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Analysis of Variance of the Heat and Mass Transfer Coefficients in an Evaporative Condenser

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Abstract. Modelling of evaporative condensers relies heavily on the correct choice of the correlations for transport coefficients to give an accurate estimate of the dissipated thermal load and evaporated water mass flowrate, yet the literature does not cover all flow arrangements, nor operating conditions. In order to avoid expensive, and often impractical, experimental campaigns devoted to the determination of these quantities for a specific piece of equipment, a statistical approach based on the analysis of variance can be adopted: by analysing the sensitivity of the dependent variables on the choice of the transport coefficients, the best set of correlations can be determined, depending on whether a more accurate estimate is desired for the heat flux or evaporated mass flowrate, or if both are equally important.

1. Introduction

Evaporative condensers are employed in industrial refrigeration, as they allow, among others, a reduction in the condensation pressure, with a decrease in electric power consumption. In industrial plants, the reference value for the evaporation pressure can vary little to keep the goods at the desired storage temperature, the condensation pressure can be modified depending on ambient conditions in order to minimize the overall energy consumption of the plant, without compromising the process. It is difficult, when not impossible, to investigate the influence of the choice of the set point on the total energy demand of a real-life, operating refrigeration loop, lest the goods be damaged during the tests, but this is not the case for numerical simulations, which can be run for several ambient and load conditions. In this perspective, the condenser is a crucial component of any control-oriented model. After defining the system's geometry, the governing equations for heat and mass transfer must be derived and the proper correlations for transport coefficients chosen. Transport coefficients are strongly dependent on the tube arrangement and flow field around them, therefore applicability of the many correlations available is normally restricted only to the conditions under which they were obtained, as e.g. a different arrangement of the fluid flow (co-flow instead of counter-flow) might yield incorrect results, [1]. On the other hand, though, obtaining the values of heat and mass transfer coefficients through experimental campaigns is usually cost-intensive and often impractical, [2].

The discussion above highlights the importance of evaluating the applicability of extant correlations under conditions different from the ones for which they were obtained, in order to use them confidently. In this paper, a sensitivity analysis of a model of an evaporative condenser to the correlations for transport coefficients is carried out through a statistical tool based on the analysis of variance (ANOVA) method. In this way, the influence on those process variables which are amenable to on-site measurement even in an industrial scenario, such as the thermal power removed and the flow rate of evaporated water, of several correlations available was evaluated. The results were compared with the nominal operating data provided by the manufacturer and some criteria for choosing the most suitable set of correlations suggested.

2. Modelling of evaporative condensers and correlations investigated

There is rather large body of studies on evaporative cooling originating with the work by Merkel [3], which laid the foundations for all later analyses and modelling. Poppe and Rögener [4] further developed Merkel's model, dropping the assumptions of unit Lewis factor, *Le*, and negligible evaporated water flowrate.

The doctoral work by Dreyer [5] used both approaches to develop a bi-dimensional, distributed-parameter model for evaporative heat transfer in a tube bundle. The work considers co-flow, counter flow and cross-flow of water and air relative to one another and to the heat and mass transfer surface. The results were compared with those obtained from a lumped-parameter model, relying on the assumption that water temperature over the tube bundle can be considered as constant, [6].

Zalewsky [7, 8] developed a one-dimensional, distributed parameter model exploiting the periodicity of evaporative heat transfer in the tube bundle. This approach allows the investigation of co-flow and counterflow of water and air, but not their cross-flow.

