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Marco Carricato and Vincenzo Parenti Castelli
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Singularity-Free Fully-Isotropic Translational Parallel Mechanisms

Marco Carricato
Department of Mechanical Engineering – DIEM
University of Bologna
Viale Risorgimento 2, 40136 Bologna, Italy
Tel. +39 051 2093459, Fax +39 051 2093446
E-mail: marco.carricato@mail.ing.unibo.it

Vincenzo Parenti-Castelli
Department of Mechanical Engineering – DIEM
University of Bologna
Viale Risorgimento 2, 40136 Bologna, Italy
Tel. +39 051 2093459, Fax +39 051 2093446
E-mail: vincenzo.parenticastelli@mail.ing.unibo.it

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Abstract

Parallel mechanisms show desirable characteristics such as a large payload to robot weight ratio, considerable stiffness, low inertia and high dynamic performances. In particular, parallel manipulators with fewer than six degrees of freedom have recently attracted researchers' attention, as their employ may prove valuable in those applications in which a higher mobility is uncalled-for.

The attention of this paper is focused on translational parallel mechanisms (TPMs), that is on parallel mechanisms whose output link (platform) is provided with a pure translational motion with respect to the frame.

It deals with the general problem of the topological synthesis and classification of TPMs and it investigates both their constraint and direct singularities. In particular, it identifies for the first time special families of fully-isotropic mechanisms. Such manipulators exhibit outstanding properties, as they are free from singularities and show a constant orthogonal Jacobian matrix throughout their workspace. As a consequence, both the direct and the inverse position problems are linear and the kinematic analysis proves straightforward.

1. Introduction

A typical *parallel mechanism* (PM) consists of a moving platform connected to a fixed base by means of several kinematic chains, called *legs*. Only some kinematic pairs are actuated, in number generally equal to the number *n* of degrees of freedom (DoFs) that the platform possesses with respect to the base; the other joints are passive. Usually, the number of legs is also equal to *n*. This makes it possible to actuate only one pair per leg, allowing all motors to be mounted close to the base. Such mechanisms show desirable characteristics like a large payload to robot weight ratio, considerable stiffness, low inertia and high dynamic performances. With respect to serial manipulators, disadvantages are a lower dexterity, a smaller workspace and more serious consequences caused by kinematic singularities (which are configurations in which the functioning of the mechanism is disrupted).

Since their first designs (Gough and Whitehall 1962; Stewart 1965), 6-DoF PMs have been extensively studied, whilst only in relatively recent times have manipulators with fewer DoFs attracted researchers' attention. As they are potentially architecturally simpler and cheaper than their 6-DoF counterparts (they require less parts and actuators), their use may be advantageous in all those applications in which less than six DoFs are required.

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In particular, different architectures of 3-DoF mechanisms have been presented in the literature. Depending on the kind of motion exhibited by the platform, PMs can be divided into mixed (MPMs), spherical (SPMs) and translational ones (TPMs).

The first allow the platform to both translate and rotate and may be employed as motion simulators, wrists of hybrid serial-parallel robots and mixed orienting/positioning systems (Lee and Shah 1988; Waldron, Raghavan and Roth 1989; Carretero et al. 2000; Di Gregorio and Parenti-Castelli 2001).

The second enable the travelling plate to rotate about a fixed point and may be used in those applications that require orienting a body in space, be it a solar panel, an antenna, a telescope, a gun, the end-effector of a robot, a human or humanoid artificial limb, etc. (Gosselin and Lavoie 1993; Innocenti and Parenti-Castelli 1993; Gosselin, Sefrioui and Richard 1994; Karouia and Hervé 2000; Vischer and Clavel 2000; Carricato and Parenti-Castelli 2001a).

The last provide the output link with a pure translational motion and may be particularly valuable in the fields of automated assembly and machine tools as alternatives to traditional serial positioning systems. The attention of this paper is focused on TPMs.

Throughout the text, the kind of kinematic pair will be addressed using the following symbols (in parentheses, the number of DoFs of the joint is also specified): P for prismatic pair (1 DoF), R for revolute pair (1 DoF), H for screw pair (1 DoF), P_A for planar parallelogram[1] (1 DoF), C for cylindrical pair (2 DoF), U for universal pair (2 DoF) and S for spherical pair (3 DoF).

In many solutions presented in the literature, all the legs comprised in a given architecture exhibit the same topological structure. In this case, the mechanism's type can be addressed by specifying the number of legs and the sequence of joints distributed along any of them, from the base to the platform.

The first TPM, called Delta Robot, was presented in the late Eighties (Sternheim 1987; Clavel 1988). It comprises three legs, each including an R-pair and a spatial parallelogram containing ball-and-socket or universal joints. Due to its outstanding dynamic performances, it was attributed a widespread success.

Hervé and Sparacino (1991) introduced a whole class of new TPMs exhibiting 4-DoF legs[2], each one of which comprises mutually parallel rotational joints and deprives the platform of two DoFs of rotation. In the literature, several TPMs have been proposed that belong to this class, which can be addressed as the Hervé family. Some have been presented by Hervé himself and his co-workers, like the Y-STAR manipulator (Hervé and Sparacino 1992), the H- and the Prism- Robots (Hervé 1995), the 3-RP_AP_AR micro finger (Arai, Hervé and Tanikawa 1996). Some others have been studied in detail by other researchers, like the University of Maryland manipulator, whose 3-RRP_AR architecture can be obtained from the Delta Robot by replacing the spatial parallelograms with planar ones (Tsai and Stamper 1996; Tsai, Walsh and Stamper 1996), and the 2- and 3- PRRR mechanisms, whose several different arrangements (in some of which the P-pair is not mounted on the base and/or is replaced by a P_A-pair) have been investigated in Hervé and Sparacino (1991), Zhao and Huang (2000), Jin and Yang (2001) and Carricato and Parenti-Castelli (2002).

Tsai (1996) presented the 3-UPU mechanism, each leg of which has five DoFs and deprives the platform of a single DoF of rotation. This mechanism is structurally similar to two Delta Robot embodiments patented by Clavel (1990), which exhibit a 3-RUU and a 3-PUU architecture respectively. Such manipulators were later recognized as particular cases of a wider class of mechanisms that can be addressed as the Clavel/Tsai family (Di Gregorio and Parenti-Castelli 1998; Tsai and Joshi 2000).

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[1] The planar parallelogram can be seen as a single kinematic pair providing two links with a relative motion of circular translation in the plane perpendicular to the parallelogram axis (which is defined as the direction of the axes of the mechanism's revolute joints).

[2] In the context of this paper, the number of DoFs of a leg is meant to be that with which the leg provides its output or terminal link, i.e. the platform, with respect to the base.

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The entire class of TPMs whose legs possess five DoFs was exhaustively studied by Frisoli et al. (2000) and, independently, by Carricato and Parenti-Castelli (2001b and 2001c). This has led to the definition of novel architectures, different from the ones derived from the 3-UPU manipulator and groupable under the name of the Frisoli/Carricato family.

It is worth mentioning that, whilst all mechanisms of the Hervé family are overconstrained, the Delta Robot and the manipulators belonging to the families of Clavel/Tsai and of Frisoli/Carricato are not.

The present paper approaches, in a systematic form, the problem of the topological synthesis and classification of TPMs and focuses on the identification of families of singularity-free mechanisms.

Both *direct* and *constraint* singularities are addressed. The former are common to any PM and represent configurations in which one or more of the existing DoFs of the platform become uncontrollable and a nonzero output motion exists even when the actuator velocities are zero. The latter are peculiar to manipulators having less than six DoFs and lead the output link to acquire additional previously constrained DoFs (Zlatanov, Bonev and Gosselin 2001). For TPMs, in particular, constraint singularities are configurations in which the platform looses its capability of purely translating and gains an instantaneous mobility of rotation. It is clear that both kinds of singularities deeply affect the kinematic behavior of the manipulator and their study is especially important.

Section 2 describes the requirements that a TPM leg must satisfy in order to, on the one hand, constrain the rotational motion of the platform, on the other, allow it a 3-DoF motion of translation. Indeed, whilst there are no particular bonds on the topological design of the legs of a 6-DoF PM, as any 6-DoF kinematic chain is consistent with a constraintless motion of the platform, when topologically designing the leg of a TPM, a fundamental question must be given a response: which architecture must the leg possess for the platform to be able to freely translate in space without altering its orientation? Three leg topologies are identified and classified on the basis of the number of rotational DoFs eliminated from the platform. How such legs must be assembled in order to generate a TPM is illustrated and, depending on the adopted leg type, corresponding families of mechanisms are identified (T_3, T_4, T_5' and T_5" TPMs). The presented discussion gathers the families of Hervé (T_4 TPMs), of Frisoli/Carricato (T_5' TPMs) and of Clavel/Tsai (T_5" TPMs) into a general and organic corpus.

Section 3 examines in detail the above families. First, the geometric conditions that the legs must meet in order to satisfy the mentioned mobility requirements are described in detail. Then, constraint singularities are investigated, proving that, with the exception of T_5" TPMs (Di Gregorio and Parenti-Castelli 1999; Parenti-Castelli and Bubani 1999; Parenti-Castelli, Di Gregorio and Bubani 2000), all other TPMs are constraint-singularity-free.

The problem of direct singularities is dealt with in Section 4, which focuses on the identification of special families of mechanisms exhibiting outstanding properties: the *fully-isotropic* TPMs. In isotropic configurations the Jacobian matrix has the condition number, as well as the determinant, equal to one and the manipulator performs very well with regard to its force and motion transmission capabilities (Yoshikawa 1990). A manipulator is defined isotropic if it possesses at least one isotropic configuration. In this work, it is defined *fully-isotropic* if it is isotropic in its entire workspace. A fully-isotropic manipulator does not show direct singularities, as the determinant of its Jacobian matrix is always equal to one. This section presents for the first time families of fully-isotropic mechanisms that exhibit a constant orthogonal Jacobian matrix throughout their workspace. As a consequence, both the direct and the inverse position problems[3], besides the velocity and acceleration ones, are linear and the kinematic analysis proves straightforward. Most of the shown manipulators are utterly *singularity-free*, since they do not even exhibit constraint singularities.

[3] In the direct kinematics, the actuated joint variables and their derivatives are given and the position of the platform and its derivatives are calculated. In the inverse kinematics, the position of the platform and its derivatives are assumed assigned and the joint variables and their derivatives are computed.

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2. Topological Classification of TPMs

Let a PM be considered, comprising n_g legs, each possessing n_i independent DoFs.

As explained in the Introduction, the architecture of a TPM leg must be devised so as to make the platform able to freely translate in space without altering its orientation. In other words, a TPM leg must:

- deprive the platform of one or more DoFs of rotation (so that, by virtue of the contribution of all legs, platform turning is completely prevented);
- be consistent with a 3-DoF translational motion of the platform.

