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ABSTRACT

Dielectric Elastomer Transducers (DETs) represent an emerging technology with great potential for mechatronic applications. DETs allow to convert electrical energy into mechanical energy and vice-versa, making it possible to design actuators, generators, and sensors. These devices show many advantages like high energy density, silent operations, and low cost, but their practical applicability is strongly affected by their reliability and lifetime, which depend on both environmental conditions and electro-mechanical loads. Theoretical and experimental studies have recently been initiated to investigate the lifetime ranges of such devices for different loading conditions (e.g., mechanical, electrical, electro-mechanical). At present, the lifetime characterization of DETs has been conducted by means of stochastic models only. In principle, a better understanding of electro-mechanical fatigue mechanism of DETs can be obtained through an appropriate analysis of their underlying physics.

In this context, this paper presents a novel modeling approach for electro-mechanical damage evolution of DETs. In order to describe the phenomena involved in the damage process in physically consistent way, a free-energy framework is adopted. Starting from well-established electro-mechanical free-energy functions, additional variables which account for both mechanical and electrical fatigue mechanisms are introduced. Singular models for damage accumulation are developed and integrated within the free-energy conservation principle, in order to dynamically simulate the life status of the dielectric material when subjected to combined electric and mechanical loads. Finally, the kinetic law for damage evolution history due to combination of different failure modes are introduced, and used to assess DETs reliability based on experimental observations.

Keywords: dielectric elastomer, continuous electro-mechanical damage, lifetime, electro-mechanical fatigue

1. INTRODUCTION

Dielectric Elastomers (DEs) are electrostatic devices that can be used as actuators, generators, or sensors. Generally, a DET consists of a highly elastic dielectric material sandwiched between compliant electrodes, forming a deformable capacitor. When a voltage is established between the two oppositely charged electrodes, an electrostatic stress proportional to the square of the electric field, known as the Maxwell stress, is developed in the material. At the same time, if the DET is subjected to deformation and then charged/discharged with proper synchronism, it allows to convert mechanical energy to electrical energy due to the electrostatic variable capacitance principle [1]. Compared to other transduction technologies, DEs features include high energy efficiency and power density, large deformations, easy manufacture and integration, low cost, good resistance to shocks and corrosion, and silent operation [1–3]. Different kinds of applications based on DE transducers (DET) have been developed over recent years, including pressure sensors [4], energy generators from natural resource as ocean waves [5] and soft actuators for robots [6], to mention a few.

Recently, commercially available rubber membranes made of silicone elastomer, natural rubber, and styrenic rubber demonstrated excellent electromechanical properties for the development of high energy density DET actuators and generators. In such applications, DETs are commonly subjected to cyclical electrical and mechanical loading. On the one hand, it is desirable to increase the DET electro-mechanical load as much as possible, in order to maximize the performance of the system. On the other hand, heavy loading conditions make the transducer highly susceptible to electro-mechanical fatigue and degradation. In particular, it is remarked how the performance of DETs are highly dependent on the maximum electric field that can be sustained by the elastic dielectric layer before breakdown, commonly referred to as dielectric strength. As an example, we report a recent experimental study on DET generators, in which inflatable membranes made it possible to consistently convert energy into electricity at an energy density per cycle, which is larger than 150 J/kg for silicone elastomers, and larger than 400 J/kg in case of styrenic rubber [5]. These experimented performances, however, can be sustained for a limited number of cycles only, after which the DET fails irreversibly. To date, very little knowledge is available concerning the effects of electro-mechanical loading on the material lifetime, thus making it not possible to find an optimal trade-off between performance and reliability in DET systems.

Among the most extensive research works published on the subject, it is worthwhile mentioning [7], where DE actuators made by an acrylic membrane (VHB-4910 by 3M), have been tested demonstrating that the maximum adopted value for electric field has a great impact on the device lifetime. Most of earlier literature focused on either mechanical or electrical failure modes of elastomeric material. More specifically, different methodologies have been presented in literature to test and model the lifetime behavior for elastomeric material under mechanical loading conditions, i.e., [8,9]. At the same time, most of the works dealing with electrical degradation and breakdown characterization of dielectrics have been developed in the context of electric insulator materials, capacitors, and power cable industry. Earlier studies focused on describing how electric treeing and conductive paths develop inside an insulator material during system operation, causing a short circuit and therefore an electric breakdown [10,11].

The experience on DETs lifetime tests has shown that if DE specimens are subject to an electric cyclical loading below the dielectric strength of the material, above which breakdown appears, the resulting number of cycles to failure will follow a stochastic distribution, [7,12,13]. From the post-processing analysis of the results, the mean cycles to failure distribution, as a function of the electric load level, can be estimated. The closer such a value to the electric breakdown level of the material, the less the average number of cycles needed to the failure of the specimen. Generally, if material lifetime is quantified by means of this approach, the final output of fatigue tests is the number of cycles to failure. Nevertheless, for DE systems the above interpretation is too restrictive, due to the fact that DEs are normally subject to multiple loading inputs at the same time (e.g., electrical, mechanical), resulting in different possible failure modes. The standard stochastic procedures for data analysis are limited to model the lifetime behavior only from final experimental results. Moreover, they also require the availability of large experimental dataset in order to interpolate the different stochastic distribution of every combination of loading conditions, each one obtained via a dedicated experimental campaign. Therefore, this process could turns out to be highly involved in terms of data collection time and data post-processing.

In order to overcome this issue and optimize the reliability assessment of DEs, their damage behavior, for both individual and coupled electro-mechanic loading, has to be better defined. By properly taking into account the cross correlation events between all the inputs in the dielectric material, a more effective prediction of material failure mechanisms can be obtained. In this way, a limited set of target experiments are only required to evaluate the lifetime behavior of the DE material under a wide range of operating conditions. In this context, the theory of continuum damage model (CDM) has been proposed [14]. This analytical methodology allows to define the damage accumulation process of a system in a physical framework, allowing to consistently describe the coupling effect among different damage mechanisms. The output of CDM is the evaluation of the damage state inside the dielectric material, which allows to estimate the amount of degradation phenomena occurring in the material, e.g., mechanical, electrical. However, we point out that the use of such a framework to characterize DET lifetime and damage evolution has not been investigated yet.

