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Numeracy at the dawn of writing: Mesopotamia and beyond

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Abstract

Numeracy and writing constitute different phenomena, whose paths of formation often appear intertwined. Here we reassess the theory that numeracy evolved universally from a concrete to an abstract concept of number, and that that shift is correlated with the invention of writing. First, we gather contemporary linguistic data and early Mesopotamian epigraphic evidence that indicates that the 'concrete' vs. 'abstract' dichotomy is not useful to understand the emergence of numbers. Then, we discuss evidence from other regions where writing was probably invented independently, in order to investigate the conceptualization and formation of early numerical notations.

Résumé

La numératie et l'écriture constituent des phénomènes différents, dont les chemins de formation apparaissent souvent inextricablement liés. Nous réévaluons ici la théorie selon laquelle la numératie est universellement passée d'un concept concret à un concept abstrait de nombre et que ce changement est corrélé à l'invention de l'écriture. Premièrement, nous rassemblons des données linguistiques contemporaines et des données épigraphiques mésopotamiennes anciennes qui indiquent que la dichotomie « concret » vs. « abstraite » n'est pas utile pour comprendre l'émergence des nombres. Ensuite, nous abordons des données provenant d'autres régions où l'écriture a probablement été inventée de manière indépendante, afin d'enquêter sur la nature de sa relation avec la numératie telle qu'elle est représentée par les premières notations numériques.

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Keywords: Numeracy; Abstract; Concrete; Writing; Tokens; Numerical notations

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1. Background

Numeracy and writing constitute different phenomena, whose paths of formation often appear intertwined. Yet their relationship, especially at their beginnings, is unclear. Did numbers always predate the birth of script? And how did the appraisal of numbers translate into tangible notations? What are we to make of how numbers were conceptualized and perceived? When were they first written down? What are their relational links to the material things that they count? Despite some progress [Chrisomalis, 2009a, 2010], all these questions are yet to receive the full attention they deserve and must be framed within a global perspective.

In this article we aim to discuss representations of numbers to the things that originally were counted and to position what has been termed as "abstract numbers" vis-à-vis "concrete" in a new critical frame. We start from the premise that the representation of numbers is contextual, situated, and generated by its specific setting. This, in turn, concurs with the view (held by some philosophers of language) that a number is a relational property of conceptual collections of objects. Like many other such features, color and measures for instance, it emerges out of the "interaction between us as subjects and the objects" [Hurford, 1987, 136-142], in this case in the form of counting.

It has been suggested that the introduction of writing, particularly in Mesopotamia, "split the notions of quality and quantity" and correlated with the emergence of abstract numbers, represented by symbols and detached from reality [Schmandt-Besserat, 1984, 48, 58]. We contend that using terms such as "abstract" does not help in singling out the quality of the representation, nor does it highlight a difference from "concrete". All numbers have an inherent relation to abstraction, even when things are materially counted. The correlation appears rather to be between the inception of writing and the introduction of numerical notations, understood as systems of notation "structured by means of powers of one or more bases"—generally 10 or a multiple of 10 [Chrisomalis, 2009a, 2010, 4, 362].

But first our context. We start with Mesopotamia, where seminal theories such as the "token model", have historically paved the way and given a substantive turn to research. Inevitably, however, this view needs to be expanded to include other regions where writing was potentially an independent invention, while theories need to stand the test of scrutiny.

The small clay and stone objects found in "pre-literate" (ca. 8000-3200 BCE) archaeological sites of southwestern Asia were taken by Schmandt-Besserat [1978, 1980, 1984, 1992, 1996, 2009], to be the earliest accounting tools for which we have evidence. She interpreted these objects, referred to as "tokens", "calculi" or "counters", as the forerunners of proto-cuneiform. The idea gave a revolutionary new twist to the narrative on how writing was born: counting, in this view, and not pictures, led to the emergence of writing.[1] Following several historians of mathematics [Dantzig, 2005 [1954], 6; Smith, 1951, 6-8; Kramer, 1970, 4-5; Flegg, 1983, 8-14], the theory hypothesized a three-step, unidirectional evolution of numeracy, as follows: (1) One-to-one correspondence: the objects counted are matched by an identical number of counters. For example, seven arrows are matched by seven counting fingers, or each cow in a herd is matched by a knot in a string. The result of the count is shown by the collection of counters. (2) Concrete counting, referring to the process by which *numbers as concepts are fused with the very things counted*. (3) Abstract counting, presented as the third and final stage, sees the rise of *numbers as independent concepts* as a consequence of their abstraction from the items counted. This would happen after the numerical words in a given language had become opaque, the specific things counted being no longer discernible [Schmandt-Besserat, 1984].