The classification of TPM legs can then be made on the basis of the number of DoFs of rotation eliminated from the platform, or, alternatively, the number of DoFs that remain: a leg will be called type T_{n_i} if it does not constrain the platform translation, has n_i DoFs and consequently removes $6-n_i$ DoFs of rotation from the platform.

Basically, any combination of type T_3, T_4 and T_5 legs that, as a whole, would prevent the platform rotating about three linearly independent axes would generate a TPM. However, a minimum number of constraints is imposed only if each one of them is not a repeated one. This can only be accomplished by using:

1. a single type T_3 leg, which prevents all platform rotations;
2. one type T_4 leg and one type T_5 leg, the former eliminating two DoFs of rotation, the latter one DoF;
3. three type T_5 legs, each one of which eliminates one rotational DoF.

However, for the manipulator to be able to take full advantage of its potentialities as a parallel mechanism (for instance, in terms of high stiffness and good dynamic performance), it should possess at least three legs, so that there would be at least two closed-loop chains and all three motors could be mounted on the base. The first two of the presented solutions clearly do not meet such a requirement. Actually, the former even represents a serial manipulator. For the TPM to have three legs, overconstrained architectures must also be taken into account, i.e., architectures in which two or more legs eliminate the same DoFs. An additional desirable feature, that would give symmetry to the design and could reduce the manufacturing cost, is making use of legs exhibiting the same topology.

According to the above specifications, the following families of TPMs are identified:

- T_3 TPMs, exhibiting three identical type T_3 legs;
- T_4 TPMs, exhibiting three identical type T_4 legs;
- T_5 TPMs, exhibiting three identical type T_5 legs.

Clearly, the mechanisms belonging to the first two families are overconstrained.

T_4 TPMs essentially coincide with those of the Hervé family and T_5 TPMs with those belonging to the families of Clavel/Tsai and of Frisoli/Carricato (see Section 1).

The above families will be studied in detail in the following sections and, in particular, new classes of TPMs exhibiting outstanding properties will be presented.

3. TPM Architectures and Constraint Singularities

In order to define the topological and geometric conditions that the legs must respect in order to provide the platform with the required mobility characteristics, what follows must be taken into consideration:

- since any DoF taken away by a leg cannot be restored by another, all legs must provide the platform with at least three DoFs of translation;
- no one of the kinematic pairs of a leg may allow a DoF the leg has to remove from the platform. For instance, let a leg that must prevent the platform rotation about the vector **n** be considered. If

such a rotation were allowed by one of the leg joints, this would be sufficient, maintaining all the other ones blocked, to make the platform rotate about **n** and hence violate the imposed constraint.

As a consequence of the last consideration, within any leg that must deprive the platform of one or more DoFs of rotation:

- S-pairs cannot exist;
- there cannot be H- or R- pairs having their axes parallel to the lines about which the rotation must be prevented;
- there are no particular constraints on the location and arrangement of P- and P_A- pairs, which are always consistent with any rotation constraint imposed on the output member.

Moreover, in order to avoid passive mobilities reducing the DoFs of the kinematic bond between the frame and the platform, some further geometrical conditions must be respected. For example, the following cannot exist: two parallel prismatic pairs, three prismatic pairs parallel to the same plane, two coaxial screw pairs with equal pitch, four screw pairs with equal pitch parallel to each other, etc.

In order to reduce the number of kinds of kinematic pairs to be taken into consideration, in the following discussion the C-pair will be considered as the ensemble of an R-pair and a P-pair mutually coaxial and the U-pair as the ensemble of two intersecting and nonparallel R-pairs. Moreover, since the functional difference between the R- and the H- pair is unessential for the aim of this section (both have one DoF and allow the relative rotation about a single axis), only the R-pair will be considered, it being understood that what is said for one applies to the other as well.

3.1. T_3 TPMs

Such manipulators possess three type T_3 legs, each one of which comprises three P- or P_A- pairs, whose axes of translation are linearly independent (since a type T_3 leg must prevent any turn of the platform, it cannot comprise any joint allowing relative rotations). Clearly, such an architecture is overconstrained, because any constraint is repeated three times.

As each leg contains only translational joints, the rotation of the platform is impossible, whatever the leg configurations are. Hence, T_3 TPMs do not exhibit constraint singularities.

3.2. T_4 TPMs

The manipulators belonging to such a family possess three identical type T_4 legs, each one of which allows the platform to rotate about a unique axis, identified by a unit vector $\mathbf{w}_i$ (i=1,2,3). Such an architecture is overconstrained, since each leg eliminates two DoFs, but, on the whole, only three DoFs of rotation are eliminated.

Since a TPM leg cannot comprise rotational joints having their axes parallel to the lines about which it prevents the platform turning, the axes of the R-pairs of a type T_4 leg must all be parallel to $\mathbf{w}_i$. As a consequence, the orientation of $\mathbf{w}_i$ remains constant throughout the motion. Moreover, the rotational joints can only be two or three (Carricato 2001).

Since the angular velocity $\boldsymbol{\omega}$ of the platform is zero, when there are two R-pairs, it must be

(1) $$\boldsymbol{\omega} = \left(\dot{\theta}_{1i} + \dot{\theta}_{2i}\right)\mathbf{w}_i = \mathbf{0} \qquad i = 1,2,3$$

and so

(2) $$\dot{\theta}_{1i} + \dot{\theta}_{2i} = 0 \qquad i = 1,2,3$$

where θ_{ji} is the angular variable relative to the j-th R-pair (j=1,2) of the i-th leg.

Analogously, when there are three rotational joints, it is

(3) $$\dot{\theta}_{1i}+\dot{\theta}_{2i}+\dot{\theta}_{3i}=0 \qquad i=1,2,3$$

If there are only two R-pairs, they provide the platform with a rotational and a translational DoF, the latter parallel to the plane Π_i perpendicular to $\mathbf{w}_i$. Hence, two P- or P_A- pairs are needed to provide the platform with the other two translational DoFs, one on and the other out of Π_i. The former may be realized by either a P-pair not having its axis parallel to $\mathbf{w}_i$ or an anyhow oriented P_A-pair (in fact, whatever the orientation of the P_A-pair axis is, at least one component of the translational movement that it provides would lie on Π_i). The latter may be realized by means of a P-pair not having its axis perpendicular to $\mathbf{w}_i$ or a P_A-pair not having its axis parallel to $\mathbf{w}_i$. Figure 1 shows a couple of examples.

If there are three R-pairs, a single translational joint is needed to permit the movement out of Π_i. If a P-pair is used, its axis must not lie on such a plane; if a P_A-pair is adopted, its axis must not be parallel to $\mathbf{w}_i$. An example is sketched in Figure 2.

Type T_4 legs were investigated for the first time by Hervé and Sparacino (1991).

Considering a whole T_4 TPM, since each leg allows the platform to rotate about $\mathbf{w}_i$ only and the orientation of this axis remains constant throughout the motion, if two legs are assembled so that $\mathbf{w}_h \neq \mathbf{w}_k$, then each one prevents the rotation allowed by the other, rendering any rotation constructively impossible, regardless of the leg configurations. It follows that T_4 TPMs do not exhibit constraint singularities.

3.3. T_5 TPMs

The manipulators belonging to this family exhibit three type T_5 legs, each one of which prevents the platform turning about a single axis identified by the unit vector $\mathbf{n}_i$ (i=1,2,3) and permits its rotation about any other line perpendicular to $\mathbf{n}_i$. Such mechanisms are not overconstrained.

All R-pairs of a type T_5 leg must have their axes perpendicular to $\mathbf{n}_i$ and their number must be equal to four or five (Carricato 2001). Type T_5' and type T_5" legs may be distinguished: the former were first investigated by Frisoli et al. (2000) and Carricato and Parenti-Castelli (2001b), the latter by Tsai (1996) and Di Gregorio and Parenti-Castelli (1998).

A type T_5' leg comprises (Figure 3):

- two contiguous rotational pairs whose axes are parallel to a unit vector $\mathbf{w}_{1i}$;
- two contiguous rotational pairs whose axes are parallel to a unit vector $\mathbf{w}_{2i}$ nonparallel to $\mathbf{w}_{1i}$;
- a translational pair, which can be placed anywhere along the kinematic chain or can be replaced by a fifth rotational joint that can be located adjacent and parallel to any one of the others (Figure 4).

A type T_5" leg comprises (Figure 5):

- two rotational pairs whose axes are parallel to a unit vector $\mathbf{w}_{1i}$;
- two contiguous rotational pairs, interposed between the previous ones, whose axes are parallel to a unit vector $\mathbf{w}_{2i}$ nonparallel to $\mathbf{w}_{1i}$;
- a translational pair, which can be placed anywhere along the kinematic chain or can be replaced by a fifth rotational joint that can be located adjacent and parallel to any one of the others (Figure 6).

For both kinds of leg, $\mathbf{n}_i$ is the vector defined as $\mathbf{w}_{1i}\times\mathbf{w}_{2i}$, γ_{12i} is the angle, different from zero, comprised between $\mathbf{w}_{1i}$ and $\mathbf{w}_{2i}$, l_{ji} is the distance between the j-th and the (j+1)-th rotational pair (j=1,2,3). If the legs are assembled so that $\mathbf{n}_1$, $\mathbf{n}_2$ and $\mathbf{n}_3$ are linearly independent, the platform rotation is prevented about any line and its instantaneous motion can only be a translation.

Depending on the employment of type T_5' or T_5" legs, T_5' and T_5" TPMs may be respectively distinguished.

3.3.1. T_5' TPMs

Let a mechanism whose legs comprise four rotational pairs be considered and let θ_{ji} be the angular variable relative to the j-th R-pair of the i-th leg (j=1,2,3,4; i=1,2,3) (Figure 3). As the angular velocity **ω** of the platform has to be zero, it is

(4) $$\left(\dot{\theta}_{1i}+\dot{\theta}_{2i}\right)\mathbf{w}_{1i}+\left(\dot{\theta}_{3i}+\dot{\theta}_{4i}\right)\mathbf{w}_{2i}=\mathbf{0} \qquad i=1,2,3$$

which infers

(5) $$\dot{\theta}_{1i}+\dot{\theta}_{2i}=0 \qquad i=1,2,3$$

and

(6) $$\dot{\theta}_{3i}+\dot{\theta}_{4i}=0 \qquad i=1,2,3$$

If the rotational pairs are five and θ_{5i} is the angular variable relative to the added joint, depending on whether this is located parallel to the first or the last two R-pairs, Eqs. (5) and (6) become alternatively

(7) $$\dot{\theta}_{1i}+\dot{\theta}_{2i}+\dot{\theta}_{5i}=0 \qquad i=1,2,3$$

and

(8) $$\dot{\theta}_{3i}+\dot{\theta}_{4i}+\dot{\theta}_{5i}=0 \qquad i=1,2,3$$

It follows from Eqs. (5) (or (7)) and (6) (or (8)) that no joint axis changes its orientation because of the movement and therefore the orientation of $\mathbf{n}_i$ remains constant. This means that, if $\mathbf{n}_1$, $\mathbf{n}_2$ and $\mathbf{n}_3$ are linearly independent at the outset, they continue to be so during the entire motion and the platform translates throughout the workspace. In other words, T_5' TPMs do not exhibit constraint singularities (Carricato and Parenti-Castelli 2001b).

