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Prevention policy in an uncertain environment

Luigi Alberto Franzoni[*†]

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Abstract

This paper investigates the case in which the benefits and the costs of prevention are subject to uncertainty. Prevention measures are taken after uncertainty has unraveled. The *conventional policy* prescribes that prevention measures are taken up to the point in which the realized marginal cost of prevention is equal to the realized marginal benefit (measured in terms of the Value of Statistical Lives saved). This policy imposes costly uncertainty on imperfectly insured parties. The *optimal ex-ante policy* mitigates this uncertainty. It deviates from the conventional policy by prescribing less prevention in those contingencies in which risk-preventers face high compliance costs and victims face a high probability of injury, and higher prevention in the opposite case. The optimal ex-ante policy supports the use of a VSL measure constant across contingencies. It dilutes the "dead-anyway" effect and it responds to the risk-preventers' level of prudence.

◦

Keywords: Value of a Statistical Life, regulation of risk; externalities; risk aversion; cost-benefit analysis; emerging risks, regulatory risk.

[*] *Correspondence to:* Luigi A. Franzoni, Department of Economics, Piazza Scaravilli 2, 40126 Bologna, Italy.

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1 Introduction

Benefits and costs of risk prevention vary in time. Epidemic outbreaks, terrorist threats, and environmental emergencies can suddenly increase the risk exposure of entire populations, thereby increasing the demand for prevention. Likewise, new treatments and technological breakthroughs can significantly decrease the cost of providing prevention. From an ex-ante perspective, benefits and costs of prevention are therefore subject to extensive uncertainty. How does this uncertainty affect the optimal prevention policy?

An example will illustrate the case. In year 2020, Covid had just erupted and experts were trying to predict its evolution. Future was highly uncertain: in year 2021, the Covid emergency might have substantially subsided thanks to the discovery of effective treatments or it might have become more acute, as a result of the diffusion of new, more dangerous, variants. In the first case, one would have expected that a mild prevention policy would be enacted, resulting in low costs for the affected businesses and in low risk for the population. If new variants would have emerged, instead, one would have expected that tough prevention policy measures would be imposed (involving extended lock-downs and closures), leading to high costs for the businesses and, probably, a non negligible risk for the population. So, in year 2020, both the businesses and the potential victims were subject to a large amount of *ex-ante* uncertainty. This paper studies the implications of this type of uncertainty on the optimal prevention policy, on the assumption that agents are averse to risk and that insurance markets are imperfect. The analysis applies to risk policy in general. It is of special relevance for environmental policy, in view of the great uncertainty surrounding the trajectory of climate change (see IPCC (2018)).

In the model, some parties - called "risk preventers" - can take measures that reduce the probability of death or physical injury for other parties, called victims.[1] The poli-

[1] The term "risk preventer" replaces the terms "polluter" (employed in environmental economics)

cymaker mandates the prevention level that risk preventers must take (for example by closing dangerous activities, forbidding the use of certain technologies, imposing emissions limits, etc.). When deciding the prevention level, the policymaker balances the cost of prevention (borne by the risk preventers) with the benefits of prevention (netted by the victims). In line with current practice in environmental and health policy, the benefits of prevention are measured in terms of the Value of the Statistical Lives (VSL) saved.[2] In the model, prevention policy is subject to (external) random events that affect the probability of injury associated to given prevention levels and the cost of prevention.[3]

The paper compares two different policies. In the first, called *conventional policy,* the policymaker balances benefits and costs of prevention when they become known (in the Covid example, in year 2021). Under this policy, the policymaker mandates the prevention level that equates the marginal cost of prevention to the marginal benefit, given the specific contingency (say, variants have emerged or not). The conventional policy maximizes social welfare ex-post. The VSL used under this policy displays the well-known "dead-anyway effect" discussed by Pratt and Zeckhauser (1996): when the baseline risk is high, money loses value in the eyes of the victims (money becomes useless upon death or injury), and victims are willing to pay more to increase prevention. It follows that, when the cost-benefit analysis is carried out, the mandated prevention level is higher when the baseline risk is larger.

and "injurer" (employed in law and economics). In contrast to the latter terms, it does not have a negative connotation.

[2]The VSL methodology has been developed by Mishan (1971), and Jones-Lee (1974). It provides the theoretical foundation for risk regulation in the environmental, health and transportation fields (see OECD (2012) and OECD (2018) for an overview). Also the Covid19 literature heavily builds on it (see Hammitt (2020) and Brodeur et al. (2021)).

[3]The model is deliberately general about these points. A change in the risk of injury can be due, for example, to variations in baseline risk (probability of injury if no prevention measures are taken), in concurrent risk (probability of injury caused by other, non controllable, factors), or in the effectiveness of prevention (ability of prevention to reduce the probability of injury). The variation in prevention costs can be due to technological breakthroughs or to changes in the price of inputs. For expositional purposes, in the what follows I will focus on changes in baseline risk.

From an ex-ante perspective (say, from the standpoint of year 2020), the conventional policy generates costly uncertainty both for the preventers and the victims. This is the uncertainty addressed by the ex-ante policy. Under the ex-ante optimal policy, the policymaker decides ex-ante what measures should be taken *conditional on the occurrence of the different contingencies*: in the Covid example, the policymaker decides in year 2020 what measures should be taken in year 2021 if new variants have emerged, and what measures should be taken if they have not emerged. This policy maximizes social welfare ex-ante: it deviates from the conventional policy because it also accounts for the burden that uncertainty places on the parties. The deviation required to cater for the uncertainty faced by the risk preventers is different from that required to cater for the uncertainty faced by the victims, so I consider them in turn.

Due to potential changes in mandated prevention levels and in prevention technology, preventers face costs of uncertain magnitude.[4] The policymaker can reduce the risk burden of the preventers by decreasing the variance of the costs arising under the different contingencies. In the Covid example, it could slightly decrease the otherwise-optimal prevention level if new virus variants emerge, and increase it if new variants do not emerge. By distorting (at the margin) the conventional policy, the policymaker generates a first order gain for the preventers - thanks to the reduction of their risk burden - with a second order loss in terms of (ex-post) social welfare (the distortion takes place in the neighborhood of the optimum).[5]

Let us consider now how the conventional policy should be modified to address the uncertainty faced by the victims. The uncertainty they face ex-ante concerns the *probability* of injury under the different contingencies (in our example, high risk if variants emerge, low risk if they do not emerge). In turn, the variation in the probability of injury affects the level of the (ex-post) VSL. From an ex-ante perspective, the vari-

[4]As noted by Shavell (2014), insurance against changes in the law is not available in the market. See Section 4 for some tentative explanations of this fact.

[5]As I explain below, this point was first made by Shavell (2014).

ability of the VSL associated with the conventional policy is ineffi cient, as it implies that victims are paying a different "price" for the same service (a reduction in the probability of injury) under the different contingencies. The policymaker can increase the victims'payoff, keeping expected expenditure constant, by shifting prevention from contingencies with a low marginal utility of money (high risk), where one dollar "buys" a small reduction in the probability of injury, to contingencies with high marginal utility of money (low risk), where one dollar "buys" a large reduction in the probability of injury.[6]

This redistribution of the demand for prevention across contingencies yields an ex-ante VSL that depends on the *expected* probability of injury, and not on the probability of injury in the specific contingency. This ex-ante VSL turns out to be equal to the *weighted harmonic mean* of the contingent VSLs. The ex-ante VSL provides the policymaker with a measure of the benefits of prevention stable across contingencies, that is not influenced by the relative uselessness of money upon death (or injury). In the Covid example, the ex-ante policy requires that the same VSL is used both in the case in which new variants have emerged, and in the case in which they have not.

Recall now that the optimal ex-ante policy should cater to the (ex-ante) uncertainty borne by both preventers and victims. Compared to the conventional policy, in each contingency the ex-ante policy might require more or less prevention, depending on the desiderata of the two sides. The main result of the paper states that if the probability of injury under the specific contingency is above its mean and the prevention costs are higher than a specific threshold (which depends on the preventer's *prudence*), then a reduction in the conventional prevention level benefits both preventers and victims. If the probability of injury is below its mean and the prevention costs are below the threshold, an increase in the prevention level benefits both preventers and victims.

[6]Equivalently, the policymaker can reduce the expected prevention expenditure, keeping the expected probability of injury unchanged, by shifting prevention from contingencies in which one point reduction in the probability of injury is expensive to contingencies in which it is cheap.

When the desiderata of preventers and victims conflict, the optimal direction of change (of the conventional policy) depends on which side stands to gain more from the change (this, in turn, depends on the relative magnitude of their marginal utility of income in the specific contingency).

Finally, it is important to note that, instead of providing insurance through a modification of prevention policy (as argued above), the policymaker could alternatively provide parties with insurance by means of direct monetary transfers. This implies that, whenever the mandated level of prevention changes, preventers are compensated (if their costs increase) and victims are subsidized (if the risk they are subject to decreases). This policy produces the same outcome that would ensue if agents had access to a perfect insurance market. I explore this more demanding case in Appendix A2. Due to the presence of income effects, the comparative statics tends to be indeterminate. I show however that, under the optimal policy, when the baseline risk of a contingency goes up, prevention in that contingency increases if the contingency is highly unlikely, prevention is highly effective in that contingency, or victims are weakly averse to risk.

