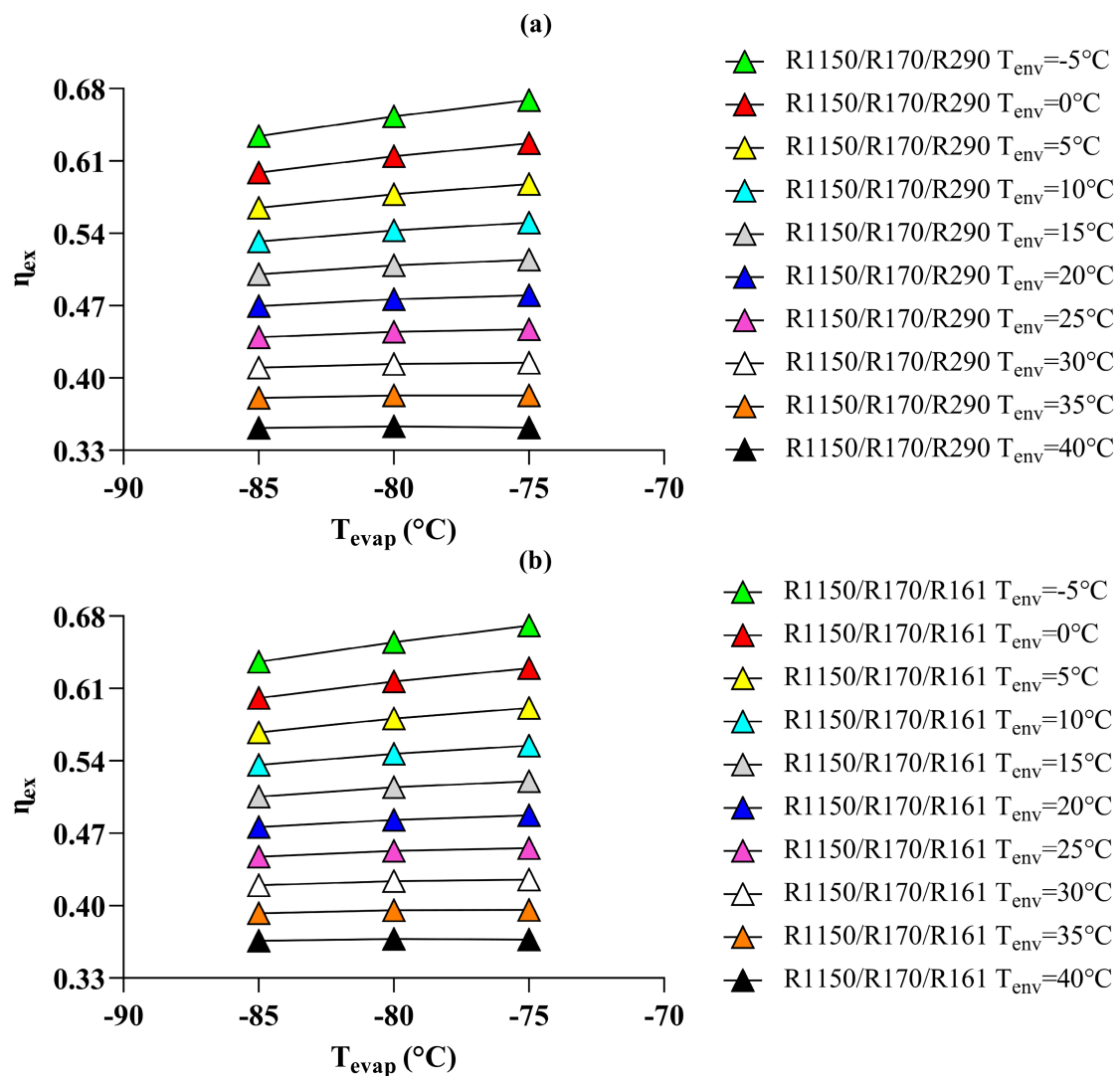


Figure 13. Influence of T_{env} and T_{evap} on η_{ex} of the three-stage configuration (a) R1150/R170/R290; (b) R1150/R170/R161.