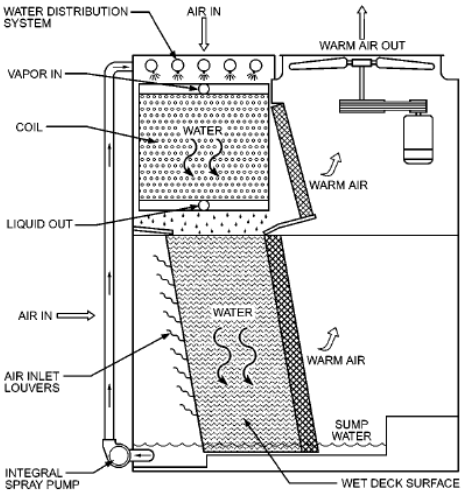


Figure 1: Evaporative condenser with fill-pack. [9]

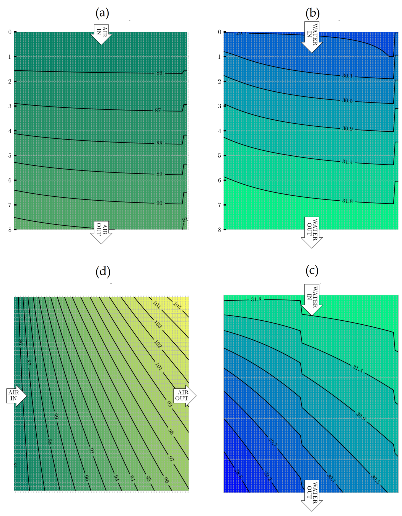


Figure 2: Moist air enthalpy and water temperature in the tube bundle (a-b) and in the fill-pack (d-c), of the condenser [10].

An extensive survey of evaporative condensers and cooling towers is offered by Kröger [11], who applied both Merkel's and Poppe's approaches to several cases, favouring the integral solution whenever possible.

Heat transfer in evaporative condensers involves three fluids at the same time: the refrigerant, water and moist air. Condensation of the refrigerant takes place inside the tube bundle, while water is sprinkled over the outer pipe wall through nozzles located above the bundle and the spent fluid is collected in a sump below, from which it is pumped back to the nozzles, Fig. 1. Contrary to the usual arrangement, air and water over the tube bundle are in co-flow.

More recent types of evaporative condensers have one more section, called fill-pack, which promotes heat transfer between the warmer water from the tube bundle and moist air, as happens in cooling towers, to further sink the water temperature, before it is collected in the sump. The fill-pack is usually made of polymeric materials and consists of a number of surfaces packed together, and shaped so as to maximise heat and mass transfer between moist air and water.

A schematic of an evaporative condenser similar to the one considered in this work is shown in Fig. 1: the fill-pack is below the tube bundle, so as to catch the water flow dripping from the tubes before it is collected in the sump below; air is sucked into the condenser by axial fans and is split into two independent streams, one flowing in co-current with water through the tube bundle, the other in cross-flow with water through the fill-pack.

A two-dimensional, distributed parameter model following Poppe's approach was chosen in this study because of two main reasons:

- the mass flowrate of evaporated water can be computed and used for validation of the model;
- the distributed-parameter model can be applied also in the case of fluids in cross-flow, as happens in the fill pack.

Evaporative heat and mass transfer over the tube bundle is computed solving the system of five differential equations below, which derive from mass and energy balances [10] :

$$dh_a = \frac{\beta_t}{\dot{m}_a}[Le(h_{asw} - h_a) - (Le - 1)(w_{asw} - w_a)h_v]dA_0 \tag{1}$$

$$dw_a = \frac{\beta_t}{\dot{m}_a}(w_{asw} - w_a)dA_o \tag{2}$$

$$d\dot{m}_w = -\dot{m}_a dw_a \tag{3}$$

$$dT_w = -\frac{1}{\dot{m}_w \cdot c_{p,w}}(\dot{m}_a dh_a + c_{pw}T_w d\dot{m}_w + \dot{m}_r dh_r) \tag{4}$$

$$dh_r = \frac{1}{\dot{m}_r}\left(\frac{1}{\alpha_r}\left(\frac{d_o}{d_i}\right) + \frac{d_o}{2\lambda}\ln\left(\frac{d_o}{d_i}\right) + \frac{1}{\alpha_w}\right)^{-1}(T_r - T_w)dA_o \tag{5}$$

A full description of the symbols is omitted due to space constraints, but is available in [10]. In the fill-pack, Eq. (5) is dropped, and in Eq. (4) the enthalpy variation of the refrigerant dh_r is zero. Finally, the heat and mass transfer area of the fill-pack is evaluated multiplying the front cross sectional area for the airflow by a surface area density coefficient, a.