Policy. The observation that prevention policy can be improved if an ex-ante perspective is taken has important policy implications.

First, it shows how cost-benefit analysis can be modified to serve an *insurance function*. This point is in line with Arrow and Lind (1970)'s observation that, for projects whose benefits and costs cannot be perfectly spread over the entire population (either because they have a private nature or because they are correlated with systemic risk), people's risk aversion should matter for the evaluation of the project.[7] In the literature that builds on Arrow and Lind (1970), the issue is how to evaluate a project that generates a random stream of benefits and costs. In my model, the policymaker

[7]The importance of this approach has been recently underlined by Baumstark and Gollier (2014), who argue that it should apply to *all* public projects. See Kind et al. (2017), and OECD (2018) for applications and an overview. Risk aversion also impinges on the discount rate to apply to benefits and costs of climate change mitigation. See, for instance, Dietz et al. (2018), and references therein.

decides the level of prevention, which is very much like a privately provided public good. So, the issue is not how to evaluate a project, but rather how to adjust the "size" of the project (the optimal quantity of the public good) to the different contingencies. In a sense, projects with high risk and high costs should be scaled down, and projects with low risk and low costs should be scaled up.

Second, addressing the long-standing issue of the costs that *regulatory uncertainty* places on affected parties, it shows how these costs can be curbed by a properly designed policy. A vast literature, surveyed by Trebilcock (2014), has focused on a variety of measures, both temporary and permanent, that can be used to mitigate the cost of legal and regulatory changes. They include waivers, subsidies, grandfathering, and phase-ins.[8] The ex-ante policy provides an additional tool to deal with this important issue. In fact, the ex-ante policy can benefit at the same time both preventers and victims without burdening the public budget (as subsidies would do).

Third, the ex-ante approach supports the use of a VSL measure that does not depend on the specific contingency. This property of the efficient policy is in line with current regulatory practice, in which policymakers fix a VSL value and use it throughout. So, this paper indirectly contributes to the debate on whether the VSL should be adjusted to the specific conditions of the victims (concerning their health status, occupation, age, etc.).[9] While the paper does not address the issue of "risk equity" (i.e., the fair distribution of risk across the population), its results imply that, in the presence of multiple identical individuals subject to random risks, ex-ante utilitarian social welfare is maximal if a constant VSL is employed.[10]

[8] Due to the size of the losses, in most countries anti-Covid measures have been accompanied by direct subsidies for adversely affected parties. Grandfathering is often used in environmental policy, where pre-existing polluting plants are subjected to weaker requirements than new ones (see, for example, Damon et al. (2019)). Great attention has been devoted, in environmental policy, also to mechanisms (like safety valves, allowance banking, and collars) able to reduce the variance of the costs that businesses have to bear to comply with emission limits (see Aldy (2017)). In this paper, I do not discuss the relative desirability of the different tools.

[9] See, for instance, Viscusi (2011). In practice, policymakers use average estimates of the VSL.

[10] Ethical justifications of invariant VSLs have also been advanced by Somanathan (2006), Baker et al.

The use of an invariant VSL should apply also to emergency situations, as in the case of the miners trapped underground, sailors lost at sea, etc.. When the risk of death gets close to unity, the (conventional) VSL of the parties at risk becomes extremely high (possibly infinity). Under the conventional policy, this effect can justify the large expenses associated with the *rules of rescue* usually employed (Hammitt and Treich (2007)). From an ex-ante perspective, however, the VSL should not change as a result of the specific contingency. Rules of rescue, with their disregard for cost effectiveness, need therefore to be grounded on moral arguments that do not depend on the victims' willingness to pay.[11]

Literature. The idea of embedding an insurance function in risk policy is due to Shavell (2014). He focusses on the costs that regulations impose on compliant parties (injurers) and shows that the risk created by variations in compliance costs can be mitigated by attenuating legal change.[12] Franzoni (2019) extends Shavell's analysis to encompass the parties that bear the risk of accident. He proves that, in order to mitigate the ex-ante risk borne by the victims, legal change should be amplified. Both authors consider accidents that cause monetary losses. In the present article, I consider risks resulting in fatal and physical injuries, and thus work within the VSL tradition.

Before getting to the model, it is important to contrast this contribution with the literature on risk policy under uncertainty. A very vast literature, inspired by Weitzman (1974), investigates optimal emission abatement when marginal costs and benefits of abatement are uncertain. In this paper, I also assume uncertainty about benefits and

(2008), and Adler et al. (2014). In this paper, I focus on Pareto effi ciency and I leave distributional issues aside. My argument resonates with the results of Pratt and Zeckhauser (1996), who argued in favor of a Rawlsian approach in which social decisions are made under "a veil of ignorance" (about individual identities and thus individual baseline risk and income).

[11]This point also touches on the highly debated issue of "identified" vs. "statistical" victims. See, for instance, McKie and Richardson (2003) and Cohen et al. (2015).

[12]In Shavell's model, an activity might turn out to be harmful. So, the policymaker decides the optimal prevention level, conditional on the harmfulness of the activity. Shavell also discusses alternative tools like liability for harm and corrective taxation.

costs. However, in sharp contrast to the aforementioned literature, I assume that policy is contingency-dependent (it is applied after uncertainty has unfolded). So the issue is not how to minimize ex-post policy mistakes (prevention levels that do not maximize ex-post social welfare), but rather how to define a policy that maximizes ex-ante social welfare (which accounts for the uncertainty born by the parties).

Similarly, it is important to distinguish this contribution from the literature that studies the implications of "scientific uncertainty" on risk policy. This literature has emphasized the costs attendant with the *irreversibility* of prevention investments and with *ambiguity aversion* (people's dislike for situations in which probabilities are not known).[13] Neither factor is relevant in my analysis. Irreversibility does not matter because I assume that prevention investments are made after uncertainty has unraveled; ambiguity aversion does not matter because victims maximize (state-dependent) expected utility and are, therefore, ambiguity neutral. My perspective emphasizes the burden that ex-ante uncertainty places on risk-averse, imperfectly insured parties.

The paper proceeds as follows. Section 2 introduces the conventional ex-post approach, in which prevention is decided once the probability of injury and the prevention cost schedules are known. Section 3 considers the optimal ex-ante policy, to be decided before uncertainty unravels. Section 4 considers several important extensions. Section 5 concludes.

2 Conventional optimal policy

There are $K \geq 2$ possible contingencies, indexed by k. The probability that contingency k occurs is π_k. The probability of injury for the victims in each contingency, $p_k(x_k)$, depends negatively on the prevention x_k taken in that contingency by the risk preventer,

[13] Both effects have been used to justify the Precautionary Principle (see, for instance, Courbage et al. (2013)). I investigate the impact of scientific uncertainty on optimal prevention in Franzoni (2017).

with $p'_k(x_k) < 0$ and $p''_k(x_k) > 0$. The cost of prevention borne by the preventer is $c_k(x_k)$, with $c'_k(x_k) > 0$, $c''_k(x_k) > 0$. The shapes of both the probability and the cost functions are affected by the contingency.

Given contingency k, the state-dependent expected utility of a representative victim is

$$EU_k^{post}(x_k) = (1 - p_k(x_k))\, u_V(y_V) + p_k(x_k)\, v_V(y_V), \tag{1}$$

where y_V is the victim's income and where $u_V(y_V) > v_V(y_V) \geq 0$: the injury causes a loss of utility. In line with the standard VSL model, I also assume (unless otherwise stated) that $u'_V(y_V) > v'_V(y_V)$ (i.e., the injury reduces the marginal utility of money) and $u''_V(y_V) < 0$, $v''_V(y_V) < 0$ for all $y_V \geq 0$ (i.e., in both states, the victim is averse to risk).[14]

The payoff of a representative preventer in contingency k is

$$U_k^P(x_k) = u_P(y_P - c_k(x_k)),$$

where u_P is her utility function, y_P her income, and $c_k(x_k)$ the cost of prevention. Note that, given contingency k, the preventer is not subject to uncertainty.

The (ex-post) Pareto efficient policy is obtained by maximizing the expected utility of the victim subject to the constraint that the utility of the preventer is held constant by means of a transfer t. Formally:

$$\begin{aligned} \max EU_k^{post}(x_k) &= (1 - p_k(x_k))\, u_V(y_V - t) + p_k(x_k)\, v_V(y_V - t) \\ \text{s.t. } U_k^P(x_k) &= u_V(y_P - c_k(x_k) + t) = \underline{U}, \end{aligned} \tag{2}$$

where $\underline{U}$ is a constant and t is the transfer. The transfer allows us to focus on effi ciency,

[14]A host of empirical research suggests that fatal injuries drastically reduce the marginal utility of income (which drops to a level close to nil), severe health injuries reduce the marginal utility of income, while mild injuries might increase or decrease it (see OECD (2012), which provids a meta-analysis of more than 800 VSL estimates). The analysis also applies to accidents that result in the loss of irreplaceable items, see Cook and Graham (1977).

leaving distributional concerns aside (see Miceli and Segerson (1995)).