3.3.2. T_5'' TPMs

In this case, if the mechanism legs comprise four rotational pairs (Figure 5), it must be

(9) $$\left(\dot{\theta}_{1i}+\dot{\theta}_{4i}\right)\mathbf{w}_{1i}+\left(\dot{\theta}_{2i}+\dot{\theta}_{3i}\right)\mathbf{w}_{2i}=\mathbf{0} \qquad i=1,2,3$$

that is

(10) $$\dot{\theta}_{1i}+\dot{\theta}_{4i}=0 \qquad i=1,2,3$$

and

(11) $$\dot{\theta}_{2i}+\dot{\theta}_{3i}=0 \qquad i=1,2,3$$

If the rotational pairs are five, depending on whether the added joint is located parallel to the first and the fourth R-pair or to the second and the third one, Eqs. (10) and (11) become alternatively

(12) $$\dot{\theta}_{1i}+\dot{\theta}_{4i}+\dot{\theta}_{5i}=0 \qquad i=1,2,3$$

and

(13) $$\dot{\theta}_{2i}+\dot{\theta}_{3i}+\dot{\theta}_{5i}=0 \qquad i=1,2,3$$

It follows from Eqs. (10) (or (12)) and (11) (or (13)) that the orientation of $\mathbf{w}_{2i}$, and therefore of $\mathbf{n}_i$, depends on θ_{1i} (and also θ_{5i}, if there is a fifth R-pair) and so it does not remain constant during the motion. This means that there can be configurations in which $\mathbf{n}_1$, $\mathbf{n}_2$ and $\mathbf{n}_3$ are linearly dependent, i.e.

(14) $$\mathbf{n}_1\cdot\left(\mathbf{n}_2\times\mathbf{n}_3\right)=0$$

Anytime Eq. (14) is satisfied, the rotation of the platform is no longer prevented and its motion

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ceases being purely translational. Such configurations are constraint singularities. This result, valid for all the mechanisms belonging to the examined family, was obtained for the first time by Di Gregorio and Parenti-Castelli (1999) for the 3-UPU manipulator (Tsai 1996): the constraint singularity locus was shown in Parenti-Castelli, Di Gregorio and Bubani (2000) and a geometric interpretation was given in Parenti-Castelli and Bubani (1999).

4. Singularity-Free Fully-Isotropic Manipulators

Let $\mathbf{x}$ be the position vector, in a given fixed reference frame, of a generic point P embedded in the platform. $\mathbf{x}$ is a function of the three displacement variables q_i ($i=1,2,3$) of the actuated joints, assumed to be distributed one per leg.

Let, for each leg, a constant direction $\mathbf{u}_i$ exist so that the displacement of P along it, with respect to a reference configuration $\mathbf{x}_0$, is determined by no other variable than q_i

(15) $$\mathbf{u}_i^{\mathrm{T}}\left(\mathbf{x}-\mathbf{x}_0\right)=f_i\left(q_i\right) \qquad i=1,2,3$$

Equation (15) can be expressed in the following form

(16) $$\mathbf{J}\left(\mathbf{x}-\mathbf{x}_0\right)=\mathbf{f}\left(\mathbf{q}\right)$$

where

(17) $$\mathbf{q}=\left[q_1 \quad q_2 \quad q_3\right]^{\mathrm{T}}$$

(18) $$\mathbf{f}\left(\mathbf{q}\right)=\left[f_1\left(q_1\right) \quad f_2\left(q_2\right) \quad f_3\left(q_3\right)\right]^{\mathrm{T}}$$

(19) $$\mathbf{J}=\begin{bmatrix}\mathbf{u}_1^{\mathrm{T}}\\ \mathbf{u}_2^{\mathrm{T}}\\ \mathbf{u}_3^{\mathrm{T}}\end{bmatrix}$$

$\mathbf{J}$ being constant ($\mathbf{u}_i$ does not vary by hypothesis), differentiating Eq. (16) with respect to time yields

(20) $$\mathbf{J}\dot{\mathbf{x}}=\frac{d\mathbf{f}}{d\mathbf{q}}\dot{\mathbf{q}}$$

where

(21) $$\frac{d\mathbf{f}}{d\mathbf{q}}=\begin{bmatrix}df_1/dq_1 & 0 & 0\\ 0 & df_2/dq_2 & 0\\ 0 & 0 & df_3/dq_3\end{bmatrix}$$

Equation (20) shows that $\mathbf{J}$ is the Jacobian matrix of the direct kinematics. The systems (16) and (20) are invertible only if $\mathbf{J}$ is not singular, that is if $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ are linearly independent. This has a clear physical meaning. Because of Eq. (15), each motorized joint basically represents the direct actuation of one of the translational DoFs of the platform, which is provided with a 3-DoF motion only if the actuated axes of translation are linearly independent. In short, if a TPM leg architecture can be devised so that Eq. (15) holds and the three legs are mounted so that their corresponding axes $\mathbf{u}_i$ are linearly independent, the resulting manipulator has a constant nonsingular Jacobian matrix and does not exhibit direct singularities.

Moreover, if the axes $\mathbf{u}_i$ are chosen mutually perpendicular, $\mathbf{J}$ becomes orthogonal and so

(22) $$\mathbf{J}^{\mathrm{T}}\mathbf{J} = \mathbf{I}$$

where **I** is the identity matrix.

Since Eq. (22) is necessary and sufficient for isotropy (Yoshikawa 1990), it follows that the examined manipulators are fully-isotropic in their whole workspace.

By virtue of Eq. (22) the inversion of the systems (16) and (20) becomes very simple

(23) $$\mathbf{x} = \mathbf{J}^{\mathrm{T}}\mathbf{f}\left(\mathbf{q}\right) + \mathbf{x}_0$$

(24) $$\dot{\mathbf{x}} = \mathbf{J}^{\mathrm{T}} \frac{d\mathbf{f}}{d\mathbf{q}} \dot{\mathbf{q}}$$

Both the direct and the inverse kinematic analyses are hence straightforward.

In addition, since $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ are mutually perpendicular, the coordinate axes of the fixed reference frame may be assumed parallel to them without loss of generality. In this case, it is

(25) $$\mathbf{J} = \mathbf{I}$$

In the following sections some families of singularity-free fully-isotropic TPMs will be presented. Such manipulators exhibit neither constraint nor direct singularities and have a constant orthogonal Jacobian matrix throughout their workspace.

4.1. Fully-isotropic T_3 TPMs

The manipulators belonging to the T_3 TPM family possess three identical type T_3 legs and do not exhibit constraint singularities (Section 3.1).

A T_3 TPM is fully-isotropic if each leg comprises three orthogonal P-pairs and the actuated axes are mutually perpendicular. An example is sketched in Figure 7. In this case, if $\mathbf{v}_{ji}$ is a unit vector along the axis of the j-th P-pair of the i-th limb (j=1,2,3; i=1,2,3) and v_{ji} is the displacement along it with respect to the reference configuration, the following relation holds for each leg

(26) $$\mathbf{x} - \mathbf{x}_0 = v_{1i}\mathbf{v}_{1i} + v_{2i}\mathbf{v}_{2i} + v_{3i}\mathbf{v}_{3i} \qquad i = 1,2,3$$

Assuming that the actuated joints are mounted on the frame, it is

(27) $$v_{1i} = q_i \qquad i = 1,2,3$$

and therefore from Eq. (26)

(28) $$\mathbf{v}_{1i}^{\mathrm{T}}\left(\mathbf{x} - \mathbf{x}_0\right) = q_i \qquad i = 1,2,3$$

In matrix form, Eq. (28) becomes

(29) $$\mathbf{J}\left(\mathbf{x} - \mathbf{x}_0\right) = \mathbf{q}$$

where

(30) $$\mathbf{J} = \begin{bmatrix} \mathbf{v}_{11}^{\mathrm{T}} \\ \mathbf{v}_{12}^{\mathrm{T}} \\ \mathbf{v}_{13}^{\mathrm{T}} \end{bmatrix}$$

is the Jacobian matrix, constant and orthogonal.

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4.2. Fully-isotropic T_4 TPMs

The manipulators of this family possess three identical type T_4 legs and do not exhibit constraint singularities (Section 3.2).

As the following will show, a fully-isotropic T_4 TPM may be designed by conceiving leg architectures that allow the "direct" actuation of either the translational DoF out of the plane Π_i perpendicular to $\mathbf{w}_i$ (type I) or one of the translational DoFs lying on such a plane (type II).

4.2.1. Type I fully-isotropic T_4 TPMs

The generic i-th leg (i=1,2,3) of a T_4 TPM must comprise, other than two or three rotational pairs whose axes are all parallel to the unit vector $\mathbf{w}_i$, at least one translational joint, here called V_i, permitting the movement out of Π_i. This can be a P-pair whose axis does not lie on Π_i, an H-pair whose axis is parallel to $\mathbf{w}_i$ or a P_A-pair whose axis is not parallel to $\mathbf{w}_i$.

Let $\mathbf{v}_i$ be a unit vector along the axis of V_i and α_i the angle that it forms with respect to $\mathbf{w}_i$. Such an angle remains constant throughout the motion, since the other pairs produce either rotations about $\mathbf{w}_i$ or translations. Let v_i also be the displacement variable relative to V_i. If V_i is a P- or an H- pair, v_i is the displacement along the joint axis with respect to the reference configuration. If V_i is a P_A-pair, v_i is the rotation of the parallelogram cranks about $\mathbf{v}_i$ with respect to the line $\mathbf{w}_i\times\mathbf{v}_i$ (the motion of one crank in the plane perpendicular to $\mathbf{v}_i$ and the projection on $\mathbf{w}_i$ of its end displacement are depicted in Figure 8, where l_i is the crank length).