The two systems of differential equations are integrated numerically with a fixed time-step Euler's method using an in-house code developed in Python; for the tube bundle, an iterative approach must be used to determine the actual refrigerant flowrate condensed, [10].

Shah's correlation, [12], was used to compute the convective heat transfer coefficient for condensation, α_r, within the tubes. To compute the evaporative heat transfer over the tube bundle, both the convective heat transfer for the water film over the tubes, α_w, and the mass transfer coefficient between water and moist air, β_t, are needed. Similarly, the mass transfer coefficient β_f must be provided to calculate the evaporative heat transfer in the fill-pack.

The correlations found in the literature and used in this work to compute α_t and β_t are reported in Table 1: none of them was devised for co-flow of water and air over the tube bundle, and only Eqs. (12) and (13) are for a cross-flow arrangement, whilst those remaining are for counterflow. This is the reason why a sensitivity analysis to the correlations employed of the model's response was deemed necessary in order to estimate their applicability outside the parametric range for which they were obtained.

For what concerns the mass transfer coefficient β_f, the surface morphology and the transport characteristics of the fill-pack are difficult to measure on site and are mostly proprietary. Since no detailed information on the issue was available, the same set of correlations was used as for the tube bundle. In the present case, the correlations are intentionally used beyond the scope of applicability provided by the authors, so the uncertainty of the calculated transfer coefficients can be evaluated only roughly with the data provided in the respective references; indeed, the aim of this work is to explore an extended field of applicability of these correlations with a statistical approach.

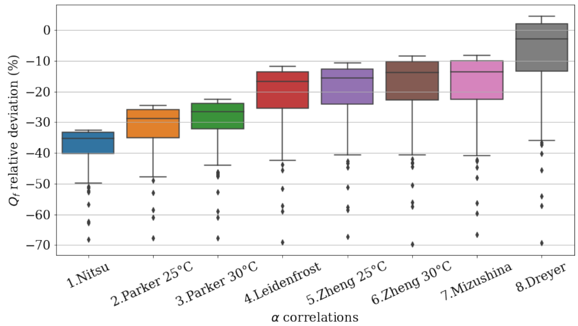
Table 1: Correlations considered.

Authors	**Heat Transfer**		**Mass Transfer**	
Parker and Treybald [13]	$\alpha_w = 704(1.3936 + 0.02214T_{wm})\left(\frac{\Gamma_m}{d_o}\right)^{\frac{1}{3}}$	(6)	$\beta_t = 0.04935\left(\frac{\dot{m}_a}{A_c}\right)^{0.905}$	(7)
Mizushina [14]	$\alpha_w = 2102.9\left(\frac{\Gamma_m}{d_o}\right)^{\frac{1}{3}}$	(8)	$\beta_t = 5.5439 \times 10^{-8} Re_{am}^{0.9} Re_{wm}^{0.15} d_o^{-1.6}$	(9)
Nitsu [15]	$\alpha_w = 990\left(\frac{\Gamma_m}{d_o}\right)^{0.46}$	(10)	$\beta_t = 0.076\left(\frac{\dot{m}_{am}}{A_c}\right)^{0.8}$	(11)
Erens and Dryer [11]	$\alpha_w = 2843\left(\frac{\Gamma_m}{d_o}\right)^{0.384}$	(12)	$\beta_t = 5.5749 \times 10^{-5}\, \mathrm{Re}_{am}^{0.64} Re_{wm}^{0.2}$	(13)
Zheng [16]	$\alpha_w = 350.3(1 + 0.0169T_w)\left(\frac{\dot{m}_a}{A_c}\right)^{0.59}\left(\frac{\Gamma}{d_o}\right)^{\frac{1}{3}}$	(14)	$\beta_t = 0.034\left(\frac{\dot{m}_a}{A_c}\right)^{0.977}$	(15)
Leidenfrost [17]	$\alpha_w = 2064\left(\frac{\Gamma_m}{d_o}\right)^{0,252}$	(16)	-	
Hasan [18]	-		$\beta_t = 0.065\left(\frac{\dot{m}_a}{A_c}\right)^{0.773}$	(17)

