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Finding damage in truss structures exploiting modal strains

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Abstract. The detection of potentially damaged elements in planar truss structures is a challenging task. Among the different methods proposed in literature, one promising procedure exploits the modal strains of the structure that are calculated from the flexibility matrix, which is, in turn, estimated from the lowest frequencies and corresponding modes of vibration. The benefit of this approach stems from the possibility of using a reduced number of mode shapes, usually available from the dynamic monitoring of the structure to perform the damage detection. In this work, a novel damage detection index based on modal strains is proposed, and its reliability in detecting stiffness reduction in elements of a planar truss is tested numerically.

Introduction

Damage detection is a key information for monitoring the status of structures and infrastructures. In the last decades, several procedures aimed at detecting structural damages and/or anomalies to prevent the degradation and avoid the collapse of the structures have been proposed. To reach this goal, a meaningful part of the available research literature makes use of the structure dynamic response. In fact, when a damage occurs, the reduction of the stiffness yields a change in the structural dynamic characteristics. Hence, vibration-based techniques able to detect the variation of such features are widely used to monitor the structural health.

Among others, the first proposed numerical strategies exploited the variation of natural frequencies and mode shapes to identify the presence of anomalies [1]. Although effective in identify the presence of damages, these methods suffer from the presence of noise which can conceal the variation of natural frequencies induced by damages. Hence, more sophisticated techniques were later proposed to locate structural damages, based, for example, on the use of modal strain energy, residual force vector, mode shape curvature and flexibility matrix [2,3,4]. Methods exploiting flexibility matrix allow to evaluate changes in the structural dynamic features using only the lowest eigenfrequencies and mode shapes [5]. This feature represents a great advantage for practical health monitoring where it is typically difficult to measure and estimate the highest modes of vibrations.

In this context, Montazer and Seyedpoor [6] recently proposed a damage index, called strain change based on flexibility index (SCBFI), which exploits the variation in strains computed from the flexibility matrix. The proposed index is proved to be particularly effective in locating the damage in truss structures.

In this paper, we stem from their approach to propose an alternative index based on the singular value decomposition of the difference between the strain matrix of the damaged and healthy structure. Following this approach, we can evaluate the bars which mostly contribute to the variation of strain in the damaged truss. The proposed index is tested numerically on a truss structure.

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Damage index

Let us consider a generic planar truss, composed by *N* nodes, connected with e bar elements. The number of degrees of freedom of the structure is equal to $n = 2N - c$, with *c* being the number of constrained DOFs.

The modal matrix $\mathbf{\Phi}$ is composed by $\mathbf{\Phi}_i$ mass-normalized mode shapes. The flexibility matrix **f** of the structure can be approximated as, [5]:

$$\mathbf{f} \approx \sum_{i=1}^{m} \frac{1}{\omega_i^2} \mathbf{\Phi}_i \mathbf{\Phi}_i^{\mathrm{T}} \tag{1}$$

where ω_i is the i-th circular frequency and m is the number of considered modes. Given the inverse proportion between flexibility matrix and the square of the circular frequencies, the expression in Eq. (1) rapidly converges to the exact value as the number of considered frequencies increases. Therefore, an accurate approximation of the flexibility matrix can be obtained using the lowest m modes of vibration.

We remind that the i-th column of the flexibility matrix collects a vector of nodal displacements corresponding to a unitary force applied in the related i-th degree of freedom. Hence, we can compute the strain matrix **SM**, composed by j-th element strain values ϵ_{ji} caused by a unitary force applied to the i-th DOF:

$$\epsilon_{ji} = \mathbf{R}_j \mathbf{u}_{ji} \tag{2}$$

where $\mathbf{R}_j$ is the topological vector of the element j and $\mathbf{u}_{ji}$ is the 1×4 nodal displacement vector of *j*-th bar, associated with the i-th column of flexibility matrix.