The general solution to this problem - that will also be used in the next section - follows some basic steps. From the constraint, we get the marginal cost of the policy. Specifically,

$$\left.\frac{\partial t}{\partial x_k}\right|_P = -\frac{\frac{\partial U_k^P}{\partial x_k}}{\frac{\partial U_k^P}{\partial t}} = -\frac{-u_P'\left(y_P + t - c_k\left(x_k\right)\right)\ c_k'\left(x_k\right)}{u_P'\left(y_P + t - c_k\left(x_k\right)\right)} = c_k'\left(x_k\right).$$

The ex-post marginal cost of prevention (from the perspective of the Preventer) is just $c_k'\left(x_k\right)$.

Next, we maximize the expected utility of the victim, keeping in mind that when x increases, also the transfer increases. The optimal policy thus solves:

$$\frac{\partial EU_k^{post}}{\partial x_k} + \frac{\partial EU_k^{post}}{\partial t}\left.\frac{\partial t}{\partial x_k}\right|_P = 0,$$

and thus

$$\left.\frac{\partial t}{\partial x_k}\right|_P = -\frac{\frac{\partial EU_k^{post}}{\partial x_k}}{\frac{\partial EU_k^{post}}{\partial t}} = \left.\frac{\partial t}{\partial x_k}\right|_V. \tag{3}$$

Under the efficient policy, prevention is required up to the point in which the marginal cost of prevention for the preventer is equal to the marginal benefit for the victim.

The (ex-post) marginal benefit $\left.\frac{\partial t}{\partial x_k}\right|_V$ is obtained from (2). For notational simplicity, it will be labled $b_k'\left(x_k\right)$. We have

$$\begin{aligned} b_k'\left(x_k\right) &= -\frac{\partial EU_k^{post}/\partial x_k}{\partial EU_k^{post}/\partial t} \\ &= -p_k'\left(x_k\right)\frac{u_V\left(y_V - t\right) - v_V\left(y_V - t\right)}{u_V'\left(y_V - t\right) - p_k\left(x_k\right)\left[u_V'\left(y_V - t\right) - v_V'\left(y_V - t\right)\right]} \\ &\equiv -p_k'\left(x_k\right)VSL_k^{post}(x_k). \end{aligned} \tag{4}$$

$VSL_k^{post}(x_k)$ is the value of a marginal reduction in the probability of jury under contingency k.

Note that an increase in prevention reduces the probability of injury, while it decreases the $VSL_k^{post}(x_k)$ at an increasing rate (omitting arguments):

$$\frac{\partial VSL_k^{post}(x_k)}{\partial x_k} = p_k' \frac{\left(u_V' - v_V'\right)\left(u_V - v_V\right)}{\left(u_V' - p_k\left[u_V' - v_V'\right]\right)^2} < 0, \tag{5}$$

$$\frac{\partial^2 VSL_k^{post}(x_k)}{\partial x_k^2} = p_k'' \frac{\left(u_V' - v_V'\right)\left(u_V - v_V\right)}{\left(u_V' - p_k\left[u_V' - v_V'\right]\right)^2} + \left(p_k'\right)^2 \frac{2\left(u_V' - v_V'\right)^2\left(u_V - v_V\right)}{\left(u_V' - p_k\left[u_V' - v_V'\right]\right)^3} > 0. \tag{6}$$

Greater prevention reduces the probability of injury and makes money more valuable (recall that the injury reduces the marginal utility of money). As a consequence, the victim is willing to pay less to be safe.[15] This follows from the "dead-anyway" effect (using the terminology of Pratt and Zeckhauser (1996)), first discussed by Jones-Lee (1974).

If policy is decided ex-post, once the contingency has occurred, Pareto efficiency requires prevention up to the level $\widehat{x}_k$ in which the marginal benefit equates the marginal cost - in line with classic analysis (see Bergstrom (1982)).[16] This is the "conventional policy."

Proposition 1 ***Conventional policy***. *If prevention levels are decided ex-post, the efficient policy requires, for all contingencies* k :

$$-p_k'\left(\widehat{x}_k\right)\ VSL^{post}\left(\widehat{x}_k\right) = c_k'(\widehat{x}_k). \tag{7}$$

The value of the lives saved at the margin should be equal to the marginal cost of prevention.

[15] These features of the VSL are consistent with the evidence presented by Viscusi and Evans (1990).

[16] This corresponds to the Samuelson condition for the optimal provision of public goods: the (sum of) the marginal rates of substitution is equal to the marginal rate of transformation. Here we have a single individual (or a set of identical individuals), and money is the private good to be transformed into prevention. See Andersson and Treich (2011) for a survey.

Equation (7) is the traditional efficiency benchmark for prevention policy.[17] Fig. 1 illustrates.

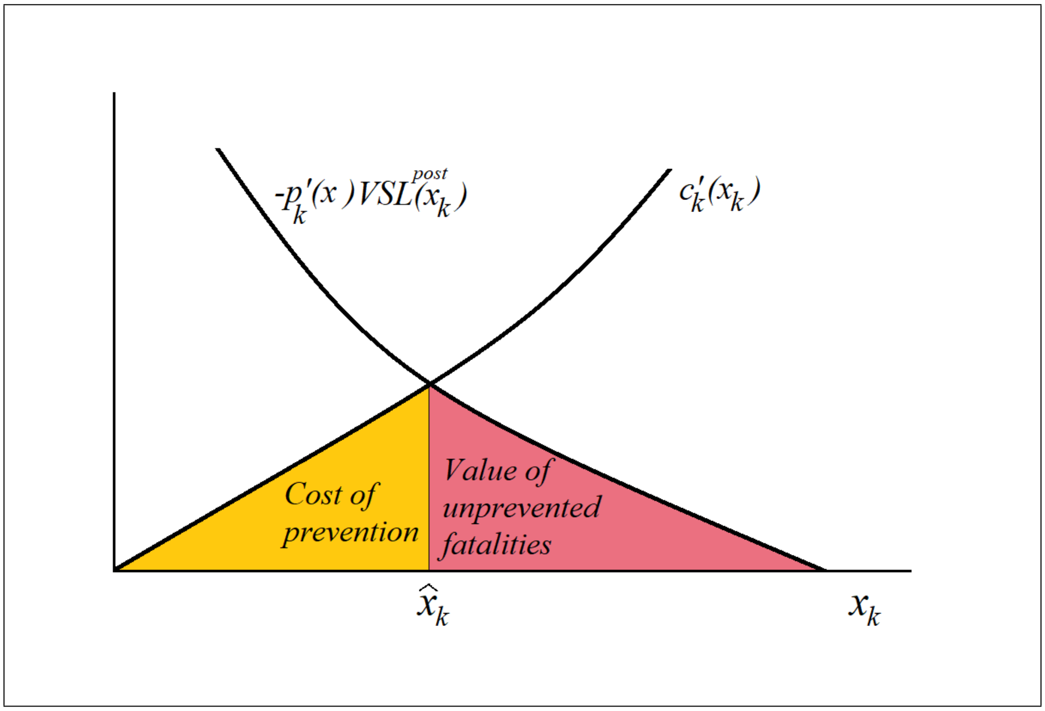

Fig. 1. Optimal prevention in the conventional policy.

In Figure 1, the optimal prevention level is obtained by balancing the impact of x_k on the (variable) cost of prevention with its impact on the value of the unprevented fatalities (the VSL of all injuries that could have been avoided). When prevention increases, the cost increases and fewer injuries occur.[18]

[17] Note that Equation (7) also defines the optimal prevention level, both ex-post and ex-ante, that victims would choose if they had to foot the prevention costs and the latter could not be insured ("self-prevention" case). See Courbage et al. (2013).

[18] The value of the unprevented fatalities is: $\int_{x_k}^{\infty} -p_{\varepsilon}'(x)\,VSL^{post}(x)\ dx$.

3 Ex-ante optimal policy

Let us now consider the ex-ante Pareto optimal policy, i.e., the optimal policy when the prevention levels x_k are decided before uncertainty unravels. The time line is as follows:

1) the policymaker decides the level of prevention for each contingency;

2) nature decides the contingency (and hence benefits and costs of prevention);

3) preventers invest in prevention in line with what was decided in stage 1; with some probability victims are injured.[19]

Pareto efficiency is obtained, once more, by maximizing the expected utility of the victim subject to the constraint that the payoff of the preventer is constant. Going through the same steps as before, we get that efficiency requires that, for all contingencies k, the marginal benefit of prevention is equal to its marginal cost. Marginal benefit and cost, however, are now calculated ex-ante (in stage 1).

