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*Research Article*

# Structural analysis of voxel-based lattices using 1D approach

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**Abstract:** Lightweight bio-inspired structures are extremely interesting in industrial applications for their known advantages, especially when Additive Manufacturing technologies are used. Lattices are composed by axial elements called ligaments: several unit cells are repeated in three directions to form bodies. However, their inherent structure complexity leads to several problems when lattices need to be designed or numerically simulated. The computational power needed to capture the overall component is extremely high. For this reason, some alternative methodologies called homogenization methods were developed in the literature. However, following these approaches, the designers do not have a local visual overview of the lattice behaviour, especially at the ligament level. For this reason, an alternative 1D modelling approach, called lattice-to-1D is proposed in this work. This method approximates the ligament element with its beam axis, uses the real material characteristics and gives the cross-section information directly to the solver. Several linear elastic simulations, involving both stretching and bending dominated unit cells, are performed to compare this approach with other alternatives in literature. The results show a comparable agreement of the 1D simulations compared to homogenization methods for real 3D objects, with a dramatic decrease of computational power needed for a 3D analysis of the whole body.

**Keywords:** lattice structure; periodic structure; homogenization; voxel; structural analysis.

## 1. Introduction

Nowadays, Additive Manufacturing (AM) is considered an important alternative to traditional processes based on chip removal, casting, milling, and lathing processes where several design constraints must be respected [1]. AM technology reveals several advantages which are highlighted in the literature, and its use is increasing in aerospace, automotive [2], biomedicine year after year [3] and even in niche applications such as musical instruments [4]. Time drop-in design-to-manufacturing cycle, design flexibility, ability to generate complex shapes in one piece and capability to imitate low-weight bio-inspired shapes are the advantages of designs based on AM [5]. On the other hand, to date the main weaknesses of AM technology can be found in: material anisotropic properties for some AM processes; high surface roughness; limitations of CAD software in the integration of design, technology, optimization, smoothing processes; limited material portfolio; problems of inspection and maintenance in complex assemblies made in one piece; high costs and slow certification process; high structural performances variability due to changes of properties in raw materials lots, changes (sometimes even small) in machines settings or environmental characteristics; behaviour of AM structure with fatiguing loads.

Bio-inspired cellular structures, also called hierarchic structures, can be included in AM components, where lightweight, stiffness and high strength to mass ratios are essential. There are different types of cellular structures such as foams, which are stochastic, honeycombs and lattices [6] which are periodic and are the object of this research. Periodic lattice structures are composed of elongated elements, such as cylindrical beams, called ligaments, beams or struts, connected with other similar elements to form a unit cell that is repeated thousands of times along the body.

Even if the density is much lower compared to a traditional fully dense part, lattices are manufactured using metallic materials such as aluminium, titanium or steel to have stiff components adopting selective laser melting (SLM) [7,8] or electron beam melting (EBM) [9] technologies. Thanks to their strength/weight ratio and good absorption energy, nowadays lattice structures replace other cellular materials such as foams used in sandwich structures in aerospace or automotive applications, due to the capacity to absorb mechanical vibrations and sound waves. The aerospace industries are interested in these types of components to increase the crashworthiness. Just to provide the reader with an example, the Boeing Model 360 helicopter is built using some sandwich structures made of lattices to achieve a lightweight design and to reduce the number of mechanical joints thus reducing maintenance costs [10]. Another interesting application of optimized lattice structure is contained in [11]: an automotive engine mounting bracket has been topologically optimized and in the following partially filled with lattice structure to reduce the weight of the component still maintaining sufficient strength and stiffness. Moreover, thanks to the ability to absorb thermal energy due to a higher surface area for heat exchange, lattice structures are suitable for application where thermal insulation is important as well.

After listing the advantages of lattices, it is important to discuss the problems arising when these structures need to be designed and simulated. Common design tools available to the engineer, such as CAD software, still show large limitations because the boundary representation technology (B-rep) is used: it is not well suited for lattices where the external surface is extremely complex [12].

Another important problem arises when lattice mechanical behaviour is investigated through numerical analysis. In literature, several contributions deal with the application of Finite Element (FE) analysis to periodic structures for different purposes, such as: material characterization of titanium alloy structures fabricated via EBM [13], prediction of fatigue behaviour applied on porous metallic biomaterials [14] and non-linear analysis of lattice structures to predict the energy absorption [15]. In case of components with a simple shape such as the beams used in the aforementioned contribution, the 3D FE analysis assures consistent and reliable results. However, to capture the lattice behaviour, especially when dealing with complex real-life components., due to its high structure complexity, FE analysis requires a huge amount of computational power to discretize the structure in billions of meshing elements, The dimension of the mesh is similar or lower than the diameter of ligaments, which is typically small. This is the reason why alternative methods, such as homogenization algorithms, have been developed in literature [16].

Several methods to cope with the time reduction of structural analyses are available in the literature, a non-inclusive list includes:

- Closed-form expression based on the Euler-Bernoulli beam [17]
- Matrix-based techniques based on Bloch's theory [18]
- Micropolar elasticity theory [19]
- High-frequency homogenization [20]
- Discrete homogenization technique [21]
- Asymptotic homogenization (AH) [22], [23]

AH shows good results in validation tests for a lot of applications, because determines accurately stress distributions in the unit cell without limitations on unit cell topology or relative density (defined as the density of a certain volume of lattice structure over the density of the material that composes the ligaments) [24].

The AH algorithm aims to speed up the mechanical analysis, and to do that, it replaces a lattice structure with a fully dense homogenous solid, with equivalent mechanical properties, same occupied volume, maintaining same loads and point of applications. In this way, the computational power needed to solve numerically the mechanical behaviour of periodic structures decreases exponentially. The most important assumption behind AH method is that each field quantity depends on two different scales: the macroscopic scale and the microscopic one.

Moreover, each field quantity (i.e. strain, displacement) varies smoothly at the macroscopic scale, while it is periodic at microscopic one. Therefore, field quantities can be expressed as an asymptotic expansion based on power series with terms that depend only on macroscopic scale and terms that describe microscopic perturbations. A detailed description of the mathematical model of AH is beyond the scope of this paper, but the reader is addressed to the source [25] in Section 2.10 for a more detailed description.