Following the framework outlined in [14], in this work we propose for the first time a CDM model for describing DET lifetime. To interpret the material damage mechanisms, two scalar continuum damage variables are defined for the electro-mechanical state of DEs in a free-energy framework. The damage evolution is then linked to an energy loss rate inside the material, in agreement with second principle of thermodynamics. Different assumptions are then performed to reflect mechanical, electrical, and coupled electro-mechanical failure mechanisms, by means of physical insights and numerical simulations. Based on such assumptions, suitable kinetic laws are then proposed for describing the damage accumulations in the material.

The remainder of this paper is organized as follows. In section 2, the CDM in literature is briefly introduced with an example of damage state definition for a general material. In section 3, the developed CDM for mechanical, electrical, and electromechanically coupled damage of DETs is presented. A practical method is also presented for estimating the maximum number of cycles to failure by means of the developed model. Finally, concluding remarks and future research directions are outlined in Section 4.

2. CONTINUUM DAMAGE MODELLING

In this section, the standard CDM framework for a generic material is presented. After discussing the idea behind CDM from a general perspective, its application to a simple example of mechanic damage on a damaged volume will be presented.

2.1 Continuum damage modelling, general framework

The basic concepts and details on CDM are discussed, based on the treatment in [14]. In CDM theory, each damage mechanism is represented via a corresponding state variable. The definition of such damage variables can be rather generic, and needs to be associated to a specific physical mechanism. Some representative example, related to mechanical damage, are given by ductile and brittle failures, creep and viscoelastic rupture modes, or crack initiation and fatigue behavior.

To quantify the damage accumulation over time, CDM theory exploits concepts from the thermodynamics of irreversible process. We start by considering a generic and reversible thermodynamic process in isothermal condition, the following relationship holds as a consequence of the first two laws of thermodynamics

$$d\psi = dW, \tag{1}$$

where $d\psi$ is the infinitesimal change in Helmholtz free-energy density of the system, and dW is the infinitesimal work density done on the system. In case N damage mechanisms are occurring, equation (1) needs to be modified as follows

$$d\psi = dW + \sum_{i=1}^{N} Y_{Ki} dD_{Ki}, \tag{2}$$

where the term $Y_{Ki}dD_{Ki}$ are representative for the energy loss due to damage. In particular, each $D_{Ki} \in [0, 1]$ defines an additional independent variable, known as the damage, on which the free-energy explicitly depends. Each D_{Ki} can be related to a specific failure mechanism. The larger D_{Ki}, the more the corresponding damage is accumulated in the system. Variables Y_{Ki}, on the other hand, represents an energy-conjugated variable to D_{Ki}, and are commonly referred to as energy release rates. Each Y_{Ki} can be computed from the Helmholtz free-energy, as follows

$$Y_{Ki} = \frac{\partial \psi}{\partial D_{Ki}}, \ i = 1, \ldots, N. \tag{3}$$

Since D_{Ki} are dimensionless variables, Y_{Ki} can be interpreted as the energy density released by loss of material properties when the damage occurs in the volume. Due to the fact that damage is modeled as an irreversible process, each additional term in (2) needs to satisfy the following inequality

$$Y_{Ki} dD_{Ki} \le 0 \ \Rightarrow \ Y_{Ki} \dot{D}_{Ki} \le 0, \ i = 1, \ldots, N. \tag{4}$$

By properly defining an explicit dependency of ψ on all D_{Ki} and the kinetic laws for each $\dot{D}_{Ki}$, in accordance with physical insights and experimental observations, the evolution of system damage based on the loading history can be modeled. Possible couplings among different damage phenomena can be represented by making suitable choices for kinetic laws $\dot{D}_{Ki}$.

2.2 Damage scalar variable for a generic material

While CDM framework has been presented on a general level in the previous section, a practical example of mechanical damage is discussed in this section, following the basis of damage theory from [14]. In order to explain how define a scalar variable which represents the i-damage state of a material, we consider a representative volume element (RVE) of a damaged body, as shown in Figure 1. Such a RVE is sufficiently small so that all its material properties can be considered as homogenized variables. Referring only to the mechanical response of the body, it is assumed that the damage generates free surfaces of discontinuities, causing different consequences for the mechanic strength and stiffness of the material in tension or compression. Thus, if we analyze its intersection face with the plane defined by its normal ***n***, the cross-section area will include also the surfaces of damaged volumes. As mentioned before, a scalar variable D_K can be associated to this mechanism, in such a way it quantifies the health state of the material. In particular, D_K can be defined as follows:

$$D_K = \frac{\delta\Omega_K^*}{\delta\Omega} = 1 - \frac{\delta\tilde{\Omega}_K}{\delta\Omega}, \tag{5}$$

where $\delta\Omega$ and $\delta\Omega_K^*$ are the undamaged and damaged RVE parts respectively. The net remaining undamaged volume, i.e., $\delta\tilde{\Omega}_K$, is obtained as the difference between $\delta\Omega$ and $\delta\Omega_K^*$. If the RVE is totally undamaged D_K is equal to zero, while D_K is equal to 1 in case the RVE is fully damaged. Within the D_K formation, the mechanic response of the system will change until critical condition for the material is reached, e.g., the yield stress. Indeed, from a simple force balance on the material element, it can be demonstrated that on the cross-section area the real stress generated from a force f_1 applied along direction **1** is:

$$\sigma_1 = \frac{f_1}{\tilde{s}_{K1}} = \frac{f_1}{s_1}\frac{s_1}{\tilde{s}_{K1}} = \sigma_1 \frac{\delta\Omega}{\delta\tilde{\Omega}_K}\frac{dl_1}{dl_1} = \frac{1}{1-D_K}\sigma_1. \tag{6}$$

In (6), σ_1 and $\tilde{\sigma}_1$ represent the nominal stress computed on the total undamaged nominal area s_1, and the true stress on the net undamaged reduced area $\tilde{s}_{K1}$, respectively. Both stresses are caused by the same applied force f_1, but are computed with respect to a different surface area. As it can be noticed, given the same external force f_1, the true stress will be higher than the nominal one, because the progressive damage in the material will decrease progressively the net resisting material. By considering the entire material body, the damage will propagate on multiple RVEs until a maximum cumulative damaged volume will be reached, after which the material could not sustains anymore the applied force. Thus, after such level is reached, mechanical rupture of the material will occur due to the fact that the real stress on the damaged area of the material will be over the material strength.

