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ERRATUM TO: INTRINSIC CURVATURE OF CURVES AND SURFACES AND A GAUSS–BONNET THEOREM IN THE HEISENBERG GROUP

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In the publication [1] there is an unfortunate computational error, which however does not affect the correctness of the main results.

Let us recall some notation from the paper. By $\gamma : [a,b] \to \mathbb{R}^3$ we denote a $\mathcal{C}^2$ smooth parametrized regular curve $t \to \gamma(t) = (\gamma_1(t), \gamma_2(t), \gamma_3(t))$. The action of the standard contact form $\omega = dx_3 - \frac{1}{2}(x_1 dx_2 - x_2 dx_1)$ on γ is denoted by

$$\omega(\dot\gamma) = \omega(\dot\gamma)(t) = \dot\gamma_3(t) - \frac{1}{2}\left(\gamma_1(t)\dot\gamma_2(t) - \gamma_2(t)\dot\gamma_1(t)\right).$$

A point $t_0 \in [a,b]$ is called horizontal if and only if $\omega(\dot\gamma)(t_0) = 0$. The mistake in the paper arises due to a statement implicitly assumed in the proof of Lemma 3.4, that at any horizontal point we also have that $\omega(\ddot\gamma)(t_0) = 0$, where

$$\omega(\ddot\gamma) = \omega(\ddot\gamma)(t) = \ddot\gamma_3(t) - \frac{1}{2}\left(\gamma_1(t)\ddot\gamma_2(t) - \gamma_2(t)\ddot\gamma_1(t)\right).$$

This fact is in general not true. As a result, various statements in the paper, including the second formula in equation (1.1), equation (3.4), the second part of equation (3.10), and the second displayed equations in both Lemma 4.8 and Proposition 4.13, do not hold for all horizontal points.

However, noticing that $\omega(\ddot\gamma) = \frac{d}{dt}\omega(\dot\gamma)$ we see that the assertion $\omega(\ddot\gamma)(t_0) = 0$ is still true for horizontal points that arise as accumulation points of other horizontal points. Since the parameterizing interval is compact, there are at most a finite number of isolated horizontal points $t_1, \ldots, t_N$ at which the quantity $\omega(\ddot\gamma)(t_i)$ may be nonzero, and hence all of the preceding formulas hold at all points of $[a,b]$ except for this finite number of isolated points.

The main result of the paper, Theorem 1.1, is not affected by these corrections since its proof is based on an approximation argument relying on the Lebesgue dominated convergence theorem. In the application of this theorem a set of countably many points can be ignored as a null set, and the proof works as indicated in Section 6 of the paper.

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References

[1] Balogh, Z. M., Tyson, J. T. and Vecchi, E., Intrinsic curvature of curves and surfaces and a Gauss-Bonnet theorem in the Heisenberg group, *Math. Z.*, **287** (2017), 1-38.

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