Table 4. Performance comparison of two- and three-stage systems.

System and Refrigerants	COP	η_{ex}	$\dot{W}_{\text{comp, total}}$ (kW)	Selected Operating Parameters
Two-stage with R170/R290	0.73	0.28	2.05	$T_{\text{evap}} = -85^\circ\text{C}$ $T_{\text{env}} = 25^\circ\text{C}$
Three-stage with R1150/R170/R290	0.81	0.44	1.86	
Two-stage with R170/R161	1.03	0.37	1.45	$T_{\text{evap}} = -80^\circ\text{C}$ $T_{\text{env}} = 10^\circ\text{C}$
Three-stage with R1150/R170/R161	1.08	0.55	1.38	

In commercially available ultra-low-temperature refrigeration units based on two-stage cascade configurations, the R170/R290 refrigerant combination is widely adopted. However, despite its reported satisfactory thermal performance, numerous studies have demonstrated that the R170/R161 refrigerant pair offers superior thermodynamic performance compared with R170/R290 and other alternative refrigerant combinations [11,33,37].

Consistent with the literature, using R161 instead of R290 in the high-temperature cycle of both the two-stage and three-stage systems improved performance by reducing compressor power consumption.

The component-wise exergy destruction of the conventional two-stage R170/R290 system and the proposed three-stage R1150/R170/R161 system was evaluated at the common operating condition of an evaporator temperature of $-75\text{ }^{\circ}\text{C}$, an ambient temperature of $30\text{ }^{\circ}\text{C}$, and a cooling capacity of 1.5 kW. The results are presented in Table 5. For the three-stage system, the largest exergy destruction occurs in the HTC compressor (20.03%), followed by the condenser (19.83%) and the HTC expansion valve (17.65%). For the two-stage R170/R290 system, the HTC compressor (22.56%) also represents the largest source of exergy destruction, followed by the condenser (19.10%), HTC expansion valve (16.37%), and the cascade heat exchanger (14.44%). Compared with the two-stage configuration, the exergy destruction in the three-stage system is distributed more uniformly among the system components. Splitting the overall temperature lift into three compression stages reduces the concentration of irreversibilities within individual components, contributing to the lower total exergy destruction and improved overall system performance.

Table 5. Component-level comparison of exergy destructions of two- and three-stage cascade refrigeration systems.

R1150/R170/R161, $\dot{Q}_{\text{evap}} = 1.5\text{ kW}$ $T_{\text{evap}} = -75\text{ }^{\circ}\text{C}$, $T_{\text{env}} = 30\text{ }^{\circ}\text{C}$		
Component	Exergy Destruction (kW)	Percentage Contribution
HTC Compressor	0.1962	20.03%
Condenser	0.1942	19.83%
HTC Valve	0.1729	17.65%
MTC Compressor	0.07522	7.68%
Cascade Heat Exchanger 1	0.07104	7.25%
Cascade Heat Exchanger 2	0.06825	6.97%
MTC Valve	0.06512	6.65%
Evaporator	0.05561	5.68%
LTC Compressor	0.05002	5.11%
LTC Valve	0.03098	3.16%
Total	0.9795	100%
R170/R290, $\dot{Q}_{\text{evap}} = 1.5\text{ kW}$ $T_{\text{evap}} = -75\text{ }^{\circ}\text{C}$, $T_{\text{env}} = 30\text{ }^{\circ}\text{C}$		
Component	Exergy Destruction (kW)	Percentage Contribution
HTC Compressor	0.2986	22.56%
Condenser	0.2529	19.10%
HTC Valve	0.2168	16.37%
Cascade Heat Exchanger	0.1912	14.44%
Evaporator	0.1607	12.14%
LTC Compressor	0.1257	9.49%
LTC Valve	0.0782	5.90%
Total	1.324	100%