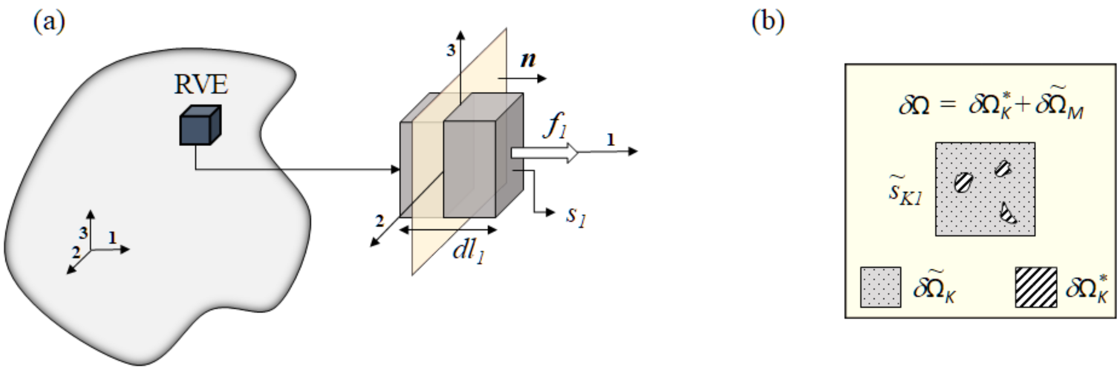

Figure 1. (a) Relevant volume element of a generic material. (b) Cross-section area of the damaged RVE with the plane defined by its normal ***n***, denoting the presence of irregularities surfaces to interpret the damage state.

3. DAMAGE MODELLING FOR DE

The novel application of CDM for describing damage in DEs represents the main objective of this section. At first, mechanical and electric damage process are described for DET as singular independent phenomena. In the last part, their coupling effects will be analyzed together to describe the total electro-mechanical damage process of DE. In the end, an example of lifetime calculation will be presented in order to demonstrate the practical use of the model.

3.1 Mechanical damage modelling for DE

In this paragraph, we consider the mechanical damage problem of a DET. In the context of DEs, the CDM can be straightforwardly applied to describe the mechanic damage process. A picture of a DE in deformed configuration is shown in Figure 2, with respect to a Cartesian reference system.

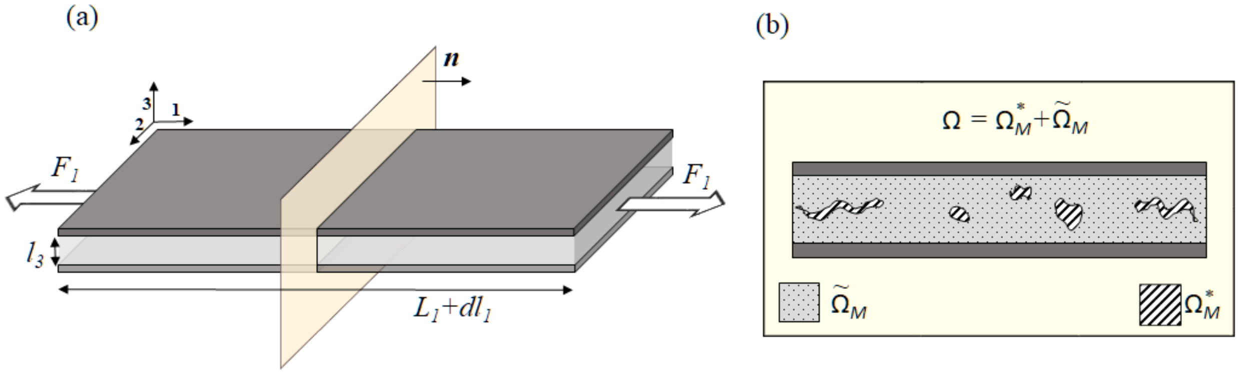

Figure 2. (a) Dielectric elastomer stretched by a force F_1. (b) The cross-section area with the plane of normal $\boldsymbol{n}$, in order to show the possible damage state of the material.

Since we are focusing on mechanical damage modeling only, we consider that the material can be described by means of a Helmholtz free-energy function comprising only a mechanical contribution. Such a free-energy function is denoted as ψ_M. For the implementation of CDM, we will introduce in the free-energy density the mechanic damage state. We start by considering an undamaged DET material. Following Figure 2, if a force F_1 is applied on a DE along direction **1**, the differential of the total Helmholtz free-energy Ψ_M is:

$$d\Psi_M = F_1 dl_1, \tag{7}$$

with dl_1 equal to the deformation along the same direction. We can define the Helmholtz free-energy density ψ_M as follows

$$\psi_M = \frac{\Psi_M}{\Omega}, \tag{8}$$

where Ω is the volume of undamaged DE material. Note that definition (8) is based on the entire DE volume Ω rather than on the RVE $\delta\Omega$, implying that an average energy density over the entire material is considered. We point out how the mechanical response of elastomers is characterized by a nonlinear elastic behavior, with a large deformation range up to 400% of elongation, [1,15]. Such large deformations make it necessary to define the stresses based on current, rather than reference geometry. This fact introduces additional nonlinearities in the model. As shown in [16], by defining material stretches λ_i and stresses σ_i, i = 1,2,3, the differential mechanic free-energy density of the undamaged DE is equal to

$$d\psi_M = \sum_{i=1}^{3} \frac{1}{\lambda_i} \sigma_i d\lambda_i. \tag{9}$$

If we assume that $\psi_M = \psi_M(\boldsymbol{\lambda})$, (9) implies that the material principal stretches can be obtained as follows

$$\sigma_i = \lambda_i \frac{\partial \psi_M}{\partial \lambda_i}, \; i = 1,2,3. \tag{10}$$

We now consider the situation in which the material undergoes a mechanical damage. In this case, according to the CDM theory, (9) is rewritten as follows

$$d\psi_M = \sum_{i=1}^{3} \frac{1}{\lambda_i} \sigma_i d\lambda_i + Y_M dD_M. \tag{11}$$

Note that ψ_M in (11) is obtained by normalizing the macroscopic Ψ_M based on the initial DE volume Ω (see (8)). On the other hand, in a mechanically damaged state the volume is defined by already mentioned total damaged volume $\tilde{\Omega}_M$, with $\tilde{\Omega}_M < \Omega$. We now define $\tilde{\psi}_M$ as the effective Helmholtz free-energy density of the material, given by

$$\tilde{\psi}_M = \frac{\Psi_M}{\tilde{\Omega}_M}. \tag{12}$$

As for a generic material, also for DEs the mechanical damage can be quantified with a variable representative for the isotropic evolution of microdefects and discontinuities over a mesoscopic material volume element. If the DE matter is damaged, the cross section area with a cutting plane will include such damaged volumes. These volumes are similar to microvoids that nucleate and grow due to the degradation processes, caused by an energy release rate dissipated in the material. Thus, as for (5), the definition of mechanical damage is a function of initial and damaged volume according to the following equation

$$D_M = 1 - \frac{\tilde{\Omega}_M}{\Omega}. \tag{13}$$

By combining (8), (12), and (13),we obtain the following relationship

$$\tilde{\psi}_M = \frac{\Psi_M}{\tilde{\Omega}_M} = \frac{\Omega}{\Omega}\frac{\Psi_M}{\tilde{\Omega}_M} = \frac{\Omega}{\tilde{\Omega}_M}\frac{\Psi_M}{\Omega} = \frac{1}{1-D_M}\psi_M. \tag{14}$$

