Transboundary pollution control under evolving social norms: a mean-field approach

Published Version:
La Torre, D., Maggistro, R., Marsiglio, S. (2025). Transboundary pollution control under evolving social norms: a mean-field approach. DECISIONS IN ECONOMICS AND FINANCE, 48(1), 437-463 [10.1007/s10203-024-00459-9].

Availability:
This version is available at: https://hdl.handle.net/11585/1047410 since: 2026-02-22

Published:
DOI: http://doi.org/10.1007/s10203-024-00459-9

Transboundary Pollution Control under Evolving Social Norms: a Mean-Field Approach*

Davide La Torre[†] Rosario Maggistro[‡] Simone Marsiglio[§]

Abstract

We analyze a dynamic game of transboundary pollution control under endogenously evolving social norms over a finite time horizon. Each player chooses their emission level in order to minimize the social cost of mitigation, which partly depends on the lack of conformity to the social norm establishing the pollution standards at the local level. We show that social norms per se are unable to favor pollution reductions, but if combined with some public reclamation effort, they become very effective in improving environmental outcomes. Indeed, provided that some minimal public reclamation takes place, social norms promote a reduction in the average of the expected value of the local pollution stocks across locations, both in the case in which players rely on an open loop and a closed loop strategy. Moreover, by explicitly characterizing the equilibrium outcome, we formally confirm the reliability of the mean-field approximation of the finite-population dynamics, despite such an approximation introduces some distortion regarding the difference between open and closed loop strategies. We also show that our results are robust to the introduction of individual abatement efforts and heterogeneity across players.

Keywords: Mean-Field Game; N-Player Game; Transboundary Pollution; Social Norms
JEL Classification: C70, Q50

1 Introduction

Over the last few decades, a growing consensus regarding the detrimental consequences of economic activity on the environment and thus the need to reduce emissions in order to contrast climate change have emerged (Oreskes, 2004; IPCC, 2014; Liu and Raftery, 2021). Despite the recommendations of scientists and international organizations not to postpone climate mitigation efforts, along with the apparent support from policymakers worldwide, the concrete results of international negotiations and agreements have been very limited (Finus and Tjotta, 2003; Ringquist and Kostadinova, 2005; Kellenberg Levinson, 2013). This is due to well-known problems related to free-riding and the transboundary features of pollution, which preclude such agreements from being able to improve environmental outcomes beyond the noncooperative scenario and from obtaining the critical mass of participation required for effective implementation (Carraro and Siniscalco, 1993; Barrett, 1997; Finus and Maus, 2008). Among the eventual strategies to improve environmental outcomes, it has recently been discussed that changes in social norms may represent a possible solution to promote individual agents to internalize part of the social cost of polluting activities (Steg et al., 2005; Nyborg et al., 2016; Sparkman et al., 2021). The concept of social norms has been heterogeneously defined across disciplines, but in widely accepted terms it refers to some type of behavioral incentive reflected

*We are grateful to Marco Tolotti and the participants in the International Conference on Sustainability, Environment, and Social Transition in Economics and Finance 2022 (Paris, France) for insightful discussions. All errors and omissions are our own sole responsibility.

[†]SKEMA Business School and Universite Cote d'Azur, Sophia Antipolis, France. Contact: davide.latorre@skema.edu.

[‡]University of Trieste, Department of Economics, Business, Mathematics and Statistics "B. de Finetti", Trieste, Italy. Contact: rosario.maggistro@deams.units.it.

[§]University of Pisa, Department of Economics and Management, Pisa, Italy. Contact: simone.marsiglio@unipi.it.

in the reputation and self-image of single agents with important implications for individual choices and actions (Hechter and Opp, 2001; Wallen and Romulo, 2017; Nyborg, 2018). To the best of our knowledge, the possible role of social norms has never been formally assessed in a game-theoretic framework of transboundary pollution control, thus in this paper, we investigate whether the social effects induced by norms may effectively drive individual emission decisions in such a way to favor environmental improvements.

The transboundary pollution control literature investigates the effectiveness of environmental policy from different perspectives and in different frameworks, such as in economic growth, uncertainty and spatial contexts (Ansuategi and Perrings, 2000; Athanassoglou and Xepapadeas, 2012; La Torre et al., 2021). The typical setting consists of a multi-player differential game which leads to the general conclusion that cooperation is essential but rather difficult to implement (Rubio and Ulph, 2007; Masoudi and Zaccour, 2013; Huang et al., 2016). However, this literature has completely neglected the role that social norms may play in driving individual behavior and determining environmental outcomes (Centola et al., 2018; Otto et al., 2020). Indeed, several studies in a number of alternative setups discuss that the need to conform to the actions taken by others may lead single individuals to make pro-environmental decisions resulting in a reduction in their ecological footprints, an increase in cooperation and an improvement in environmental health (Brekke et al., 2003; Marsiglio and Tolotti, 2020; Szekely et al., 2021). Such beneficial effects have been extensively documented in different contexts, ranging from increasing resource conservation to adopting sustainable technologies (Bollinger and Gillingham, 2012; Abrahamse and Steg, 2013), from reducing littering to promoting waste sorting (Keizer et al., 2008; Fornara et al., 2011). These effects may take place because social norms help in spreading information at the collective level, augmenting intrinsic motivation at the individual level and increasing concerns at the societal level (Sparkman et al., 2021). It is thus natural to wonder whether and how accounting for the presence of social norms in a transboundary pollution control framework may change our conclusions regarding the long run sustainability of economic activities.

In order to address this question we analyze an N-players differential game of transboundary pollution control in which players' payoff partly depends on social norms, which endogenously evolve over time according to players' actions. Specifically, we consider a framework in which the economic activities of each player generate polluting emissions with localized effects (such as carbon monoxide, sulfur dioxide, and particulate matter) and the stock of local pollution is affected also by the pollution flows from surrounding locations because of transboundary externalities, along with public reclamation activities and random shocks. Each player needs to choose their emissions level to minimize the social cost of mitigation, which partly depends upon social norms determining the pollution standards at the local level. In particular, the pollution standard each player wishes to conform to is quantified by the average stock of local pollution. As players' emission decisions may change over time also the average pollution stock will change, meaning that the local pollution standard each player wishes to conform to is endogenously determined as an equilibrium outcome of the strategic interactions among players. This setting allows us to analyze how endogenously evolving social norms may affect individual player's emission decisions and how the interaction between social norms and individual incentives may determine long run environmental outcomes. We show that despite such an interaction between social norms and individual incentives is not enough to favor a reduction in pollution standards, combining social norms with some reclamation effort proves very effective in improving environmental outcomes. Indeed, whenever some minimal reclamation takes place, social norms allow for a reduction in the average of the expected value of the local pollution stocks across locations, both in the case in which players rely on an open loop and a closed loop strategy. Moreover, by explicitly characterizing the equilibrium outcome we formally confirm the reliability of the mean-field approximation of the finite-population dynamics, despite such an approximation introduces some distortion regarding the difference between open and closed loop strategies. Furthermore, by extending the analysis to introduce individual abatement and heterogeneity in mitigation efforts and conformism across players, we show that our main conclusions hold true even in more complicated and realistic setups.

Our paper relates thus to two branches of the economics literature, dealing with transboundary pollution control and mean-field games respectively. The transboundary pollution control literature typically focuses on the evolution of global pollutants (i.e., greenhouse gases such as carbon dioxide, methane and nitrous oxide) to which each player directly contributes with their individual emissions (Carraro and Siniscalco, 1993; Masoudi and Zaccour, 2013). In our local pollution setup instead it is the pollution flows from surrounding locations that contribute to the evolution of pollution, thus players' choices affect one another indirectly through their implications of different stocks of local pollution. While theoretical works have mainly discussed global pollutant dynamics, empirical studies have put more emphasis on local pollutants due to the difficulty to disentangle the impact of individual polluters on global environmental problems (Missfeldt, 1999; Silva and Zhu, 2009). Thus, it seems interesting to explore also from a theoretical perspective the transboundary implications of local pollutants, and in this context, we can show that the interaction between players' desire to conform to social norms and public reclamation activities ensures that long run environmental improvements effectively take place. In the mean-field games literature typically the absence of a closed-form solution of the finite-population model precludes the possibility to make any explicit comparison between the two solutions. Due to the specific linear-quadratic structure of our framework, we can explicitly solve even the finite-population game (see e.g., Carmona et al. 2015, for a similar resolution procedure) and thus we can analytically assess the reliability of the approximation provided by the mean-field solution. This comparison, in relation to how we have characterized the Nash equilibria of the finite-population game, shows that the mean-field approximation well describes the behavior of the finite-population dynamics, apart from the fact it hides the non-negligible difference between closed and open loop solutions.

The paper proceeds as follows. Section 2 introduces our stochastic differential game of transboundary pollution and endogenous social norms. Section 3 analyzes on the N-players stochastic game in which the number of players is finite, determining in closed-form its solution both in an open and closed loop setting. Section 4 focuses on the asymptotic game and determines its solution under a deterministic mean-field approximation in which the number of players is infinitely large. Section 5 presents a numerical example which clarifies our theoretical results. Section 6 presents an extension of our baseline model to account for the implications of heterogeneity across players focusing on the deterministic mean field approximation. Section 7 presents concluding remarks and highlights directions for future research. The proofs of our results are postponed to the appendix A.

2 The Model

We consider a dynamic game of transboundary pollution under endogenously evolving social norms over a finite time horizon. The local pollution stock in venue $i = \{1, 2, ..., N\}$, $X_i(t)$, which is measured on a log-scale for the sake of analytical tractability (Carmona et al., 2013, 2015), is affected by local emissions and transboundary externalities associated with the pollution flows from surrounding locations. Each player (i.e., the local regulator) is located in a different venue i and determines their own emission level, $y_i(t)$, in order to minimize the social cost of climate mitigation, which partly depends on the existing social norms determining the pollution standards at the local level. Specifically, local pollution increases with the spatial flows from surrounding locations $F_i(t)$ and local emissions, while it decreases with publicly-funded reclamation activities $R_i(t)$, and it is subject to random shocks $Z_i(t)$ as follows: $\dot{X}_i(t) = F_i(t) + y_i(t) - R_i(t) + Z_i(t)$. The spatial pollution flows are given by the difference between the average stock of pollution across locations $m_N(t) = \frac{1}{N}\sum_{i=1}^{N} X_i(t)$ and the stock in location i, $F_i(t) = m_N(t) - X_i(t)$. This means that if in location i pollution is lower (higher) than the average pollution then location i will receive some incoming (send out some outgoing) pollution flows which will increase (decrease) its local stock. This term captures the spatial properties of pollution which tends to diffuse across space from locations in which it is relatively more abundant to others in which it is relatively scarce (La Torre at al., 2021, Boucekkine et al., 2022). Reclamation activities aimed at reducing the local pollution stock are assumed to be homogeneous across locations and to be publicly funded (i.e., a global regulator finances such activities), and thus given by the following expression: $R_i(t) = \tau$, where $\tau > 0$ quantifies the amount of pollution cleaned up. This term is consistent with current environmental policy and legislation which demands reclamation efforts to restore the deteriorated land to a viable state, and with the fact that reclamation often involves the active participation of public authorities in the development or implementation of clean-up projects (Lappi, 2018; Marsiglio and Masoudi, 2022). Apart from the effects of spatial flows and reclamation, the dynamics of local pollution is also affected by exogenous random shocks driven by a Brownian motion $Z_i(t) = \sigma \mathrm{d}B_i(t)$, where $\sigma \geq 0$ measures the standard deviation of such shocks.