3.2. Long-Term Dynamic Simulation Results

Figure 14 compares double- and triple-cascade systems, using different refrigerators, in terms of monthly energy consumption. For the double cascade system, the pairs R170/R161 and R170/R290 are investigated, while for the triple cascade system, attention is paid to R1150/R170/290 and R1150/R170/161. Figure 14 shows that the triple-cascade system performs better in all cases, and that the best-performing refrigerant combination is R1150/R170/161. Moreover, the figure shows that under milder climate conditions, the difference between the considered refrigerant combination becomes sharper, especially in the summer. So, the use of R1150/R170/161 could lead to significant savings in both money and carbon dioxide emissions.

Quantitative savings in terms of annual energy consumption are reported in Table 6. The table highlights a significant reduction in energy consumption (20.5% in Warsaw, 26.6% in Mugla) when using three-cascade refrigerators with R1150/R170/R161 refrigerants compared to the conventional double-cycle system with R170/R290.

Figures 15 and 16 show the annual energy consumption for a single refrigerator when a fan is installed to introduce fresh air from outside into the room where the refrigerators are located. In particular, Figure 15 shows the trend in annual energy consumption for the given configuration (R1150/R170/R161), assuming the rooms are located in Mugla, Milan, and Warsaw. The figure reports across all three cities under investigation a significant reduction in energy consumption resulting from the introduction of outdoor air into the installation room. This reduction becomes progressively less significant as the airflow rate increases.

Figure 16 analyses the trend in annual electricity consumption for the city of Milan as the airflow rate is varied for the four types of refrigeration units. The highest energy consumption is always associated with the installation of a two-stage system with R170/290, while the lowest is associated with a three-stage system with R1150/R170/R161.

As the airflow rate in the environment increases, the energy-saving benefit associated with higher-performance systems (three-stage vs. two-stage) tends to decrease.

In both Figures 15 and 16, the values shown on the y-axis refer exclusively to the energy consumption of a single refrigerator and do not include any data regarding the fan's consumption. For reference, at an airflow rate of 750 m³/h, the fan power can be assumed to be 100 W. Continuous operation of the fan results in an annual energy consumption of 876 kWh, which corresponds to the five refrigerators installed in the room, or 175.2 kWh per refrigerator.

It is evident that introducing outdoor air to cool the installation room for refrigerators provides significant energy benefits by lowering the ambient temperature at which the refrigerators exchange heat, thereby increasing the COP.

To assess any further potential for energy savings, additional simulations were conducted by cooling the air supply during the summer (from 1 May to 30 September) using a commercial refrigeration unit. The analysis was carried out for the city of Milan at an airflow rate of 750 m³/h in two configurations: (i) Summer air cooling, operating 24 h a day; (ii) Summer air cooling, operating from 10:00 to 20:00. Table 7 reports the energy consumption of three-stage system with R1150/R170/R161 in which the air in the installation room is supplied at a flow rate of 750 m³/h under three different conditions: (1) exclusively outdoor air; (2) during the summer months (from 1 May to 30 September), air cooled by a chiller operating 24 h a day; (3) during the summer months (from 1 May to 30 September), air cooled by a chiller operating from 10:00 to 20:00 each day (10 h).