Thanks to AH, it's possible to obtain a closed-form expression for the equivalent stiffness matrix for several unit cell topologies, exploiting the results available in the literature [26,27]. Then it is possible to analyze the fully dense solid, using a finite element method, with same dimensions but with equivalent mechanical properties compared to the original object, obtaining a mesh with a lower number of nodes and decreasing the FE problem dimensions. This reflects on the speed-up of the numerical process, with a confined error on the numerical results. When FE analysis is applied to components made by isotropic materials, 5% error with experimental data can be expected with a proper modelling and accurate material properties definition [28]. When dealing with orthotropic or anisotropic materials and complex shapes such as lattice structures, a higher difference between numerical and experimental tests can be noticed. However, the L1D approach has been developed to be applied during preliminary and conceptual design phases, where FE assures the capability to explore many design scenario in short times, and higher errors than what introduced by [28] are accepted. However, the main drawback of AH algorithm, that is its high computational cost in case of complex shapes due to a multiscale problem that must handle a lot of variables in case of non-closed-form results. This is the reason why, in case of complex unit cell topologies, the research community still has to develop different simplification methods to speed up the material analysis.

Moreover, this kind of procedure moves away from the real object towards a fully dense object and the designer may lose the geometrical lattice characteristics by using a bulk component instead of a periodic structure.

The aim of this research is a comparison of three techniques which can be used for FEM analyses on lattice structures: innovative 1D representation and simulation, 3D analyses of the lattice structure and asymptotic homogenization. The first method tries to solve this problem by modelling the tridimensional lattice with 1D elements to decrease the computational effort (nearly 90% less compared to the full 3D model), with limited estimation error (about 15-20%) together with the capability of giving to the designer a visual idea of the periodic lattice.

Such an approach hasn't been yet investigated in detail, but it can offer to the designer several advantages such as light computation efforts, better lattice geometry understanding in a context of preliminary design phase where several scenarios have to be investigated in a fast way with reasonable results and limited estimation error compared to the real 3D lattice behaviour. Using the terminology of [29], the herein investigated approach is still limited to uniform strut-and-node arrangement lattices and not for Triply Periodic Minimal Surface (TPMS) structures [30]. The strut-and-node arrangement perfectly fits the approximation, which is at the basis of this work, where the ligaments are approximated with their beam axis.

A numerical analysis scenario has been implemented to assess the differences in terms of maximum and mean deformation estimation and understand if this approach can be a valid alternative to AH and 3D lattice analysis. The performance of the innovative approach has been tested in terms of maximum and mean deformation and not regarding the comparison of stresses (e.g. Von Mises criterion for isotropic materials) because in this latter case possible numerical instabilities can introduce fictitious values of maximum stress for the FE analysis of 3D and 1D models. This aspect will be better investigated in the further, but it is out of the scope of the present paper.

After this initial introduction, the second section presents the methodology based on the innovative approach. Then, the results of finite element analysis will be shown in section three. In section four the results are discussed. Finally, section 5 lists conclusions and future developments.

## 2. Methodology

In this section, the methodology to generate the 1D model of the lattice structure and to compare the mechanical behaviour of 3D lattice, 1D lattice and AH approach is described.

To assess the suitability of this alternative approach, which is called L1D (Lattice to 1D) and described in this work, several numerical analyses are performed using Patran/Nastran software with a tensile load case, applied on a cantilevered rectangular beam which is filled with uniform lattice (Figure 1).

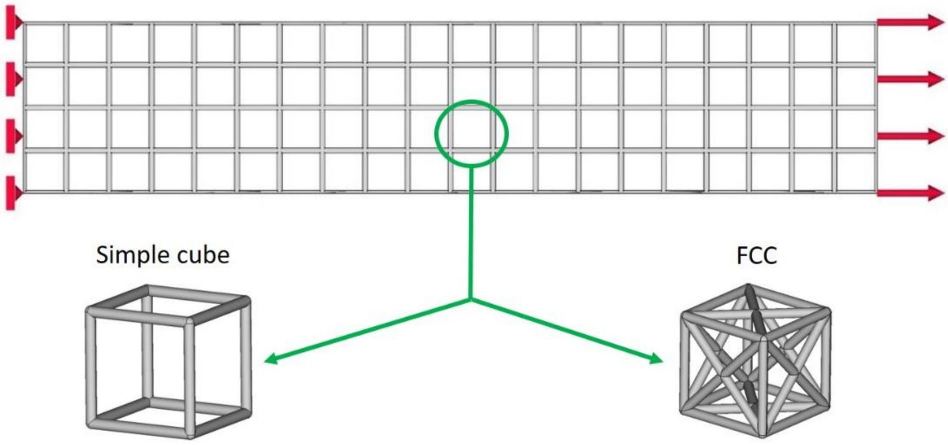



**Figure 1.** 3D cantilevered beam filled with a uniform lattice; a tensile load is applied on the free end. Two different unit cells are examined in this work: simple cube unit cell which is bending dominated; FCC unit cell which is stretching dominated.

These preliminary simulations have been used to assess the capability of the L1D approach and investigate how some design parameters (unit cell type, cross-section type) may affect the results. Then, the L1D approach is applied on a real-life object such as an aircraft engine bracket designed with uniform and periodic lattice. The material chosen for these simulations is the Ti6Al4V ELI-0406 powder for AM applications with high specific strength (strength to weight ratio) which makes it an ideal choice where weight saving load structures are required [31]. The unit cells of the periodic structure are based on cubic shape. Two different unit cells are used for the scope of this paper in the cantilevered beam example: the simple cube unit cell and the Face Centered Cubic one (FCC). A similar approach could be carried out with other lattice structures.

This choice comes from the necessity to investigate the efficiency of the L1D for both bending (simple cube) and stretching dominated (FCC) lattice unit cells. In this way, it is possible to understand how this kind of modelling lattice structures influences the approach performances in terms of accuracy of results and computational time. Stretch-dominated unit cells are characterized by high stiffness while bending dominated cells have lower stiffness but achieve higher strain values that make them appropriate for energy-absorbing applications [32]. Moreover, for the sake of this research, the ligaments composing the lattice are modelled using both square and circular beam cross-section to investigate all the possible settings which may affect the lattice design. For simplicity, only the simple cube unit cell is used in the engine bracket simulation.

To obtain the mono-dimensional lattice structure, only the periodic uniform lattices were considered in this research. At first, a dense 3D model of the part in lattice structure is sketched, and saved in STL format. In the following, an algorithm has been developed to fill the dense part with a periodic structure which is obtained thanks to a 1D wireframe modelling: the orientation and size of this 1D modelling are equal to the lattice cell properties. In this way, the 3D dense part is converted into a 1D lattice using the axis of ligaments through a voxel-based approach; the geometrical cross-section data and material properties are given to the solver in a second moment inside the software itself. The resulting geometry is only made by mono-dimensional geometrical entities as lines connected to different points according to the unit cell geometry. It is worth noting that, thanks to this methodology, is not necessary to model the complete 3D part with lattice structure, but only a dense part is sketched. In the following, the designer can do simulations to assess the behaviour as if the body was in lattice structure. Different cell size, ligaments shape and dimensions and orientation can be tested to tune the model depending on the final application. Finally, the body can be automatically sketched in lattice structure for manufacturing purposes.