The expected utility of the victims is now

$$
\begin{aligned}
EU_V^{ante}(\mathbf{x}) &= E_k\left[(1-p_k(x_k))\,u_V(y_V-t)+p_k(x_k)\,v_V(y_V-t)\right] \\
&= u_V(y_V-t)-E_k\left[p_k(x_k)\right]\left[u_V(y_V-t)-v_V(y_V-t)\right], \qquad (8)
\end{aligned}
$$

where $\mathbf{x}$ is the vector of the prevention levels: $\mathbf{x}=\{x_1,x_2,...,x_K\}$, E_k represents the expectation operator over the different contingencies, and t the transfer to the preventer. The payoff of the victim depends on the *expected* probability of injury.[20]

[19] It is clear that the policymaker should have the ability to commit to the levels fixed in stage 1. The ex-ante policy should be regarded as a "long term policy," i.e., a policy that defines the way in which conventional cost-benefit analysis should be modified so as to account for ex-ante uncertainty.

[20] Due to the uncertainty about the probability of injury, the agent is subject to ambiguity. In the present model, agents maximize their expected state-dependent utility and are, therefore, ambiguity neutral. The impact of ambiguity aversion on the VSL is studied by Treich (2010) and Bleichrodt et al. (2019).

The ex-ante marginal benefit of prevention x_k (to be taken in state k) is

$$\begin{aligned} B_k'(\mathbf{x}) &= -\frac{\partial EU_V^{ante}/\partial x_k}{\partial EU_V^{ante}/\partial t} \\ &= -\pi_k\ p_k'(x_k)\,\frac{u_V(y_V-t)-v_V(y_V-t)}{u_V'(y_V-t)-E_k[p_k(x_k)]\,[u_V'(y_V-t)-v_V'(y_V-t)]}. \end{aligned} \tag{9}$$

If prevention in contingency k is increased, the expected probability of injury decreases by $\pi_k p_k'(x_k)$. A reduction in the probability of injury generates a utility gain equal to $u_V(y_V-t)-v_V(y_V-t)$ (this is the same under all contingencies). This utility gain is converted in monetary terms by dividing it by the *expected* marginal utility of income (the denominator), since the benefit is calculated ex-ante before uncertainty has unfolded.

So,

$$VSL^{ante}(\mathbf{x}) \equiv \frac{u_V(y_V-t)-v_V(y_V-t)}{u_V'(y_V-t)-E_k[p_k(x_k)]\,[u_V'(y_V-t)-v_V'(y_V-t)]}. \tag{10}$$

From the ex-ante perspective, the value of a statistical life is equal to the income that the victim is willing to forfeit ex-ante to reduce the probability of injury in contingency k.[21] $VSL^{ante}(\mathbf{x})$ is equal to the conventional VSL (eq. 4) for a probability of injury equal to the *mean* probability of injury. Note that the VSL for contingency k now depends on the prevention levels of all contingencies.

We have (omitting arguments):

$$\frac{\partial VSL^{ante}(\mathbf{x})}{\partial x_k} = \pi_k p_k' \frac{u_V - v_V}{\left(u_V' - E_k[p_k(x_k)]\,[u_V'-v_V']\right)^2}[u_V'-v_V'] < 0.$$

The prospect of being injured under contingency k has an impact on the expected

[21] Since the ex-ante willingness to pay is determined before uncertainty unravels, it is calculated as the amount of income to be (equally) forfeited under all contingencies. Ideally, the victim would prefer to contribute more to the overall cost of prevention in contingencies in which the marginal utility of income is lower. This could be done, for instance, by means of an ex-ante insurance contract. The case with insurance is analyzed in Section 4.

marginal utility of income proportional to π_k, the probability that contingency k arises. The dead-anyway effect is still present, but it is substantially diluted.

Given the probabilities of injury, the relationship between the ex-ante and the ex-post VSL takes a relatively simple shape. Note, from (4) and (10), that

$$
\begin{aligned}
VSL^{ante}(\mathbf{x}) &= \frac{u_V(y_V - t) - v_V(y_V - t)}{E_k\left[(1 - p_k(x_k))\, u'_V(y_V - t) + p_k(x_k)\, v'_V(y_V - t)\right]} \\
&= \frac{1}{E_k\left[\frac{(1-p_k(x_k))u'_V(y_V-t)+p_k(x_k)v'_V(y_V-t)}{u_V(y_V-t)-v_V(y_V-t)}\right]} = \frac{1}{E_k\left[\frac{1}{VSL^{post}(x_k)}\right]}.
\end{aligned}
$$

So, the *ex-ante VSL is the weighted harmonic mean of the ex-post VSLs.*

Since $\frac{1}{VSL^{post}(x)}$ is a convex function of $VSL^{post}(x)$, for Jensen inequality: $E_k\left[\frac{1}{VSL^{post}(x_k)}\right] > \frac{1}{E_k[VSL^{post}(x_k)]}$, and thus $VSL^{ante}(\mathbf{x}) < E_k\left[VSL^{post}(x_k)\right]$.

To sum up, the ex-ante VSL is the same under all contingencies and is less than the mean ex-post VSL.

We can now turn to the preventer. Her expected utility is

$$EU_P^{ante}(\mathbf{x}) = E_k\left[u_P(y_P + t - c_k(x_k))\right]. \tag{11}$$

The preventer is now subject to risk: the costs that she bears vary across contingencies.

Note that the payoff of the preventer can also be formulated in terms of a certainty equivalent:

$$EU_P^{ante}(\mathbf{x}) = u_P(y_P + t - E_k[c_k(x_k)] - RP(\mathbf{x})), \tag{12}$$

where $RP(\mathbf{x})$ is the risk premium associated to the variability in prevention costs. The preventer bears the expected cost of prevention $E_k[c_k(x_k)]$ and the cost of uncertainty $RP(\mathbf{x})$.

The ex-ante marginal cost of prevention, for contingency k, is, from either (11) or

(12) :

$$C'_k(\mathbf{x}) = -\frac{\partial EU_P^{ante}/\partial x_k}{\partial EU_P^{ante}/\partial t} = \frac{u'_P(y_P + t - c_k(x_k))]}{E_k[u'_P(y_P + t - c_k(x_k))]}\pi_k c'_k(x_k), \qquad \text{or} \tag{13}$$

$$C'_k(\mathbf{x}) = \pi_k c'_k(x_k) + RP'_k(\mathbf{x}). \tag{14}$$

An increase in prevention x_k has two effects: it increases the expected cost by $\pi_k c'_k(x_k)$, and it affects the ex-ante risk burden by $RP'_k(\mathbf{x})$ (where the latter stands for $\frac{\partial RP(\mathbf{x})}{\partial x_k}$).

We have $RP'_k(\mathbf{x}) > 0$ if, and only if, $u'_P(y_P - c_k(x_k)) > E_k[u'_P(y_P - c_k(x_k))]$ (from 13). Since u'_P is decreasing in net income, the latter inequality is met if $c_k(x_k)$ is relatively large: $c_k(x_k) > \tilde{c}_k$, where $\tilde{c}_k$ is such that

$$u'_P(y_P + t - \tilde{c}_k) = E_k[u'_P(y_P + t - c_k(x_k))]. \tag{15}$$

If the preventer is *prudent* (i.e., if her marginal utility of income is convex), then by Jensen inequality: $E_k[u'_P(y_P + t - c_k(x_k))] > u'_P(y_P + t - E_k[c_k(x_k)])$.[22] Thus, $u'_P(y_P + t - \tilde{c}_k) > u'_P(y_P + t - E_k[c_k(x_k)])$, and $\tilde{c}_k > E_k[c_k(x_k)]$.

If prevention under contingency k goes up, expected costs increase by the amount $\pi_k c'_k(x_k)$. The distribution of the perspective expenses across contingencies also changes, and this increases the risk burden of the preventer if the cost in that contingency is relatively high ($c_k(x_k) > \tilde{c}_k$), while it reduces the risk burden of the preventer if the cost is relatively low ($c_k(x_k) < \tilde{c}_k$). The threshold $\tilde{c}_k$ is higher if the preventer is more prudent (i.e., if her marginal utility of income is more convex).

We can now determine the effi cient ex-ante prevention levels. For all contingencies, the marginal benefit of prevention should be equal to its marginal cost [see (9) and

[22] An agent is prudent if an increase in future risk in the sense of Rothschild-Stiglitz raises the marginal value of income (see Kimball (1990)).

(13)]:[23]

$$
\begin{aligned}
& B_k'(\mathbf{x}) = C_k'(\mathbf{x}) \iff \\
& -p_k'(x_k)\,\pi_k \frac{u_V(y_V - t) - v_V(y_V - t)}{u_V'(y_V - t) - E_k[p_k(x_k)]\,[u_V'(y_V - t) - v_V'(y_V - t)]} = \\
& \frac{u_P'(y_P + t - c_k(x_k))]}{E_k[u_P'(y_P + t - c_k(x_k))]} c_k'(x_k)\,\pi_k.
\end{aligned}
\tag{16}
$$

For both sides, the utility effect of an increase in prevention is weighted by the inverse of the expected marginal utility of income.