As well known, the presence of one or more damaged bars in a truss produces a reduction in the structural stiffness and, in turn, an increase of flexibility and strain. For this reason, it is possible to evaluate and compare the strain matrix **SM** for both the healthy and for the damaged structure, hereinafter labelled as $\mathbf{SM}^{\mathrm{H}}$ and $\mathbf{SM}^{\mathrm{D}}$, respectively.

From the strain matrices, we compute the matrix collecting the strain variation between damaged and healthy structures as:

$$\mathbf{\Delta SM} = \mathbf{SM}^{\mathrm{D}} - \mathbf{SM}^{\mathrm{H}} \tag{3}$$

At this stage, we assume that only the damaged elements contribute significantly to the variation of the modal strain. Following this assumption, we propose to identify such damaged elements by performing a singular value decomposition (SVD) of the matrix $\mathbf{\Delta SM}$:

$$\mathbf{\Delta SM} = \mathbf{USV} \tag{4}$$

where **U** is a $e \times e$ matrix collecting the left-singular vectors of $\mathbf{\Delta SM}$, **S** is a $e \times n$ rectangular diagonal matrix whose diagonal entries are the singular values s_i of $\mathbf{\Delta SM}$ and **V** is a $n \times n$ matrix collecting the right-singular vectors of $\mathbf{\Delta SM}$. We remark that the norm of the matrix $\mathbf{\Delta SM}$ can be approximated by exploiting only the first $v \ll e$ singular values and vectors:

$$\|\mathbf{\Delta SM}\|^2 \approx \sum_{i=1}^{v} \mathbf{U}_i s_i^2 \mathbf{U}_i^* \tag{5}$$

where * indicate the complex conjugate.

In particular, the first singular value and the related left-singular vector provides the largest contribution to the strain difference $\mathbf{\Delta SM}$. Hence, we utilize the first left-singular vector $\mathbf{U}_1$ to define the damage index indicated as $\mathbf{SVD}_1$:

$$\mathbf{SVD}_1 = |\mathbf{U}_1| \tag{6}$$

being $|\,.\,|$ the absolute value. Similarly, taking into consideration both the first and the second singular values and the corresponding left-singular vectors, we can compute a second damage index defined as the weighted sum of the two lowest singular vectors, $\mathbf{U}_1$ and $\mathbf{U}_2$, of the $\mathbf{\Delta SM}$ matrix multiplied by the corresponding singular values, s_1 and s_2, as follows:

$$\mathbf{SVD}_2 = s_1|\mathbf{U}_1| + s_2|\mathbf{U}_2| \tag{7}$$

In the next section, the proposed indexes $\mathbf{SVD}_1$ and $\mathbf{SVD}_2$ are applied to detect and locate the presence of damages in a two-dimensional planar truss and their reliability in finding damage is compared.

Numerical results

Single damage scenario. For the numerical investigation, we consider the planar truss shown in Fig. 1, which was previously used in literature as a numerical benchmark for other damage identification methods [7]. The truss is organized in 6 square bays, for an overall of e = 31 bars and $n = 25$ degrees of freedom. Bars are made of aluminium, with an elastic modulus of 70 GPa and a density equal to 2770 kg/m^3.

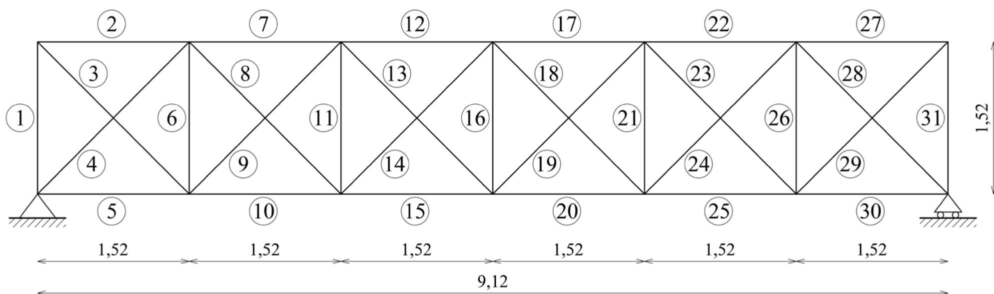

***Fig. 1.** Planar truss composed of 31 bars and 6 square bays.*

The natural frequencies and mode shapes of the truss are computed using a standard Finite Element code where each element is modelled as linear elastic truss element. The damage in the bars is simulated reducing the element stiffness, namely by decreasing the Young's modulus.