### *2.1. Methodology to get the 1D geometry*

To simulate the 3D lattice, an original MATLAB code was written by authors. The code is capable to read a .STL file describing a fully dense 3D object and get the facet and the coordinates of the vertices composing the triangle mesh of the external surface. Then, using the ray intersection method [33], a voxelized representation of the original object is achieved and a logical matrix made by 0 and 1 (B/W representation) is available to the user, knowing the voxel resolution the designer want (which imitates the lattice structure features). The ray intersection method is the more diffused and easy method to voxelize a 3D object. In particular, the mesh is ray-traced in each of the $x, y, z$ directions, with the overall result being a combination of the result from each direction (Figure 2). The voxelization approach applied for each triangle of the .STL mesh can be divided into these steps:

- Take each edge of the facet in turn
- Find the position of the opposing vertex to that edge
- Find the position of the ray relative to that edge
- Check if the ray is on the same side of the edge as the opposing vertex
- If this is true for all three edges, then the ray passes through the facet.

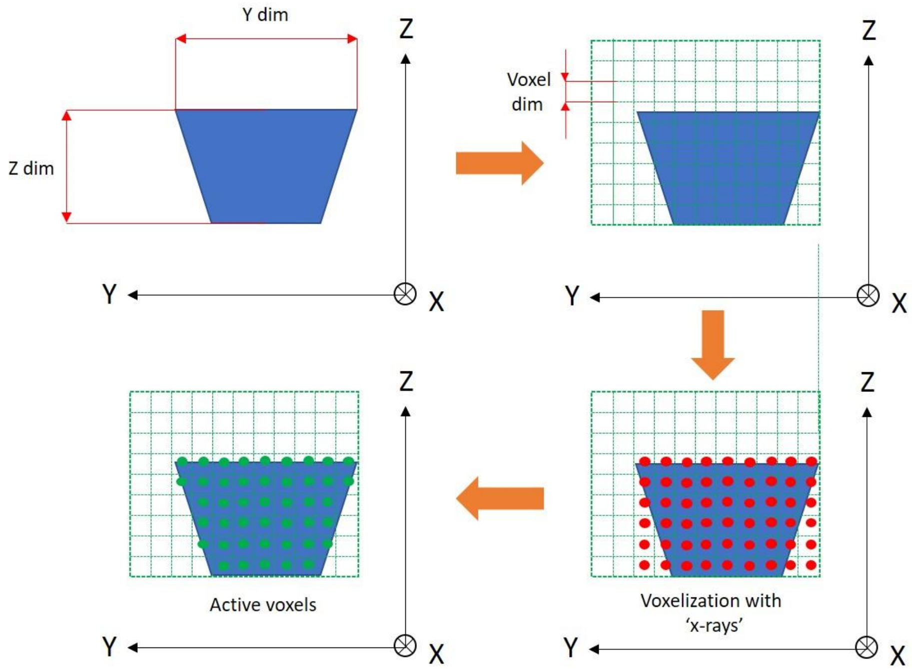



**Figure 2.** Ray intersection method for object's voxelization: in this picture only the rays in the x-direction are shown; the algorithm passes sorted rays along the X-axis incrementing Y and Z coordinates and finds their intersections with the facets. Adapted from [33]

The proposed methodology matches the unit cell of the lattice structure with the voxels used to represent the 3D object, by setting size and orientation. After the unit cell topology is set, the own code is capable to generate two distinct matrices which describe the uniform and periodic lattice. The first matrix of dimensions $n \times 3$ contains all the coordinates of the vertices of the lattice cells belonging to active voxels, while the second matrix of dimensions $m \times 2$ contains the information regarding the IDs of two vertices composing a ligament. The final step of the code is to write a .out neutral file. The neutral file contains the coordinates of the lattice points and the index of vertices linked together, according to the neutral file format [34] structure. Several types of geometry file formats were investigated, but the neutral file format of Patran software has been set as the best one due to importation, geometrical description, and formatting easiness. A flowchart describing the methodology developed to get the 1D geometry is shown in Figure 3, in which it is possible to see different stages of this approach, applied to a model of an aircraft engine bracket in lattice structure. The conversion from 3D to 1D is performed in MATLAB on a workstation with 32GB RAM and an Intel Zeon CPU @ 3.50 GHz. The overall time needed for the conversion of the sample part given by the aircraft engine bracket is listed in Table 1.

As it can be expected, the more the dimension of the discretization element decreases, the more the number of active voxels and the computational time increase. However, values of computational time around seconds or minutes in case of complex shapes suits the scenarios where the L1D approach is expected to be used: the preliminary and conceptual design stages.