Two energy density functions appear in (14). While $\tilde{\psi}_M$ represents the true Helmholtz free-energy density of the material, since it is obtained as the macroscopic Helmholtz free-energy divided the actual volume $\tilde{\Omega}_M$, ψ_M represents an “apparent” Helmholtz free-energy density computed on the basis of the original volume Ω. By means of equation (14), we can link the two energy functions via the damage D_M. The larger D_M, the smaller the apparent Helmholtz free-energy density ψ_M for the same true energy $\tilde{\psi}_M$. This means that if the material is damaged mechanically, an apparent decrease in stored energy is deduced if we still use the original volume Ω for computing ψ_M. Since the true material behavior is described by $\tilde{\psi}_M$, we can assume that $\tilde{\psi}_M = \tilde{\psi}_M(\boldsymbol{\lambda})$ only, where vector $\boldsymbol{\lambda}$ is defined as follows $\boldsymbol{\lambda} = [\lambda_1 \quad \lambda_2 \quad \lambda_3]^T$

Therefore, by combining this assumption with (14), we obtain

$$\psi_M(\boldsymbol{\lambda}, D_M) = (1 - D_M)\tilde{\psi}_M(\boldsymbol{\lambda}). \tag{15}$$

By combining (15) with (11), we obtain the following equations

$$\begin{cases} \tilde{\sigma}_i = \dfrac{1}{1-D_M}\sigma_i = \lambda_i \dfrac{\tilde{\psi}_M(\boldsymbol{\lambda})}{\partial \lambda_i} \quad i = 1,2,3, \\ Y_M = -\tilde{\psi}_M(\boldsymbol{\lambda}) \end{cases} \tag{16}$$

where $\tilde{\sigma}_i$ are the true stresses computed on the undamaged area, previously defined by equation (6). The second law of thermodynamics, together with (16), implies the following:

$$Y_M dD_M \le 0 \Rightarrow -\tilde{\psi}_M(\boldsymbol{\lambda}) dD_M \le 0 \Rightarrow \dot{D}_M \ge 0, \tag{17}$$

since energy $\tilde{\psi}_M$ is commonly chosen as a non-negative and convex function. Physically, (17) implies that the damage can only increase over time. This is consistent with the nature of the phenomenon, due to the fact that the damage is an intrinsically irreversible process and material self-healing is not allowed.

Different approaches have been proposed in literature, to find suitable shapes for the kinetic law of D_M. In this work, based on experience on fatigue behavior and experimental work on DE lifetime, the kinetic law of damage can be represented by

$$\begin{cases} \dot{D}_M = \left(-\dfrac{Y_M + \bar{\psi}_M}{a_1}\right)^{a_2} & -Y_M > \bar{\psi}_M \\ \dot{D}_M = 0 & -Y_M \le \bar{\psi}_M \end{cases}, \tag{18}$$

where a_1 and a_2 are parameters estimated by experimental results and $\bar{\psi}_M$ is a threshold level for the energy density, below which no damage is accumulated. Such an equation is standard for fatigue problem, because is always positive and, most importantly, it can be easily compared with lifetime results on a logarithmic scale. The unknown parameters a_1 and a_2 characterize the log-linear relation of the data, and need to be tuned via experiments.

3.2 Electrical damage model for DE

Whenever a voltage V is applied between the electrodes of a DET, the dielectric material is subjected to an electric loading. Under certain conditions, this electric loading could potentially affect the dielectric properties of the material. As already discussed, different damage processes can be triggered in the DE, and each one of them could theoretically lead to the dielectric breakdown failure mode. This phenomenon can be interpreted as the creation of a conduction path between the electrodes that allows to their charges to runaway and flow in the device [11,17]. Based on results on electric breakdown for dielectric materials, it can be inferred that electric failure occurs in the DET due to the decay of its electrostatic properties, in analogy to the mechanic damage accumulation process. Indeed, also in this case, the DE can be subjected to a degradation process similar to fatigue behavior as already demonstrated from previous experiments. The electric damage accumulation evolves in time until the electric breakdown is established. Nevertheless, the causes and the effects of electrical damage have completely different physic reasons with respect to the mechanical one. Whenever a minimum conductive path connects the two DE electrodes, electric failure of the system occurs, even though the rest of the matter is perfectly undamaged. This is induced by an excessive electrostatic energy density which the DE material cannot absorbs, causing local deterioration of its dielectric properties when damage takes place.

For a generic dielectric system as the DE reported in Figure 3, the vector-valued quantities that characterize the electrostatic constitutive relations are polarization $\boldsymbol{P}$, electrical displacement $\boldsymbol{D}$, and electric field $\boldsymbol{E}$. For a linear and isotropic dielectric material, these quantities are related by the following equations

$$\boldsymbol{P} = \varepsilon_0 \chi_E \boldsymbol{E}, \qquad \boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_0 \left(1 + \chi_E\right) \boldsymbol{E} = \varepsilon_0 \varepsilon_r \boldsymbol{E} = \varepsilon \boldsymbol{E}. \tag{19}$$

Additional parameters appearing in (19), i.e., ε_0, ε_r, ε, and χ_E, represent the vacuum permittivity, the relative DE permittivity, the total material permittivity, and the electric susceptibility of the material, respectively. For a DE as in Figure 3, the electric field is only applied along direction **3**, i.e., the direction perpendicular to the electrodes. As a result, vectors $\boldsymbol{P}$, $\boldsymbol{D}$, and $\boldsymbol{E}$ are defined as follow:

$$\boldsymbol{P} = \begin{bmatrix} 0 & 0 & P_3 \end{bmatrix}^T, \qquad \boldsymbol{D} = \begin{bmatrix} 0 & 0 & D_3 \end{bmatrix}^T, \qquad \boldsymbol{E} = \begin{bmatrix} 0 & 0 & E_3 \end{bmatrix}^T, \tag{20}$$

where $D_3 = -\sigma_f$, with σ_f representing the free charges surface distribution over the electrodes due to the imposed voltage, and P_3 and E_3 following from equations (19).