To evaluate the capabilities and robustness of the proposed damage index, we have tested several configurations of single damaged bars. Our aim is to locate the presence of damage along the truss for increasing damage intensities. To this purpose, we computed natural frequencies and mode shapes of the truss structure considering 5 different levels of damage in each bar, with a damage intensity varying between 10% and 90%.

Additionally, we polluted each damaged mode shape components $\varphi_{mn}^{\mathrm{D}}$, related to the m-th mode of vibration and n-th DOF, considering a given level of noise p. This noise aims at replicating the inherent variability induced by environmental and operational conditions of the structure in the field. The components $\varphi_{mn}^{\mathrm{D}}$ utilized to build the flexibility matrix are thus obtained as the mean of 200 random polluted $\hat{\varphi}_{mn}$ as follows:

$$\hat{\varphi}_{mn} = \varphi_{mn}(1 + p \cdot \mathrm{rand}) \tag{8}$$

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where p is the intensity of a uniformly distributed random noise rand in the interval [-1;1]. In the described analysis, we considered noise value p between 0% (without noise) and 3%. For the approximation of flexibility matrix, Eq. 1, we used the first m = 4 mode shapes.

Table 1.** Number of correct identifications over 100 attempts in single damage scenarios using* ***SVD$_1$ *with noise p = 1%.*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
10%	100	100	100	100	100	99	100	100	100	100	96	100	100	100	100	99	100	100	100	100	78	100	100	100	100	27	100	100	100	100	100
30%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
50%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
70%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
90%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Table 2.** Number of correct identifications over 100 attempts in single damage scenarios using* ***SVD$_2$ *with noise p = 1%.*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
10%	100	100	100	100	100	97	100	99	98	100	88	100	92	100	100	86	100	86	92	100	69	100	98	100	100	58	100	100	99	100	100
30%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
50%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
70%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
90%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Table 1 collects the number of correctly identified damage scenarios using the index **SVD$_1$**, while Table 2 shows the same results obtained exploiting **SVD$_2$**. Each column of the tables represents a different damaged bar of the truss and each row an increasing level of damage. Note that for each damage level and bar, we performed 100 simulations and damage identifications. The reader can appreciate that even with the presence of noise, the results with the proposed indexes are robust, since the greatest part of the damage cases are correctly identified by the algorithm.

Table 3.** Number of correct identifications over 100 attempts in single damage scenarios using* ***SVD$_1$ *with noise p = 3%.*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
10%	74	76	79	98	100	20	100	48	63	100	33	100	18	21	100	32	100	14	31	100	20	100	47	62	100	6	73	95	71	88	76
30%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	97	100	100	100	100	47	100	100	100	100	100
50%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	99	100	100	100	100	100
70%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
90%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Table 4.** Number of correct identifications over 100 attempts in single damage scenarios using* ***SVD$_2$ *with noise p = 3%.*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
10%	43	32	71	56	100	42	100	29	48	100	46	100	44	41	100	45	100	38	34	100	34	100	42	64	100	12	54	74	66	68	58
30%	100	100	100	100	100	99	100	100	100	100	91	100	97	100	100	93	100	95	99	100	83	100	98	100	100	65	100	100	100	100	100
50%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	99	100	100	100	100	100
70%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
90%	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

In particular, to ease the interpretation of the tables, we report with a green background all the scenarios with a damage identification success rate greater than 85%.