Let V_i be the only joint responsible for the platform movement out of Π_i. This implies that, if another translational joint is present in the leg, it only allows movements parallel to Π_i (and that there are no H-pairs other than the one that, if present, provides the translation along $\mathbf{w}_i$). Therefore, if V_i is a P- or an H- pair, the following relation must hold

(31) $$\mathbf{w}_i^{\mathrm{T}}\left(\mathbf{x}-\mathbf{x}_0\right)=v_i\cos\alpha_i \qquad i=1,2,3$$

or, if V_i is a P_A-pair,

(32) $$\mathbf{w}_i^{\mathrm{T}}\left(\mathbf{x}-\mathbf{x}_0\right)=l_i\sin v_i\sin\alpha_i \qquad i=1,2,3$$

If V_i is assumed actuated, then

(33) $$v_i=q_i \qquad i=1,2,3$$

and Eqs. (31) and (32) can be written in the form

(34) $$\mathbf{w}_i^{\mathrm{T}}\left(\mathbf{x}-\mathbf{x}_0\right)=f_i\left(q_i\right) \qquad i=1,2,3$$

where

(35) $$f_i\left(q_i\right)=\begin{cases}\left(\cos\alpha_i\right)q_i, & \text{if } V_i \text{ is a P- or an H-pair}\\ \left(l_i\sin\alpha_i\right)\sin q_i, & \text{if } V_i \text{ is a PA-pair}\end{cases}$$

In matrix form, Eq. (34) becomes

(36) $$\mathbf{J}\left(\mathbf{x}-\mathbf{x}_0\right)=\mathbf{f}\left(\mathbf{q}\right)$$

where

(37) $$\mathbf{J} = \begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}$$

is the constant Jacobian matrix. If $\mathbf{w}_1$, $\mathbf{w}_2$ and $\mathbf{w}_3$ are chosen linearly independent **J** is always nonsingular; if they are chosen mutually perpendicular **J** is also orthogonal. Figure 9 shows some leg architectures that make the manipulator fully-isotropic, provided that the legs are assembled so that $\mathbf{w}_1$, $\mathbf{w}_2$ and $\mathbf{w}_3$ are mutually perpendicular. Figure 10 shows the entire manipulator in the case in which the linear actuation is provided by means of H-pairs mounted on the frame with α_i equal to zero.

4.2.2. Type II fully-isotropic T_4 TPMs

A type II fully-isotropic T_4 TPM may be designed by conceiving a leg architecture that allows the "direct" actuation of one of the two DoFs that permit translation on the plane Π_i. As shown in Section 3.2, such translation may be realized by means of either

- two rotational pairs, or
- one rotational pair and a P-pair not having its axis parallel to $\mathbf{w}_i$ (or a however oriented P_A-pair).

Because each rotational pair (be it R- or H-) necessarily affects both translational DoFs on Π_i, the former realization is not applicable: both DoFs would always depend on the displacement variables of both pairs in a coupled way.

Therefore, let the latter solution be considered. On the whole, the leg comprises two rotational pairs (whose relative rotation variables θ_{1i} and θ_{2i} satisfy Eq. (2)) and two translational ones. Let $\mathbf{v}_i$ be a constant unit vector perpendicular to $\mathbf{w}_i$, Γ_i the vector plane containing $\mathbf{w}_i$ and $\mathbf{v}_i$, and $\mathbf{u}_i = \mathbf{w}_i \times \mathbf{v}_i$. Also, let the translational joints be arranged so as to span Γ_i. For instance, they can be:

- two nonparallel P-pairs whose axes are parallel to Γ_i (Figure 11a);
- one P-pair whose axis is parallel to Γ_i and one P_A-pair whose axis is perpendicular to it (Figure 11b);
- two P_A-pairs whose axes are perpendicular to Γ_i (Figure 11c).

Since $\mathbf{v}_i$ and $\mathbf{u}_i$ must be constant vectors, the translational joints must be attached to either the frame or the platform. Only a P-pair whose axis is parallel to $\mathbf{w}_i$ can be interposed between the rotational pairs, for it would not change its orientation.

Because of such an arrangement, any movement along $\mathbf{u}_i$ can only depend on the rotational pairs. Therefore, referring to Figure 12, it is

(38) $$\mathbf{u}_i^{\mathrm{T}} \left(\mathbf{x} - \mathbf{x}_0 \right) = l_i \sin\theta_{1i} \qquad i = 1,2,3$$

where l_i is the distance between the axes of the rotational pairs (clearly, with a simple sign change, θ_{2i} could be used in the place of θ_{1i}).

If θ_{1i} (or θ_{2i}) is assumed actuated, then

(39) $$\theta_{1i} = q_i \qquad i = 1,2,3$$

and Eq. (38) becomes

(40) $$\mathbf{u}_i^{\mathrm{T}} \left(\mathbf{x} - \mathbf{x}_0 \right) = f_i\left(q_i\right) \qquad i = 1,2,3$$

where

(41) $$f_i\left(q_i\right) = l_i \sin q_i$$

In matrix form, Eq. (40) can be written as

(42) $$\mathbf{J}(\mathbf{x}-\mathbf{x}_0)=\mathbf{f}(\mathbf{q})$$

where

(43) $$\mathbf{J}=\begin{bmatrix}\mathbf{u}_1^{\mathrm{T}}\\ \mathbf{u}_2^{\mathrm{T}}\\ \mathbf{u}_3^{\mathrm{T}}\end{bmatrix}$$

is the constant Jacobian matrix. As in the previous sections, if $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ are chosen linearly independent **J** is always nonsingular. If they are chosen mutually perpendicular **J** is also orthogonal, like in the example shown in Figure 13.

4.3. Fully-isotropic T_5 TPMs

T_5 TPMs possess three identical type T_5' or T_5" legs.

4.3.1. Fully-isotropic T_5' TPMs

The generic i-th leg (i=1,2,3) of a T_5' TPM is sketched in Figure 3; as explained in Section 3.3, the P-pair can be substituted by fifth rotational joint. It is worth noting that, unlike l_{1i} and l_{3i}, l_{2i} can be equal to zero. Manipulators of this kind do not exhibit constraint singularities and the orientations of $\mathbf{w}_{1i}$ and $\mathbf{w}_{2i}$ remain constant throughout the motion (Section 3.3.1).

A fully-isotropic T_5' TPM may be designed by conceiving leg architectures that allow the "direct" actuation of the movement along either $\mathbf{w}_{2i}$ (type I) or $\mathbf{w}_{1i}$ (type II).

Let the former case be considered. If the translational joint is chosen so that it does not produce motion along $\mathbf{w}_{2i}$, any displacement along this axis can only depend on the rotational pairs whose axes are parallel to $\mathbf{w}_{1i}$. As illustrated in Figure 14 (in which θ_{1i} is negative), such a displacement is equal to

(44) $$\mathbf{w}_{2i}^{\mathrm{T}}(\mathbf{x}-\mathbf{x}_0)=-l_{1i}\sin\gamma_{12i}\sin\theta_{1i} \qquad i=1,2,3$$

Assuming l_{1i} constant and θ_{1i} actuated (q_i=θ_{1i}), and defining the function

(45) $$f_i(q_i)=-l_{1i}\sin\gamma_{12i}\sin q_i \qquad i=1,2,3$$

Equation (44) yields

(46) $$\mathbf{J}(\mathbf{x}-\mathbf{x}_0)=\mathbf{f}(\mathbf{q})$$

where

(47) $$\mathbf{J}=\begin{bmatrix}\mathbf{w}_{21}^{\mathrm{T}}\\ \mathbf{w}_{22}^{\mathrm{T}}\\ \mathbf{w}_{23}^{\mathrm{T}}\end{bmatrix}$$

Since the orientation of $\mathbf{w}_{2i}$ does not change during the motion, **J** is a constant matrix. Figure 15 shows three examples of leg architectures consistent with Eq. (44), whilst Figure 16 provides the sketch of a type I fully-isotropic T_5' TPM, in which the legs have been arranged so that $\mathbf{w}_{21}$, $\mathbf{w}_{22}$ and $\mathbf{w}_{23}$ are mutually perpendicular.

In type II fully-isotropic T_5' TPMs, the translational joint is chosen so as not to produce motion along $\mathbf{w}_{1i}$, so that any displacement along this axis only depends on the rotational pairs whose axes are parallel to $\mathbf{w}_{2i}$. As illustrated in Figure 17, such a displacement is equal to

(48) $$\mathbf{w}_{1i}^{\mathrm{T}}\left(\mathbf{x}-\mathbf{x}_0\right)=l_{3i}\sin\gamma_{12i}\sin\theta_{3i} \qquad i=1,2,3$$

Assuming l_{3i} constant and θ_{3i} actuated ($q_i=\theta_{3i}$), and defining the function

(49) $$f_i\left(q_i\right)=l_{3i}\sin\gamma_{12i}\sin q_i \qquad i=1,2,3$$

Equation (48) yields

(50) $$\mathbf{J}\left(\mathbf{x}-\mathbf{x}_0\right)=\mathbf{f}\left(\mathbf{q}\right)$$

where

(51) $$\mathbf{J}=\begin{bmatrix}\mathbf{w}_{11}^{\mathrm{T}}\\ \mathbf{w}_{12}^{\mathrm{T}}\\ \mathbf{w}_{13}^{\mathrm{T}}\end{bmatrix}$$

As in the former case, **J** is a constant matrix and can be easily made orthogonal. The disadvantage of such a solution is that the motors cannot be mounted on the frame.

4.3.2. Fully-isotropic T_5" TPMs

The generic *i*-th leg (i=1,2,3) of a T_5" TPM is sketched in Figure 5; as explained in Section 3.3, the P-pair can be substituted by a fifth rotational joint. It is worth noting that, unlike l_{2i}, l_{1i} and l_{3i} can be equal to zero. As proved in Section 3.3.2, $\mathbf{w}_{2i}$ and $\mathbf{n}_i$ change orientation during the motion and constraint singularities exist.

The projection of the platform displacement on any line perpendicular to $\mathbf{w}_{1i}$ always depends, in general, on both θ_{1i} and θ_{2i}. Therefore, let the projection on $\mathbf{w}_{1i}$ be considered. Since any rotation about the joints having their axes parallel to $\mathbf{w}_{1i}$ cannot provide a contribution, such a projection is equal to (Figure 18)

(52) $$\mathbf{w}_{1i}^{\mathrm{T}}\left(\mathbf{x}-\mathbf{x}_0\right)=l_{2i}\sin\gamma_{12i}\sin\theta_{2i} \qquad i=1,2,3$$

where it has been assumed that the translational joint is chosen so as not to produce motion along $\mathbf{w}_{1i}$ (as in the example provided in Figure 5). Assuming l_{2i} constant, θ_{2i} actuated ($q_i=\theta_{2i}$) and defining the function

(53) $$f_i\left(q_i\right)=l_{2i}\sin\gamma_{12i}\sin q_i \qquad i=1,2,3$$

Equation (52) yields

(54) $$\mathbf{J}\left(\mathbf{x}-\mathbf{x}_0\right)=\mathbf{f}\left(\mathbf{q}\right)$$

where

(55) $$\mathbf{J}=\begin{bmatrix}\mathbf{w}_{11}^{\mathrm{T}}\\ \mathbf{w}_{12}^{\mathrm{T}}\\ \mathbf{w}_{13}^{\mathrm{T}}\end{bmatrix}$$

Even though **J** is constant and may be easily made orthogonal, the manipulators belonging to this class are not singularity-free, because of the existence of constraint-singularities. Moreover, the motors cannot be mounted on the frame. Such disadvantages make the presented architecture less significant than the ones proposed in the previous sections.