Each player seeks to minimize the social cost of mitigation, which is given by the weighted sum of two discounted ($\lambda > 0$ is the rate of time preference) terms: the sum of the instantaneous losses during the duration of the mitigation program and the final damage at the end of the program. The instantaneous loss function increases with local emissions and with the lack of social conformity $S_i(t)$, $\ell(S_i(t), y_i(t))$. The lack of social conformity measures the extent to which deviating from extant social norms increases the individual cost of mitigation through a reputational effect, and depends on the excess of pollution locally with respect to the average stock of pollution across locations, $S_i(t) = X_i(t) - m_N(t)$. The two arguments enter additively the loss function, which is assumed to take a quadratic form as follows: $\ell(S_i(t), y_i(t)) = [S_i(t) + y_i(t)]^2$. The final damage function increases with the lack of social conformity and is assumed to take a quadratic form as follows: $d(S_i(t)) = S_i(t)^2$. The relative importance of the final damage in terms of the instantaneous losses is quantified by $\phi > 0$, which measures the degree of sustainability concerns (La Torre et al., 2017). Accounting for social norms in the definition of the social cost is consistent with several studies which discuss the crucial role played by social norms in driving individuals' decisions in environmental contexts (Brekke et al., 2003; Marsiglio and Tolotti, 2020). Some even suggest that the compelling need to adhere to such norms at the individual level may induce positive effects at the social level allowing eventually to solve environmental problems (Steg et al., 2005; Nyborg et al., 2016). Thus, it is natural to wonder whether social norms may effectively allow for more favourable outcomes in a transboundary pollution context.

Formally, let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ be a filtered probability space with the filtration $\mathbb{F} = \{\mathcal{F}_t\}_{0 \leq t \leq T}$ satisfying the usual conditions of right continuity and completeness, and is such that $\mathcal{F}_0$ is $\mathbb{P}$-trivial. Given the initial pollution stock, which we assume to be homogeneous across locations for the sake of simplicity, namely $X_i(0) = X^0, \forall\, i$, the problem faced by player i can be summarized as follows:

$$\min_{y_i} \quad \mathcal{C}^i(y_1, \ldots, y_N) = \mathbb{E}\left[\int_0^T e^{-\lambda t}\Big([X_i(t) - m_N(t)] + y_i(t)\Big)^2 dt + e^{-\lambda T}\phi[X_i(T) - m_N(T)]^2\right] \tag{1}$$

$$s.t. \quad \mathrm{d}X_i(t) = [(m_N(t) - X_i(t)) + y_i(t) - \tau]\,\mathrm{d}t + \sigma\,\mathrm{d}B_i(t), \tag{2}$$

where $y_i(t) \in \mathbb{H}^2$ is a real-valued, square-integrable adapted process and $B_i(t)$ is an independent Brownian motion adapted to the filtration.

The problem above clearly states that the emission decisions of the player in location i are critically affected by the environmental outcomes in other locations as well. Apart from its role in determining transboundary effects, the average pollution level across locations represents the social norm quantifying the pollution standard each player wishes to conform to. Note that the instantaneous losses non-trivially depend on the interaction between the lack of social conformity and local emissions. If in location i the pollution stock exceeds the average pollution then it will be optimal minimizing emissions, which means producing almost nothing. If instead, local pollution falls short of the average pollution it will be possible to increase emissions (and thus economic output). Thus, the local regulator needs to balance economic (i.e., output) and environmental (i.e., pollution) objectives accounting for the role of social effects in driving their own optimal emission decisions, by considering also the dynamic evolution of the social norm. Indeed, the pollution standard at the local level endogenously changes over time as the average pollution across locations reflects the aggregate result of individual players' emission decisions, which in turn depend on free-riding and strategic interaction considerations.

The problem above characterizes a stochastic differential game of interacting players under random dynamics. As explicit solutions of similar problems generally cannot be found, a typical approach employed in literature consists of analyzing the model through a deterministic mean-field approximation by assuming that the population of players is infinitely large (Lopez–Pintado, 2008; Carmona and Delarue, 2018). Since the mean-field approximation is more analytically tractable it may be possible to obtain a closed-form solution from which we may infer what the stochastic solution of the true finite-population game might look like. Indeed, provided that certain suitable assumptions are verified, the N-player game in which all players adopt the mean field strategy gives rise to an approximate equilibrium (referred to as ε-Nash equilibrium where ε measures the error induced by the mean field approach) allowing to assess the accuracy of the approximation by determining a precise quantification of the relation between N and ε (Carmona and Delarue, 2013). This type of argument has been also used in a number of works aiming to compare the N-player game and the mean field game solutions in different settings (Bensoussan et al., 2016; Cardaliaguet 2013). However, given the specific formulation of our problem, we can even be more precise in such a comparison since by relying on Carmona's (2016) approach (see section 5.5 at p. 198 and section 6.4.1 at p. 243), we can solve in closed form the finite-population model and hence obtain an explicit comparison between the two solutions. This allows us to confirm, as it is well known in the literature, that the mean-field approximation provides an accurate representation of the behavior of the finite-population solution, but also to stress that the the mean-field approximation makes the differences between closed and open loop solutions fade away.

3 The Finite-Population Game

In this section, we analyze the N-players stochastic game by determining the noncooperative Nash equilibria, distinguishing between open and closed loop Nash equilibria. Let us first give the following definitions.

Definition 3.1. *The strategy profile $\widehat{y} = (\widehat{y}_1, \widehat{y}_2, \ldots, \widehat{y}_N)$, with every $\widehat{y}_i \in \mathbb{H}^2$, is an open loop Nash equilibrium if*

$$\forall\, i \in \{1, \ldots, N\}, \quad \forall\, \widehat{y}_i \in \mathbb{H}^2, \qquad \mathcal{C}^i(\widehat{y}) \leq \mathcal{C}^i(y_i, \widehat{y}_{-i}), \tag{3}$$

where $(y_i, \widehat{y}_{-i})$ stands for the strategy profile $(\widehat{y}_1, \ldots, \widehat{y}_{i-1},\, y_i,\, \widehat{y}_{i+1}, \ldots, \widehat{y}_N)$ in which the player i chooses the strategy y_i while the others $k \neq i$ keep the original ones $\widehat{y}_k$.

Definition 3.2. *The strategy profile $\widetilde{y} = (\widetilde{y}_1, \widetilde{y}_2, \ldots, \widetilde{y}_N)$, with every $\widetilde{y}_i \in \mathbb{H}^2$, is a closed loop Nash equilibrium [1] if satisfies the condition (3) of Definition 3.1 with the restriction that the strategies $\widetilde{y}_{-i}$ and y_i are deterministic functions in feedback form of time, initial state and state trajectory at time t.*

Now, by relying on the Pontryagin stochastic maximum principle (PSMP) it is possible to characterize in closed-form both the open and closed loop solutions of the N-players game, as summarized in the next two propositions.

Proposition 1. *An open loop equilibrium outcome for player i is the pair $\left(\widehat{y}_i(t), \widehat{X}_i(t)\right)$, where the optimal emission strategy $\widehat{y}_i(t)$ is given by:*

$$\widehat{y}_i(t) \;\; = \;\; \left[1 + \left(1 - \frac{1}{N}\right)\eta(t)\right](m_N(t) - \widehat{X}_i(t)) \tag{4}$$

and the optimal local pollution stock $\widehat{X}_i(t)$ solves the following stochastic differential equation:

$$d\widehat{X}_i(t) \;\; = \;\; \left[\left(2 + \left(1 - \frac{1}{N}\right)\eta(t)\right)(m_N(t) - \widehat{X}_i(t)) - \tau\right]dt + \sigma\, dB_i(t), \tag{5}$$

[1] Actually this is the definition of Markovian Nash equilibrium; however, we rely on the closed loop equilibrium terminology, which is most commonly employed in economics.

with

$$\eta(t) = \frac{\phi\left(4+\lambda-\frac{1}{N}\right)}{\left(4+\lambda-\frac{1}{N}\right)e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)} + \phi\left(1-\frac{1}{N}\right)\left(e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}-1\right)}. \tag{6}$$

Proposition 2. *A closed loop equilibrium outcome for player i is the pair $\left(\widetilde{y}_i(t), \widetilde{X}_i(t)\right)$, where the optimal emission strategy $\widetilde{y}_i(t)$ is given by:*

$$\widetilde{y}_i(t) = \left[1 + \left(1 - \frac{1}{N}\right)\varphi(t)\right](m_N(t) - \widetilde{X}_i(t)) \tag{7}$$

and the optimal local pollution stock $\widetilde{X}_i(t)$ solves the following stochastic differential equation:

$$d\widetilde{X}_i(t) = \left[\left(2 + \left(1 - \frac{1}{N}\right)\varphi(t)\right)(m_N(t) - \widetilde{X}_i(t)) - \tau\right]dt + \sigma\, dB_i(t) \tag{8}$$

with

$$\varphi(t) = \frac{(4+\lambda)\phi}{(4+\lambda)e^{(4+\lambda)(T-t)} + \phi\left(1-\frac{1}{N^2}\right)\left(e^{(4+\lambda)(T-t)}-1\right)}. \tag{9}$$