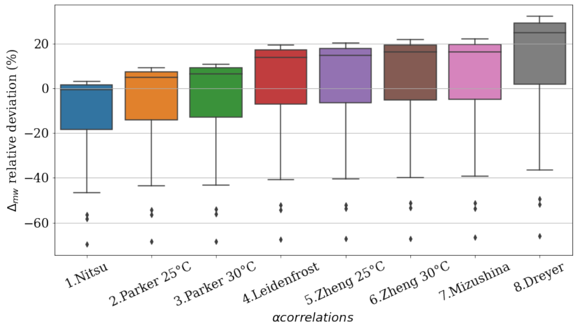
The model sensitivity to the various parameters was investigated through the analysis of variance (ANOVA) [19], a method which assesses the influence of one or more independent variables on a dependent variable. For the latter, the thermal power removed from the refrigerant and the amount of water evaporated under steady-state conditions were chosen, whose value were supplied by the manufacturer at two operating points, which had the same condensation temperature $T_{cond} = 35\,°C$, but different wet and dry bulb temperatures, namely $T_{wb,1} = 27\,°C$, $T_{wb,2} = 26\,°C$, $T_{db,1} = 37\,°C$, $T_{db,2} = 36\,°C$. The values of the dependent variables were consequently different: $\dot{Q}_{f,1} = 1610\,kW$, $\dot{Q}_{f,2} = 1783\,kW$, $\Delta\dot{m}_{w,1} = 0.655\,kg \cdot s^{-1}$ and $\Delta\dot{m}_{w,2} = 0.706\,kg \cdot s^{-1}$; no uncertainties were provided for the data, but this is a minor drawback, given the nature of the analysis carried out. The first operating point was used to evaluate the sensitivity of the two quantities, the second as verification.

3. Results and discussion

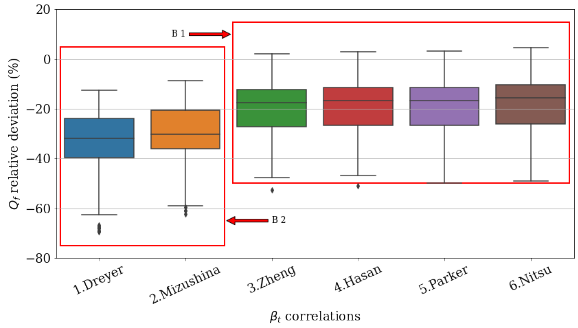
The first stage in the statistical analysis of the data computed was carried out using so-called box-plots, which relate the effects of the correlations employed for transport coefficients on the two dependent variables. Box-plots are a means of representing a data distribution: the central, horizontal line indicates the median, the upper and lower side of the box correspond to the 25$^{\text{th}}$ and the 75$^{\text{th}}$ quantile, with the box thus containing 50% of all data; the whiskers cover the extent of the dispersion in the remaining data which are not classified as outliers. The latter, where present, are plotted as lozenges.



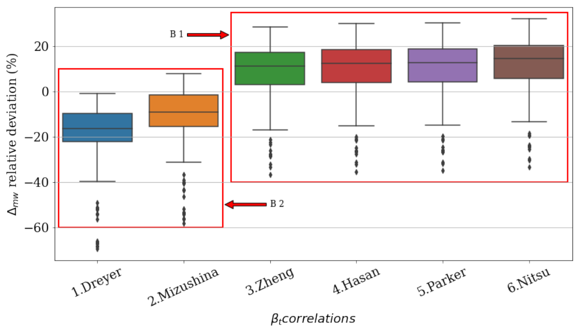
(a) Influence of α_w on heat flux.