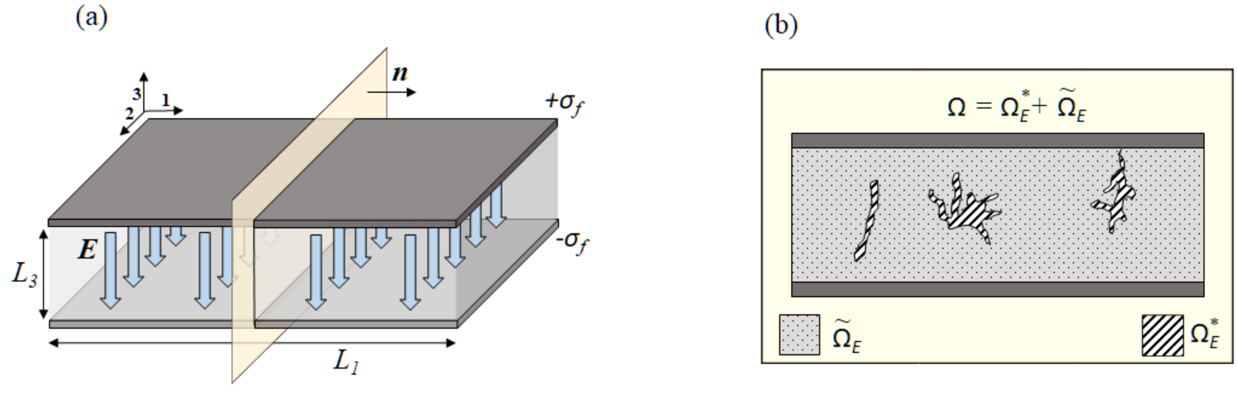

Figure 3. (a) The electric field is a constant electric field in the opposite direction 3, established due to the charge distribution over the opposite compliant electrodes. (b) Cross section area where are reported the supposed damaged conductive volumes.

The differential of total electrostatic Helmholtz free-energy Ψ_E when placing free charges q_f on the electrodes of the DE it is known to be

$$d\Psi_E = Vdq_f. \tag{21}$$

Similarly to the mechanical case, equation (21) can be properly normalized by introducing the electrical Helmholtz free-energy density, as follows

$$\psi_E = \frac{\Psi_E}{\Omega}. \tag{22}$$

By accounting for the non-constant geometry of DEs, and by considering that the electric field and the dielectric displacement vectors are nonzero only along direction **3**, and by assuming a linear dielectric material, normalization of (21) over the volume Ω implies results in the following [16]

$$d\psi_E = \sum_{i=1}^{3} \frac{\sigma_{E,i}}{\lambda_i} d\lambda_i + E_3 dD_3. \tag{23}$$

Terms $\sigma_{E,i}$ appearing in (23) are the diagonal components of the Maxwell stress tensor, due to the fact that the electric field $\boldsymbol{E}$ is only in the direction **3** [18]. It is possible to demonstrate that, for a principal system of coordinates (as in Figure 3), the Maxwell stress tensor T is given as follows

$$T_{i,j} = \varepsilon\left(E_i E_j - \frac{1}{2}\delta_{i,j}\|\boldsymbol{E}\|^2\right); \qquad i,j = 1,2,3; \qquad \begin{cases} \delta_{i,j} = 0, & i \neq j \\ \delta_{i,j} = 1, & i = j \end{cases}. \tag{24}$$

Under the assumption of linear dielectric, we also have

$$\psi_E = \psi_E(\boldsymbol{D}) = \frac{1}{2\varepsilon}\|\boldsymbol{D}\|^2. \tag{25}$$

By combining (23) and (25), the linearity between $\boldsymbol{D}$ and $\boldsymbol{E}$ can readily be verified.

Based on these considerations, a proper scalar variable for electric damage D_E has to be define in the CDM framework. In analogy with the mechanical case, D_E is chosen as the density of conductive material volumes inside the DE which do not contribute anymore to the dielectric strength of the media. A depiction of such a behavior is shown in Figure 3 (b). More formally, DE is defined as follows

$$D_E = 1 - \frac{\tilde{\Omega}_E}{\Omega}, \tag{26}$$

where $\tilde{\Omega}_E$ represents the amount of electrically undamaged material volume. Several earlier studies reported that the evolution of electrical damage follows electric treeing formation, which propagate according to a stochastic process, see, e.g., [10]. From literature on electric breakdown failure modes and previous lifetime experiments on dielectric materials, the electric damage in a solid material mainly occurs and evolves in the following locations [10,11,17]:

- Where the electric field is above a certain threshold, below which no damage is generated. As consequence, the electrostatic energy will modify the dielectric properties of the material, ionizing a RVE where damage appears and conferring electric conductive properties. At the breakdown level the failure would be so rapid to be considered instantaneous;
- On the edges of inclusions and defects in the material. As a matter of fact, at these locations an electric field concentration factor is established, inducing the initiation and evolution of an electric conductive paths. The boundaries of previous damaged conductive volumes appear to be favorite locations for D_E initiation, which is a reasonable statement that explains the formation of a continuum conductive path. Therefore, the electric degradation progress has a favorite gradient direction of evolution;
- At the electrodes irregularities and borders, i.e., where there is a fringing field effect. Also these locations are considered as concentration factor for the electric field, thus the behavior would be as the previous points.

While for the mechanical damage there are several methods to experimentally verify the undergoing phenomenon, e.g., through material Young's modulus evaluations or through direct micro inspection of cross section areas of the device, for the electric damage this is not trivial. Indeed, it would be necessary to determine the effective electric field inside the DE matter. This analysis would be too detailed and complicated for the purpose, because it implies to model and compute the superposition effects of the electric fields generated from the polarized dipoles, bound or free charges, and other possible microscale electric phenomena in the material [18]. On the other hand, through the dielectric constitutive equations (19), it is possible to analyze the electric effects of damaged zones inside the DE.