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5. Conclusions

This paper focused on translational parallel mechanisms (TPMs), that is on parallel manipulators whose platform is provided with a pure translational motion with respect to the frame.

The problem of the topological synthesis and classification of TPMs was dealt with in a systematic and exhaustive form, providing previously published material with a general and organic frame. It was seen how a TPM leg must be designed in order to allow the platform to be able to freely translate in space without altering its orientation: more precisely, which topological and geometric conditions such a leg must satisfy in order to, on the one hand, deprive the platform of one or more DoFs of rotation (so that, by virtue of the contribution of all legs, platform turning is completely prevented) and, on the other, be consistent with a 3-DoF translational motion of the platform. How such legs must be assembled in order to generate a TPM was illustrated.

Then the problem of singularities and the identification of classes of singularity-free TPMs were addressed. Constraint singularities, which are configurations in which the platform looses its capability of purely translating and acquires an instantaneous mobility of rotation, were investigated for all families of TPMs and it was shown that only some of them are constraint-singularity-free. Finally, direct singularities were addressed and whole families of singularity-free fully-isotropic mechanisms were presented for the first time. These manipulators show outstanding properties. In particular:

- they exhibit a constant orthogonal Jacobian matrix throughout their workspace;
- they do not present either constraint or direct singularities;
- both the direct and the inverse kinematic analyses are straightforward.

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Figures

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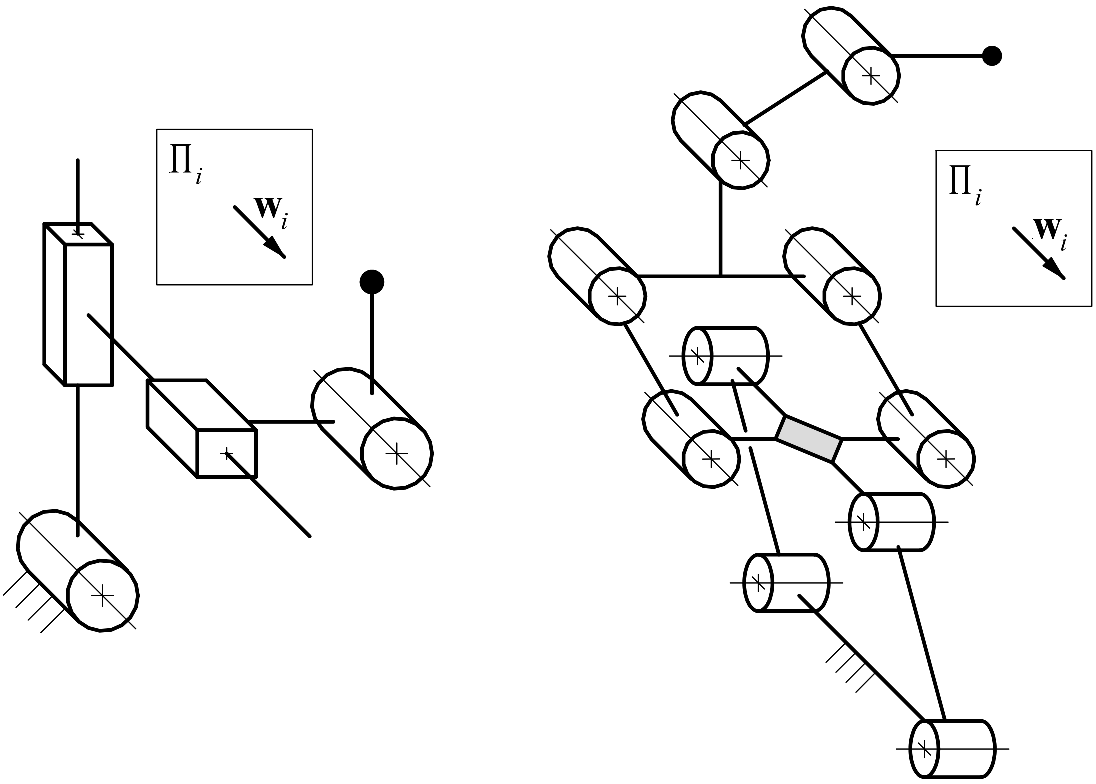

Fig. 1: Type T_4 legs with two R-pairs.

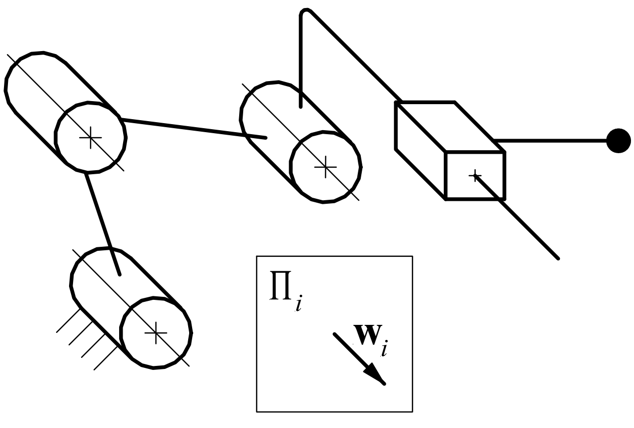

Fig. 2: A type T_4 leg with three R-pairs.

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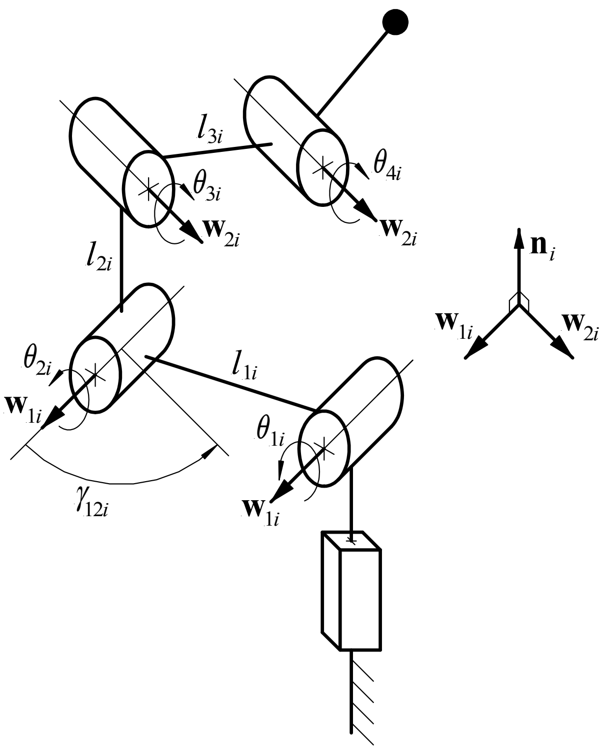

Fig. 3: A type T_5' leg with four R-pairs.

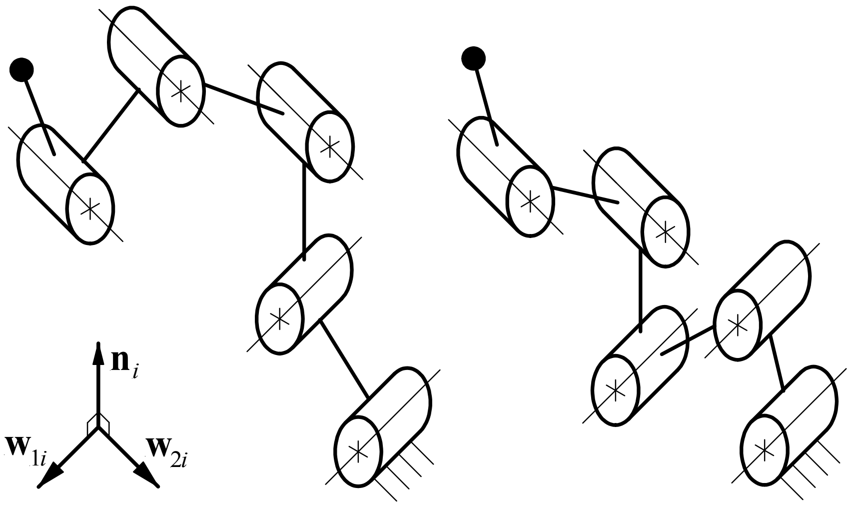

Fig. 4: Type T_5' legs with five R-pairs.

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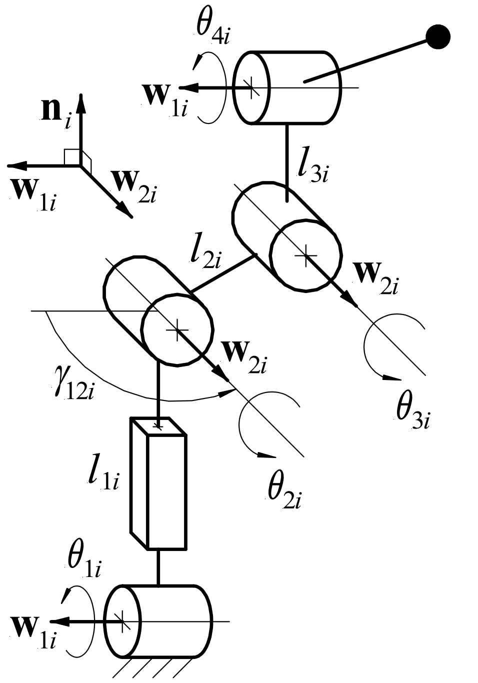

Fig. 5: A type T_5” leg with four R-pairs.