Proposition 1 determines an open loop equilibrium outcome for player i and shows that the optimal strategy requires to set the emission level, $\widehat{y}_i(t)$, proportionally to the spatial pollution flow in location i, $F_i(t) = m_N(t) - \widehat{X}_i(t)$. Specifically, the factor of proportionality, which we shall refer to as the "emission intensity" for expositional simplicity, depends on a time-varying function $\eta(t)$ and it monotonically increases over time, critically depending on the key parameters λ, ϕ, N and T. In particular, it decreases with the end-of-time-horizon date and the discount factor, it increases with the degree of sustainability concern, while it depends ambiguously on the number of players. These results are to a large extent intuitive. A further end-of-time-horizon date increases the instantaneous losses and a higher discount factor makes current environmental decisions more and more relevant, favoring both a reduction in the emission intensity. A higher degree of sustainability concern increases the relative impact of the final damage (decreasing thus that of the instantaneous losses) in each player's environmental considerations, promoting a higher emission intensity. The effect of the number of players on individual emission decisions is instead more complicated, as it depends on two competing forces: the desire to adhere to the social norm tends to reduce the emission intensity, while the expectation that other players may free ride tends to increase it. In an open loop equilibrium in which players pre-commit to a certain strategy at time 0 without the possibility to modify it according to what other players do or what happens to local pollution later on, it may happen that the former or the latter force dominates according to the specific parameters configuration, thus it is not possible to state a priori whether the emission intensity will increase or decrease with the number of players. Note that the results just discussed refer to the emission intensity, represented by the factor of proportionality in the square brackets in (4), while we cannot say exactly how the parameters affect emissions since this depends on how the emission intensity interacts with the spatial pollution flow in location i. It may happen for example that the emission intensity is monotonically increasing and the spatial pollution flow monotonically decreasing over time, so that the time evolution of the emission level may not be unambiguously determined. Note also that the additive form of local emissions in the dynamics of local pollution given in (2) implies that, apart from the effects of reclamation activities and random shocks, the dynamic of the optimal local pollution stock, $\widehat{X}_i(t)$, mimics the evolution of emissions. In particular, pollution clean-up due to reclamation does not affect either the emission intensity or emissions, but it results in reducing the optimal local pollution stock.

In a similar vein, Proposition 2 determines a closed loop equilibrium outcome for player i showing that also in this case the optimal strategy requires to set the emission level, $\widetilde{y}_i(t)$, proportionally to the spatial pollution flow in location i. However, in this case, the emission intensity depends on another time-varying function, $\varphi(t)$, and it still monotonically increases over time and depends on the key parameters exactly as the open-loop emission intensity, with the exception of N. Indeed, in the closed loop equilibrium, the factor of proportionality unambiguously increases with the number of players, as linking the emission strategy to the local pollution stock makes the conformism motive less relevant and the free-riding incentive becomes predominant. Also in this case because of the additive form of local emissions, the dynamic of the local pollution stock, $\widetilde{X}_i(t)$, closely mimics the evolution of emissions, apart from the reclamation and shock effects.

By comparing Propositions 1 and 2, it is straightforward to note that the departure between the open and closed loop equilibrium solutions is entirely driven by the difference between the functions $\eta(t)$ and $\varphi(t)$, which drive the emission strategy and thus the evolution of the local pollution stock in the open and closed loop settings, respectively. In particular, it is possible to show that $\eta(t) - \varphi(t) > 0$ for every $t \in [0, T)$, that is the open loop Nash equilibrium prescribes larger emission levels than the closed loop one, and this is intuitively due to the fact that in a closed loop setting the conformism incentive is stronger than in an open loop one. However, this difference gradually decreases over time and at the end of the planning horizon it becomes null, $\eta(T) - \varphi(T) = 0$, that is the open and closed loop strategies dictate the same emission levels at T. Even if at the end of the planning horizon the open and closed loop emissions coincide, the local pollution stocks may not, as emissions differ for every $t \in [0, T)$ such that their accumulated effects will result in different local pollution stocks. Moreover, at each moment in time the difference $\eta(t) - \varphi(t)$ tends to vanish as the number of players increases, and in the case of an asymptotically large population of players, $N \to \infty$, $\eta(t)$ and $\varphi(t)$ converge to the same limit. Then the following result holds true.

Proposition 3. *When $N \to +\infty$ the open and closed loop equilibria outcomes coincide since*

$$\lim_{N\to\infty} \eta(t) = \lim_{N\to\infty} \varphi(t) = \frac{(4+\lambda)\phi}{(4+\lambda)e^{(4+\lambda)(T-t)} + \phi\left(e^{(4+\lambda)(T-t)} - 1\right)}. \tag{10}$$

Apart from the implications of open and closed loop strategies for the emission level of the single individual player, it is also interesting to understand their environmental consequences. Since the local pollution stock is a random variable it is not possible to say much about its exact value, but we can nevertheless discuss more precisely the behavior of its expected value. In particular, we can determine in closed-form the average of the expected value of the local pollution stocks across locations, $\overline{X}(t) \equiv \frac{1}{N}\sum_{i=1}^{N} \mathbb{E}[\, X_i(t)]$, which both in open and the closed loop cases turns out to be $\overline{X}(t) = X^0 - \tau t$. This suggests that, by pushing single players to conform to the behavior of others, social norms allow for environmental improvements only to the extent to which they interact with public reclamation activities. Indeed, with no reclamation efforts (i.e., $\tau = 0$), the conformism motive does not translate into an environmental improvement since single players do not reduce their emissions and consequently the average of the expected value of the local pollution stocks across locations tends to remain constant over time (i.e., $\overline{X}(t) = X^0$). However, with some even minimal reclamation efforts (i.e., $\tau > 0$) conformism promotes a reduction in emissions which over time leads to a gradual improvement in environmental standards resulting in a lower average of the expected value of the local pollution stocks across locations (i.e., $\overline{X}(t) < X^0$). This result can be summarized as follows.

Proposition 4. *Social norms promote environmental improvements by interacting with reclamation, and local pollution reduction will occur if and only if some public reclamation activities take place.*

Proposition 4 states that social norms are not a straightforward solution to environmental problems per se, as their effectiveness largely depends on the presence of pollution clean-up activities. Indeed, what really matters to generate environmental improvements is the interaction between individual players' conformism motives and public reclamation efforts, and this conclusion is independent of the number of players and the length of the time horizon. Therefore, the desire of single players to adhere to social norms combined with some public pollution clean-up is capable to favor environmental improvements even without cooperation incentives. Even if cooperation is not essential, public authorities play an important role in allowing for pollution reductions by taking the lead in generating the required behavioural changes at the individual level through their reclamation efforts.

4 The Mean Field Approximation

Since it is quite rare to solve in closed-form the N-player game as we have done in the previous section, in order to characterize some of the Nash equilibria of a dynamic game similar to ours it is generally convenient to consider its mean-field approximation. This consists of analyzing the mean behavior of the players in a situation in which the number of players is infinitely large and players are assumed to be identical (symmetry assumption). Since by Proposition 3 follows that differences between the open and closed loop solutions fade away when $N \to +\infty$, we now characterize the solution of the mean-field approximation of our N-player game without distinguishing between open and closed loop one in order to assess whether the mean-field solution approximates well its N-player equivalents. Note that in the following, under the symmetry assumption, we drop the player index i, as the same results apply to all players.
The mean field game (MFG) strategy is based on the following three steps:

1. Fix $m(t)$, $t \in [0, T]$ as a candidate for the limit of $m_N(t)$ as $N \to +\infty$, that is:

$$\lim_{N \to +\infty} m_N(t) = m(t).$$

2. Solve the standard control problem for the representative player:

$$\min_{y(t) \in \mathbb{H}^2} \mathbb{E}\left[\int_0^T e^{-\lambda t}\Big(\big(X(t) - m(t)\big) + y(t)\Big)^2 dt + e^{-\lambda T}\phi(X(T) - m(T))^2\right] \tag{11}$$

subject to:

$$\begin{cases} \mathrm{d}X(t) = [(m(t) - X(t)) + y(t) - \tau]\,\mathrm{d}t + \sigma\,\mathrm{d}B(t), \quad t \in (0, T] \\ X(0) = X^0, \end{cases} \tag{12}$$

where $B(t)$ is a Brownian motion independent of the initial value X^0.

3. Solve the fixed point problem, that is find $m(t)$ so that $m(t) = \mathbb{E}[X_m(t)]$ for all $t \in [0, T]$, where $X_m(t)$ is the optimally controlled state process related to $m(t)$.

Definition 4.1. *The deterministic function $m : [0, T] \to \mathbb{R}$ is said to be a mean field equilibrium if solves the fixed point equation, i.e., $m(t) = \mathbb{E}[X_m(t)]\ \ \forall\, t \in [0, T]$.*

Given the definition above, we can now determine the optimal MFG strategy.

Proposition 5. *A mean field optimal emission strategy $\widehat{y}(t)$ is given by:*

$$\widehat{y}(t) = [1 + \psi(t)](m(t) - \widehat{X}(t)), \tag{13}$$

while the optimal local pollution stock $\widehat{X}(t)$ solves the following stochastic differential equation:

$$d\widehat{X}(t) = \Big[(2 + \psi(t))\,(m(t) - \widehat{X}(t)) - \tau\Big]\,dt + \sigma\,dB(t), \tag{14}$$

with:

$$\psi(t) = \frac{(4 + \lambda)\phi}{(4 + \lambda)e^{(4+\lambda)(T-t)} + \phi\left(e^{(4+\lambda)(T-t)} - 1\right)}. \tag{15}$$

Similar to what we have discussed in the previous section, Proposition 5 shows that the optimal emission level, $\widehat{y}(t)$ is proportional to the spatial pollution flow. However, in this case, the emission intensity depends on the time-varying function, $\psi(t)$, it still monotonically increases over time and depends on the key parameters exactly as the emission intensity in Propositions 1, with the sole exception that it is independent of N. As before, the dynamic of the local pollution stock, $\widehat{X}(t)$, closely mimics the evolution of emissions, apart from the effects of reclamation and random shocks. Moreover, the function $\psi(t)$ represents exactly the limit as $N \to +\infty$ of the functions $\eta(t)$ and $\varphi(t)$, given by (10), clarifying thus the source of the equivalence between the open and closed loop solutions in mean-field games. This fact is also consistent with the folk theorem (see Fudenberg and Levine, 1988) stating that the differences between the open and closed loop equilibria disappear in the limit for $N \to \infty$ of large games, but it also introduces an important discrepancy between what happens in our N-players game and in its mean-field equivalent, as in a finite player population setting the open and closed loop solutions generally differ.

Proposition 5, in conclusion, shows that the mean field optimal emission strategy, as expected, coincides with the N-limit of the (open and closed-loop) equilibrium emission strategy in the N-players game. This result similarly extends also the local pollution stock. In order to look at this, by recalling that the local pollution stock is a random variable and thus we cannot say much about its value, we can focus on its expected value, $\overline{X}(t)$, which is linear in time and given by the following expression: $\overline{X}(t) = X^0 - \tau t$ (see appendix A.6), showing, similar to what we have discussed in the previous section, that the expected value of the pollution stock across locations decreases over time, provided that some reclamation effort takes place. This implies that also in a mean-field setting Proposition 4 applies.

