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Stackelberg leadership and managerial delegation under hyperbolic demand*

Flavio Delbono§ and Luca Lambertini#

Department of Economics, University of Bologna

§ Piazza Scaravilli 2, 40126 Bologna, Italy; flavio.delbono@unibo.it

Via San Giacomo 3, 40126 Bologna, Italy; luca.lambertini@unibo.it

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Abstract

We revisit the traditional Stackelberg model considering a hyperbolic demand function. We show that, in duopoly, there exists no incentive to acquire leadership or to separate ownership and control by hiring a manager. The reason is that best replies are orthogonal in a complete neighbourhood of the Nash equilibrium. The unilateral incentive either to lead or to hire a manager is restored if the industry is at least triopolistic. This holds irrespective of the specific delegation contract being adopted.

Keywords: strategic delegation; first mover advantage; hyperbolic demand

JEL: D43, L13

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1 Introduction

The widespread use of linear demand functions and full symmetry across firms made us accustomed to think of the Stackelberg model as one where moving first is better than playing the simultaneous Nash equilibrium, in particular in the baseline Cournot setting with a linear demand for a homogeneous good. This also holds for non linear demand functions such as $p = (a - Q)^{\gamma}$, with $\gamma > 0$, as in Anderson and Engers (1992, 1994). However, adopting a lattice-theoretic approach, Amir and Grilo (1999) identify sufficient conditions for Nash and Stackelberg equilibria to arise at the subgame perfect equilibrium of extensive games with endogenous timing *à la* Hamilton and Slutsky (1990), for both log-concave and log-convex inverse demand function and accounting for cost asymmetries.

We are about to consider a specific case of log-convex demand and a common technology being at least quasi-convex in output, to show that Nash and Stackelberg outcomes coincide in duopoly if demand is hyperbolic. The driver of this result can be found in reaction functions being locally orthogonal in the neighbourhood of their intersection identifying the Nash equilibrium. Since we know the Stackelberg outcome can be replicated as a Nash equilibrium at the market stage if a firm delegates control to a manager, then we also draw the logical implication that such a duopoly cannot host unilateral strategic delegation. We prove this result taking into account the prevailing managerial contracts investigated in the related literature. The incentive to move first or to unilaterally delegate reappears when the market is supplied by at least three firms.[1]

The next section presents the general model and the coincidence between Nash and Stackelberg outcomes in a duopoly with entrepreneurial firms. The

[1] The orthogonality of reaction functions for a generic number of firms emerges if objective functions are parabolic cylinders (Delbono and Lambertini, 2018).

oligopoly game is in section 3, while section 4 contains the analysis of strategic delegation contracts. Implications are outlined in section 5.

2 Problem statement

We consider a homogeneous Cournot oligopoly with $n \geq 2$ firms competing along a hyperbolic inverse demand function $p = a/Q$, $Q = \sum_{i=1}^{n} q_i$, where q_i is the output of firm i, and a is a positive parameter. The technology being used is summarised by the individual cost function $C_i = f_i(q_i)$, with $\partial f_i(q_i)/\partial q_i > 0$ and $\partial^2 f_i(q_i)/\partial q_i^2 \geq 0$. Hence, the profit function of the i-th firm is $\pi_i = pq_i - f_i(q_i)$.

The game takes place under complete and symmetric information. To begin with, we illustrate a key property of Cournot competition under hyperbolic demand. The first order condition (FOC) of the generic firm is

$$\frac{\partial \pi_i}{\partial q_i} = \frac{a \sum_{j \neq i} q_j}{Q^2} - \frac{\partial f_i(q_i)}{\partial q_i} = 0 \tag{1}$$

while the concavity condition is

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{2a \sum_{j \neq i} q_j}{Q^3} - \frac{\partial^2 f_i(q_i)}{\partial q_i^2} \leq 0 \tag{2}$$

which is always met as a strict inequality. Now, the sign of the slope of the best reply function is the same as the sign of the following (Bulow *et al.*, 1985):

$$\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} = \frac{a\left(q_i - \sum_{j \neq i} q_j\right)}{Q^3} \tag{3}$$