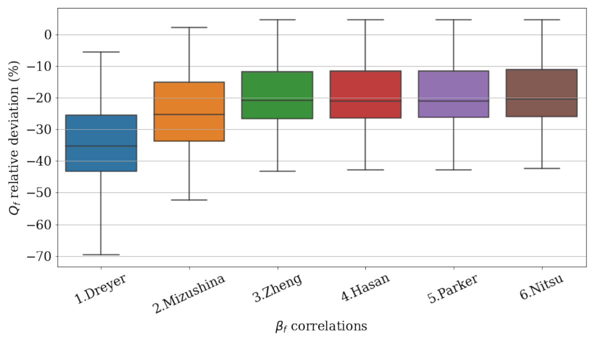
(b) Influence of α on evaporated water flowrate.



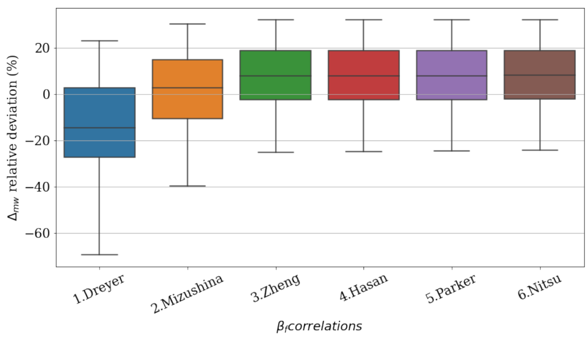
(c) Influence of β_t on heat flux.



(d) Influence of β_t on evaporated water flowrate.



(e) Influence of β_f on heat flux.



(f) Influence of β_f on evaporated water flowrate.

Figure 3: Influence of the correlations chosen on cooling power and evaporated water flowrate.

Figure 3(a) shows the influence of the convective heat transfer coefficient in the film on the dissipated heat flux: it turns out that all correlations but Dreyer's underestimate it. The effect of this correlation on heat transfer from the tube bundle is markedly different from that of the remaining ones, which can be divided into three groups of similar effects.

This behaviour is confirmed by two further tests not shown here, namely the single-factor

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ANOVA, consisting in the analysis of variance for a data set related to one dependent variable (dissipated heat flux or flowrate of evaporated water) under the influence of a single independent variable, and the Tukey's HSD (honestly significant difference) test, which allow multiple comparisons in order to pinpoint any significant difference due to the factor applied on the dependent variable.

The correlations used for α_w does not appear to influence the mass flowrate of evaporated water (see Fig. 3(b)); the values obtained for all correlations fall at 50% within a $\pm 20\%$ relative error on the expected value; Dreyer's correlation overestimates the flowrate the most.

The choice of the correlation used to compute β_t is shown in Fig. 3(c-d); these can be grouped into two sets, $B1$ and $B2$. The name value of the dissipated thermal power falls within the upper quartile of group $B1$. For the evaporated water flowrate, the data sets within the box-plots fall short of the name value of little less than $\pm 20\%$ or more. Correlations of group $B1$ on the other hand, tend to overestimate the actual evaporated mass flowrate.

Figures 3(e-f) show the effect of the correlations used for the mass transfer coefficient in the fill-pack. Dreyer's correlation stands out among the others in this case too, for both dependent variables. All correlations generally underestimate the actual dissipated heat flux, and the name value is attained in the upper quartile at most. For the evaporated water flowrate, 50% of the data lie within $\pm 20\%$ of the name value; Dreyer correlation tends to underestimate it, whilst the other overestimate it.