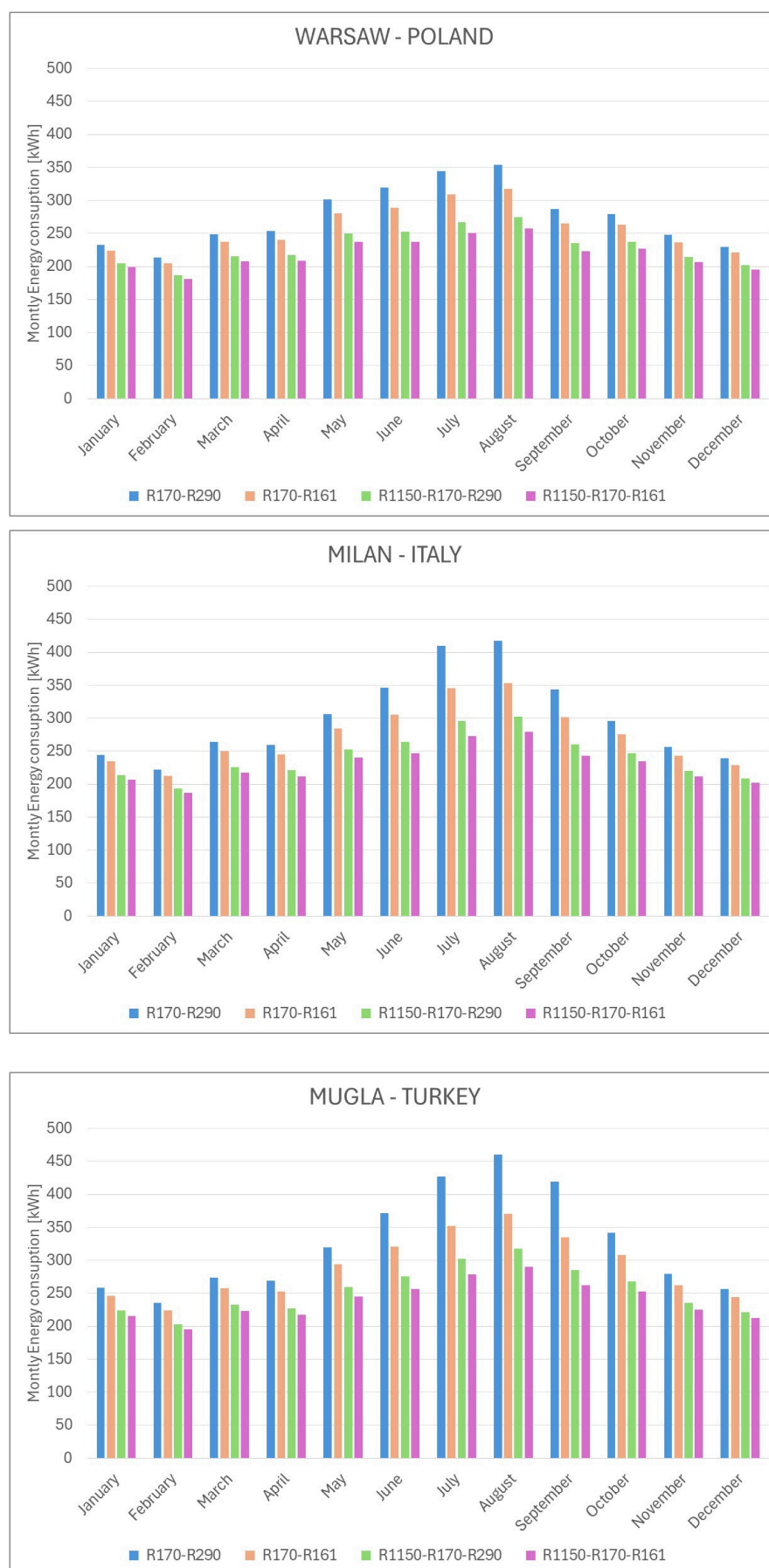


Figure 14. Monthly energy consumption for a single refrigerator across the three considered locations: comparison between double- and triple-cascade systems.

Table 6. Annual energy consumption for a single refrigerator R170/R290, R1150/R170/R290, R170/R161 and R1150/R170/R161 for different locations.