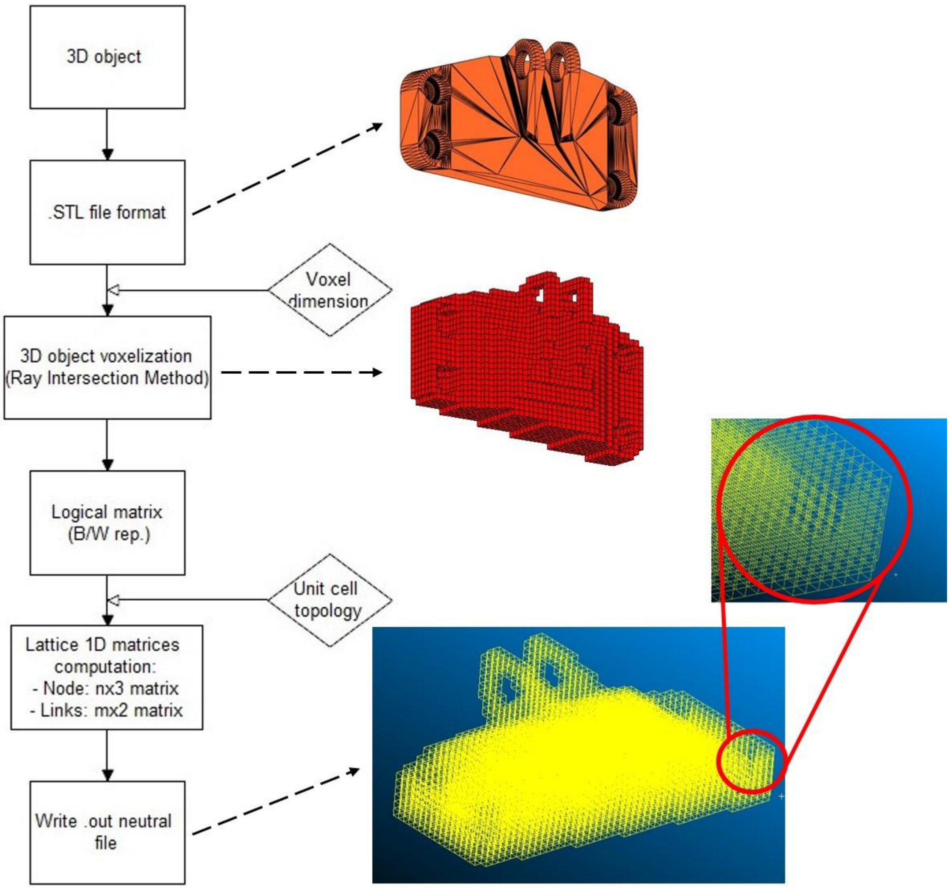



**Figure 3.** Flowchart describing the algorithm to get the 1D model in Patran from a 3D object in .STL file format. The L1D methodology is applied to an aircraft engine bracket: from the .STL mesh of the dense part, the algorithm obtains a voxelized model of the object; finally, the 1D lattice structure in Patran is depicted.

*2.2. Methodology to compare the 3D, 1D and AH analyses*

After importing the mono-dimensional lattice geometry as .out format file in Patran/Nastran, it is possible to provide the software with the material properties which are the effective ones and not the equivalent properties coming from AH methods of a hypothetic fully dense object. Moreover, the user can define in the solver the beams cross-section characteristics inside the Patran "Properties" menu since the neutral file does not contain this information. For the scope of this work, the "beam" 1D property was chosen to characterize all the ligaments which make the lattice. A rod option would neglect the bending effects. The great advantage of Patran software is the capability to show the resulting structure by selecting the element properties display option: going from the "1D: Line" option to the "3D: FullSpan" option, the user can understand if the lattice properties are correct without increasing the computational power needed to model a 3D structure, as can be seen in Figure 4 where a close to reality 3D lattice structure visualization is obtained without efforts.

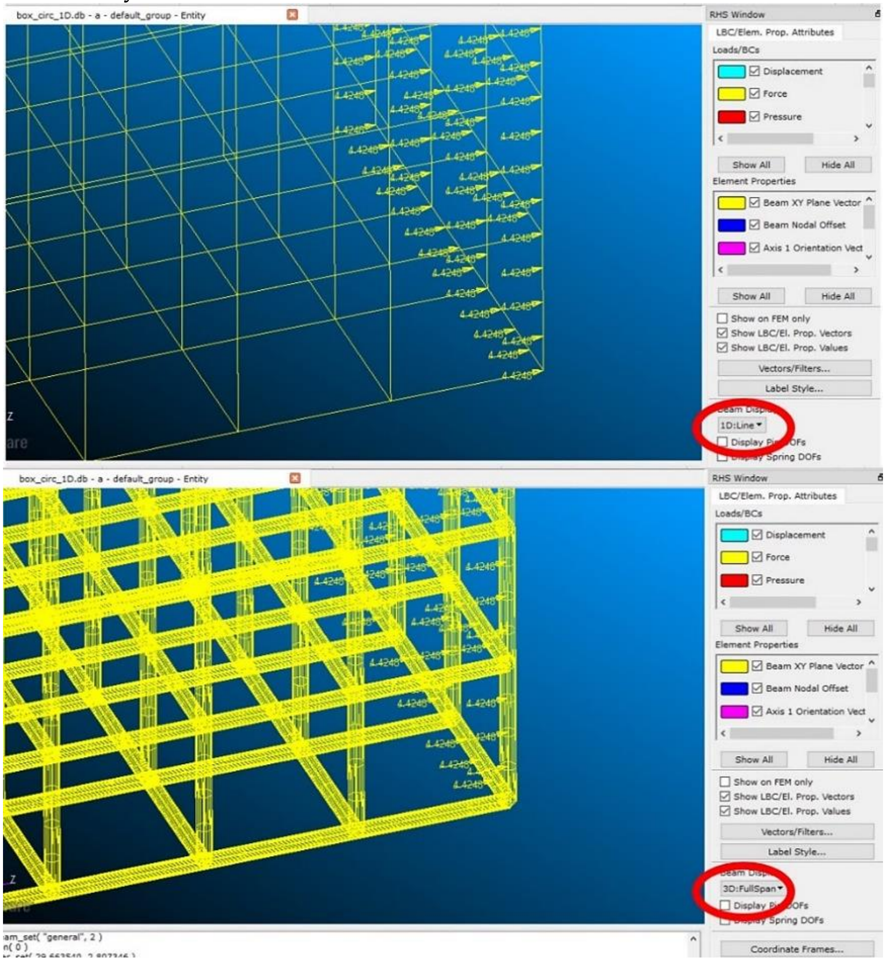



**Figure 4.** Same 1D lattice with two different displaying options in Patran

In this research, the lattice 3D models of the cantilever beam used for the simulation herein described were designed using a lattice structure exploiting the LSWM workbench of FreeCAD [35], developed by authors. On the other hand, the 3D full lattice model of the engine bracket was designed using the commercial Element software by nTopology [36]. This software can be used to obtain a 3D model based on lattice structures and save it in CAD formats. The geometry is then imported as .step file in Patran. Other commercial software codes could be used for lattice modelling as well. This to highlight that nowadays alternative software to design 3D models, filled by lattice structures with properties set by the user, are available to the community. However, several problems arise when such complex structures need to be structurally investigated through mechanical simulations, as already cited in the previous sections.

As last methodology useful to understand the goodness of the L1D approach, AH closed-form results coming from two different literature contributions [26,27] are used as entries for the stiffness matrix of the equivalent 3D fully dense homogenized material. In the following, AH results are compared with 1D analysis. The source [26] uses a multiscale approach to determine the macroscopic stiffness of different lattice topologies. The results are given as different stiffness matrices depending on the geometrical unit cell characteristics, as ligament length, cross-sectional dimension, cross-sectional area and material characteristics as Young's modulus ($E$) and Poisson ratio ($\nu$). These results can be applied only if a slenderness ratio (ligament length over cross-sectional dimension) of 10 at least is guaranteed to verify the slender beam assumption at the basis of this method. On the other hand, the research [27] developed a MATLAB code by using a voxel-based approach to analyze different unit cell lattice topologies, based on the procedure described by Andreassen [37]. The code is capable to return the lattice stiffness matrix knowing the unit cell dimensions, the unit cell topology in terms of point position and relative links, the material properties ($E$ and $\nu$). However, this method is limited in terms of cross-sectional topologies and only circular ones are modelled.