Table 3 and Table 4 report the number of correct identifications with a level of noise p equal to 3%. As expected, using both **SVD$_1$** and **SVD$_2$** the success rate decreases as the level of noise increases and as the damage intensity decreases. However, the missed identified bars correspond to the lowest level of damage (10% reduction in the elastic modulus) and, likely, less harmful for the structure. We remark that the indexes **SVD$_1$** and **SVD$_2$** have similar results in terms of success rate in the analysed single damage scenarios, showing that the second singular value contributes marginally to the reconstruction of the strain matrix in Eq. (5).

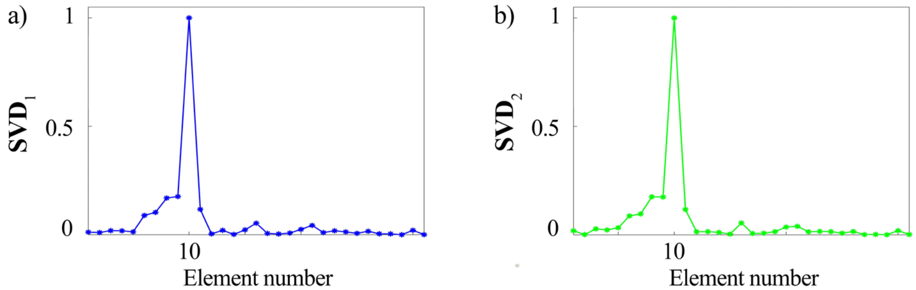

Fig. 2. *a)* $\mathbf{SVD}_1$ *and b)* $\mathbf{SVD}_2$ *graphs with a damage of 30% in bar 10 and noise level p of 3%.*

As an example, we report in Fig. 2 the value of the damage indices as computed from a single simulation of a damage scenario corresponding to a reduction of 30% in the elastic modulus of bar 10, with a noise level p equal to 3%. The indexes are normalized with respect to their maximum value. The reader can appreciate how the proposed damage indexes successfully identify the damaged bar, as highlighted by the peaks in the graphs.

Double damage scenario. Motivated by the satisfactory performance of the proposed indexes, we investigated the capabilities of the identification algorithm against damage scenarios with multiple damaged bars. In particular, we considered a stiffness reduction of 50% in bar 16 and of 30% in bar 5. In Fig. 3 and Fig. 4 a comparison between the indexes $\mathbf{SVD}_1$ and $\mathbf{SVD}_2$ as obtained for the considered damage scenario with noise level p = 1% and p = 3%, respectively, is shown.

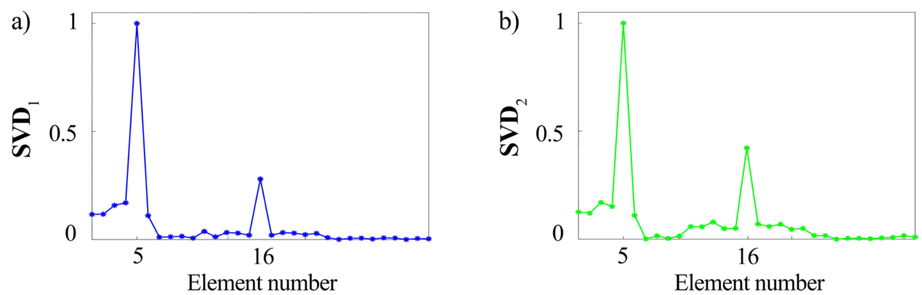

Fig. 3. *a)* $\mathbf{SVD}_1$ *and b)* $\mathbf{SVD}_2$ *graphs with an elastic modulus reduction of 50% in bar 16 and damage of 30% in bar 5, polluted by a noise p equal to 1%.*