Then, if we impose symmetry across all firms, whereby $q_j = q_i = q$, the above condition reads

$$\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} = \frac{a(n-2)}{n^3 q^2} \tag{4}$$

On the basis of (4), we may claim

Lemma 1 *In a Cournot duopoly with hyperbolic demand, Nash and Stackelberg equilibria coincide, irrespective of the specific shape of the cost function.*

Proof. The expression appearing on the r.h.s. of (4) becomes nil in correspondence of $n = 2$, which implies that, in duopoly, the best replies are orthogonal in the neighbourhood of the their intersection. This, in turn, entails that the relevant isoprofit curve of each firm is tangent to the rival's best reply. The ultimate consequence is that the Nash and the Stackelberg solutions indeed coincide.

Concerning the second part of the claim contained in the Lemma, it suffices to observe that the partial cross derivative appearing in (3) is independent of the cost function. ■

That is to say that in such a market, if it is a duopoly, Stackelberg leadership cannot arise - or, it is indistinguishable from the Nash equilibrium because the first mover advantage ceases to exist.

Hence, we shall illustrate the Stackelberg solution with more than two firms in order to show that the incentive for a single firm to acquire the leadership indeed exists for all $n \geq 3$, and characterise the relevant properties of this setting.

3 Stackelberg leadership in oligopoly

In order to solve the game explicitly, we pose $f_i(q_i) = cq_i$, $c > 0$, for all $i = 1, 2, ...n$. Suppose firm i holds the first mover advantage against the other $n - 1$ firms. The leader's problem is

$$\max_{q_i} \pi_i = q_i \left(\frac{a}{q_i + \sum_{j \neq i} q_j} - c \right) \tag{5}$$

subject to

$$q_j = \arg\max_{q_j} \pi_j = q_j \left(\frac{a}{q_i + q_j + \sum_{\ell \neq i,j} q_\ell} - c \right) \forall\, j \neq i, \ell \tag{6}$$

Proceeding by backward induction, we write the system of FOCs of the followers:

$$\frac{\partial \pi_j}{\partial q_j} = \frac{a\left(q_i + \sum_{\ell \neq i,j} q_\ell\right)}{Q^2} - c = 0 \tag{7}$$

Then, imposing symmetry across followers' output levels, we may write the single follower's best reply as

$$q_F^{br} = \frac{a(n-2) - 2c(n-1)q_i + \sqrt{a\left[a(n-2)^2 + 4c(n-1)q_i\right]}}{2c(n-1)^2} \tag{8}$$

Now, plugging $(n-1)q_F^{br}$ into the leader's profit function, which simplifies as follows:

$$\pi_L = \frac{\sqrt{a\left[a(n-2)^2 + 4c(n-1)q_L\right]} - a(n-2) - 2cq_L}{2} \tag{9}$$

which reaches its maximum at $q_L^* = a(2n-3)/[4c(n-1)]$. The corresponding equilibrium profits are $\pi_L^* = a/[4(n-1)]$. The next step amounts to comparing the performance of the Stackelberg leader with that of a generic firm at the Nash equilibrium, which is easily attained by imposing symmetry on outputs in (7) and solving w.r.t. the individual output, to get $q_N^* = a(n-1)/(cn^2)$ and $\pi_N^* = a/n^2$. Trivial algebra suffices to show that $q_L^* > q_N^*$ and $\pi_L^* > \pi_N^*$ for all $n \geq 3$, so that a strict incentive to acquire the leadership exists in any market at least as competitive as a triopoly.