For both selected contributions ([26,27]), the resulting lattice stiffness matrix has a different scheme concerning common isotropic materials, where the matrix is defined using only two parameters, namely Young's modulus and Poisson ratio. For periodic structures, the stiffness matrix $K_{lat}$ can be written as a function of three parameters $\alpha, \beta$ and $\gamma$ (Eq. 1) [38]. These are functions of geometric characteristics of the unit cell topology and the beam elements. Each unit cell type will have different eigenvalues that bring to different mechanical characteristics. It's important to underline that this method can be applied only if the Euler-Bernoulli beam assumptions are valid, that reflects on a limited unit cell edge slenderness ratio. Otherwise, some instabilities could arise in case of local compressive loads, as well described by [39].

$$K_{lat} = \begin{bmatrix} \alpha & \beta & \beta & 0 & 0 & 0 \\ \beta & \alpha & \beta & 0 & 0 & 0 \\ \beta & \beta & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma \end{bmatrix} \tag{1}$$

As previously said, the scope of this work is to compare and understand if the L1D approach can be a valid alternative to AH methodology for lattice structural analysis. To do that a set of simulations has been carried out in Patran/Nastran MSC software following the layout proposed in Figure 5: the maximum and mean displacement, the mesh size, the meshing time, and the solving time are compared for three cases (AH, 1D, 3D).

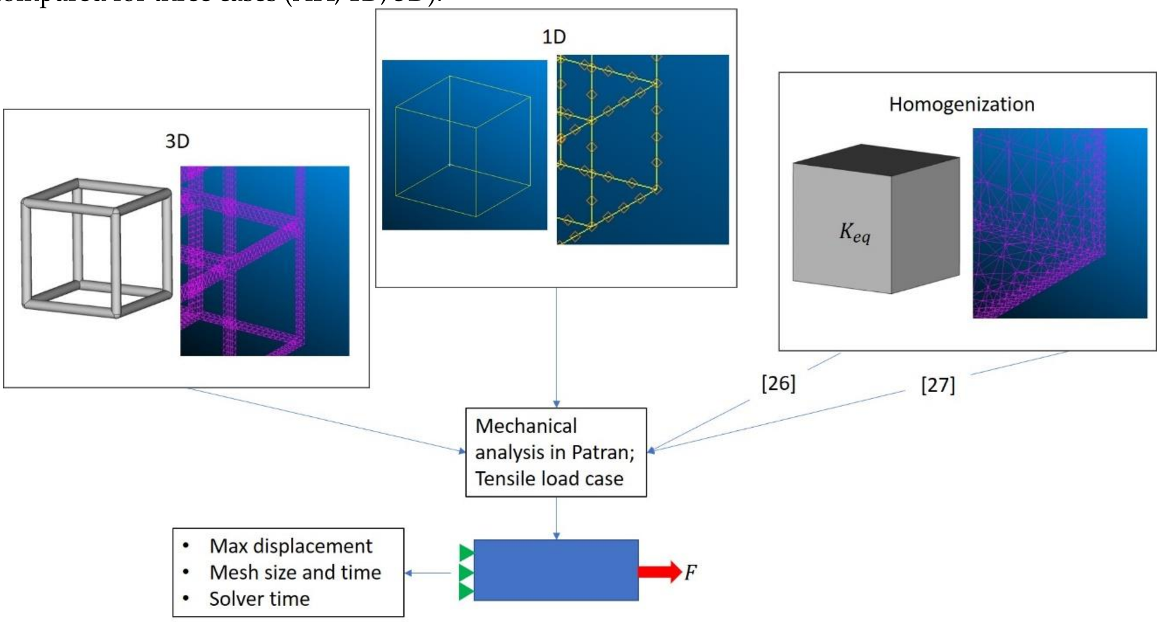



**Figure 5.** Methodology layout used to compare and estimate in terms of accuracy the performances of the alternative 1D modelling for uniform periodic structures along with a detailed view of meshing elements of 3D full model, 1D model and 3D homogenized model.

### *2.3. Case studies: geometry, load material definition*

The first part used to assess the L1D methodology capabilities is a cantilevered beam with a rectangular cross-section with a 500N axial load to the free extremity applied on the overall surface (Figure 1). The beam dimensions are 30mm (thickness), 40mm (height) and 200mm (length): this slenderness has been set to comply with the hypothesis of the classic beam theory [40]. The applied load is at first modelled as a set of nodal forces distributed on the structure nodes of the free end. Additional simulations have been performed by applying nodal forces in the mesh nodes instead of structure ones to detect possible differences in terms of structural deformation. As an alternative, kinematic link (e.g. RBE2 or RBE3) could have been used to distribute loads within the model in a similar manner to what suggested in [41].

As previously said, two different unit cell topologies are investigated to understand if both stretching (FCC) and bending dominated (simple cube) lattices are well modelled by the L1D approach. For each unit cell type, both squared and circular cross-section are modelled; in this way, even the cross-section variable is investigated for the scope of the research. On the one hand, the unit cell characteristics are chosen to guarantee a slenderness ratio of 10 to comply with the Euler-Bernoulli assumption. On the other hand, the size is set to limit the computational power needed to design and model the 3D lattice structure. In fact, by increasing the ligament length, the meshing procedure can take longer time or evenly fail or return a low-quality discretization. For the above-mentioned reasons, the unit cell topology characteristics set for the cantilever beam case study can be summarized as:

- Ligament length: 10mm
- Cross-section dimension:
  - squared: edge of 1mm;
  - circular: radius of 0.5mm

The average mesh size for the full 3D model is 0.5mm, while for the homogenized one is 1mm made by Tet10 elements. The 1D models have a mesh average size of 2.5mm, made of Bar2 elements. The 3D homogenized component is modelled as a dense part with an orthotropic material according to eq.1 with matrix entries coming from [26,27] references. An isotropic material using the bulk characteristics is employed to model the full 3D lattice and the 1D wireframe.