Summarizing, the mean-field approximation provides us with an accurate representation of the behavior of the finite-population solution, apart from the distortion related to the fact it makes all the differences between closed and open loop solutions fade away. This implies thus that even in the absence of an analytical solution for dynamic games with a finite population it is possible to infer equilibrium outcomes from their mean-field approximations, but it may be more difficult to infer policy conclusions since it is not possible to distinguish between open and closed loop settings in which players' action typically differ.

5 A Numerical Example

We now present some numerical examples to graphically illustrate and clarify our results. We set the parameter values as follows: $\lambda = 0.04$, $\phi = 1$, $\sigma = 0.2$, $T = 100$, $X^0 = 10$, while we change the number of players $N \in \{10, 100, 1000, 10000\}$ to show how the results vary according to the players' population size, and we consider either $\tau = 0$ or $\tau = 0.01$ to clarify the implications of reclamation. In order to approximate the trajectories of $\widehat{X}_i$ and $\widetilde{X}_i$, we proceed by discretizing the stochastic differential equations 5 and 8 using a classical discrete difference method and by recalling that $dB_i(t)$ is normally distributed with zero mean and variance dt. The numerical results have been obtained by first simulating the Brownian motions and then plugging them into the stochastic differential equations.

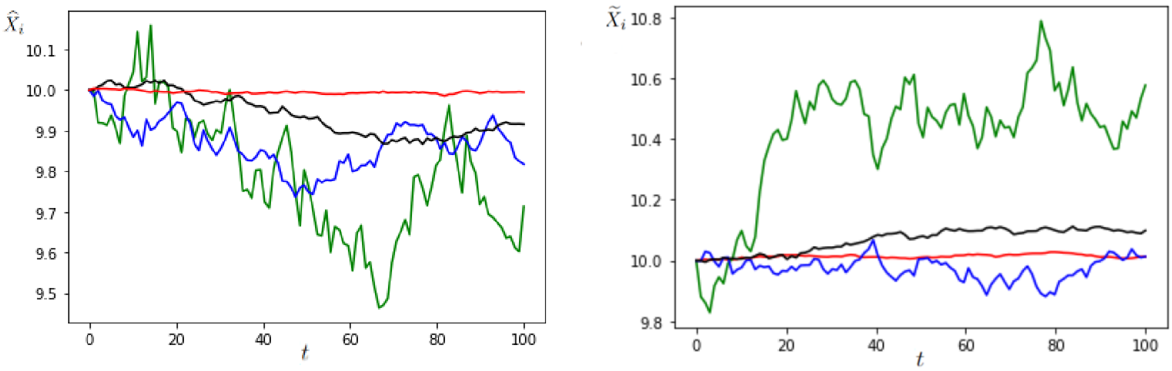

Figure 1: Path evolution in the open (left) and closed (right) loop scenarios with $N = 10$ (green), $N = 100$ (blue), $N = 1000$ (black), $N = 10000$ (red) without reclamation ($\tau = 0$).

Figure 1 and Figure 2 show the time evolution of the local pollution stock of a given player i in the open (left panel) and closed (right panel) loop settings for $N = 10$ (green), $N = 100$ (blue), $N = 1000$ (black),

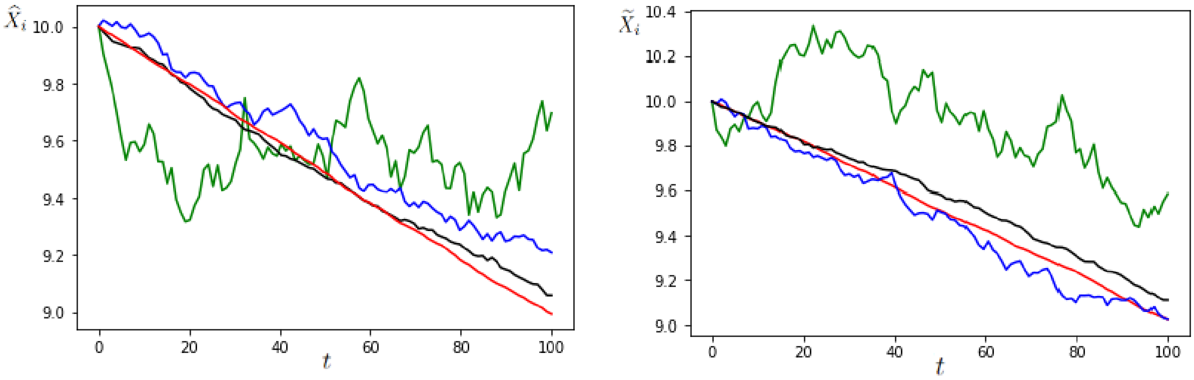

Figure 2: Path evolution in the open (left) and closed (right) loop scenarios with $N = 10$ (green), $N = 100$ (blue), $N = 1000$ (black), $N = 10000$ (red) with reclamation ($\tau = 0.01$).

and $N = 10000$ (red), in the absence and presence of reclamation respectively. In the absence of reclamation (Figure 1, where $\tau = 0$), we can observe that for a large number of players ($N = 10000$) the finite-population trajectory closely mimics the mean-field one (which also represents the trajectory of the single player i, due to the symmetry assumption) which predicts that both in an open and closed loop setting the local pollution stock will remain equal to its initial value, $\overline{X}(t) = X^0 = 10$. In the presence of reclamation instead (Figure 2, where $\tau = 0.01$), all the trajectories for a large enough N show a reduction in the local pollution stock which decreases with respect to its initial value, both in an open and closed loop setting, such that at the end of the time horizon the local pollution is lower in the former scenario. Note also that, as the number of players decreases the discrepancy between the finite-population and mean-field trajectories increases and for a small number of players ($N = 10$) the finite-population trajectory might substantially depart from the mean-field one and can show different patterns in the open and closed loop scenarios. This confirms our theoretical conclusions regarding the accuracy of the mean-field approximation when the players' population is large enough apart from the distortion that it introduces regarding the difference between open and closed loop strategies.

6 An Extended Model

By abstracting from heterogeneity across players and individual players' efforts to reduce the local pollution stock, our setup oversimplifies the nature of transboundary pollution control and its relation with social norms. Indeed, in our analysis, we have assumed that reclamation activity is performed at the global level, while in reality policymakers may rely on abatement activities at the local level in order to reduce the social cost of pollution. Moreover, we have assumed that environmental preservation activities take place homogeneously across locations, while in reality local policymakers may respond to global reclamation standards or implement purposeful abatement with different intensities according to their individual preferences for environmental vis-a-vis economic outcomes. Furthermore, we have assumed that the desire to adhere to social norms is homogeneous across players, while in reality local policymakers may be characterized by different degrees of conformism leading them to attach a heterogeneous evaluation of the lack of social conformity at the end of the time horizon.

In order to account for these issues, we now present an extension of our baseline model to introduce individual abatement activities $A_i(t)$ and to account for the role of heterogeneity in reclamation and abatement efforts. Each player i abates a share $0 < \theta_i(t) < 1$ of the excess of local pollution with respect to the average stock of pollution across locations, $S_i(t)$: $A_i(t) = \theta_i(t)S_i(t)$, and such an abatement represents an extra cost which introduces an additional term in the instantaneous loss function which, by maintaining our previous quadratic additive form, now becomes: $\ell(S_i(t), y_i(t), A_i(t)) = [S_i(t) + y_i(t) + A_i(t)]^2$. Both the share of excess local pollution abated $\theta_i(t)$ and the amount of pollution cleaned up $\tau_i(t)$ are assumed to be time-varying and heterogeneous across players. The degree of sustainability concern, which quantifies the weight attach to the lack of social conformity at the end of the mitigation program, is instead assumed to be constant but heterogeneous across players. Therefore, the problem faced by player i can be stated as follows:

$$\min_{y_i} \quad \mathcal{C}^i(y_1, \ldots, y_N) = \mathbb{E}\left[\int_0^T e^{-\lambda t}\Big((1+\theta_i(t))\,[X_i(t) - m_N(t)] + y_i(t)\Big)^2 dt + e^{-\lambda T}\phi_i[X_i(T) - m_N(T)]^2\right] \quad (16)$$

$$s.t. \quad \mathrm{d}X_i(t) = [(m_N(t) - X_i(t)) + y_i(t) - \tau_i(t) - \theta_i(t)(X_i(t) - m_N(t))]\,\mathrm{d}t + \sigma_i\,\mathrm{d}B_i(t), \quad (17)$$

where $\theta_i(t) : [0,T] \to \mathbb{R}$ and $\tau_i : [0,T] \to (0, +\infty)$ are continuous function in $[0,T]$ for every i. The evolution of the pollution stocks in each location is driven by the interaction between the heterogeneous levels of reclamation and abatement across locations. Note that whenever $\theta_i(t) = 0$ and $\tau_i(t) = \tau$ for all i, our extended model boils down to our baseline specification.

Note that the presence of heterogeneity in (16) - (17) does not allow for a closed form solution of the problem above. However, as shown in our previous analysis and as it is well known from literature (see, e.g., Carmona and Delarue, 2018), the mean-field model provides a good approximation of the solution of the finite-population model thus in the following we shall focus on the former. Consistent with extant literature (see, e.g. Cardaliaguet 2013), under the assumption of symmetry in which we drop the player index i as the same properties apply to all players, we can state the mean-field problem as follows:

$$\min_{y} \quad \mathcal{C}(y) = \mathbb{E}\left[\int_0^T e^{-\lambda t}\Big((1+\theta)\,[X(t) - m(t)] + y(t)\Big)^2 dt + e^{-\lambda T}\phi[X(T) - m(T)]^2\right] \quad (18)$$

$$s.t. \quad \mathrm{d}X(t) = [(1+\theta)(m(t) - X(t)) + y(t) - \tau(t)]\,\mathrm{d}t + \sigma\,\mathrm{d}B(t), \quad (19)$$

Similar to what was discussed in section 4, we can explicitly derive the optimal MFG strategy.