Working Fluids	Warsaw	Milan	Mugla
R170/R290	3312	3614	3913
R1150/R170/R290	2759	2907	3053
R170/R161	3089	3281	3467
R1150/R170/R161	2634	2754	2873
SAVINGS	20.5%	23.6%	26.6%

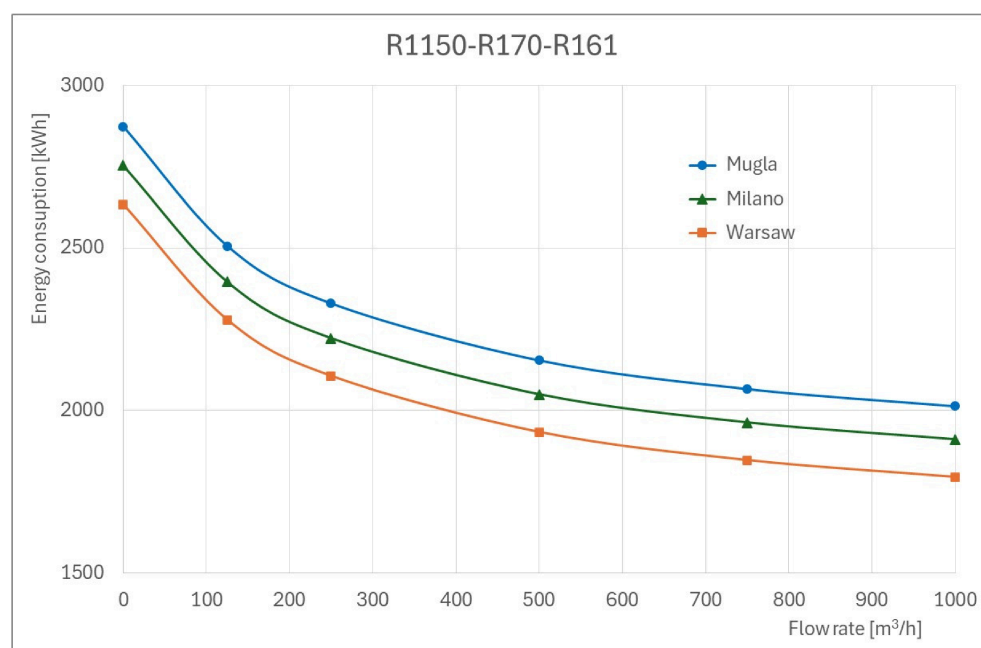


Figure 15. Effect of flow rate on energy consumption for a single refrigerator R1150/R170/R161 for the three cities.

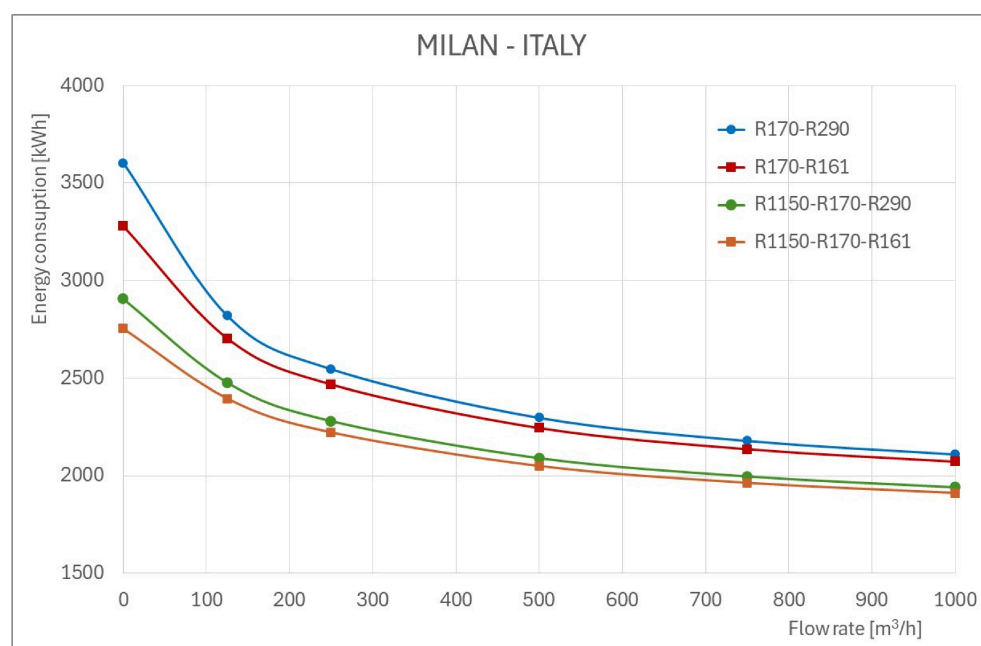


Figure 16. Effect of flow rate on energy consumption for a single refrigerator using all refrigerants for the city of Milan.