The L1D approach is applied in the following also on a real component, namely an aircraft engine bracket. This bracket has been designed with uniform and periodic lattice in other to understand the L1D performances in a significant industrial engineering context. The investigated component is a bracket sketched for a famous challenge organized by General Electric and Grabcad [42]. About FE analysis, for sake of simplicity, the constraints applied on the bracket consist of four holes on a base, fully constrained for all the 6 degrees of freedom. In the opposite part of the bracket, there are two vertical holes where a tensile load is applied in the form of nodal force distributed on the cylindrical hole surface. The simulated load is composed of a vertical component of 200N and a horizontal one of the same amount (Figure 6).

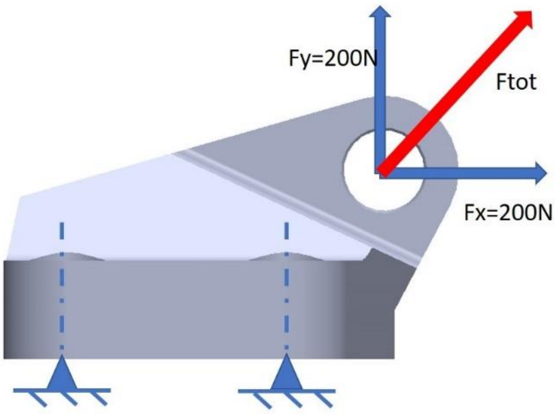



**Figure 6.** Schematic view of loads and constraints applied on the aircraft engine bracket

The bracket is filled with uniform and non-conformal lattice, using for simplicity only the simple cube unit cell topology. To verify the slenderness ratio imposed by the investigated AH methodologies, the unit cell characteristics are:

- Ligament length: 4mm

- Cross-section dimension:
    - squared: edge of 0.4mm;
    - circular: radius of 0.2mm

The average mesh size for the homogenized models is 2mm made by Tet10 elements, while the meshing procedure failed for the full 3D model. On the other hand, the 1D models have a mesh average size of 2mm, made of Bar2 elements. In this case study, the 3D homogenized component is modelled with an orthotropic material according to [26,27]; an isotropic material using the bulk characteristics is employed to model the 1D wireframe material as well.

Such unit cell dimension has been set to have at least two layers of lattice cells in the support region where the tensile load is applied: a trade-off has been carried out using a voxel size which is not highly demanding for the computational power available to us. As for the cantilevered beam, the material used for these simulations is Ti6Al4V ELI-0406 powder for AM applications with a Young's Modulus of 126 GPa and a Poisson ratio of 0.3.

The unit cell size changes in the two case studies due to computational power burden limitation. In the cantilever beam example, the unit cell size has been set to limit the computational power as much as possible, thus limiting the number of unit cells, also satisfying the imposed limitations in the lattice geometry in terms of slenderness ratio.

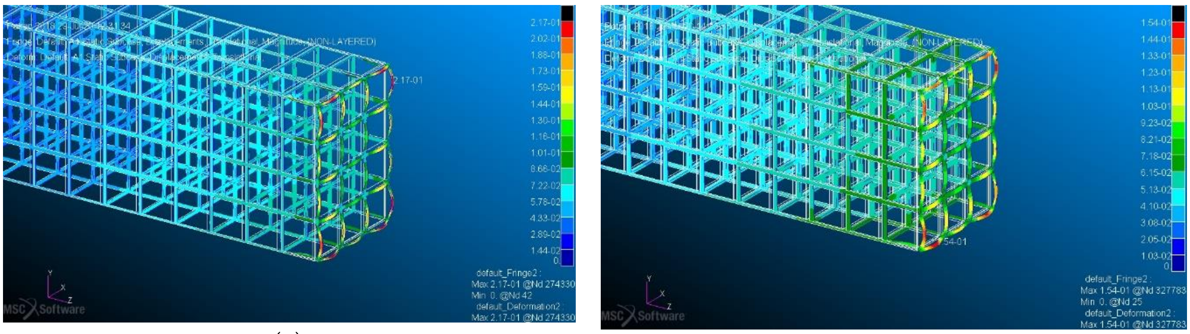
The lattice dimension used to fill the bracket has been set to limit the computational power, thus limiting the overall number of unit cells, and satisfying at the same time two conditions: the geometry limitations to the slenderness ratio, and to guarantee at least two layers of unit cells in the region where the load is applied.

## 3. Results

In this section, the features of the mechanical simulations and the results are collected, leaving Sec. 4 for its discussion. The linear elastic numerical simulations have been performed in Patran/Nastran on a workstation with 32GB RAM and an Intel Zeon CPU @ 3.50 GHz.

### *3.1. Cantilever beam*

*Simple cube unit cell*. In this paragraph, all the simulation results are collected. Starting from the simple cube unit cell topology, two distinct simulations are completed to analyze the periodic structure modelled with the complete 3D geometry for the circular (Figure 7a) and square cross-sections (Figure 7b).



(**a**) (**b**)

**Figure 7.** View of strain field of 3D period structure with simple cube unit cells: (**a**) zoom view for the circular cross-section; (**b**) detailed view for the square cross-section

Before doing that, a complete mesh size convergence study was performed to optimize the minimum element size of the discretization process on a simplified (less lattice unit cells) version of the 3D full model, which is the more critical one in terms of computational requirements (Figure 8).

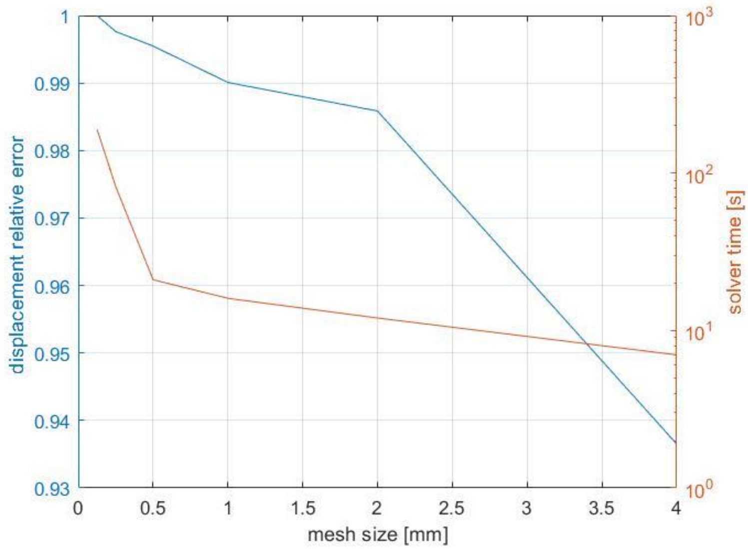
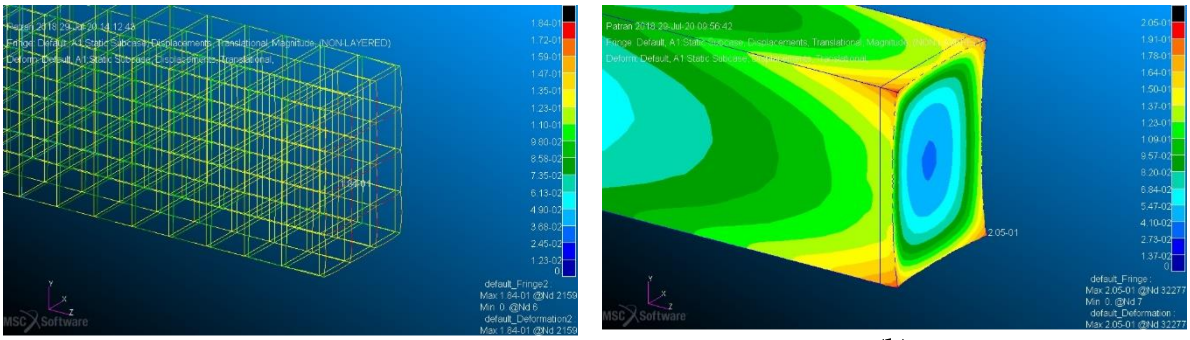