Proposition 6. *A mean field optimal emission strategy $y^*(t)$ is given by:*

$$y^*(t) = [1 + \theta(t) + \Psi(t)](m(t) - X^*(t)), \quad (20)$$

while the optimal local pollution stock $X^*(t)$ solves the following stochastic differential equation:

$$dX^*(t) = [(2\,(1+\theta(t)) + \Psi(t)) + (m(t) - X^*(t)) - \tau(t)]\,dt + \sigma\,dB(t), \quad (21)$$

with:

$$\Psi(t) = \frac{\phi}{e^{(4+\lambda)(T-t)+\int_t^T 4\theta(\xi)d\xi} + \phi\, e^{-(4+\lambda)(t-1)-\int_1^t 4\theta(\xi)d\xi}\left(\int_t^T e^{(4+\lambda)(s-1)+\int_1^s 4\theta(\xi)d\xi}\, ds\right)}.$$

Results similar to those earlier discussed (see Proposition 5) apply, and Proposition 6 shows that the optimal emission level, $y^*(t)$, is proportional to the spatial pollution flow, but different from what we have earlier seen the emission intensity depends on two time-varying functions, $\Psi(t)$ and $\theta(t)$. The dynamic of the local pollution stock, $X^*(t)$, closely mimics the evolution of emissions apart from the effects of time-dependent reclamation efforts and random shocks. As before, since the local pollution stock is a random variable, we can focus on its expected value, $\overline{X}(t)$, which is given by $\overline{X}(t) = X^0 - \int_0^t \tau(s)\mathrm{d}s$ (see appendix A.8), which suggests that the expected value of the pollution stock across locations decreases over time, provided that some reclamation effort takes place. This implies that also in this mean-field extension Proposition 4 applies, thus the conclusions outlined in our baseline model are robust to the introduction of individual abatement activities and heterogeneity. This confirms that despite its simplicity, our baseline model represents a reliable benchmark framework for understanding the relationship between transboundary pollution and social norms.

7 Conclusion

The transboundary nature of pollution and free-riding effects represent probably the major obstacles to the success of international climate negotiations. Among the possible strategies to improve environmental outcomes, it has recently been advanced that changes in social norms may be an important solution. We thus assess whether the social effects induced by social norms may be effective in modifying individuals' emission incentives in such a way to achieve desirable environmental outcomes even in the absence of cooperation enforcement. Specifically, we analyze an N-players differential game of transboundary pollution control in which the local pollution stock is affected by the pollution flows from surrounding locations and players' payoff partly depends on social norms, which endogenously evolve over time according to players' actions. We show that social norms per se are unable to favor a reduction in local pollution but, provided that some public reclamation activities aimed at pollution clean-up are in place, they allow for long run environmental improvements. Indeed, the interaction between social norms and reclamation generates a reduction in the average of the expected value of the local pollution stock across locations, independently of whether players rely on an open loop or a closed loop strategy. Moreover, by characterizing the equilibrium outcome similar to Carmona et al. (2015), the reliability of the mean-field approximation of the finite-population dynamics is confirmed, despite such an approximation introducing some distortion regarding the difference between open and closed loop strategies. Our main conclusions turn out to be robust to the introduction of individual abatement and heterogeneity in mitigation efforts and conformism across players.

To the best of our knowledge, no other paper has thus far either analyzed the role of social norms in transboundary pollution control problems or explicitly compared the equilibrium outcomes of the N-players and the mean-field game solutions. We thus believe that our work presents an important contribution in the environmental economics and mathematical economics literature, illustrating how the theory of the stochastic differential games can be applied to the study of relevant problems in economics. For future research, it would be interesting to analyze the implications of social norms in contexts of global rather than local pollution dynamics, to understand whether our conclusions regarding the effectiveness of social norms in promoting pro-environment behavioral changes are robust to the type of pollution considered. This is left for future research.

A Technical Appendix

A.1 Proof of Proposition 1

The proof follows the scheme proposed in Carmona et al. (2015; see section 3.1). Since $\sigma > 0$ is constant, we can rely on the reduced Hamiltonian that for the agent i is given by:

$$H^i(x_1, \dots, x_N, p^{i,1}, \dots, p^{i,N}, y_1, \dots, y_N) = \sum_{j=1}^{N} [(m_N - x_j) + y_j - \tau] p^{i,j} + e^{-\lambda t}[(x_i - m_N)^2 + 2y_i(x_i - m_N) + y_i^2].$$

By the necessary condition of the Pontryagin stochastic maximum principle (PSMP) the value of y_i minimizing the reduced Hamiltonian when all the other variables (including y_j for $j \neq i$) are fixed, is given by:

$$\frac{\partial H^i}{\partial y_i} = 0 \quad \longrightarrow \quad \widehat{y_i} = -\frac{p^{i,i}}{2e^{-\lambda t}} + (m_N - x_i). \tag{22}$$

Given an admissible strategy $y = (y_1, \dots, y_N)$ and the corresponding controlled state, the adjoint processes associated with y are the processes $P^i(t) = (P^{i,j}(t) : j = 1, \dots, N)$ and $Q^i(t) = (Q^{i,j,k}(t) : j = 1, \dots, N,\ k =$

$0,\ldots,N$) solving the system of backward stochastic differential equations (BSDEs):

$$\begin{cases} \mathrm{d}P^{i,j}(t) = -\dfrac{\partial H^i}{\partial x_j}\mathrm{d}t + \displaystyle\sum_{k=0}^{N} Q^{i,j,k}(t)\,\mathrm{d}B_k(t) \\ \qquad = -\left[\displaystyle\sum_{k=1}^{N}\left(\frac{1}{N}-\delta_{k,j}\right)P^{i,k}(t) + e^{-\lambda t}\left(2\left(m_N(t)-X_i(t)\right)\left(\frac{1}{N}-\delta_{i,j}\right) - 2y_i(t)\left(\frac{1}{N}-\delta_{i,j}\right)\right)\right]\mathrm{d}t \\ \qquad + \displaystyle\sum_{k=0}^{N} Q^{i,j,k}(t)\,\mathrm{d}B_k(t), \\ P^{i,j}(T) = 2e^{-\lambda T}\phi\left(m_N(T)-X_i(T)\right)\left(\dfrac{1}{N}-\delta_{i,j}\right) \end{cases} \tag{23}$$

Note that standard existence and uniqueness results for BSDEs apply to (23) and the existence of the adjoint processes is guaranteed (see Chapter 2 in Carmona, 2016, and references therein). According to the strategy outlined earlier, we replace all the occurrences of the controls $y_i(t)$ in the forward equation (2) and the backward equation (23), by:

$$\widehat{y}_i(t) = -\frac{P^{i,i}(t)}{2e^{-\lambda t}} + \left(m_N(t) - X_i(t)\right). \tag{24}$$

If the resulting forward-backward system is solvable then the strategy (24) will characterize the optimal open loop emission level. However, that system is in general extremely difficult to solve, hence we make the following ansatz (i.e., we search for a solution of the form):

$$P^{i,j}(t) = 2e^{-\lambda t}\eta(t)\left(m_N(t)-X_i(t)\right)\left(\frac{1}{N}-\delta_{i,j}\right) \tag{25}$$

for some smooth deterministic function $\eta : [0,T] \to \mathbb{R}$ to be determined. Using (24) and (25) the forward equation (2) becomes:

$$\mathrm{d}X_i(t) = \left[\left(2 + \left(1-\frac{1}{N}\right)\eta(t)\right)\left(m_N(t)-X_i(t)\right) - \tau\right]\mathrm{d}t + \sigma\mathrm{d}B_i(t), \tag{26}$$

which by summation gives:

$$\mathrm{d}m_N(t) = -\tau\,\mathrm{d}t + \sigma\frac{1}{N}\sum_{i=1}^{N}\mathrm{d}B_i(t),$$

and hence:

$$\mathrm{d}\left(m_N(t)-X_i(t)\right) = -\left(2+\left(1-\frac{1}{N}\right)\eta(t)\right)\left(m_N(t)-X_i(t)\right)\mathrm{d}t + \sigma\left(\frac{1}{N}\sum_{k=1}^{N}\mathrm{d}B_k(t) - \mathrm{d}B_i(t)\right). \tag{27}$$

Using instead (24) and (25) in (23) we get:

$$\mathrm{d}P^{i,j}(t) = 2e^{-\lambda t}\left(\frac{1}{N}-\delta_{i,j}\right)\left(m_N(t)-X_i(t)\right)\left(2-\frac{1}{N}\right)\eta(t)\mathrm{d}t + \sum_{k=0}^{N}Q^{i,j,k}(t)\,\mathrm{d}B_k(t). \tag{28}$$

Differentiating the ansatz (25) and using (27), it follows that:

$$\begin{aligned} \mathrm{d}P^{i,j}(t) &= 2e^{-\lambda t}\left(\frac{1}{N}-\delta_{i,j}\right)\left(m_N(t)-X_i(t)\right)\left[-\lambda\eta(t) + \dot{\eta}(t) - \eta(t)\left(2+\left(1-\frac{1}{N}\right)\eta(t)\right)\right]dt \\ &\quad + 2e^{-\lambda t}\left(\frac{1}{N}-\delta_{i,j}\right)\eta(t)\,\sigma\left(\frac{1}{N}\sum_{k=1}^{N}\mathrm{d}B_k(t)-\mathrm{d}B_i(t)\right), \end{aligned} \tag{29}$$

where $\dot{\eta}(t)$ is the time-derivative of $\eta(t)$. Comparing (28) and (29) we get the process $Q^{i,j,k}(t)$:

$$Q^{i,j,0}(t) = 0, \qquad Q^{i,j,k}(t) = 2e^{-\lambda t}\left(\frac{1}{N} - \delta_{i,j}\right)\eta(t)\,\sigma\left(\frac{1}{N} - \delta_{i,k}\right), \qquad k = 1,\dots,N$$

which turns out to be deterministic and hence adapted. Identifying the drift terms, it follows that $\eta(t)$ must satisfy the following Bernoulli equation:

$$\dot{\eta}(t) = \left(4 + \lambda - \frac{1}{N}\right)\eta(t) + \left(1 - \frac{1}{N}\right)\eta^2(t) \tag{30}$$

with terminal condition $\eta(T) = \phi$, obtained applying the ansatz (25) to the the final condition in (23). Equation (30) has a unique solution given by:

$$\eta(t) = \frac{\phi\left(4 + \lambda - \frac{1}{N}\right)}{\left(4 + \lambda - \frac{1}{N}\right)e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)} + \phi\left(1 - \frac{1}{N}\right)\left(e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)} - 1\right)}. \tag{31}$$