Table 7. Annual energy consumption for a single refrigerator R1150/R170/R161, considering air cooling from 1 May to 30 September for 24 h or 10 h a day.

Energy Consumption [kWh]	R1150/R170/R161	Fan	Cooling Air
Only fan	1964	175	--
Air cooling 24 h	1904	175	368
Air cooling 10 h	1929	175	213

The conducted analysis highlights the lack of benefit from further cooling the supply air using a chiller, even when limited to daytime operation. In both cases considered (24 h or 10 h per day), the reductions in energy consumption associated with the additional air-cooling amount to only a few tens of kWh, whereas the increase in consumption due to the chiller is on the order of several hundred kWh.

4. Conclusions

In the present paper, firstly, two-stage and three-stage cascade refrigeration systems were thermally designed using an Engineering Equation Solver (EES), followed by detailed energy and exergy analyses. Evaporator temperatures of $-85\text{ }^{\circ}\text{C}$, $-80\text{ }^{\circ}\text{C}$, and $-75\text{ }^{\circ}\text{C}$ and ambient temperatures between $-5\text{ }^{\circ}\text{C}$ and $40\text{ }^{\circ}\text{C}$ were examined. For the two-stage system, the refrigerant pair R170/R161 was evaluated as an alternative to the conventional R170/R290 combination. For the three-stage system, R1150/R170/R290 and R1150/R170/R161 configurations were analyzed. Results showed that replacing R290 with R161 in the high-temperature cycle improved system performance in both configurations. Additionally, under identical operating conditions, the three-stage system demonstrated lower compressor power consumption and exergy destruction, along with higher COP and exergy efficiency, compared to the two-stage system. In the second part of the paper, long-term dynamic energy simulations were conducted in TRNSYS under varying ambient conditions. The findings confirmed that R161 significantly reduced energy consumption compared to R290, in both two- and three-stage systems.

The superior performance of the three-stage cascade system compared to the two-stage configuration can be explained by fundamental thermodynamic principles. In a three-stage system, the overall temperature lift between the evaporator and condenser is divided into smaller temperature intervals across multiple stages. This reduction in temperature lift per stage results in lower compression ratios, thereby significantly reducing total compressor power consumption. As a result, the work required by each compressor is reduced, and the cumulative compressor energy demand becomes lower than that of the two-stage system. Therefore, the thermodynamic advantage of the three-stage system arises from the combined effects of reduced temperature lift and lower total compressor work.

From a practical perspective, the results suggest that hospital facility managers operating ultra-low-temperature vaccine storage rooms should prioritize improving the refrigeration system efficiency before considering additional room cooling equipment. Replacing the conventional two-stage R170/R290 system with the three-stage R1150/R170/R161 configuration can reduce annual energy consumption by approximately 20–27%, depending on climatic conditions. Moreover, providing adequate fresh-air ventilation to the equipment room can further reduce energy consumption, whereas installing an additional chiller for room conditioning is not recommended based solely on energy-saving considerations, as the resulting reduction in electricity consumption is insufficient to justify the additional investment.

The present study has several limitations that should be acknowledged. The intermediate temperatures were maintained at representative design values throughout the parametric analyses to enable a consistent comparison between the investigated system

configurations. Furthermore, the long-term simulations focused solely on refrigeration system energy consumption and did not account for equipment degradation, refrigerant leakage, advanced control strategies, or economic factors such as capital investment and maintenance costs. These aspects should be addressed in future studies to provide a more comprehensive techno-economic assessment.

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