Using (10) and (12), we get:

Proposition 2 ***Ex-ante optimal policy.*** *If prevention levels are decided ex-ante, the efficient policy requires, for all contingencies* k :

$$
-p_k'(x_k^*)\ \ VSL^{ante}(\mathbf{x}^*) = c_k'(x_k^*) + \frac{1}{\pi_k} RP_k'(\mathbf{x}^*), \tag{17}
$$

where $VSL^{ante}(\mathbf{x}^*)$ *is the same for all contingencies and where* $RP_k'(\mathbf{x}^*)$ *is positive if, and only if,* $c_k(x_k^*)$ *is larger than a threshold* $\tilde{c}_k$ *that depends on the victim's prudence.*

Since the ex-ante VSL is the same for all contingencies, eq. (17) implies that the efficient prevention level will be higher in those contingencies in which $c_k'(x_k) + RP_k'/\pi_k$ is lower (marginal costs are smaller, net of the risk effect) and $|p_k'(x_k)|$ is higher (prevention is more effective at reducing the probability of injury). Note that, since the ex-post optimal prevention levels $\widehat{x}_k$ can still be chosen, the ex-ante policy can only improve upon the conventional policy.

Figure 2 illustrates the marginal benefit and cost of prevention under contingency k, given the prevention levels in the other contingencies.

[23] I assume that the social welfare function is strictly quasi-concave. Note that x_i affects marginal benefits B_j and marginal costs C_j, with $i \neq j$, only through an income effect. So, it the number of contingencies is large, cross effects tend to be small and, given our assumptions on the probability of injury and prevention costs, quasi-concavity is easily satisfied.

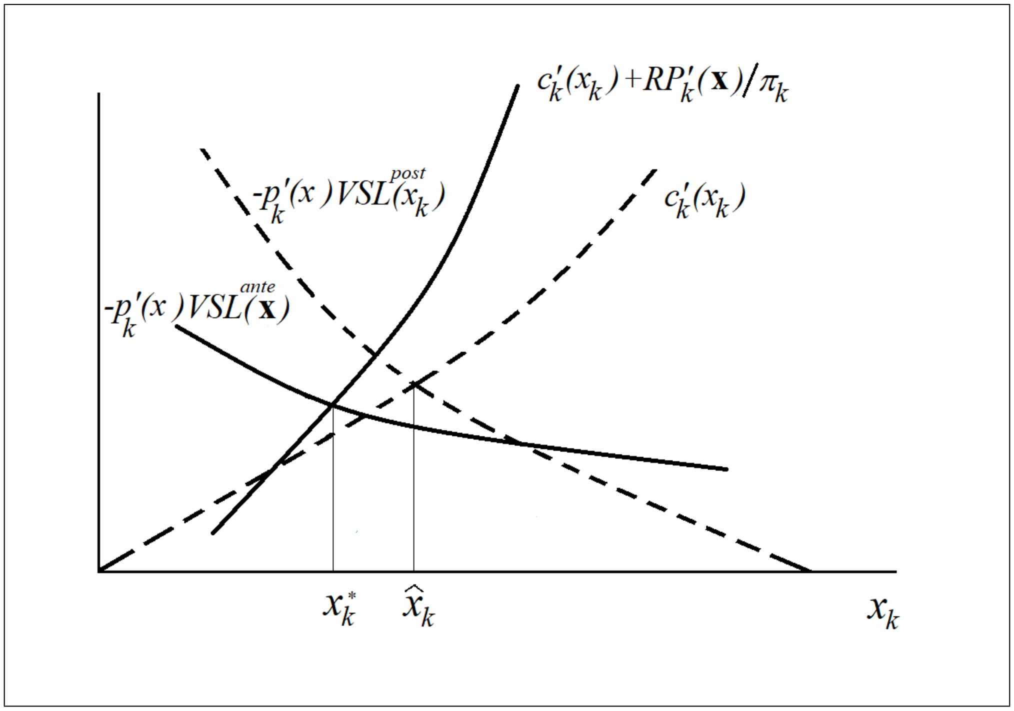

Fig. 2. Ex-ante marginal benefit and cost of prevention.

In the case depicted in Fig. 2, we have $RP_k'(\mathbf{x}) > 0$ if x_k and $c_k(x_k)$ are sufficiently large (with respect to other contingencies). The ex-ante VSL is lower than the ex-post VSL for low levels of prevention, for which the probability of injury tends to be relatively high. The higher marginal costs and the lower VSL push the optimal prevention level to the left of $\widehat{x}_k$. This is, in fact, one possible case among the many.

In order to clarify the direction in which the ex-ante perspective modifies conventional cost-benefit analysis, let us consider the impact of a marginal change in prevention at the ex-post efficient levels $\hat{x}_k$ (see eq. (7)). Quasi-concavity of the optimization problem guarantees that the local directions of change effectively point to the global optimum.

For contingency k, we have $B_k'(\hat{x}_k) > C_k'(\hat{x}_k)$ if, and only if (using the ex-post optimality condition (7) and omitting arguments):

$$-p_k' \frac{u_V - v_V}{u_V' - E_k\left[p_k\left(x_k\right)\right]\left[u_V' - v_V'\right]} > \frac{u_P'}{E_k\left[u_P'\right]} c_k' \qquad \Leftrightarrow$$

$$-p_k' \frac{u_V - v_V}{\left[\left(1-p_k\right) u_V' + p_k v_V'\right]} \frac{\left(1-p_k\right) u_V' + p_k v_V'}{\left(u_V' - E_k\left[p_k\left(x_k\right)\right]\left[u_V' - v_V'\right]\right)} > \frac{u_P'}{E_k\left[u_P'\right]} c_k' \quad \Leftrightarrow$$

$$-p_k' VSL^{post}\left(\widehat{x}_k\right) \; \frac{\left(1-p_k\right) u_V' + p_k v_V'}{u_V' - E_k\left[p_k\left(x_k\right)\right]\left[u_V' - v_V'\right]} > \frac{u_P'}{E_k\left[u_P'\right]} c_k' \qquad \Leftrightarrow$$

$$\frac{\left(1-p_k\right) u_V' + p_k v_V'}{u_V' - E_k\left[p_k\left(x_k\right)\right]\left[u_V' - v_V'\right]} > \frac{u_P'}{E_k\left[u_P'\right]},$$

that is, subtracting unity from both sides:

$$\frac{\left[E_k\left[p_k\left(\widehat{x}_k\right)\right] - p_k\left(\widehat{x}_k\right)\right]\left[u_V' - v_V'\right]}{u_V' - E_k\left[p_k\left(x_k\right)\right]\left[u_V' - v_V'\right]} > \frac{u_P' - E_k\left[u_P'\right]}{E_k\left[u_P'\right]}. \tag{18}$$

The ex-post prevention level $\hat{x}_k$ should be increased if, and only if, (18) is met.

Ex-ante, preventers would like to transfer prevention (resources) from contingencies in which prevention costs are high to contingencies in which they are low. Victims would like to transfer prevention from contingencies with a low marginal utility of income (high risk) to contingencies with a high marginal utility of income (low risk). Depending on the sign and the intensity of these desiderata, prevention levels should be adjusted upwards or downwards.

If the desiderata of both sides go in the same direction, the resulting policy correction is easily determined. Specifically, note that the LHS of (18) is positive if, and only if,

$$\left(E_k\left[p_k\left(\widehat{x}_k\right)\right] - p_k\left(\widehat{x}_k\right)\right)\left[u_V'\left(y_V - t\right) - v_V'\left(y_V - t\right)\right] > 0, \tag{19}$$

while the RHS is positive if, and only if, $c_k\left(\hat{x}_k\right) > \tilde{c}_k$ - see eq. (15).

So, since we have assumed $u_V'\left(y_V - t\right) > v_V'\left(y_V - t\right)$, an increase in $\hat{x}_k$ improves the welfare of the victims if $p_k(\widehat{x}_k) < E_k[p_k(\widehat{x}_k)]$, while it reduces the ex-ante risk exposure of the preventers if $c_k(\hat{x}_k) < \tilde{c}_k$. If both conditions hold, the increase in $\hat{x}_k$ is unambiguously beneficial. Similarly, a reduction in $\hat{x}_k$ improves the welfare of both parties if $p_k(\widehat{x}_k) > E_k[p_k(\widehat{x}_k)]$ and $c_k(\hat{x}_k) > \tilde{c}_k$. The following diagram illustrates.

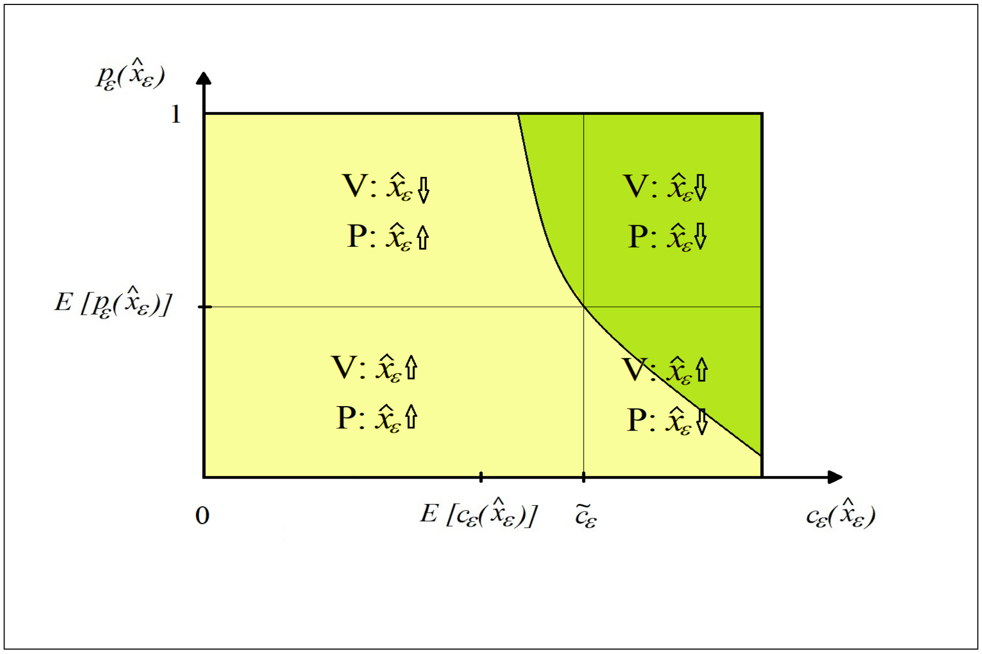

Fig. 3. Optimal change of conventional prevention levels for Victim and prudent Preventer.