With (31) in hand, the sufficient part of PSMP implies that an optimal strategy profile is given by:

$$\widehat{y}_i(t) = \left[1 + \left(1 - \frac{1}{N}\right)\eta(t)\right](m_N(t) - \widehat{X}_i(t)), \tag{32}$$

obtained by plugging (25) in (24) and where we denote by $\widehat{X}_i$ the state of the player i only to stress the fact we are using an open loop equilibrium. We remark that even though the control (32) is in feedback form (since it only depends upon the current value of the state $\widehat{X}_i(t)$) we can only claim that it is in an open loop form. Note also that in equilibrium, the state $\widehat{X}_i(t)$ is Markovian for every $i = 1,\dots,N$ and satisfies the following:

$$\begin{cases} \mathrm{d}\widehat{X}_i(t) = \left[\left(2 + \left(1 - \frac{1}{N}\right)\eta(t)\right)(m_N(t) - \widehat{X}_i(t)) - \tau\right]\mathrm{d}t + \sigma\,\mathrm{d}B_i(t), \quad t \in (0,T] \\ \widehat{X}_i(0) = X_i^0. \end{cases} \tag{33}$$

A.2 Proof of Proposition 2

Following the proof given by Carmona et al. (2015; see section 3.2), we characterize in closed-form a closed-loop solution in which players at time t have complete information of the states of all the other players at the same time. Hence when all the other players $k \neq i$ have chosen strategies in feedback form given by deterministic functions $y_k(t,x)$ of time and state $x = (x_1,\dots,x_N)$, player i needs to solve a control problem to find their best response to these choices. The reduced Hamiltonian of their control problem is given by:

$$\begin{aligned} &H^i(x, p^{i,N}, \dots, p^{i,N}, y_1(t,x), \dots, y_i(t), \dots y_N(t,x)) \\ &= \sum_{k=1,k\neq i}^{N} [(m_N - x_k) + y_k(t,x) - \tau]p^{i,k} + [(m_N - x_i) + y_i - \tau]p^{i,i} + e^{-\lambda t}\left[(x_i - m_N)^2 + 2y_i(x_i - m_N) + y_i^2\right]. \end{aligned} \tag{34}$$

As in the proof of Proposition 1, by the necessary condition of the PSMP we get that the value of $\widetilde{y}_i$ minimizing the reduced Hamiltonian is as in (22). The adjoint processes $P^i(t) = (P^{i,j}(t) : j = 1,\dots,N)$ and $Q^i(t) = (Q^{i,j,k}(t) : j = 1,\dots,N,\ k = 0,\dots,N)$, are the solutions of the same equation (23) with H^i as in (34), while the state dynamics are again as in (2). As before, replacing all the occurrences of the control $\widetilde{y}_i(t)$ both in the state dynamics and the adjoint equation by (24), gives a forward-backward system which, if solved, provides the optimal closed loop emission level because of the sufficient part of the PSMP. To solve that system we use the ansatz:

$$P^{i,j}(t) = 2e^{-\lambda t}\varphi(t)\,(m_N(t) - X_i(t))\left(\frac{1}{N} - \delta_{i,j}\right) \tag{35}$$

for some smooth deterministic function $\varphi : [0,T] \to \mathbb{R}$ to be determined. This choice guarantees that (24) is a feedback control. In this way, the state dynamics is as in (26) (apart from replacing η with φ), while the backward equation becomes:

$$\begin{cases} \mathrm{d}P^{i,j}(t) = 2e^{-\lambda t}\left(m_N(t) - X_i(t)\right)\left(\frac{1}{N} - \delta_{i,j}\right)\left[2\varphi(t) + \frac{1}{N}\left(1 - \frac{1}{N}\right)\varphi^2(t)\right]\mathrm{d}t + \sum_{k=0}^{N} Q^{i,j,k}(t)\,\mathrm{d}B_k(t), \\ P^{i,j}(T) = 2e^{-\lambda T}\phi\left(m_N(T) - X_i(T)\right)\left(\frac{1}{N} - \delta_{i,j}\right) \end{cases} \tag{36}$$

Differentiating the ansatz (35) we get:

$$\begin{aligned} \mathrm{d}P^{i,j}(t) &= 2e^{-\lambda t}\left(\frac{1}{N} - \delta_{i,j}\right)\left(m_N(t) - X_i(t)\right)\left[-\lambda\varphi(t) + \dot{\varphi}(t) - \varphi(t)\left(2 + \left(1 - \frac{1}{N}\right)\varphi(t)\right)\right] dt \\ &\quad + 2e^{-\lambda t}\left(\frac{1}{N} - \delta_{i,j}\right)\varphi(t)\,\sigma\left(\frac{1}{N}\sum_{k=1}^{N}\mathrm{d}B_k(t) - \mathrm{d}B_i(t)\right). \end{aligned} \tag{37}$$

Next, by identifying term by term the first equation of (36) and (37), we obtain:

$$Q^{i,j,0}(t) = 0, \qquad Q^{i,j,k}(t) = 2e^{-\lambda t}\left(\frac{1}{N} - \delta_{i,j}\right)\varphi(t)\,\sigma\left(\frac{1}{N} - \delta_{i,k}\right), \qquad k = 1,\dots,N$$

which are deterministic and adapted, while from the drift terms it follows that $\varphi(t)$ must satisfy the Bernoulli equation:

$$\dot{\varphi}(t) = (4+\lambda)\varphi(t) + \left(1 - \frac{1}{N^2}\right)\varphi^2(t) \tag{38}$$

with terminal condition $\varphi(T) = \phi$, obtained by applying the ansatz (35) to the the final condition in (36). Equation (38) has a unique solution given by:

$$\varphi(t) = \frac{(4+\lambda)\phi}{(4+\lambda)e^{(4+\lambda)(T-t)} + \phi\left(1 - \frac{1}{N^2}\right)\left(e^{(4+\lambda)(T-t)} - 1\right)} \tag{39}$$

Then, an optimal closed loop emission strategy is given by

$$\widetilde{y}_i(t) = \left[1 + \left(1 - \frac{1}{N}\right)\varphi(t)\right]\left(m_N(t) - \widetilde{X}_i(t)\right), \tag{40}$$

where we denote by $\widetilde{X}_i$ the state of the player i to stress the fact we are using a closed loop equilibrium. Then, in equilibrium, the state $\widetilde{X}_i(t)$ satisfies, for every $i = 1,\dots,N$:

$$\begin{cases} \mathrm{d}\widetilde{X}_i(t) = \left[\left(2 + \left(1 - \frac{1}{N}\right)\varphi(t)\right)\left(m_N(t) - \widetilde{X}_i(t)\right) - \tau\right]\mathrm{d}t + \sigma\,\mathrm{d}B_i(t), \quad t \in (0,T] \\ \widetilde{X}_i(0) = X_i^0. \end{cases} \tag{41}$$

A.3 Proof of Proposition 3

Notice that the two functions $\eta(t)$ and $\varphi(t)$, given by (31) and (39) respectively, converge to the same limit as $N \to +\infty$, i.e.,

$$\psi(t) = \frac{(4+\lambda)\phi}{(4+\lambda)e^{(4+\lambda)(T-t)} + \phi\left(e^{(4+\lambda)(T-t)} - 1\right)}$$

which solves the following equation:

$$\dot{\psi}(t) = (4+\lambda)\psi(t) + \psi^2(t), \qquad \psi(T) = \phi.$$

Consequently, when $N \to +\infty$ the open and closed loop emission strategies, given by (32) and (40) respectively, coincide. And it follows that also the open and closed loop optimal states, given by (33) and (41), coincide.

A.4 Proof of Proposition 4

Consider the optimal state (41) (which is the same as (33) apart from substituting φ with η). By applying the sum over i and the ratio over N to both sides of (41) we get

$$\mathrm{d}\left(\frac{1}{N}\sum_{i=1}^{N}\widetilde{X}_i(t)\right) = -\tau\mathrm{d}t + \frac{\sigma}{N}\sum_{i=1}^{N}\mathrm{d}B_i(t). \tag{42}$$

Now, by taking the expectation of both sides of (42) we have that

$$\mathrm{d}\left(\frac{1}{N}\sum_{i=1}^{N}\mathbb{E}[\widetilde{X}_i(t)]\right) = -\tau\mathrm{d}t,$$

form which follows

$$\overline{X}(t) = \frac{1}{N}\sum_{i=1}^{N}\mathbb{E}[\,\widetilde{X}_i(t)] = \frac{1}{N}\sum_{i=1}^{N}\Big(-\tau t + \mathbb{E}[\,\widetilde{X}_i(0)]\Big) = X^0 - \tau t. \tag{43}$$

These considerations hold, as just observed, also for the optimal state (33) in which there is the function η instead of φ.

A.5 Proof of Proposition 5

The proof follows the same scheme presented in Carmona et al. (2015; see section 5.1).
In order to characterize the optimal emission strategy, we minimize the reduced Hamiltonian given by:

$$H(t,x,p,y) = [(m(t)-x)+y-\tau]\,p + e^{-\lambda t}\Big((x-m(t))^2 + y^2 + 2y(x-m(t))\Big)$$

which admits minimum at:

$$\widehat{y}(t) = -\frac{p}{2e^{-\lambda t}} + (m(t)-x).$$

The corresponding adjoint forward-backward equations are given by:

$$\mathrm{d}X(t) = \left[2(m(t)-X(t)) - \frac{P(t)}{2e^{-\lambda t}} - \tau\right]\mathrm{d}t + \sigma\,\mathrm{d}B(t), \tag{44}$$

$$\begin{aligned} \mathrm{d}P(t) &= 2P(t)\,\mathrm{d}t + Q(t)\,\mathrm{d}B(t), && (45)\\ P(T) &= 2e^{-\lambda T}\phi\left(X(T)-m(T)\right), \end{aligned}$$

where $Q(t)$ is an adapted square integrable process. To show the existence of a solution of the above affine forward-backward system we follow the result of Carmona, Delarue, and Lachapelle (2013). In particular, we denote by $m^X(t) = \mathbb{E}[X(t)]$ and $m^P(t) = \mathbb{E}[P(t)]$. Using the fact that in equilibrium (i.e., after solving for the fixed point) $m^X(t) = m(t)$ for every $t \le T$ which in turn implies $m^P(T) = 2e^{-\lambda T}\phi(m^X(T) - m(T)) = 0$ and considering what has been done with the corresponding finite player game, we take the ansatz:

$$P(t) = -2e^{-\lambda t}\psi(t)\left(m(t) - X(t)\right) \tag{46}$$

for some smooth deterministic function $\psi : [0,T] \to \mathbb{R}$ to be determined. Using (46), equation (44) becomes:

$$\mathrm{d}X(t) = \left[(2+\psi(t))\left(m(t)-X(t)\right) - \tau\right]\,\mathrm{d}t + \sigma\,\mathrm{d}B(t). \tag{47}$$