It can be demonstrated, indeed, that the electric field would be distorted only close to the boundaries of the conductive path or other inclusions in the matter. For a proper explanation, a simple yet representative example is proposed. We consider a DET in steady state conditions as in Figure 3, subjected to a constant level of electric $\boldsymbol{E}$ along direction **3** in the presence of a sphere-shape inclusion inside the dielectric material. A schematic depiction is reported in Figure 4.With respect to the inclusion volume, three cases are possible:

I. The inclusion is made of air with its proper permittivity ε_{air}, that is lower than the DE permittivity
II. The inclusion has the same property of the surrounding material, but with higher permittivity ε_{in}
III. The inclusion is an electrical conductor, thus with null internal electric field

All these situations are illustrated in Figure 4. For the three cases, the problem is solved by calculating the potential V from the Laplace equation of the electrostatic system. Successively, the electric field is calculated as the negative gradient of V:

$$\begin{cases} \nabla^2 V_{\text{I-III}} = 0 \\ \boldsymbol{E}_{\text{I-III}} = -\nabla V_{\text{I-III}} \end{cases} \tag{27}$$

Subscripts I to III stand for the three different situations mentioned above. By means of numerical simulations performed with COMSOL software, the solution of (27) is computed for the electric field in the described electrostatic problem for all the cases of potential distribution. In Figure 4, the contour plots of the electric fields in direction **3** are reported.

The differences between the cases are due to the effects of the induced charge distributions on the inclusion boundaries. In fact, those charge distributions will generate their own electric field, which modifies the external one in the DE matter. Moreover, from Figure 4 it can be noted that the larger the permittivity of the spherical inclusion, the more the inclusion volume behaves like a conductor. This is consistent with the physical explanation of the phenomenon. In fact, by increasing the permittivity ε of a material is equivalent in increasing its electric susceptibility χ_E and, in turn, its polarization $\boldsymbol{P}$. Therefore, if this locally polarized matter is above a maximum electric strength value, the dielectric saturates and the dipoles reach a maximum displacement. As a result, conductivity properties are conferred to the dielectric. This is, indeed, the physical explanation behind D_E. This deterioration process can be modeled as a gradual phenomenon where portion of dielectric material volumes gradually increase their electric susceptibility to finally lose all dielectric properties and became a conductor [11,17]. All these physical mechanisms support the definition of electric damage as equation (26), in analogy with the mechanical damage definition. The larger the electrical damage accumulated in the matter, the larger the amount of conductive volume that does not contribute anymore to the electrostatic energy.

As a consequence, in analogy to the mechanical case, we can define the true electrical Helmholtz free-energy density of the damaged material as follows

$$\tilde{\psi}_E = \frac{\Psi_E}{\tilde{\Omega}_E}. \tag{28}$$

By comparing (22) and (28), and by using the definition in (26), we obtain

$$\tilde{\psi}_E = \frac{\Psi_E}{\tilde{\Omega}_E} = \frac{\Omega}{\Omega}\frac{\Psi_E}{\tilde{\Omega}_E} = \frac{\Omega}{\tilde{\Omega}_E}\frac{\Psi_E}{\Omega} = \frac{1}{1-D_E}\psi_E. \tag{29}$$

By considering (29), and by assuming that the true electric energy $\tilde{\psi}_E$ only depends on electrical displacement $\boldsymbol{D}$, we obtain the following

$$\psi_E\left(\boldsymbol{D}, D_E\right) = \left(1-D_E\right)\tilde{\psi}_E\left(\boldsymbol{D}\right) = \left(1-D_E\right)\frac{1}{2\varepsilon}\left\|\boldsymbol{D}\right\|^2. \tag{30}$$

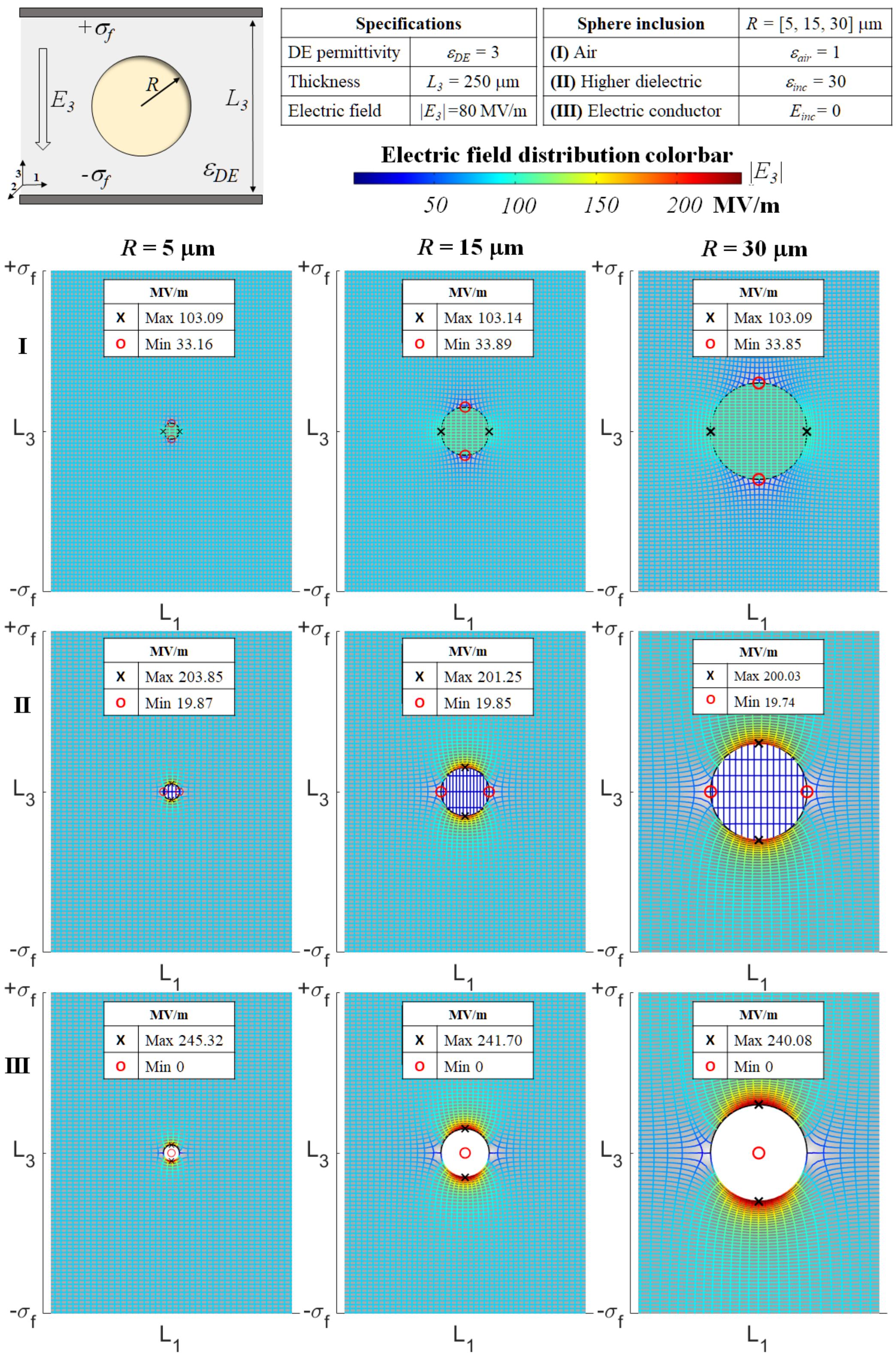

Figure 4. The three different cases (I)-(III) of sphere inclusion in a DE with constant electric field, or as a perfect conductor. The three column represent different values of internal radius of the sphere.