**Figure 8.** Mesh size convergence study on a simplified version of the full 3D lattice model of the cantilever beam with simple cube and square cross-section geometry

In particular, reducing the average mesh size from 4mm to 0.125mm, it was found out that an average mesh size of 0.5mm is detailed enough to capture the mechanical behaviour of the component with confined computational costs (less than 0.5% of error and 89% of saved time compared to 0.125mm average mesh size). The simulation of the 3D complete model is the most critical since it gives benchmarking results to compare alternative approaches to study periodic structures.

The same is done for the 1D model of the periodic structure (i.e. circular cross-section type shown in Figure 9a) and for the 3D fully dense equivalent model (see as example Figure 9b for the case with circular cross-section using closed-form results coming from [26]). Two distinct simulations of the 1D model with the same geometry are performed to assess the behaviour of the L1D approach in case of nodal forces applied on FE nodes or structure ones, as previously described.



(**a**) (**b**)

**Figure 9.** View of strain field of period structure with simple cube unit cells: (**a**) detailed view for circular cross-section for the 1D lattice model; (**b**) zoom view for the 3D equivalent fully dense material using AH method from [26]

It is important to pinpoint that [27] does not provide closed-form results in terms of stiffness matrix when the square beam section is used so that only one simulation is performed using the closed-form entries for the stiffness matrix coming from this contribution.

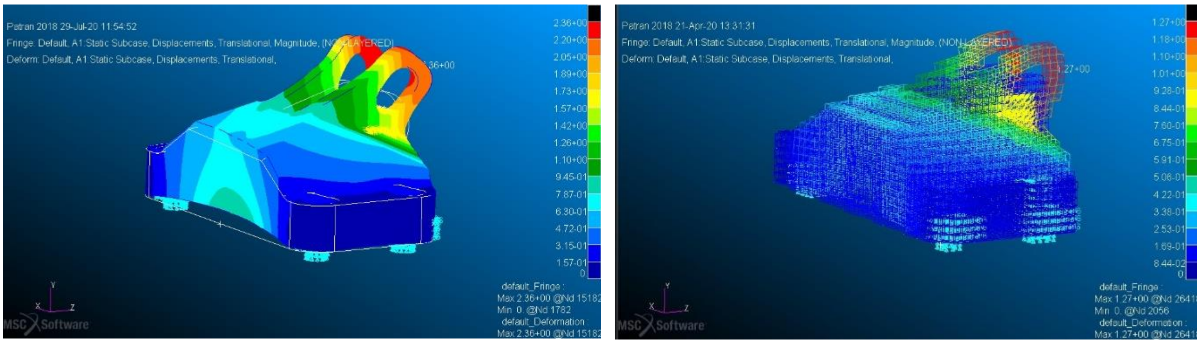
To have a comprehensive view of all the results, Table 2 and Table 3, respectively for circular and square cross-section, contain the results coming from the simulations performed using the simple cube unit cell for the periodic structure under investigation. The 3D analysis of lattice structure has been considered the reference benchmark for other techniques.

*Face centred cubic unit cell*. The same procedure was followed to investigate the periodic structure behaviour when a stretching dominated unit cell is chosen to fill the component. Same boundary conditions and same scheme of numerical simulations are used: 2 distinct simulations for the 3D complete model (one for circular cross-section and one for the square type), 2 simulations using the 1D model coming from the L1D approach (respectively for circular cross-section and square) and 3 simulations with the 3D fully dense beam (two simulations using the results coming from [26] for circular and square topologies and one using the results from [27] for the circular cross-section). Also in this case, two distinct simulations of the 1D model are performed depending on the nodal forces point of application. All the results are collected in Table 4 and Table 5.

### 3.2. G.E. Aircraft engine bracket

The same approach to compare the L1D methodology against AH methods and 3D full model has been followed in the case study represented by the GE bracket. However, at this step, we encountered some issues to numerically simulate the 3D full model of the engine bracket filled with uniform lattice. The resulting lattice structure is made by 28712 ligaments and using a discretization with a minimum element size of half of the lattice cross-section dimension, the computational power at our disposal was not enough and at the end, the meshing procedure failed. This is exactly the context where alternative methods need to be used to simulate the mechanical behaviour of a complex lattice structure because of limited computational power. Knowing the number of nodes, elements and the meshing time applied to a single 3D ligament, it was possible to estimate the number of elements and the meshing time of the entire lattice structure. Thanks to this evaluation, assuming a linear trend, we were able to estimate about 30 million of elements in the mesh and 4 hours of meshing time for the 3D full model with squared cross-section ligaments.

Since the results coming from the 3D full model were not available, the AH methodologies were used as a benchmark to evaluate the L1D capabilities applied on a real-life object. AH methods [26] and [27] are used with the same scheme to simulate the engine bracket using the simple cube unit cell with both circular and squared cross-section (Figure 10a). L1D method is applied to the 1D model of the same object (Figure 10b).




(**a**) (**b**)

**Figure 10.** View of strain field of engine bracket filled with period structure with simple cube unit cells: (**a**) view for the 3D equivalent fully dense material using AH method from [26], (**b**) view for circular cross-section for the 1D lattice model.