Taking the expectation of both sides of (47) we get $\mathrm{d}m(t) = -\tau\,\mathrm{d}t$, from which it follows that:

$$m(t) = X^0 - \tau t = m^X(t), \qquad m^P(t) = 0, \qquad t \in [0,T]. \tag{48}$$

Moreover, we have:

$$\mathrm{d}\left(m(t) - X(t)\right) = -\left(2 + \psi(t)\right)\left(m(t) - X(t)\right)\mathrm{d}t - \sigma \mathrm{d}B(t). \tag{49}$$

Using (46) in (45) we get:

$$\mathrm{d}P(t) = -4e^{-\lambda t}\psi(t)\left(m(t) - X(t)\right)\mathrm{d}t + Q(t)\,\mathrm{d}B(t). \tag{50}$$

Differentiating the ansatz (46) and using (49), it follows that:

$$\mathrm{d}P(t) = 2e^{-\lambda t}\left[\lambda\psi(t) - \dot{\psi}(t) + \psi(t)(2+\psi(t))\right]\left(m(t) - X(t)\right)\mathrm{d}t + 2e^{-\lambda t}\sigma\psi(t)\,\mathrm{d}B(t). \tag{51}$$

By the comparison of (50) with (51) we get that $Q(t) = 2e^{-\lambda t}\sigma\psi(t)$ and $\psi(t)$ must satisfy the Bernoulli equation:

$$\dot{\psi}(t) = (4+\lambda)\psi(t) + \psi^2(t), \qquad \psi(T) = \phi \tag{52}$$

whose solution is

$$\psi(t) = \frac{(4+\lambda)\phi}{(4+\lambda)e^{(4+\lambda)(T-t)} + \phi\left(e^{(4+\lambda)(T-t)} - 1\right)},$$

In conclusion, as consequence of Proposition 3, the optimal control is:

$$\widehat{y}(t) = (1+\psi(t))(m(t) - X(t)). \tag{53}$$

A.6 The Optimal Mean-Field State

The optimal state correspondent to the optimal strategy (53), reads as:

$$\begin{cases} \mathrm{d}\widehat{X}(t) = \left[(2+\psi(t))\left(m(t) - \widehat{X}(t)\right) - \tau\right]\mathrm{d}t + \sigma\,\mathrm{d}B(t), \quad t \in (0,T] \\ \widehat{X}(0) = X^0. \end{cases} \tag{54}$$

By applying the expectation to both members of the first equation of (54) (and using (48)) we get:

$$\overline{X}(t) = \mathbb{E}[\widehat{X}(t)] = X^0 - \tau t. \tag{55}$$

A.7 Proof of Proposition 6

The proof follows the same reasoning as Proposition 5, so we will avoid repetitions where possible. To characterize the optimal emission strategy, we minimize the reduced Hamiltonian given by:

$$H(t,x,p,y) = \left[(1+\theta(t))(m(t)-x) + y - \tau(t)\right]p + e^{-\lambda t}\Big((1+\theta(t))^2(x-m(t))^2 + y^2 + 2y(1+\theta(t))(x-m(t))\Big)$$

which admits minimum at:

$$y^*(t) = -\frac{p}{2e^{-\lambda t}} + (1+\theta(t))(m(t) - x). \tag{56}$$

The corresponding adjoint forward-backward equations are given by:

$$\mathrm{d}X(t) = \left[2(1+\theta(t))(m(t) - X(t)) - \frac{P(t)}{2e^{-\lambda t}} - \tau(t)\right]\mathrm{d}t + \sigma\,\mathrm{d}B(t), \tag{57}$$

$$\mathrm{d}P(t) = 2P(t)(1+\theta(t))\,\mathrm{d}t + Q(t)\,\mathrm{d}B(t), \tag{58}$$

$$P(T) = 2e^{-\lambda T}\phi\left(X(T) - m(T)\right),$$

where $Q(t)$ is an adapted square integrable process. The existence of a solution of the above affine forward-backwards system follows the result of Carmona, Delarue, and Lachapelle (2013). In particular, with the same considerations made above (46), we take the ansatz:

$$P(t) = 2e^{-\lambda t}\Psi(t)\left(X(t) - m(t)\right) \tag{59}$$

for some smooth deterministic function $\Psi : [0,T] \to \mathbb{R}$ to be determined. Using (59), equation (57) becomes:

$$\mathrm{d}X(t) = \left[(2(1+\theta(t)) + \Psi(t))\,(m(t) - X(t)) - \tau(t)\right]\,\mathrm{d}t + \sigma\,\mathrm{d}B(t). \tag{60}$$

Taking the expectation of both sides of (60) we get $\mathrm{d}m(t) = -\tau(t)\,\mathrm{d}t$, from which it follows that:

$$m(t) = X^0 - \int_0^t \tau(s)\,\mathrm{d}s = m^X(t), \qquad m^P(t) = 0, \qquad t \in [0,T]. \tag{61}$$

Moreover,

$$\mathrm{d}\,(m(t) - X(t)) = -\,(2(1+\theta(t)) + \Psi(t))\,(m(t) - X(t))\,\mathrm{d}t - \sigma\mathrm{d}B(t). \tag{62}$$

Using (59) in (58) we get:

$$\mathrm{d}P(t) = -4e^{-\lambda t}\Psi(t)(1+\theta(t))\,(m(t) - X(t))\,\mathrm{d}t + Q(t)\,\mathrm{d}B(t). \tag{63}$$

Differentiating the ansatz (59) and using (62), it follows that:

$$\mathrm{d}P(t) = 2e^{-\lambda t}\left[\lambda\Psi(t) - \dot{\Psi}(t) + \Psi(t)(2(1+\theta(t)) + \Psi(t))\right](m(t) - X(t))\,\mathrm{d}t + 2e^{-\lambda t}\sigma\Psi(t)\,\mathrm{d}B(t). \tag{64}$$

By the comparison of (63) with (64) we get that $Q(t) = 2e^{-\lambda t}\sigma\Psi(t)$ and $\Psi(t)$ must satisfy the Bernoulli equation:

$$\dot{\Psi}(t) = (4(1+\theta(t)) + \lambda)\Psi(t) + \Psi^2(t), \qquad \Psi(T) = \phi$$

whose solution is:

$$\Psi(t) = \frac{\phi}{e^{(4+\lambda)(T-t)+\int_t^T 4\theta(\xi)\mathrm{d}\,\xi} + \phi\, e^{-(4+\lambda)(t-1)-\int_1^t 4\theta(\xi)\mathrm{d}\,\xi}\left(\int_t^T e^{(4+\lambda)(s-1)+\int_1^s 4\theta(\xi)\mathrm{d}\,\xi}\,\mathrm{d}\,s\right)}.$$

In conclusion, by applying the ansatz (59) to (56) the mean field optimal emission strategy is given by:

$$y^*(t) = (1 + \theta(t) + \Psi(t))(m(t) - X(t)). \tag{65}$$

A.8 The Optimal Mean-Field State of the Extended Model

The optimal state correspondent to (65), is:

$$\begin{cases} \mathrm{d}X^*(t) = \left[(2(1+\theta(t)) + \Psi(t))\,(m(t) - X^*(t)) - \tau(t)\right]\mathrm{d}t + \sigma\,\mathrm{d}B(t), \quad t \in (0,T] \\ X^*(0) = X^0. \end{cases} \tag{66}$$

By applying the expectation to both members of the first equation of (66) (and using (61)) we get:

$$\overline{X}(t) = \mathbb{E}[X^*(t)] = X^0 - \int_0^t \tau(s)\,\mathrm{d}s, \qquad t \in [0,T].$$

Declarations

Competing interests

There is no conflict of interest.

Acknowledgements

This work was partially supported by the Gruppo Nazionale per l'Analisi Matematica e le loro Applicazioni (GNAMPA-INdAM).

References

1. Abrahamse, W., Steg. L. (2013). Social influence approaches to encourage resource conservation: a meta-analysis, Global Environmental Change 23, 1773-–1785
2. Ansuategi, A., Perrings, C.A. (2000). Transboundary externalities in the environmental transition hypothesis, Environmental and Resource Economics 17, 353–373
3. Athanassoglou, S., Xepapadeas, A. (2012). Pollution control with uncertain stock dynamics: when, and how, to be precautious, Journal of Environmental Economics and Management 63, 304–320
4. Barrett, S. (1997). Heterogeneous international environmental agreements, in (Carraro, C., Ed.) "International environmental negotiations: strategic policy issues" (Cheltenham: Elgar)
5. Bensoussan, A., Sung, K.C.J., Yam, S.C.P., Yung, S.P. (2016). Linear quadratic mean field games, Journal of Optimization Theory and Applications 169, 469–529
6. Bollinger, B., Gillingham, K. (2012). Peer effects in the diffusion of solar photovoltaic panels, Marketing Science 31, 900-–912
7. Boucekkine, R., Fabbri, G., Federico, S., Gozzi, F. (2022), A dynamic theory of spatial externalities, Games and Economic Behavior 132, 133–165
8. Brekke, K.A., Kvemdokk, S., Nyborg, K. (2003). An economic model of moral motivation, Journal of Public Economics 87, 1967—1983
9. Cardaliaguet, P. (2013). Notes on mean field games (from P.L. Lions' lectures at the Collège de France), available at: https://www.ceremade.dauphine.fr/ cardaliaguet/MFG20130420.pdf
10. Carmona, R. (2016). Lectures on BSDEs, stochastic control, and stochastic differential games with financial applications (SIAM: Philadelphia, PA)
11. Carmona, R., Delarue, F. (2013). Probabilistic analysis of mean field games, SIAM Journal on Control and Optimization 51, 2705– 2734
12. Carmona, R., Delarue, F. (2018). Probabilistic theory of mean field games with applications I: mean field FBSDEs, control, and games (Springer)
13. Carmona, R., Delarue, F., Espinosa, G.E., Touzi, N. (2013). Singular forward-backward stochastic differential equations and emissions derivatives, Annals of Applied Probability 23, 1086-–1128
14. Carmona, R., Delarue, F., Lachapelle, A. (2013). Control of McKean-Vlasov versus mean field games, Mathematics and Financial Economics 7, 131–166
15. Carmona, R., Fouque, J-P., Sun, L-H. (2015). Mean field games and systemic risk, Communications in Mathematical Sciences 13, 911–933
16. Carraro, C., Siniscalco, D. (1993). Strategies for the international protection of the environment, Journal of Public Economics 52, 309–328
17. Centola, D., Becker, J., Brackbill, D., Baronchelli, A. (2018). Experimental evidence for tipping points in social convention, Science 360, 1116—1119
18. Finus, M., Maus, S. (2008). Modesty may pay!, Journal of Public Economic Theory 10, 801–826
19. Finus, M., Tjøtta, S. (2003). The Oslo Protocol on sulfur reduction: the great leap forward? Journal of Public Economics 87, 2031–2048
20. Fornara, F., Carrus, G., Passafaro, P., Bonnes, M. (2011). Distinguishing the sources of normative influence on proenvironmental behaviors: the role of local norms in household waste recycling, Group Processes & Intergroup Relations 14, 623—635
21. Fudenberg, D., Levine, D. (1988). Open-loop and closed-loop equilibria in dynamic games with many players, Journal of Economic Theory 44, 1–18
22. Huang, X., He, P., Zhang, W. (2016), A cooperative differential game of transboundary industrial pollution between two regions, Journal of Cleaner Production 120, 43–52