Furthermore, in analogy to the mechanical case, we can define a damaged version of equation (23), as follows

$$d\psi_E = \sum_{i=1}^{3} \frac{\sigma_{E,i}}{\lambda_i} d\lambda_i + E_3 dD_3 + Y_E dD_E, \tag{31}$$

where Y_E is an electrical energy release rate. Finally, by combining (30) and (31), we obtain the following expression for Y_E,

$$Y_E = \frac{\partial \psi_E(\boldsymbol{D}, D_E)}{\partial D_E} = -\tilde{\psi}_E(\boldsymbol{D}) = -\frac{1}{2\varepsilon}\|\boldsymbol{D}\|^2. \tag{32}$$

Since Y_E in (32) is always negative, in order to satisfy the second law of thermodynamics the rate of change of DE must be always positive, as expected. The following model is then proposed for its time evolution

$$\begin{cases} \dot{D}_E = \left(-\dfrac{Y_E + \overline{\psi}_E}{b_1}\right)^{b_2} & -Y_E > \overline{\psi}_E \\ \dot{D}_E = 0 & -Y_E \le \overline{\psi}_E \end{cases}. \tag{33}$$

Such an equation is similar to (18). Indeed, all the considerations done before are also true for only electric failure lifetime behavior, which still follows the classic fatigue degradation process. Thus, also in here the unknown parameters b_1 and b_2 need to be estimated from experimental results and characterize the log-linear relation of the data.

3.3 Coupling effects for electro-mechanical damage model for DE

So far, the processes of damage evolution have been discussed for the cases in which the DE is subjected to mechanical or electrical loading independently. Nevertheless, coupled electro-mechanical effects on DE lifetime have been experimentally verified in previous works, such as DE electric breakdown level changes that with different mechanic prestretch [19], or different geometric configurations of the compliant electrodes lead to different lifetime values under similar test conditions [13]. This may introduce cross correlation between the damage kinetic laws (18) and (33), resulting into an acceleration (or deceleration) of damage evolution. The free-energy formulation discussed above permits to naturally combine mechanical and electrical damage mechanisms in a thermodynamically consistent fashion. In fact, different energy functions can be naturally combined together in an additive way, allowing to describe more complex and coupled phenomena. This approach has allowed, in the past years, to develop several models of different DET configurations DETs, e.g., out-of-plane and in-plane deformation [20,21]. Following such references, the constitutive relations of a DET are obtained from the total free-energy density of the system. This total Helmholtz free-energy density ψ can be obtained as the sum of a mechanical contribution ψ_M and an electrical contribution ψ_E. By considering the damaged version of free-energy as in (15) and (30), we have then

$$\psi(\boldsymbol{\lambda}, \boldsymbol{D}, D_M, D_E) = \psi_M(\boldsymbol{\lambda}, D_M) + \psi_E(\boldsymbol{D}, D_E). \tag{34}$$

We point out how some authors also consider an additional free-energy contribution related to the DE viscoelasticity (e.g., [16]). However, since viscoelasticity mostly affects dynamic applications, it will not be considered in this work. The additive decomposition of DE free-energy in (34), permits to combine results obtained in mechanical and electrical cases in a complete model. In particular, by combining results in (11) and (31), we have

$$d\psi = \sum_{i=1}^{3} \frac{\sigma_i + \sigma_{E,i}}{\lambda_i} d\lambda_i + E_3 dD_3 + Y_M dD_M + Y_E dD_E. \tag{35}$$

Finally, by combining (34) and (35) with (15) and , we obtain the following

$$Y_M = \frac{\partial \psi(\boldsymbol{\lambda}, \boldsymbol{D}, D_M, D_E)}{\partial D_M} = \frac{\partial \psi_M(\boldsymbol{\lambda}, D_M)}{\partial D_M} = -\tilde{\psi}_M(\boldsymbol{\lambda}) \le 0, \tag{36}$$

$$Y_E = \frac{\partial \psi\left(\boldsymbol{\lambda}, \boldsymbol{D}, D_M, D_E\right)}{\partial D_E} = \frac{\partial \psi_E\left(\boldsymbol{D}, D_E\right)}{\partial D_E} = -\frac{1}{2\varepsilon}\left\|\boldsymbol{D}\right\|^2 \leq 0. \tag{37}$$

In order to describe electro-mechanically coupled damage, we need to find kinetic laws for D_M and D_E such that the following conditions hold

$$Y_M dD_M \leq 0,\ Y_E dD_E \leq 0 \Rightarrow \dot{D}_M \geq 0,\ \dot{D}_E \geq 0. \tag{38}$$

While the combination of mechanical and electrical free-energy has been performed in a straightforward way in (34) and (35), the definition of kinetic damage laws need to be properly redefined with respect to (18) and (33), since coupling mechanisms have to be accounted as well. To properly select the damage kinetic laws, it is assumed that:

a) The microvoids that determine D_M are essentially air volumes, characterized by a dielectric permittivity different from the one of the DE. Therefore, they behave like inclusions with different dielectric properties with respect to the surrounding DE media, Figure 4;

b) The volume of electric conductive inclusions generated while D_E occurs do not change the mechanical properties. Thus, from a mechanic response point of view, the entire material remains unchanged.

Regarding the electric effects on the mechanic damage, the Maxwell stress in (24) already explains the influence of the electric field on the total stress tensor in DEs. Generally, for an incompressible DET subjected to $\boldsymbol{E}$, the Maxwell stress will reduce the actual stress needed to stretch the material, making it softer. As explained previously, within D_E it is implied a certain distribution of $\boldsymbol{E}$, with distorted direction and module with respect to the rest of the dielectric volume. Thus, in such cases the DE stiffness will not be uniform in the material but rather dependent on the electric field, producing regions inside the material having a different local stiffness. With cyclic electro-mechanic loading of the DETs, such a phenomenon could enhance the fatigue of the material.