The following proposition summarizes.

Proposition 3 ***Ex-ante vs. conventional policy.*** *Prevention should be* ***increased*** *above the conventional level if both the probability of injury and the prevention costs are low:* $p_k(\widehat{x}_k) < E_k[p_k(\widehat{x}_k)]$ *and* $c_k(\hat{x}_k) < \tilde{c}_k$*, while it should be* ***decreased*** *if both the probability of injury and the prevention costs are high:* $p_k(\widehat{x}_k) > E_k[p_k(\widehat{x}_k)]$ *and* $c_k(\hat{x}_k) > \tilde{c}_k$*.*

If the probability of injury and prevention costs do not meet the conditions listed above, the conventional prevention level should be changed in a direction that depends on the intensity of the "shocks" parties are subject to - see ineq. (18). Specifically, whether the ex-post level of prevention should be increased or decreased depends on which of the two parties experiences, in the specific contingency, a greater (relative) deviation of his/her marginal utility of income from its mean.

Clearly, if the (physical) injury increased the marginal utility of income ($u'_V(y_V) < v'_V(y_V)$), for the victims the desirable variation in prevention would go in the opposite way (see eq. (19)): prevention should be increased when $p_k(\widehat{x}_k) < E_k[p_k(\widehat{x}_k)]$ and decreased when $p_k(\widehat{x}_k) > E_k[p_k(\widehat{x}_k)]$. So, while for fatal and severe physical injuries we are reasonably certain that Proposition 3 applies, with minor injuries it may not apply: from the victim's perspective, an increase in prevention might be desirable when the risk is low.[24]

In order to better grasp the ex-ante logic, two special cases are analyzed.

■ *Additive risk.* Let us consider the case in which only the baseline risk varies across contingencies (both the effectiveness and the costs of prevention are unaffected). Let us assume, for the time being, that the preventer is risk neutral. Let $p_k(x_k) = p(x_k) + \varepsilon_k$, where ε_k is a continuous random variable independent of $p(x_k)$ such that $E_k[\varepsilon_k] = 0$ and $0 \leq p(x_k) + \varepsilon_k \leq 1$ for all k. Given the realization of ε_k, the conventional (ex-post efficient) policy $\widehat{x}_k$ entails:

$$-p'(\widehat{x}_k)\frac{u_V(y_V) - v_V(y_V)}{u'_V(y_V) - [p(\widehat{x}_k) + \varepsilon_k]\left[u'_V(y_V) - v'_V(y_V)\right]} = c'(\widehat{x}_k).$$

When ε_k is positive, $VSL^{post}(x_k)$ increases and $\widehat{x}_k$ goes up. The increase in prevention partially counters the increase in the baseline risk.

[24]When the injury causes a monetary loss, the marginal utility of income increases. So, the ex-ante perspective calls for greater prevention when the victim is subject to above-average risk and lower prevention when the victim is subject to below-average risk, as shown for the binary case by Franzoni (2019).

The optimal ex-ante policy $\mathbf{x}^*$ requires instead, for each contingency:

$$-p'\left(x_k^*\right)\frac{u_V\left(y_V\right)-v_V\left(y_V\right)}{u_V'\left(y_V\right)-p\left(x_k^*\right)\left[u_V'\left(y_V\right)-v_V'\left(y_V\right)\right]}=c'\left(x_k^*\right). \tag{20}$$

The optimal prevention level is the same under all contingencies (recall that in this example, both the effectiveness and the costs of prevention do not vary across contingencies). So, when the baseline risk goes up, prevention does not change and the victim is exposed to the full increase in the probability of injury. In a dynamic setup in which ε_k smoothly evolves in time, the victim is subject to greater risk fluctuations because, ex-ante, the VSL is not affected by k.

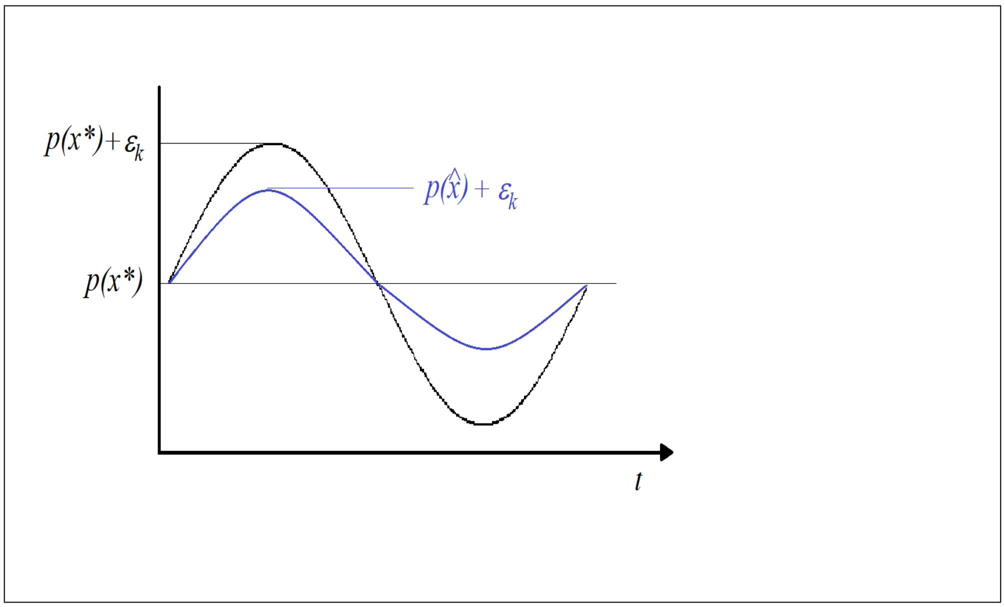

Fig. 4. Risk fluctuation under the ex-ante $(x^*)$and the conventional $(\widehat{x})$policy.

Note that, since the prevention level is the same across contingencies, no risk is borne by the preventer. This implies that solution (20) is optimal also when the preventer is averse to risk.

■ *Prevention costs' shocks.* Let us consider the case in which prevention costs are $c\left(x_0\right)$ with probability π, and $c\left(x_1\right)+z$ with probability $1-\pi$, with $z>0$. Nothing else changes across contingencies. The preventer is averse to risk.

The ex-post efficient policy $\widehat{x}_k$ requires:

$$-p'(\widehat{x}_k)\frac{u_V(y_V)-v_V(y_V)}{u_V'(y_V)-p(\widehat{x}_k)\left[u_V'(y_V)-v_V'(y_V)\right]}=c'(\widehat{x}_k)$$

under both contingencies. In line with conventional cost-benefit analysis, the preventer's net income does not affect the efficient policy: $\widehat{x}_0=\widehat{x}_1$.

The optimal ex-ante policy x_k^* requires instead, from (17) ,(omitting arguments):

$$\begin{cases} -p'(x_0^*)\frac{u_V-v_V}{u_V'-E[p(x_k^*)][u_V'-v_V']}=c'(x_0^*)+\frac{1}{\pi}RP_0'(\mathbf{x}), \\ -p'(x_1^*)\frac{u_V-v_V}{u_V'-E[p(x_k^*)][u_V'-v_V']}=c'(x_1^*)+\frac{1}{1-\pi}RP_1'(\mathbf{x}). \end{cases} \tag{21}$$

The preventer's risk premium is approximately equal to $RP(\mathbf{x})\simeq\frac{1}{2}\pi(1-\pi)\left[c(x_1)+z-c(x_0)\right]^2\rho_P$, where ρ_P is her absolute degree of risk aversion. Thus, at the conventional levels:

$$\begin{aligned} RP_0'(\widehat{\mathbf{x}}) &\simeq -\pi(1-\pi)\ z\ \rho_P\ c'(\widehat{x}_0)<0, \\ RP_1'(\widehat{\mathbf{x}}) &\simeq \pi(1-\pi)\ z\ \rho_P\ c'(\widehat{x}_1)>0. \end{aligned}$$

The variance of the prevention costs decreases if $\widehat{x}_0$ is increased and if $\widehat{x}_1$ is decreased. The optimal ex-ante policy entails therefore: $x_0^*>x_1^*$. It provides the preventer with partial insurance against the fluctuations of her expenses.