23. Hechter, M., Opp, K.D. (2001). Social norms (Russell Sage Foundation: New York)
24. Intergovernmental Panel on Climate Change, IPCC (2014). Climate change 2014: synthesis report summary for policymakers. IPCC, Geneva, available at: https://www.ipcc.ch/pdf/assessment-report/ar5/syr/AR5_SYR_FINAL_SPM.pdf
25. Kellenberg, D., Levinson, A. (2013). Waste of effort? International environmental agreements, Journal of the Association of Environmental and Resource Economists 1, 135–169
26. Keizer, K., Lindenberg, S., Steg, L. (2008). The spreading of disorder, Science 322, 1681—1685
27. La Torre, D., Liuzzi, D., Marsiglio, S. (2017). Pollution control under uncertainty and sustainability concern, Environmental and Resource Economics 67, 885–903
28. La Torre, D., Liuzzi, D., Marsiglio, S. (2021). Transboundary pollution externalities: think globally, act locally?, Journal of Mathematical Economics 96, 102511
29. Lappi, P. (2018). Optimal clean-up of polluted sites, Resource and Energy Economics 54, 53-68
30. Liu, P.R., Raftery, A.E. (2021). Country-based rate of emissions reductions should increase by 80% beyond nationally determined contributions to meet the 2°C target, Communications Earth & Environment 2, 29
31. Lopez–Pintado, D. (2008). Diffusion in complex social networks, Games and Economic Behavior 62, 573–590
32. Marsiglio, S., Masoudi, N. (2022). Reclamation of a resource extraction site: a differential game approach, Metroeconomica 73, 770–802
33. Marsiglio, S., Tolotti, M. (2020). Motivation crowding-out and green-paradox-like outcomes, Journal of Public Economic Theory 22, 1559–1583
34. Masoudi, N., Zaccour, G. (2013). A differential game of international pollution control with evolving environmental costs, Environment and Development Economics 18, 680–700
35. Missfeldt, F. (1999). Game-theoretic modelling of transboundary pollution, Journal of Economic Surveys 13, 287–321
36. Nyborg, K. (2018). Social norms and the environment, Annual Review of Resource Economics 10, 405—423
37. Nyborg, K., Anderies, J.M., Dannenberg, A., Lindahl, T. Schill, C., Schlüter, M., Adger, W.N., Arrow, K.J., Barrett, S., Carpenter, S., F.S., Chapin III, Crepin, A.S., Daily, G., Ehrlich, P., Folke, C., Jager, W., Kautsky, N., Levin, S.A., Madsen, O.J., Polasky, S., Scheffer, M., Walker, B., Weber, E.U., Wilen, J., Xepapadeas, A.,de Zeeuw, A. (2016). Social norms as solutions, Science 354, 42–43
38. Oreskes, N. (2004). The scientific consensus on climate change, Science 306, 1686
39. Otto, I.M., Donges, J.F., Cremades, R., Bhowmik, A., Hewitt, R.J., Lucht, W., Rockstrom, J., Allerberger, F., McCaffrey, M., Doe, S.S.P., Lenferna, A., Moran, N., van Vuuren, D.P., Schellnhuber, J. (2020). Social tipping dynamics for stabilizing Earth's climate by 2050, Proceedings of the National Academy of Sciences of the USA 117, 2354-–2365
40. Ringquist, E.J.,, Kostadinova, T. (2005). Assessing the effectiveness of international environmental agreements: the case of the 1985 Helsinki Protocol, American Journal of Political Science 49, 86–102
41. Rubio, S.J., A. Ulph, A. (2007). An infinite-horizon model of dynamic membership of international environmental agreements, Journal of Environmental Economics and Management 54, 296–310
42. Silva, E.C.D., Zhu, X. (2009). Emissions trading of global and local pollutants, pollution havens and free riding, Journal of Environmental Economics and Management 58, 169-–182
43. Sparkman, G., Howe, L., Walton, G. (2021). How social norms are often a barrier to addressing climate change but can be part of the solution, Behavioural Public Policy 5. 528-–555
44. Steg, L., Lindenberg, S., Keizer, K. (2005). Intrinsic motivation, norms and environmental behaviour: the dynamics of overarching goals, International Review of Environmental and Resource Economics 9, 179—207

45. Szekely, A., Lipari, F., Antonioni, A., Paolucci, M., Sánchez, A., Tummolini, L., Andrighetto, G. (2021). Evidence from a long-term experiment that collective risks change social norms and promote cooperation, Nature Communications 12, 5452
46. Wallen, K.E., Romulo, C.L. (2017). Social norms: more details, please, Proceedings of the National Academy of Sciences of the USA 114, E5283–E5284

B Online Appendix [NOT FOR PUBLICATION]

B.1 Comparative Statics

Denoting the open loop emission intensity by $\widehat{A}(t,N) = \left[1 + \left(1 - \frac{1}{N}\right)\eta(t)\right]$, and taking into account the form of $\eta(t)$ (6), tedious algebra allows us to prove the following:

$$\frac{\partial \widehat{A}}{\partial \lambda} = \frac{\phi(-1+N)\left[-e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}(-1+(4+\lambda)N)^2(T-t) + \phi(-1+N)\left(-N + e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}\left((T-t)(1-N(4+\lambda))+N\right)\right)\right]}{N(\phi(1-N) + e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}(-1-\phi+(4+\phi+\lambda)N))^2} < 0$$

$$\frac{\partial \widehat{A}}{\partial \phi} = \frac{e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}(-1+N)(-1+(4+\lambda)N)^2}{N\left(\phi(1-N) + e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}(-1-\phi+(4+\phi+\lambda)N)\right)^2} > 0$$

$$\frac{\partial \widehat{A}}{\partial T} = -\frac{\phi e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}(-1+N)(-1+(4+\lambda)N)^2(-1-\phi+(4+\phi+\lambda)N)}{N^2\left(\phi(1-N) + e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}(-1-\phi+(4+\phi+\lambda)N)\right)^2} < 0$$

$$\frac{\partial \widehat{A}}{\partial N} = \frac{\phi\left[e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}\left((T-t)(1-(4+\lambda)N)\left((1-(4+\lambda)N) + \phi(-1+N)^2\right) + N((1-(4+\lambda)N)^2 + \phi(-1+N)^2)\right) - \phi N(-1+N)^2\right]}{N^3\left(\phi(1-N) + e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}(-1-\phi+(4+\phi+\lambda)N)\right)^2}$$

where only the last derivative has an ambiguous sign.

Denoting the closed loop emission intensity by $\widetilde{B}(t,N) = \left[1 + \left(1 - \frac{1}{N}\right)\varphi(t)\right]$, and exploiting the form of $\varphi(t)$ (9), we can prove the following results:

$$\frac{\partial \widetilde{B}}{\partial \lambda} = \frac{\phi(-1+N)N\Big(-N^2 e^{(4+\lambda)(T-t)}(4+\lambda)^2(T-t) + \phi(-1+N^2)\Big(-1 + e^{(4+\lambda)(T-t)}(1-(4+\lambda)(T-t))\Big)\Big)}{(\phi(1-N^2) + e^{(4+\lambda)(T-t)}(-\phi+(4+\phi+\lambda)N^2))^2} < 0$$

$$\frac{\partial \widetilde{B}}{\partial \phi} = \frac{e^{(4+\lambda)(T-t)}(4+\lambda)^2(-1+N)N^3}{(\phi(1-N^2) + e^{(4+\lambda)(T-t)}(-\phi+(4+\phi+\lambda)N^2))^2} > 0$$

$$\frac{\partial \widetilde{B}}{\partial T} = -\frac{\phi e^{(4+\lambda)(T-t)}(4+\lambda)^2(-1+N)N(-\phi+(4+\phi+\lambda)N^2)}{(\phi(1-N^2) + e^{(4+\lambda)(T-t)}(-\phi+(4+\phi+\lambda)N^2))^2} < 0$$

$$\frac{\partial \widetilde{B}}{\partial N} = \frac{\phi(4+\lambda)\Big(-\phi(-1+N)^2 + e^{(4+\lambda)(T-t)}(\phi(1-2N) + (4+\phi+\lambda)N^2)\Big)}{(\phi(1-N^2) + e^{(4+\lambda)(T-t)}(-\phi+(4+\phi+\lambda)N^2))^2} > 0$$

The difference between $\eta(t) - \varphi(t)$ is given by:

$$\eta(t) - \varphi(t) = \frac{\phi(-1+(4+\lambda)N))}{\phi(1-N) + e^{\left(4+\lambda-\frac{1}{N}\right)(T-t)}(-1-\phi+(4+\phi+\lambda)N)} - \frac{\phi(4+\lambda)N^2}{\phi(1-N^2) + e^{(4+\lambda)(T-t)}(-\phi+(4+\phi+\lambda)N^2)},$$

which is greater than zero for every $t \in [0,T)$, while $\eta(t) - \varphi(t) = 0$ when $t = T$.

The derivatives of the mean-field emission intensity in (53) with respect to λ, ϕ and T are:

$$\frac{\partial \psi}{\partial \lambda} = \frac{\phi\Big(-\phi + e^{(4+\lambda)(T-t)}\left(\phi - (4+\lambda)(T-t)(4+\phi+\lambda)\right)\Big)}{(-\phi + e^{(4+\lambda)(T-t)}(4+\phi+\lambda))^2} < 0,$$

$$\frac{\partial \psi}{\partial \phi} = \frac{e^{(4+\lambda)(T-t)}(4+\lambda)^2}{(-\phi + e^{(4+\lambda)(T-t)}(4+\phi+\lambda))^2} > 0,$$

$$\frac{\partial \psi}{\partial T} = -\frac{\phi\, e^{(4+\lambda)(T-t)}(4+\lambda)^2(4+\phi+\lambda)}{(-\phi + e^{(4+\lambda)(T-t)}(4+\phi+\lambda))^2} < 0.$$