However, note that each follower obtains $\pi_F^* = \pi_L^*/(n-1)$ by selling $q_F^* = q_L^*/(n-1)$. This automatically entails that, in presence of a hyperbolic demand, the performance of the leader, in terms of market share and profits, is equivalent to that of the entire population of followers, irrespective of the size of the latter, including the duopoly case. This fact, by the way, also explains the conundrum illustrated in Lemma 1 above: since the leader's and the follower's profits must coincide in duopoly, they must also coincide with the symmetric Nash profits, which in turn drives the consequence that, if and only if $n = 2$, the reaction functions are locally orthogonal.[2]

This discussion boils down to the following

Proposition 2 *If market demand is hyperbolic, the first mover advantage for a Cournot firm arises if and only if* $n \geq 3$. *Moreover,* $q_L^* = (n-1)\, q_F^*$ *and* $\pi_L^* = (n-1)\, \pi_F^*$ *for all* $n \geq 2$.

Now there remains to illustrate the two-stage game where firms play simultaneously at the market stage and the first stage is for the strategic choice of delegating control to managers.

4 Strategic delegation

The results appearing in Lemma 1 and Proposition 2 drive a consequence as for the possibility of delegating control to managers in this setting. The literature on strategic delegation has discussed four main incentive schemes for managers, based upon production (Vickers, 1985), revenues (Fershtman, 1985; Fershtman and Judd, 1987; Sklivas, 1987), market shares (Jansen *et al.*, 2007; Ritz, 2008) and comparative performance (Salas Fumas, 1992; Reitman, 1993; Lundgren, 1996; Miller and Pazgal, 2001). While the outcome of industry-wide delegation to managers is affected by the nature of the delegation contract, that associated with unilateral delegation by a single firm is not, as in this case delegation systematically delivers the Cournot-Stackelberg outcome irrespective of the magnitude accompanying firm's own profits in the

[2] Example 2 in Amir and Grilo (1999, pp. 14-15) illustrates a Cournot duopoly with constant but asymmetric marginal costs in which demand is hyperbolic, $p = 1/(1 + q_1 + q_2)$. Due to the presence of a constant at the denominator, best replies cannot be orthogonal in the neighbouhood of the intersection and therefore Nash and Stackelberg equilibria do not coincide.

definition of the managerial objective function (see Berr, 2011; Lambertini, 2017).

Accordingly, now we illustrate the solution of the two-stage game in which, say, firm i is (at least potentially) managerial and any other firms remain entrepreneurial. We will confine ourselves to the cases of linear and quadratic costs as the presence of a generic convex cost function does not allow the replication of Lemma 1. Adopting the delegation scheme dating back to Vickers (1985), the relevant objective functions at the market stage are $M_i = \pi_i + \theta_i q_i$ and π_j, respectively. If the cost function is linear, the resulting first order conditions are the following:

$$\begin{aligned} \frac{\partial M_i}{\partial q_i} &= \frac{a \sum_{j \neq i} q_j}{\left(q_i + \sum_{j \neq i} q_j\right)^2} - (c - \theta_i) = 0 \\ \frac{\partial \pi_j}{\partial q_j} &= \frac{a\left(q_i + \sum_{\ell \neq i,j} q_\ell\right)}{\left(q_i + q_i + \sum_{\ell \neq i,j} q_\ell\right)^2} - c = 0 \end{aligned} \tag{10}$$

Imposing symmetry across the output levels of the entrepreneurial firms, $q_j = q_\ell = q$ for all $j, \ell \neq i$, and solving the above system, we obtain the expression of the profit function which the owner of firm i is supposed to maximise w.r.t. the contractual variable at the first stage, i.e.,

$$\pi_i = \frac{a\,(c - \theta_i)\,[c + (n-2)\,\theta_i]}{(\theta_i - nc)^2} \tag{11}$$

Maximising (11) w.r.t. θ_i delivers $\theta_\iota^* = c\,(n-2)\,/\,(2n-3)\,$, which is nil at $n = 2$.