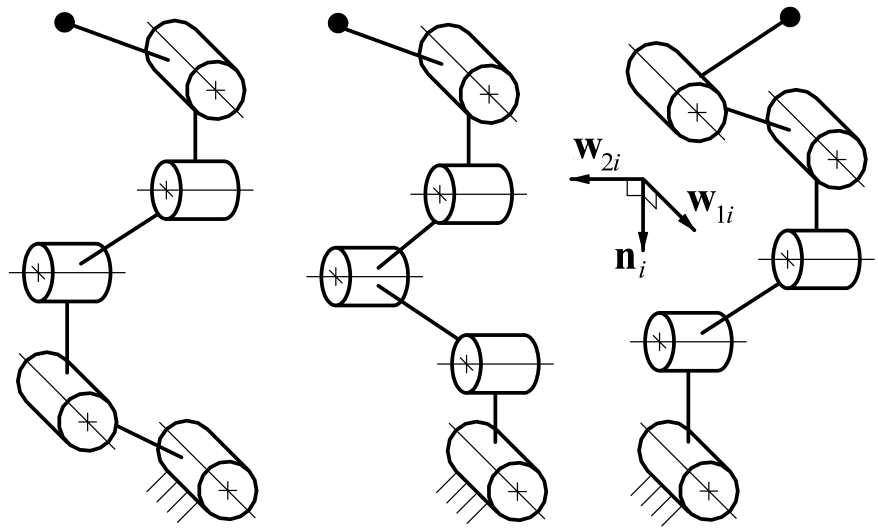

Fig. 6: Type T_5” legs with five R-pairs.

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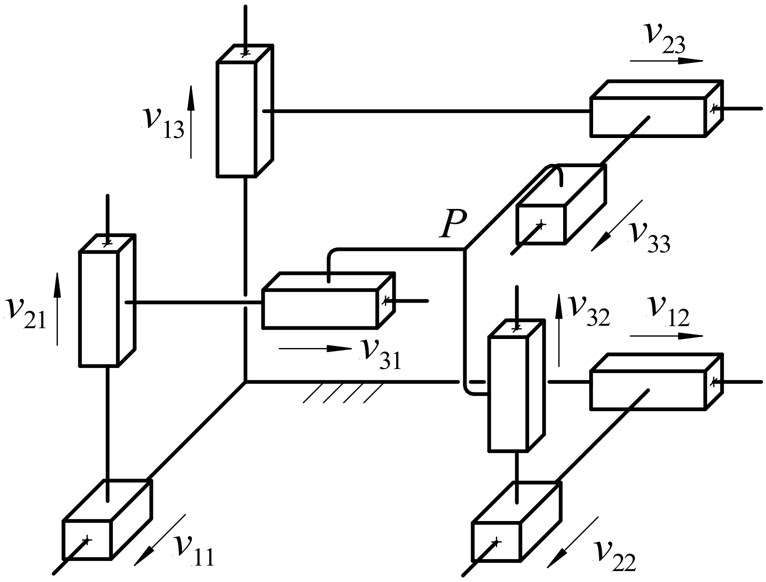

Fig. 7: A fully-isotropic T_3 TPM.

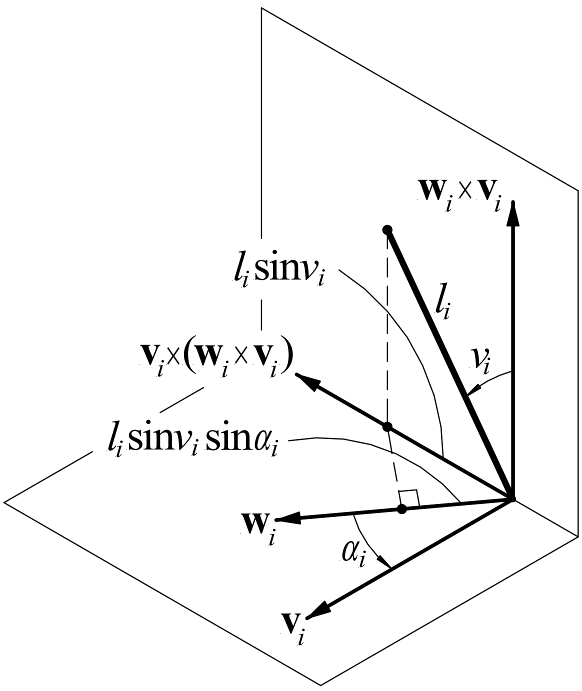

Fig. 8: Motion of one of the cranks of a parallelogram that permits, in a type T_4 leg, the platform's movement out of the plane perpendicular to $\mathbf{w}_i$.

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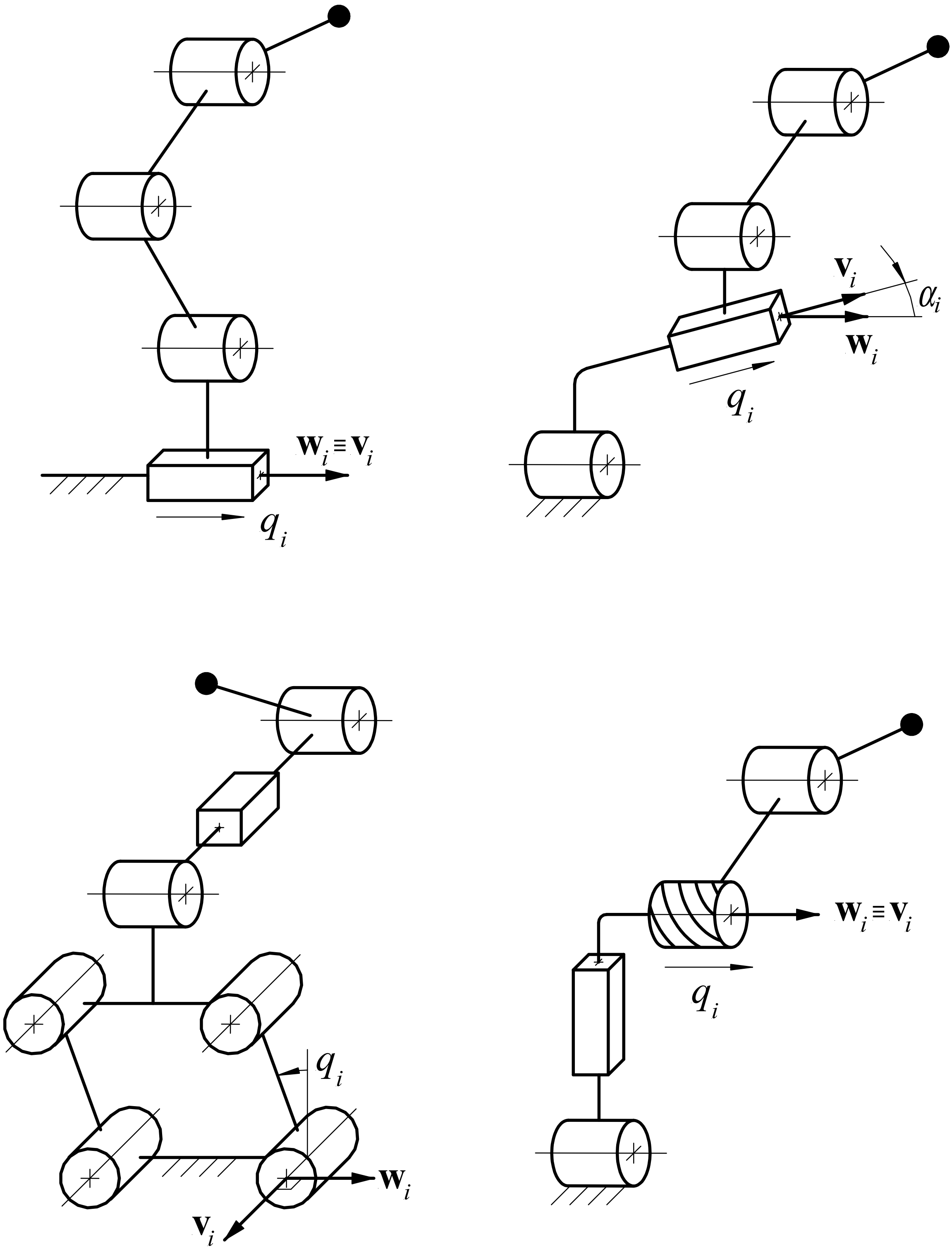

Fig. 9: Examples of architectures of type I fully-isotropic T_4 TPM legs.

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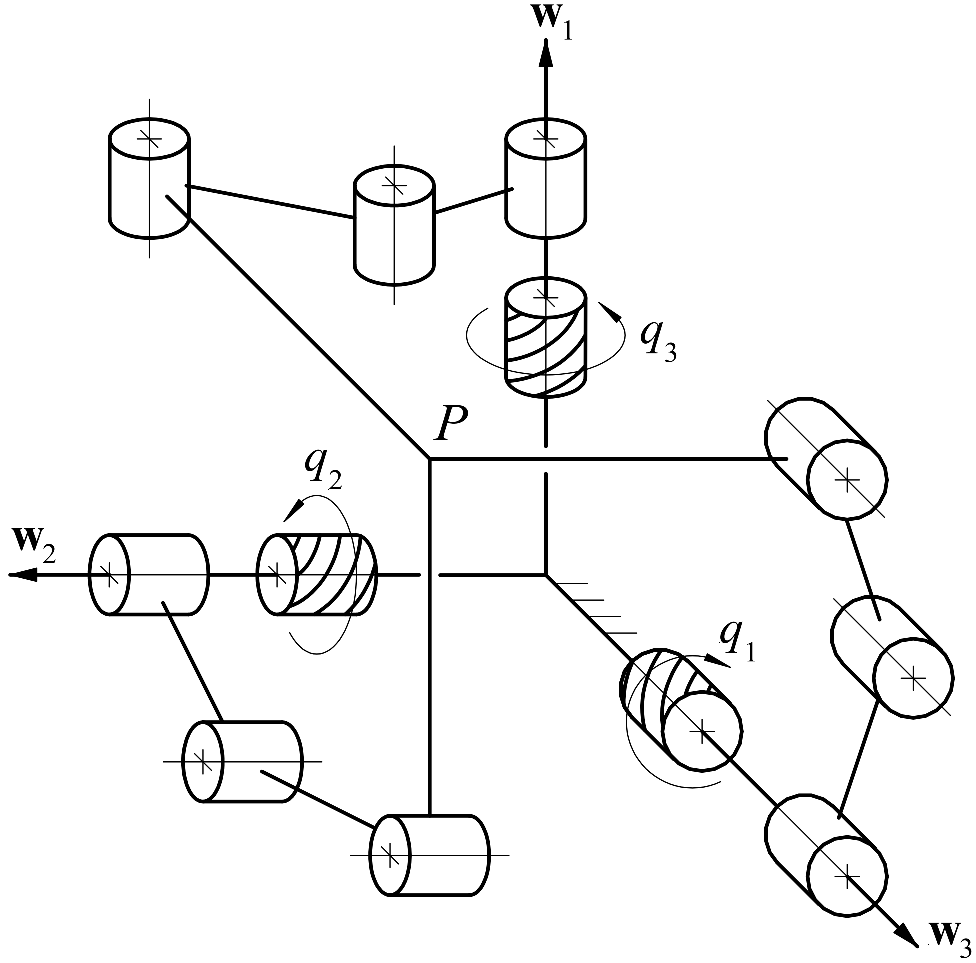

Fig. 10: A type I fully-isotropic T_4 TPM.