4 Extensions

■ **Life insurance.** As noted above, the only case in which ex-post and ex-ante optimal prevention levels coincide (from the victim's perspective) is when the marginal utility of income is not affected by the injury. So, one may wonder if the ex-ante and the ex-post prevention policies differ in the case in which the victim can buy insurance in every contingency, so as to equate the marginal utility of income in the "injury" and "non-injury" states.

Appendix A1 shows that Proposition 3 survives, suitably modified, also in this case. When people can insure against injury, they will buy an insurance policy that increases their income when their marginal utility of money is higher. With respect to fatal injuries (that reduce the marginal utility of money), this can be achieved through life annuities, pension schemes, and tontines.[25] The insurance premium depends on the probability of injury, so it varies across contingencies. Thus, the scope for ex-ante insurance remains. A similar argument applies to the case in which the injury increases the marginal utility of money and the victim purchases standard accident insurance.

■ **Ex-ante insurance.** The main assumption underlying the ex-ante policy approach is that imperfections in the insurance market prevent individuals from equalizing the marginal utility of income across contingencies. Such imperfections might be due to the administrative cost of insurance, missing contingencies, correlation across injuries, and moral hazard (see Shavell (2014) for an overview).

In Appendix A2, I consider the impact of ex-ante uncertainty on victims that are insured (so they can equalize marginal utility across contingencies). This case is equivalent to the case in which insurance is provided by the policymaker itself by means of contingent transfers. The comparative statics here is generally indeterminate. The victim is willing to contribute more to the overall prevention expenditure in those contingencies in which baseline risk is larger. This induces a modification of income levels that can increase or decrease prevention in the specific contingency. The following suffi cient condition, however, can be obtained. *An increase in the baseline risk of a contingency increases prevention in that contingency if: i) the contingency is highly unlikely, ii) prevention is highly effective in that contingency, or iii) victims are weakly averse to risk.*

[25] After Lorenzo Tonti, who popularized this insurance instrument in 17th-century France. Tontines collect funds from subscribers and, in time, provide returns only to the survivors (see Breyer and Felder (2005)).

5 Conclusions

The VSL model represents the theoretical backbone of risk prevention around the world in environment, health, and transport policy. In most cases, lawmakers and regulators use fixed VSLs that they apply throughout. One point made by this paper is that the use of an invariant VSL benefits the victims, once an ex-ante perspective it taken. The ex-ante VSL is the same for all contingencies and is equal to the weighted harmonic mean of the contingent VSLs.

Since also the preventers'marginal utility of income is affected by variations in risk and prevention costs, whether the ex-ante perspective calls for a greater or smaller prevention than the level that would arise from the conventional application of cost-benefit analysis depends on which of the two sides stands to gain more from a deviation from it. In those contingencies in which the probability of injury and the prevention costs are high, a reduction in prevention provides an ex-ante gain to both sides. The opposite applies to those contingencies in which the probability of injury and the prevention costs are low. In those contingencies in which the desiderata of the parties pull in opposite directions, the optimal change should cater to the side that is facing a higher marginal utility of income (in relation to its mean).

A large number of scholars have advocated the inclusion of redistributive weights in risk prevention policy (and, more generally, in cost-benefit analysis, see OECD (2018), ch. 11, and references therein). This paper argues not in favor of redistribution across agents, but across contingencies. A suitably devised long-term policy incorporating insurance elements can benefit, at the same time, both victims and preventers.

The ex-ante analysis applies to all situations in which risk policy is subject to significant uncertainty.

Appendix

A.1 Ex-ante efficiency with ex-post insurance. Let us consider the case in which individuals have access to an insurance market. Once the probability of injury is known, they can stipulate a fair insurance contract which specifies transfers to be made or received when an injury occurs. Depending on whether the injury reduces or increases their marginal utility of income, they will opt for "tontine insurance" (they lose money in case of injury) or a standard "accident insurance" (they receive money in case of injury).

The welfare level of the agent is now:

$$W_k^T = (1 - p_k(x_k))\, u_V(y + z_k) + p_k(x_k)\, v_V(y_V - d_k),$$

where z_k is the amount that the individual receives in case of no-injury and let d_k be the amount that she forfeits in case of injury. Both z_k and d_k can be negative.[26]

The insurance contract is actuarially fair, so:

$$\begin{aligned}(1 - p_k(x_k))\, z_k &= p_k(x_k)\, d_k, \text{ or} \\ z_k &= \frac{p_k(x_k)}{1 - p_k(x_k)} d_k.\end{aligned}$$

To fix ideas, let us consider the case of positive z_k and d_k.

Individual welfare can be written as

$$W_k^T = (1 - p_k(x_k))\, u_V\left(y + \frac{p_k(x_k)}{1 - p_k(x_k)} d_k\right) + p_k(x_k)\, v_V(y_V - d_k).$$

Given, $p_k(x_k)$, the optimal insurance contract solves (assuming an interior solution):

$$\frac{\partial W_k^T}{\partial d_k} = (1 - p_k(x_k))\, u_V'\left(y + \frac{p_k(x_k)}{1 - p_k(x_k)} d_k\right) \frac{p_k(x_k)}{1 - p_k(x_k)} - p_k(x_k)\, v_V'(y_V - d_k) = 0,$$

that is

[26] See Breyer and Felder (2005) for the general case with bequeathable and not bequeathable wealth.

$$u_V'\left(y+\frac{p_k(x_k)}{1-p_k(x_k)}d_k\right)=v_V'(y_V-d_k). \tag{22}$$

Under the optimal contract, the marginal utility of income is the same in both states.

We have

$$\frac{\partial d_k}{\partial p_k(x_k)}=-\frac{u_V''d_k\frac{1}{(1-p_k)^2}}{u_V''\frac{p_k}{1-p_k}+v_V''}<0. \tag{23}$$

If the probability of injury increases, one dollar lost in the injury state yields a larger return in the no-injury state. So, a smaller transfer is enough to equate marginal utilities. So, for fatal injures, the net income upon injury increases.

When $u_V'(y_V)<v_V'(y_V)$, the agent buys accident insurance. As $p_k(x_k)$ increases, a larger premium should be paid to get the same level of indemnity. So the indemnity d_k decreases, and the net income upon injury decreases.

The ex-post marginal benefit of prevention is (keeping in mind that $\partial W_k/\partial d_k=0$) :

$$\begin{aligned}
b^{T\prime}(x_k) &= \frac{\partial W_k/\partial x_k+\partial W_k/\partial d_k\ \partial d_k/\partial x_k}{\partial W_k/\partial y+\partial W_k/\partial d_k\ \partial d_k/\partial y}\\
&= -p_k'\frac{\left[u_V\left(y+\frac{p_k}{1-p_k}d_k\right)-v_V(y_V-d_k)\right]-(1-p_k)u_V'd_k\frac{1}{(1-p_k)^2}}{(1-p_k)u_V'\left(y+\frac{p_k}{1-p_k}d_k\right)+p_kv_V'(y_V-d_k)}\\
&= -p_k'\frac{\left[u_V\left(y+\frac{p_k}{1-p_k}d_k\right)-v_V(y_V-d_k)\right]-u_V'd_k\frac{1}{1-p_k}}{v_V'(y_V-d_k)}=-p_k'\ VSL_k^T.
\end{aligned}$$

The presence of the insurance contract affects the contingent VSL, which now includes an income effect $\left(u_V'd_k\frac{1}{1-p_k}\right)$, negative with tontine insurance and positive with accident insurance. The wedge between the utilities in the injury and no-injury case is larger in the presence of tontine insurance, and smaller in the presence of accident insurance.

Before uncertainty about the contingency unravels, the marginal benefit of prevention x_k

is (omitting arguments)

$$\begin{aligned} B^{T\prime}(x_k) = \frac{\partial W_k^T/\partial x_k}{\partial W_k^T/\partial y} &= -p_k' \frac{u_V - v_V - (1-p_k)\, u_V' d_k \frac{1}{(1-p_k)^2}}{E_k\left[(1-p_k)\, u_V' + p_k v_V'\right]} \\ &= -p_k' \frac{u_V - v_V - (1-p_k)\, u_V' d_k \frac{1}{(1-p_k)^2}}{E_k\left[v_V'\right]} = -p_k' VSL_k^{T\ ante}(\mathbf{x}). \end{aligned}$$

At the ex-post efficient level, we have (omitting arguments), $VSL_k^{T\ ante}(\widehat{\mathbf{x}}) > VSL_k^{T\ post}(\widehat{x}_k)$ if and only if:

$$\frac{u_V - v_V - (1-p_k)\, u_V' d_k \frac{1}{(1-p_k)^2}}{E_k\left[v_V'\right]} > \frac{u_V - v_V - (1-p_k)\, u_V' d_k \frac{1}{(1-p_k)^2}}{v_V'(y_V - d_k)} \iff$$
$$v_V'(y_V - d_k) > E_k\left[v_V'(y_V - d_k)\right].$$

Recall that an increase in p_k increases the net "injury" income with tontine insurance, while it decreases it with accident insurance. It follows that: *When the injury reduces the marginal utility of income, the agent purchases tontine insurance. From the victim's point of view, the ex-post optimal level of prevention should be increased if, and only if, the contingent probability of injury is small. When the injury increases the marginal utility of income, the agent purchases accident insurance. From the victim's point of view, the ex-post optimal level of prevention should be increased if, and only if, the contingent probability of injury is large.*

A2. Full efficiency (or ex-ante insurance). Let us consider the case with risk neutral (or perfectly insured) preventers. Their welfare level is: $W^P = E_k(y_P - c_k(x_k))$. The efficient policy is obtained by solving

$$\begin{aligned} &\max EU^V_{x_1,\ \ldots\ x_k; t_1,\ \ldots\ t_k} = E_k\left[(1-p_k(x_k))\, u_V(y_V - t_k) + p_k(x_k)\, v_V(y_V - t_k)\right], \\ &\text{s.t. } E_k(y_P - c_k(x_k) + t_k) = \underline{U}, \end{aligned}$$

where $\underline{U}$ is a fixed welfare level and t_k are the contingent-specific transfers from the victim to the preventer. This formulation implies that the expected level of the transfers should cover expected prevention costs (the preventer is risk neutral).