Concerning the mechanic effects on the electric damage, the coupled electro-mechanical effect are obtained from assumption (a). This case has already been analyzed in the example of an inclusion inside the DE material in Figure 4, case (I), i.e., when the permittivity of the inclusion is lower than in the surrounding media (as for air and DE material) the distribution of $\boldsymbol{E}$ changes. These phenomena affect both evolution of both D_M and D_E.

These assumptions reported above represents some examples of the possible cross-correlation events between the electro-mechanic loadings upon the damage state of the material. What it can be asses at this stage of experience and model is that equations (18) and (33) have to be modified to properly take in account the effects of those phenomena on the dynamics of damage evolution. To this end, parameters a_1, a_2, b_1, and b_2 can be chosen as functions h_1, h_2, g_1, and g_2 which take in account these cross effects depending from on the damage states D_M and D_E and the rate of energy loss. The total electro-mechanic CDM can be therefore specified as:

$$\begin{cases} \dot{D}_M = \left(-\dfrac{Y_M + \overline{\psi}_M}{h_1\left(Y_M, Y_E\right)}\right)^{h_2\left(Y_M, Y_E\right)} \\ \dot{D}_E = \left(-\dfrac{Y_E + \overline{\psi}_E}{g_1\left(Y_M, Y_E\right)}\right)^{g_2\left(Y_M, Y_E\right)} \\ -Y_M \leq \overline{\psi}_M \Rightarrow \dot{D}_M = 0,\ g_1\left(Y_M, Y_E\right) = b_1,\ g_2\left(Y_M, Y_E\right) = b_2 & (i) \\ -Y_E \leq \overline{\psi}_E \Rightarrow \dot{D}_E = 0,\ h_1\left(Y_M, Y_E\right) = a_1,\ h_2\left(Y_M, Y_E\right) = a_2 & (ii) \\ h_2\left(Y_M, Y_E\right) > 0,\ g_2\left(Y_M, Y_E\right) > 0 & (iii). \end{cases} \tag{39}$$

Model (39) must be in agreement with the thermodynamical conditions in (38). The electro-mechanic effects introduced in model through functions h_1, h_2, g_1, and g_2 need to be properly mapped based on experimental data with different combinations of electro mechanical loading conditions, in agreement with (*i*)-(*iii*). If needed, dependency of h_2 and g_2 on the previous damage state can be also implemented. For example, when the DE is subjected to only one kind of load condition or in a combination of electro-mechanic loading and one of them is not above the damage threshold, the resulting (39) must agree with the models mentioned before, i.e., (18) or (33), to let (*i*) and (*ii*) in (39) hold true.

3.4 Model validation process

In this section, it is explained how to practically use the model for the evaluation of lifetime and reliability of DE. The lifetime of DEs depends on several factors and phenomena, each one having a certain probability of occurrence. This means that if several DE samples are tested under the same environmental conditions and electro-mechanical loading, each one will exhibit a different number of cycles before failure. The results will follow a certain stochastic distribution which is determined by post processing analysis [22]. An example of a procedure for estimating parameter from these kind of processes is already explained in [13]. By using a similar procedure, unknown parameters and functions of the CDM of DEs of equation (39) can be estimated. From the kinetic law of damage evolution, a number of cycles to failure N_f can be computed, representing an estimation of the mean cycle to failure (MCTF) of the results distribution. For example, for the case of electric damage evolution in the equation (39), from the evolution of the damage over time, by considering uniform loading applied over the lifetime T_f, by assuming constant cycling rate f_c, and under the hypothesis that the g_1 and g_2 do not depends on the number of cycles, N_f is calculated as follows

$$\int_0^1 dD_E = \int_0^{T_f} \left(-\frac{Y_E + \overline{\psi}_E}{g_1} \right)^{g_2} dt \Rightarrow N_f = f_c \left(-\frac{Y_E + \overline{\psi}_E}{g_1} \right)^{-g_2}. \tag{40}$$

Generally, the experimental comparison between the number of cycles and load inputs experimental results is performed in logarithmic scale. If this is the case, the lifetime relation will be the following linear equation, which reminds the classic linear logarithmic curve for fatigue problem, with abscissa $\log(N_f)$ and ordinate $\log\left(\overline{Y}_E\right)$:

$$\begin{cases} \log\left(N_f\right) = -g_2 \log\left(\overline{Y}_E\right) + \left[\log\left(f_c \Omega_{Ec}\right) - g_2 \log\left(g_1\right)\right] \\ \overline{Y}_E = -\left(Y_E + \overline{\psi}_E\right) \end{cases}. \tag{41}$$

By comparing (41) with the experimental results, and by supposing that the phenomenon follows a certain stochastic distribution such as a Weibull curve, the unknown parameters can be estimated together with their confidence [13]. The form of the two unknown functions $h_i(Y_M ,Y_E,)$ and g_i $(Y_M ,Y_E,)$, i = 1, 2, will be dependent from the experimental result fitting. For example, if there are no correlation effect between electro and mechanic loading in the experiment, the two functions will coincide with constant parameters a_i and b_i. Otherwise, if no linear relationship such as in (41) is observed, h_i and g_i can be adapted in order to fit the behavior of the experiment as best as possible, e.g., based on universal or linear-in-parameter approximators. After a proper experimental campaign, these two unknown functions will be totally estimated, and the electro-mechanical coupling effects will be fully identified by the model.

4. CONCLUSION

This work has presented a continuum electro-mechanical damage model for dielectric elastomer transducers. By extending the free-energy framework commonly used for modeling DE systems, a continuous damage model has been developed for both mechanic and electric degradation of the transducer. Two scalar variables representing the damage states of such dielectric devices have been defined for each loading input, and have been related to corresponding failure mechanisms. The kinetic law of their evolution over time has been proposed in agreement with the second principle of thermodynamics. Then, the cross correlation phenomenon between the two completely different failure modes have been presented and discussed, and a coupled electro-mechanic model has finally been defined. The obtained model will allow to evaluate the reliability of DETs still at the design procedure stage, enhancing the industrial application of the technology. In future research, the developed model will be validated by means of an extensively experimental campaign. As soon as the model will be fully estimated, it will be possible to perform reliability tests for DET regarding the specific application of the technology. For example, it will be used to estimate the failure probability of DE devices in several applications, such as valves or wave energy generators.

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