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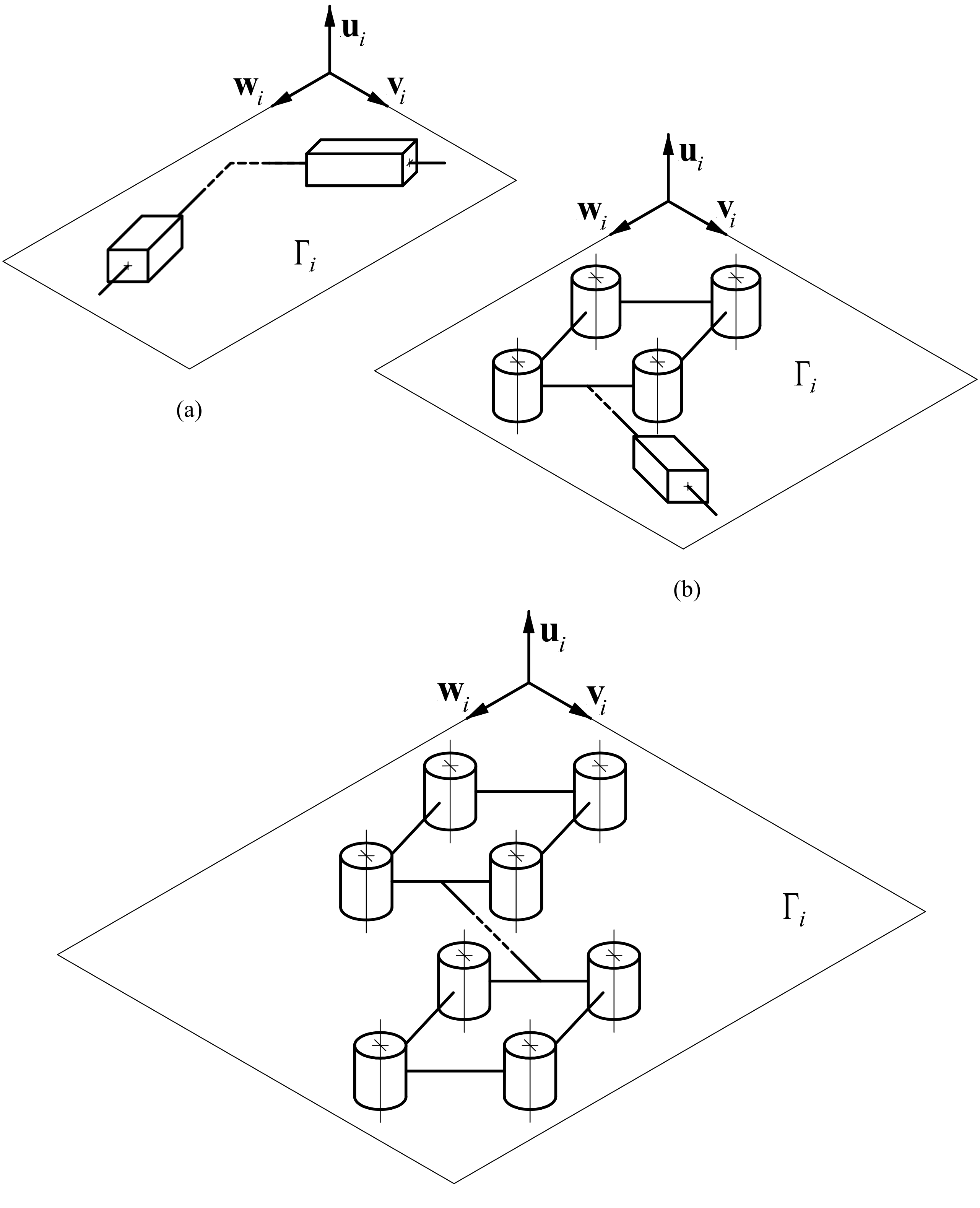

Fig. 11: Arrangement of the translational joints in a type II fully-isotropic T_4 TPM leg.

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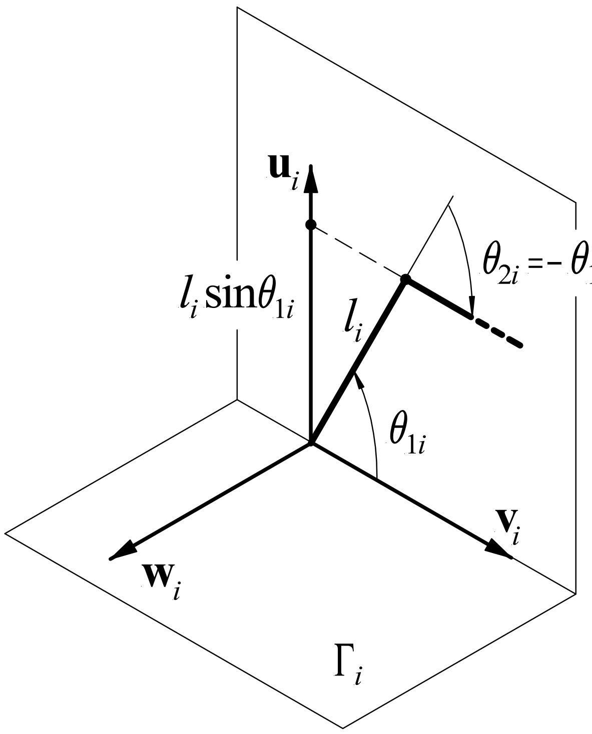

Fig. 12: Platform displacement along the constant direction $\mathbf{u}_i$ in a type II fully-isotropic T_4 TPM.

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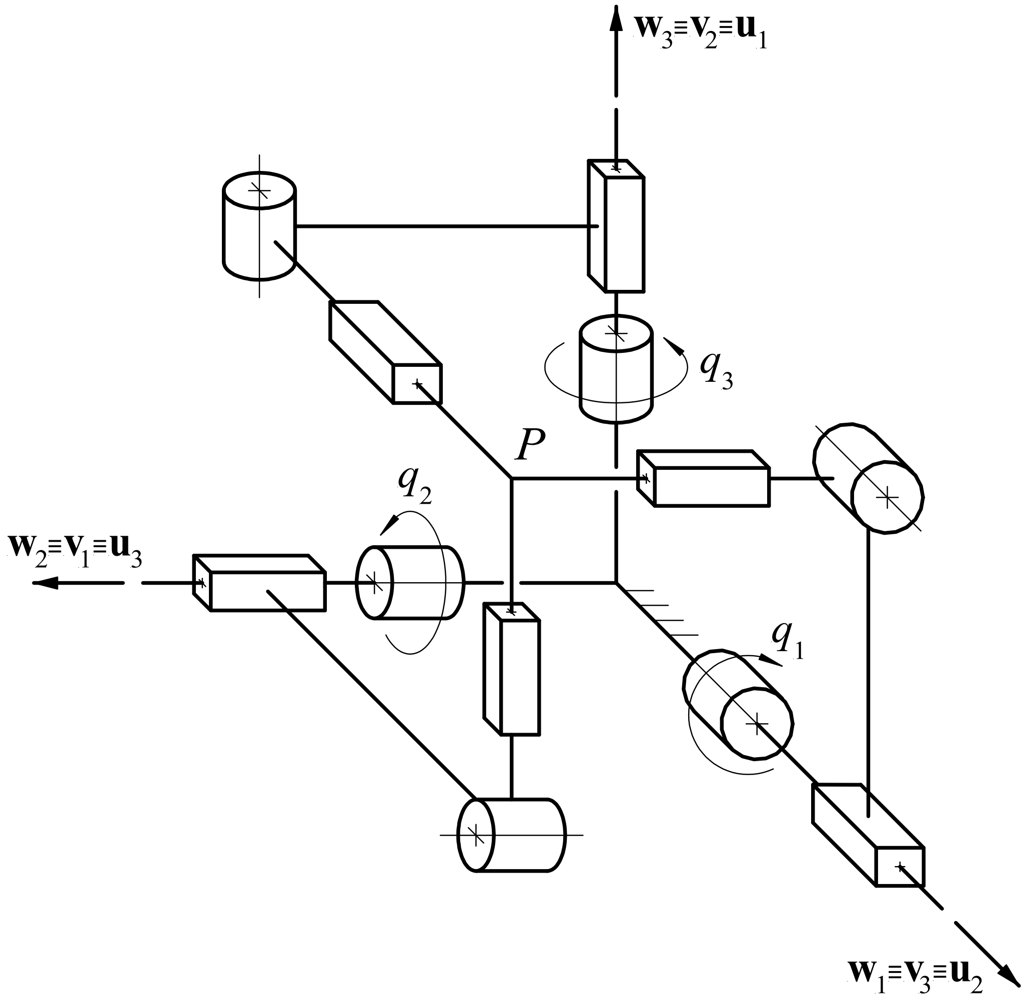

Fig. 13: A type II fully-isotropic T_4 TPM.

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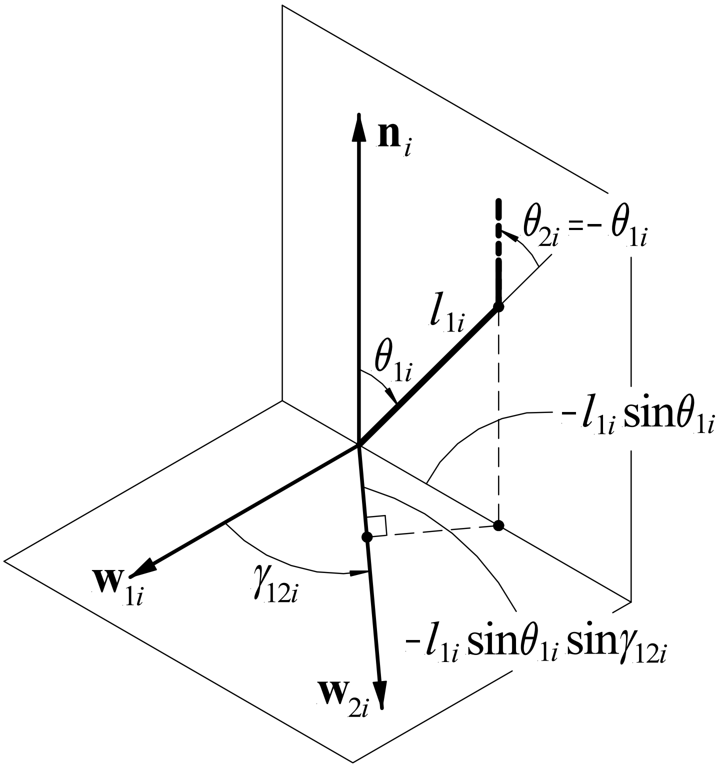

Fig. 14: Platform displacement along the constant direction $\mathbf{w}_{2i}$ in a type I fully-isotropic T_5' TPM.