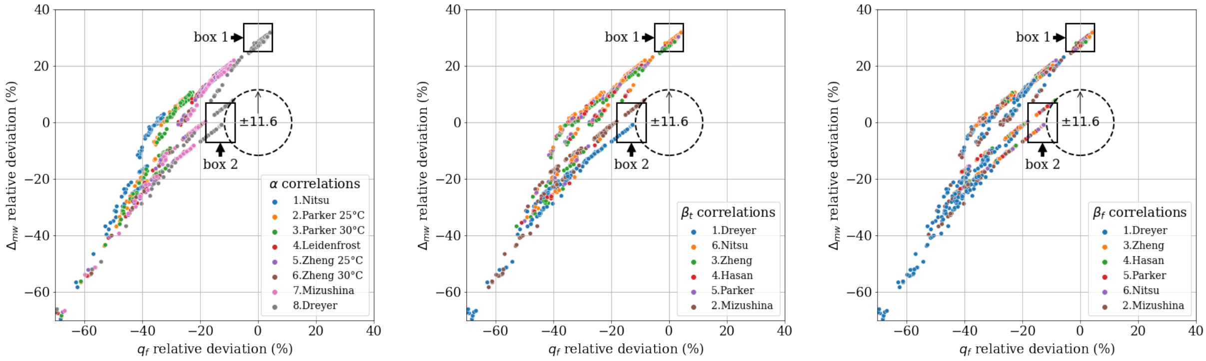
Figure 4 shows three plots of the relative error in the data for the estimate of the dissipated heat flux and evaporated mass flowrate. Colours refer to the correlation used. It is seen that no combination of correlations was able to achieve a relative error below $\pm 10\%$ for both dependent variables. Yet data cluster can be collected, which minimise the relative error for either of the dependent variables.

Box 1 encircles the tests which yield a low error ($\pm 5\%$ from the name value) on the dissipated heat flux, whilst the evaporated mass flowrate is captured with a relative error between 25% and 35%. Box 2, on the other hand, represents the data which minimise the error on the evaporated mass flowrate ($\pm 7\%$ of name value), with the corresponding dissipated heat flux falling within a relative error between -18% and -8%.

Also, all data in both boxes share Dreyer's correlation to compute α_w: it can therefore be concluded that this is the one suggested to obtain data accurate to within a reasonable error. From Fig. 4(b), it can be noticed that the correlations used to compute the mass transfer coefficient in the tube bundle are clearly distinguishable. Indeed, box 1 contains correlations from the $B1$ group only, which ANOVA detected, whilst box 2 only has data obtained using the correlations by Mizushina and Dryer, which were referred to as group $B2$. The choice of Dreyer's correlation for α_w in combination with one correlation from group $B1$ for β_t grants the minimum deviations on the dissipated heat flux, at the expenses of a somewhat overestimated evaporated mass flowrate. Conversely, using a correlation from group $B2$ for β_t underestimates the dissipated thermal power, whilst yielding the best results for the evaporated mass flowrate.

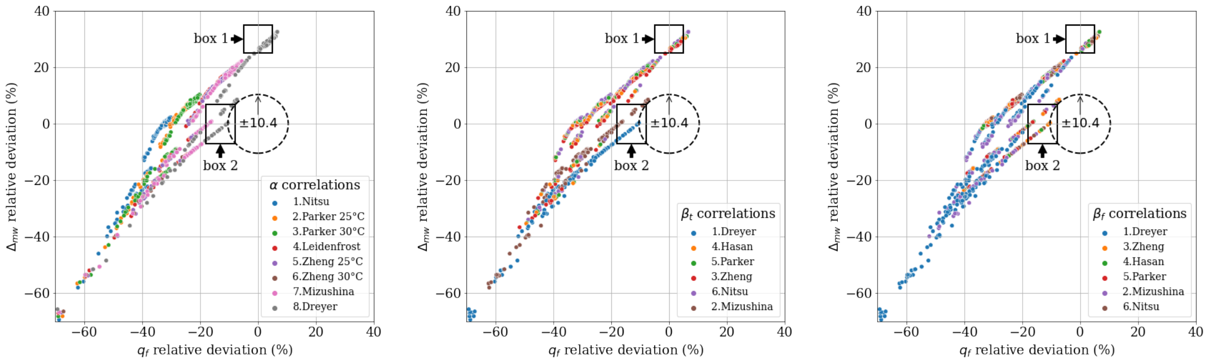
Lastly, concerning the choice of the correlation for β_f in the fill-pack, Fig. 4(c) shows that all correlations but Dreyer's are contained within the two boxes: as demonstrated in Fig. 5 (c)-(d), the latter is the only correlation which underestimates both dependent variables in the fill-pack. To wrap it all up, the set of correlations recommended for the distributed-parameter of the type of evaporative condenser considered should use Dreyer's correlation to compute α_t in Eq. (12), but the correlation by same author should not be employed to determine β_f, Eq. (13). For β_t over the tube bundle, two options are available:

- if an accurate estimate of the dissipated heat flux is sought, the correlation should be chosen among those of group $B1$, thus obtaining one of the points in box 1;
- if the evaporated mass flowrate is more important, correlations from group $B2$, which keep the deviation in $\dot{Q}_f$ to $\pm 10\%$, and one of the data in box 2 is obtained.



(a) Influence of α_w (b) Influence of β_t (c) Influence of β_f

Figure 4: Influence of the correlations chosen on cooling power and evaporated water flowrate, operating point 1.



(a) Influence of α_w (b) Influence of β_t (c) Influence of β_f

Figure 5: Influence of the correlations chosen on cooling power and evaporated water flowrate, operating point 2.

The results for operating point 2, Fig. 5, show a similar behaviour.

Three optimal sets of correlations for each of the two operating points, and the results are reported in Table 2 and Table 3 respectively. Set 1 minimises the deviation of $\dot{Q}_f$ and belongs to box 1, set 2, which relates to box 2, minimizes the deviation of both $\Delta\dot{m}_w$ and $\dot{Q}_f$. Set 3, finally, minimizes the mean deviation of both dependent variables and is computed as

$$\delta_{tot} = \sqrt{\delta_{mw}^2 + \delta_{qf}^2} \tag{18}$$

If the results for both operating points according to the latter criterion are averaged, the optimal deviation obtained is $\delta_m \simeq 11.4\%$ for the choice of Dreyer, Mizushina and Nitsu for α, β_t and β_{fp} respectively.

Table 2: Optimal correlation sets for operating point 1.

Set of correlations Deviation considered	Set 1 $\delta_{qf} \simeq 0.02\%$	Set 2 $\delta_{mw} \simeq 0.8\%$, $\delta_{qf} \simeq 12.6\%$	Set 3 $\delta_{tot} \simeq 11.6\%$
α	Dreyer	Dreyer	Dreyer
β_t	Zheng	Dreyer	Mizushina
β_f	Hasan	Nitsu	Nitsu

Table 3: Optimal correlation sets for operating point 2.

Set of correlations Deviation considered	Set 1 $\delta_{qf} \simeq 0.35\%$	Set 2 $\delta_{mw} \simeq 0.09\%$, $\delta_{qf} \simeq 10.6\%$	Set 3 $\delta_{tot} \simeq 10.4\%$
α	Dreyer	Dreyer	Dreyer
β_t	Hasan	Dreyer	Dreyer
β_f	Hasan	Nitsu	Zheng

4. Conclusions

This paper suggested some criteria to choose the most suitable set of correlations for transport coefficients to be used in the distributed-parameter model of an evaporative condenser without the need of a complex and expensive experimental campaign to obtain them on site. The model, which uses the approach by Poppe and Rögener was tested with correlations available from the literature, none of which had been obtained for the same fluid flow arrangement. The suggested statistical investigation, based on the analysis of variance, evidenced the model sensitivity to the correlations used with respect to two dependent variables, namely the dissipated heat flux and the evaporated water mass flowrate. Based on the results, different criteria were suggested to choose the triplet of correlations which minimise the absolute deviation from the reference value for either $\dot{Q}_f$ or $\Delta\dot{m}_w$ or from both. With these choices, the maximum deviations between the model's computations and the reference values are slightly larger than $\pm 11\%$. While the numerical results pertain to a specific realisation, the method proposed can be extended to any other evaporative coolers, provided a suitable model for their description is available.

5. Referecens

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