Using the values of the relative errors found with the cantilevered beam example, for simplicity it was assumed a linear trend for the displacement estimation error, just to have a numeric reference for the 3D model. In this way, it was possible to estimate the maximum displacement of the 3D full model, knowing the maximum displacements of the AH models of both [26] and [27], according to Eq. 2:

$$\frac{Umax_{3D}}{1} = \frac{Umax_{[25]}}{1 - err_{[25]}} = \frac{Umax_{[26]}}{1 - err_{[26]}} \quad (2)$$

To have a complete overview of all the results, Table 6 and Table 7, respectively for circular and square cross-section, contain the results coming from the simulations performed using the simple cube unit cell for the aircraft engine bracket under investigation.

**4. Discussion**

In this section, we will refer to the results reported in the previous Tables. The reader can immediately notice that both 1D modelling and AH approaches decrease more than 90% the time needed for meshing and more than 80% time needed to the solver to converge to a solution whatever is the unit cell or the cross-section type for both cantilevered beam and engine bracket. This result is extremely important in the context of conceptual/preliminary design when several scenarios need to be investigated in a fast way. Along with time reduction, also the number of elements needed to discretize the geometry decreases dramatically. However, this computational power reduction must be combined with good accuracy of the numerical results to provide consistent data.

Focusing on the simple cube unit cell topology applied to cantilevered beam and engine bracket, the AH method proposed by [26] estimates with more accuracy the lattice deformations (from 3 to 6% of error for both maximum and mean displacement), but the L1D approach shows good results too. Depending on the application of the nodal forces on the 1D simulations, the L1D estimates the structure maximum and mean displacement with sufficient accuracy. In particular, when the forces are applied on the structure nodes, the ligament bending is lower, decreasing the overall maximum displacement of some percentage points, while estimating the mean displacement with higher accuracy compared to 1D simulations with nodal forces applied on mesh nodes (respectively 9% and 23% error compared to 3D full model). The main advantage of the L1D approach is that the designer can have a good overview of the structure geometry and behaviour in terms of single beam element, which is not possible with AH methodologies (See Figures 9 and 10). The AH approach proposed in [27] overestimates the structural behaviour of about 13%, which is a figure still acceptable in the case of conceptual/preliminary design. In case of objects with complex geometry, such as the GE engine bracket, simplification methods such as AH or L1D must be used to decrease the computational time and power needed for mechanical simulations. On the one hand, the 3D full lattice model was so complex and computationally demanding that led to the simulation failure. On the other hand, both AH and L1D approaches decrease up to 99% the time for meshing and the number of elements, while maintaining a satisfactory degree of accuracy which is extremely important in a context of conceptual/preliminary design.

Concerning the FCC topology implemented only in the cantilevered beam, the reader can immediately conclude that a complex structure leads to higher computational power and time when the real 3D object is analyzed. This is again a context where approximation methods are extremely important. For example, by using the AH methodology or the 1D modelling, the computational power reduces almost 96%, along with the reduction of the number of elements of the mesh. Even for FCC unit cell type, L1D approach seems a valid alternative because the deformations are overestimated with a limited error for both types of nodal force application points (18%) with respect both AH methodologies in which the maximum deformations are underestimated (50% of error). Even in FCC topology, using the L1D approach the mean displacement is estimated with high accuracy in case the nodal forces are applied on the structure points instead of mesh nodes (respectively 9% and 13% of error).

To summarize, the 1D approach can estimate the maximum and mean deformations with a limited error, low computational power, but still giving to the designer an overview of the lattice geometry and behaviour, even at the ligament level for both stretching and bending dominated unit cells, while AH approaches are more precise for bending dominated topologies compared to stretching ones. Moreover, comparing both AH methods, [26] has outstanding estimation performances compared to [27] in all the studied contexts.

Both AH and 1D methodologies have similar computational requirements both for meshing and results in convergence if compared to the 3D object.

From this study, it is also possible to understand that the cross-section topology does not affect the computational costs and the results accuracy for both AH and 1D approaches, confirming the results obtained in [43]. The only main difference is that when the square cross-section is used (whatever is the unit cell type), the computational cost increases for the 3D model.

Moreover, from the above results coming especially from the GE engine bracket simulations, it is clear that a simplification method, such as the L1D approach described in this work, is compulsory nowadays in case of mechanical investigation through numerical analysis of 3D lattice structures: this kind of analyses are usually carried out when designing lightweight structures for industrial applications such as aerospace, automotive and automatic machines.

The analysis of the results of the simulations carried out in this research, confirms that the L1D approach is a good alternative to homogenization approaches because comparable performances can be achieved, or even better, still giving to the designer the quasi-real geometry view. However, the developed approach still shows some limitations because only node-strut arrangement lattices can be modelled. In the future, it could be of interest comparing the 3 methods explained in this work varying the slenderness ratio of the lattice structure and using different loading conditions. Moreover, only uniform and non-conformal periodic structures can be analyzed.

## 5. Conclusions

This paper aimed to investigate alternative methods for numerical simulations of periodic structures. In literature, several contributions use the asymptotic homogenization approach by substituting the lattice with a fully dense material with equivalent mechanical characteristics. However, this approach does not give an overview of the structure behaviour in terms of deformation at ligament levels since a bulk material is used.

To fill the gap, this research focused on a 1D modelling for uniform periodic lattices which can give to the designer a good idea of how is behaving the structure in case of static loads. Several linear elastic simulations were performed to compare the maximum deformation of the 1D modelling with the complete 3D model of the lattice structure, which was taken as a benchmark, and two valid AH methods available in the literature. Two different unit cell topologies and two different cross-section types were investigated to understand how these variables influence the performance of the alternative 1D approach. Results show good agreement with the 3D complete model, but with a drastic decrease of the time and computational power required, with comparable or even better performance compared to AH approaches here analyzed. The benefits of the 1D modelling are amplified in case of complex unit cells, while the results show that the cross-section does not influence the accuracy of the methodology herein investigated.

However, this approach still shows some limitations because it can model only uniform, non-conformal periodic structures with a strut-and-node arrangement. Moreover, only the tensile load case was investigated; other load scenarios and other slenderness ratios of the lattice structure must be investigated in the future to declare this alternative approach mature for periodic structure analysis. Finally, the conversion from the 3D model to the 1D is based on the voxelization procedure, which is an approximation of the real geometry. For this reason, a certain % of error in terms of maximum displacement can be attributed to the L1D methodology.

It is worth noting that the development of methodologies to compute periodic structures in complex geometries is tightly connected to the evolution of computational power which can open new possibilities in coming years. However, nowadays it is not feasible the analysis of complex lattice structures using 3D meshing, using standard computational devices such as a PC. In this research, a solution to address the structural analysis of lattice structures in a typical industrial environment where large PC clusters are not available to each designer is proposed.

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