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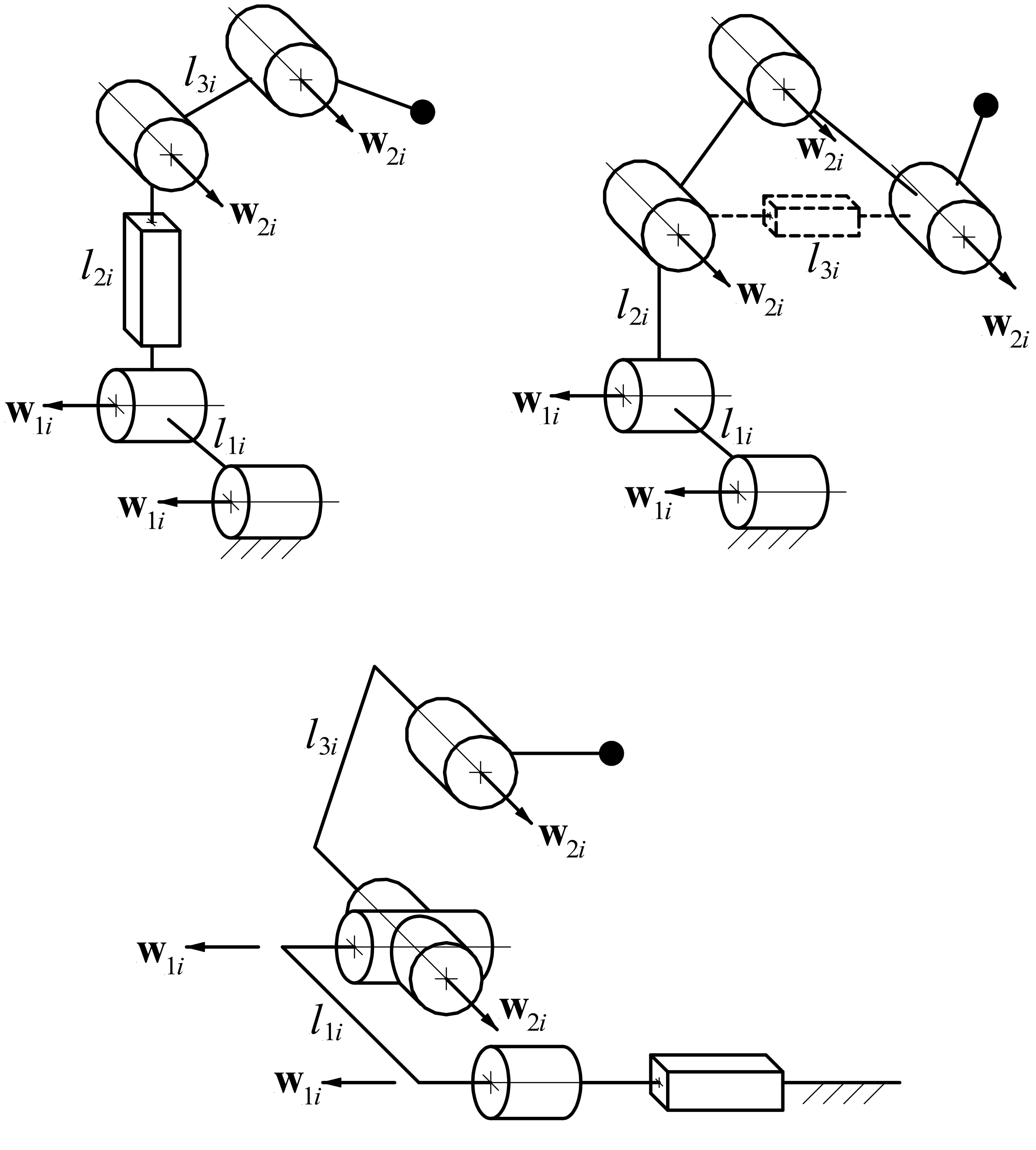

Fig. 15: Examples of leg architectures of type I fully-isotropic T_5' TPMs.

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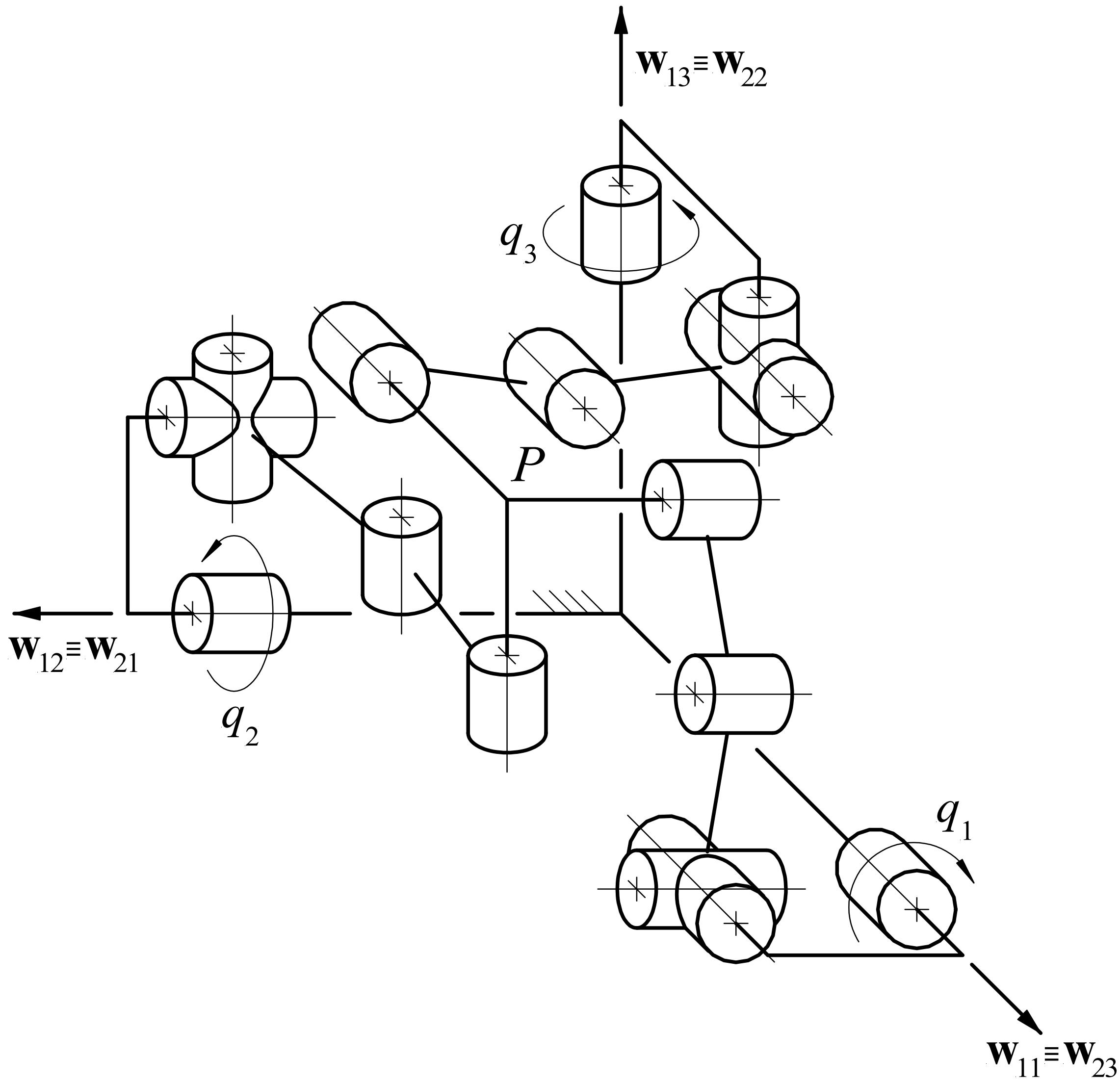

Fig. 16: A type I fully-isotropic T_5' TPM.

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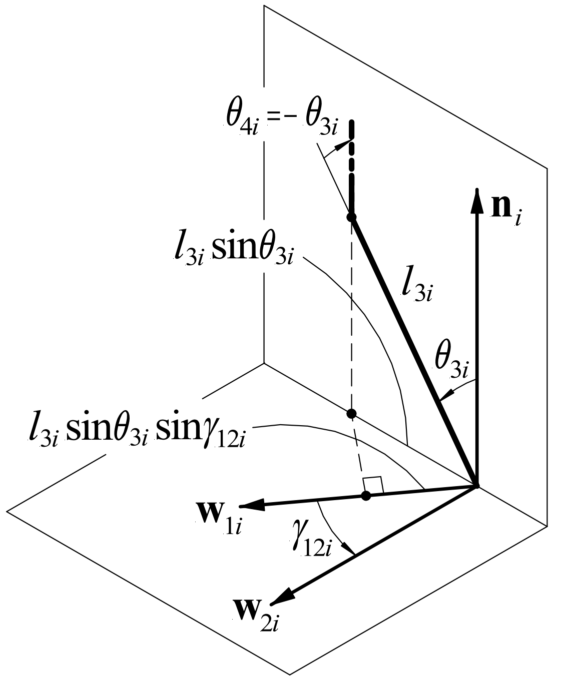

Fig. 17: Platform displacement along the constant direction $\mathbf{w}_{1i}$ in a type II fully-isotropic T_5' TPM.

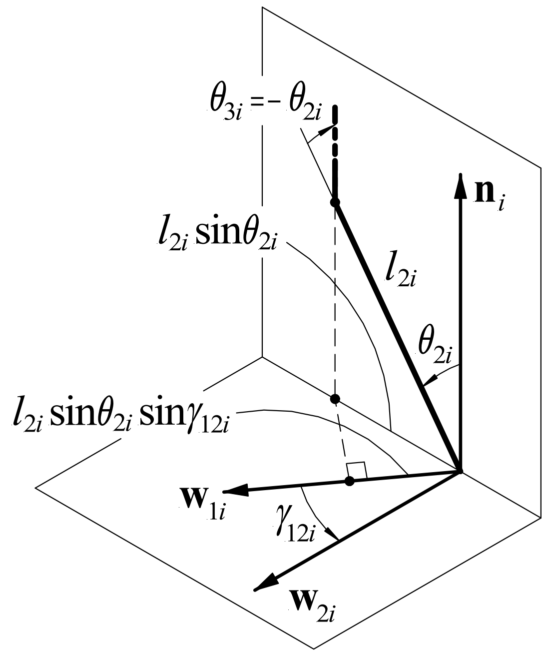

Fig. 18: Platform displacement along the constant direction $\mathbf{w}_{1i}$ in a fully-isotropic T_5'' TPM.