For simplicity, let us posit that there are only two states, 0 and 1, with probabilities π

and $(1-\pi)$, respectively. The constraint is thus

$$\pi\left(y_P - c_0\left(x_0\right) + t_0\right) + (1-\pi)\left(y_P - c_1\left(x_1\right) + t_1\right) = \underline{U}, \quad \text{i.e.}$$
$$t_0 = \tfrac{w+\bar{c}-(1-\pi)t_1}{\pi},$$

where $w = \underline{U} - y_P$ and $\bar{c} = \pi c_0\left(x_0\right) + (1-\pi)\,c_1\left(x_1\right)$.

So, the maximization problem can be rewritten as

$$\max_{x_0,x_1,t_1} W^V = \pi\left[\left(1-p_0\left(x_0\right)\right)u_V\left(y_V - \tfrac{w+\bar{c}-(1-\pi)t_1}{\pi}\right) + p_0\left(x_0\right)v_V\left(y_V - \tfrac{w+\bar{c}-(1-\pi)t_1}{\pi}\right)\right] +$$
$$(1-\pi)\left[\left(1-p_1\left(x_1\right)\right)u_V\left(y_V - t_1\right) + p_1\left(x_1\right)v_V\left(y_V - t_1\right)\right],$$

which yields the first order conditions (using deponents instead of arguments):

$$\begin{cases} \frac{\partial W}{\partial x_0} = & \pi\left[-p_0'\left(u_0 - v_0\right) - \left(\left(1-p_0\right)u_0' + p_0 v_0'\right)c_0'\right] = 0, \\ \frac{\partial W}{\partial x_1} = & -\pi\left[\left(1-p_0\right)u_0' + p_0 v_0'\right]c_0'\frac{1-\pi}{\pi} - (1-\pi)\,p_1'\left(u_1 - v_1\right) = 0, \\ \frac{\partial W}{\partial t_1} = & \pi\left[\left(1-p_0\right)u_0' + p_0 v_0'\right]\frac{1-\pi}{\pi} - (1-\pi)\left[\left(1-p_1\right)u_1' + p_1 v_1'\right] = 0, \end{cases} \tag{24}$$

which can also be rewritten, upon substitution, as:

$$\begin{cases} -p_0'\frac{u_0 - v_0}{(1-p_0)u_0' + p_0 v_0'} = c_0', \\ -p_1'\frac{u_1 - v_1}{(1-p_1)u_1' + p_1 v_1'} = c_1', \\ \left[\left(1-\ p_0\right)u_0' + p_0 v_0'\right] = \left[\left(1-\ p_1\right)u_1' + p_1 v_1'\right]. \end{cases}$$

The first two equations represent the usual conditions for effi cient prevention: $-\,p_k'\left(x_k\right)VSL_k = c_k'\left(x_k\right)$. Note that the denominator of the VSL is the same under all contingencies, in view of the third equation, but the numerator is not because the victim's net income now differs across contingencies. The last equation of the system requires that the marginal utility of money be the same under all contingencies. This implies that in contingencies with high probability of injury, the victim's net income must be lower (the transfer is higher). Note that the same set of equations would obtain if the victim could buy an insurance policy allowing her to shift resources across contingencies.

Given to the simultaneous determination of x_0, x_1, and t_1, comparative statics tends to be indeterminate. Below, I analyze the impact of a change in background risk. To simplify the exposition (without loss of generality), let $c_0(x_0) = x_0$, $c_1(x_1) = x_1$, and let the probability of injury in state 1 be $p_1(x_1) + \varepsilon$ (thus, ε is the background risk in state 1).

From system (24), we get

$$\begin{aligned}
\frac{\partial^2 W}{\partial x_0^2} &= \pi\left[-p_0''(u_0 - v_0) + 2p_0'\left(u_0' - v_0'\right) + (1-p_0)u_0'' + p_0 v_0''\right] \equiv \pi a < 0, \\
\frac{\partial^2 W}{\partial x_0 \partial x_1} &= (1-\pi)\left[p_0'\left(u_0' - v_0'\right) + (1-p_0)u_0'' + p_0 v_0''\right] \equiv (1-\pi)b < 0, \\
\frac{\partial^2 W}{\partial x_0 \partial t_1} &= (1-\pi)\left[-p_0'\left(u_0' - v_0'\right) - (1-p_0)u_0'' - p_0 v_0''\right] \equiv -(1-\pi)b > 0, \\
\frac{\partial^2 W}{\partial x_1 \partial t_1} &= (1-\pi)\left[-(1-p_0)\frac{1-\pi}{\pi}u_0'' - p_0\frac{1-\pi}{\pi}v_0'' + p_1'\left(u_1' - v_1'\right)\right] \equiv (1-\pi)c, \\
\frac{\partial^2 W}{\partial t_1^2} &= (1-\pi)\left[\frac{1-\pi}{\pi}\left[(1-p_0)u_0'' + p_0 v_0''\right] + (1-p_1-\varepsilon)u_1'' + (p_1+\varepsilon)v_1''\right] \equiv (1-\pi)d < 0, \\
\frac{\partial^2 W}{\partial x_0 \partial \varepsilon} &= 0,\ \frac{\partial^2 W}{\partial x_1 \partial \varepsilon} = 0,\ \frac{\partial^2 W}{\partial t_1 \partial \varepsilon} = (1-\pi)\left(u_1' - v_1'\right) > 0.
\end{aligned}$$

We have

$$\frac{\partial x_1}{\partial \varepsilon} = \frac{\det\begin{bmatrix} \frac{\partial^2 W}{\partial x_0^2} & \frac{\partial^2 W}{\partial x_0 \partial \varepsilon} & \frac{\partial^2 W}{\partial x_0 \partial t_1} \\ \frac{\partial^2 W}{\partial x_1 \partial x_0} & \frac{\partial^2 W}{\partial x_1 \partial \varepsilon} & \frac{\partial^2 W}{\partial x_1 \partial t_1} \\ \frac{\partial^2 W}{\partial t_1 \partial x_0} & \frac{\partial^2 W}{\partial t_1 \partial \varepsilon} & \frac{\partial^2 W}{\partial t_1^2} \end{bmatrix}}{\det[H]}, \tag{25}$$

where H is the Hessian matrix, which has a negative determinant at the optimum (the determinant is equal to the product of 3 negative eigenvalues). So,

$$\begin{aligned}
\frac{\partial^2 x_1}{\partial \varepsilon} > 0 \Leftrightarrow \det &\begin{bmatrix} \pi a & 0 & -(1-\pi)b \\ (1-\pi)b & 0 & (1-\pi)c \\ -(1-\pi)b & (1-\pi)(u_1' - v_{V1}') & (1-\pi)d \end{bmatrix} < 0 \\
\Leftrightarrow (1-\pi)^3 \det &\begin{bmatrix} \frac{\pi}{(1-\pi)}a & 0 & -b \\ b & 0 & c \\ -b & (u_1' - v_{V1}') & d \end{bmatrix} < 0 \\
\Leftrightarrow -\tfrac{\pi}{(1-\pi)}ac\,(u_1' - v_1') - b^2\,(u_1' - v_1') &< 0 \Leftrightarrow b^2 > -\tfrac{\pi}{(1-\pi)}ac,
\end{aligned}$$

where $a < 0$.

So, if $c < 0$, then $\frac{\partial x_1}{\partial \varepsilon} > 0$, where $c < 0$ iff

$$-\left(1-p_0\right)\frac{1-\pi}{\pi}u_0'' - p_0\frac{1-\pi}{\pi}v_0'' < -p_1'\left(u_1' - v_1'\right).$$

This leads us to the following result: $\frac{\partial x_1}{\partial \varepsilon} > 0$ *if i) victims are weakly averse to risk in state* 0*, or ii) prevention* x_1 *has a great impact on the probability of harm* p_1, *or iii) state 1 is very unlikely (*$\pi \to 1$*).*

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