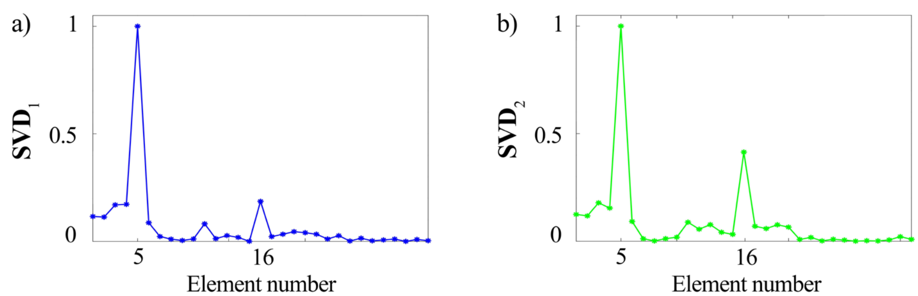

Fig. 4. *a)* $\mathbf{SVD}_1$ *and b)* $\mathbf{SVD}_2$ *graphs with an elastic modulus reduction of 50% in bar 16 and damage of 30% in bar 5, polluted by a noise p equal to 3%.*

Outputs in Fig. 3 and Fig. 4 show that the addition of the second singular value, as considered in $\mathbf{SVD}_2$ index, is beneficial in the multiple case scenario, as indicated by the peaks in Fig. 3b and Fig. 4b, considering both a level of noise p of 1% and of 3%. In fact, while the $\mathbf{SVD}_1$ still succeeds in locating properly the damaged bars with noise equal to 1%, Fig. 3a, it shows some limitations in the scenario with 3% of noise, where extra peaks are found by the damage index, Fig. 4a.

Finally, we remark that the proposed index cannot quantify the damage extent, namely there is no direct correlation between the value of the damage index and the intensity of the damage.

Conclusions

In this paper, we proposed a novel damage index which exploits the variation in modal strains to detect damages in truss structures. The index is based on the Singular Value Decomposition of the variation strain matrix $\mathbf{\Delta SM}$ between the healthy and damaged structure. The performance and reliability of the method in detecting the damaged elements were numerically tested on a 31 bars planar truss considering several single damage scenarios, even in presence of noise polluting the structures modal characteristics. As expected, the identification success rate increases as the damage extent increases and decreases with higher noise level. The proposed index is applied with promising results also to a double case scenario. Future research efforts will be devoted to extend the proposed algorithm to quantify the damage extent.

References

[1] J.-T. Kim, Y.-S. Ryu, H.-M. Cho, N. Stubbs, Damage identification in beam-type structures: frequency-based method vs mode-shape-based method, Engineering Structures 25 (2003) 57–67. https://doi.org/10.1016/S0141-0296(02)00118-9

[2] A. Alvandi, C. Cremona, Assessment of vibration-based damage identification techniques, Journal of Sound and Vibration 292 (1) (2006) 179–202. https://doi.org/10.1016/j.jsv.2005.07.036

[3] O. Avci, O. Abdejlaber, S. Kiranyaz, M. Hussein, M. Gabbouj, D.J. Inman, A review of vibration-based damage detection in civil structures: From traditional methods to machine learning and deep learning applications, Mechanical Systems and Signal Processing 147 (2021) 107077. https://doi.org/10.1016/j.ymssp.2020.107077

[4] S. Doebling, C. Farrar, M. Prime, A summary review of vibration-based damage identification methods, The Shock and Vibration Digest 30 (1998) 91–105. https://doi.org/10.1177/058310249803000201

[5] A. Pandey, M. Biswas, Damage detection in structures using changes in flexibility, Journal of Sound and Vibration 169 (1) (1994) 3–17. https://doi.org/10.1006/jsvi.1994.1002

[6] M. Montazer, S.M. Seyedpoor, A new flexibility based damage index for damage detection of truss structures, Shock and Vibration (2014) article ID 460692. https://doi.org/10.1155/2014/460692

[7] M. Nobahari and S. M. Seyedpoor, Structural damage detection using an efficient correlation-based index and a modified genetic algorithm, Mathematical and Computer Modelling (2011), vol. 53, no. 9-10, pp. 1798–1809. https://doi.org/10.1016/j.mcm.2010.12.058