Concerning the alternative delegation contracts, we briefly recollect the relevant managerial objective functions and summarise the resulting equilibrium levels of the delegation variables without an extensive exposition of the related calculations, which can be easily replicated.

Take first the delegation mechanism adopted in Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987). In this approach, the single managerial firm, at he market stage, chooses its output to maximise a weighted average of profits and revenues defined by $M_i = \alpha_i \pi_i + (1 - \alpha_i) p q_i$ As we know from Lambertini and Trombetta (2002), this delegation scheme is equivalent to Vickers (1985) with a linear demand, and indeed this also applies if market demand is hyperbolic because the equilibrium value of the delegation variable is $\alpha_i^* = (n-1) / (2n-3)$ which delivers the same quantities and profits as in the previous case. If $n = 2$, then $\alpha_i^* = 1$, confirming that in duopoly delegation is not profitable and firm i's owner decides to remain in control.

The third case is that where delegation is based on comparative performance, as in Miller and Pazgal (2001) and several others. Here, the relevant maximand for the manager of firm i is $M_i = \pi_i + \beta_i \sum_{j \neq i} \pi_j$. Again the outcome is the same, with $\beta_i^* = (2-n) / (n-1) \leq 0$ for all $n \geq 2$.

The fourth and last case (Jansen *et al.*, 2007; Ritz, 2008) contemplates a managerial incentive relying upon a mix of profits and market share, $M_i = \pi_i + \lambda_i q_i / Q$. This game also delivers the same outputs and profits at equilibrium, as in the previous cases, and $\lambda_i^* = a(n-2) / (n-1)$, which vanishes if $n = 2$.

If $f_i(q_i) = c q_i^2$, the Nash equilibrium at the market stage with firm i being managerial, is identified by

$$\begin{aligned} q_i^N &= \frac{\theta_i^2 (\theta_i + \Psi) + 2ac \left[(2 + n(2n-3)) \theta_i + n\Psi \right]}{4c \left(2acn^2 + \theta_i^2 \right)} > 0 \\ q_j^N &= \frac{a \left[n\Psi - (n-2) \theta_i \right]}{4c \left(2acn^2 + \theta_i^2 \right)} > 0 \end{aligned} \tag{12}$$

with $\Psi \equiv \sqrt{8ac(n-1) + \theta_i^2}$. If $n = 2$, the optimal contract at the first stage must solve

$$\frac{\partial \pi_i}{\partial \theta_i} = -\frac{\theta_i \left[\theta_i^4 \left(\theta_i + \sqrt{\Psi} \right) + 4ac\theta_i^2 \left(5\theta_i + 4\sqrt{\Psi} \right) + 8a^2c^2 \left(12\theta_i + 7\sqrt{\Psi} \right) \right]}{4c \left(8ac + \theta_i^2 \right)^{5/2}} = 0 \tag{13}$$

which is nil only in $\theta_i = 0$. If $n \geq 3$, the optimal contract replicates the corresponding Stackelberg equilibrium. Again, the same conclusion may be reached over the aforementioned range of delegation schemes.

All of this can be summarised in

Corollary 3 *In a Cournot duopoly with hyperbolic demand, unilateral delegation cannot arise at equilibrium, irrespective of the shape of managerial incentives.*

5 Implications

Here we have shown that, in presence of a hyperbolic demand function, the unilateral incentive to go managerial or incorporate the rival's reaction function does not arise. However, such incentive is restored with at least three firms. Now imagine a sequential entry process leading to $n \geq 3$. The first entrant, being an ad interim monopolist, will not delegate control, and will not react either way to the arrival of the second firm.

However, starting from the entry of the third, we do have an issue about whether one firm - which is not necessarily the first - is smarter and faster than the others. If this is the case, then at least for a while we will observe the emergence of a dominant position. Yet, under symmetric and complete information, all firms are aware that separating control from ownership is incentive compatible (while incorporating rivals' best replies is not), and therefore the entire industry must be expected to be populated by managerial firms from the triopoly onwards.

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