Tokens were specifically associated with stages (1) and (2): allegedly, one of the most "archaic" features of the token system was that it was "used in one-to-one correspondence. Two jars of oil were shown by two

[1] See already Amiet [1966, 66].

ovoids and three jars of oil were marked by three ovoid tokens". But eventually they came to reflect a "concrete" concept of number, as each kind of thing "was counted with a special numeration or special number words specific to that particular item". Again, for instance, "small and large units of grain were counted with cones and spheres, oil with ovoids, animals with cylinders and the units of labor with tetrahedrons" [Schmandt-Besserat, 2009, 149]. Finally, and still according to her account, the invention of writing entailed a major shift regarding the token system: the first written tablets of the Late Uruk period (3200-3000 BCE) supposedly introduced the use of abstract counting i.e. stage (3) above.

This view of "concreteness" and "abstractness" as two distinct concepts of number, and as unidirectional steps in evolving human numeracy, has inevitably percolated through the literature. For instance, according to Damerow [1996], the multiplicity and semantic divergence of the numerical notation systems of proto-cuneiform indicate that late 4th-millennium Mesopotamians had an incomplete concept of abstract number. This view has been brought into question [Chrisomalis, 2009b; Brown, 2010; Overmann, 2018], especially regarding several aspects linked to the formulations of Schmandt-Besserat [Michalowski, 1993; Zimansky, 1993; Friberg, 1994; Englund, 1998, 46-47], but there has been no in-depth treatment of the problem from a comprehensive and comparative perspective.[2] Reassessing the role played by concrete and abstract numbers, as separate or successive stages in the development of numeracy, is therefore a pressing issue. Is "concrete" even an essential or useful category in the study of the emergence of numbers? In this article we contend that not only are these categories not useful, but they also appear not to follow an evolutionary order, as has been suggested heavily in the literature.

Concrete counting has been used to refer to the supposed conception of *numbers as fused with the very things counted* before they became abstract. It is true, as we will see shortly, that certain counting words in some languages, as well as written numbers in some societies or groups, are used only in specific contexts. But are we to infer that users of these spoken and graphic devices cannot perceive numbers independently from context? Does the use of words such as "twins" mean that the speaker cannot distinguish the concept of "two" (quantity) from the concept of "baby" or "person" (object)?

With these questions in mind, first we demonstrate that the linguistic evidence used as a basis for the three-step evolution towards abstraction actually contradicts it. Second, we show that the idea of numbers fused with counted objects is not just contradicted by modern linguistic data, it is also (following the trail of previous works) unsubstantiated by the very material evidence and early number notations from 4th-millennium BCE Mesopotamia. Third, our excursus encompasses evidence from other regions where writing was probably invented independently, to investigate the nature of its relationship with numeracy as represented by early numerical notations.

2. Deconstructing unidirectional evolutions of numbers

Historians of mathematics have defended the unidirectional evolution of numbers with data from spoken numerals in certain languages of the Northern Pacific and Polynesia, in which "words for numbers change according to the things counted". This approach has already been questioned [Chrisomalis, 2009b, 501-502], because it relies on the false premise that such lexical peculiarities can only be expressed by people incapable of abstract mathematical thought. Yet it is useful to expose exactly what the argument entails linguistically and why it is based on a fallacy.

Schmandt-Basserat, for instance, provided examples from two languages: one is Nivkh (also called Gilyak), spoken around the Amur River (Outer Manchuria) and on the island of Sakhalin; the other is

[2] It is worth noting that the distinction between concrete and abstract numbers, as successive stages in the development of numeracy and with implications to the emergence of writing, was convincingly refuted from the perspective of neuroscience and cognitive analysis by Karenleigh Overmann, in two conferences whose proceedings were not published (Basel 2016 "The Idea of Writing", and Pisa 2017 "The Beyond the Text. The Materiality of Ancient Egyptian and Near Eastern Texts").

Table 1
Tsimshian numeral systems [Conant, 1896, 87].[a]

	Counting	Flat objects	Round objects	Men	Long objects	Canoes	Measures
1	**gyak**	**gak**	**g'**erel	**k'**al	**k'**awustkan	**k'**amaet	**k'**al
2	**t'epqat**	**t'epqat**	goupel	**t'epqad**al	gaopskan	g'alpēeltk	gulbel
3	guant	guant	**gu**tle	**gul**al	**gal**tskan	**gal**tskantk	**gul**eont
4	**tqalpq**	**tqalpq**	**tqalpq**	**tqalpq**dal	**tqaap**skan	**tqalp**sk	**tqalpq**alont
5	**kctōnc**	**kctōnc**	**kctōnc**	**kcenec**al	**k'etoent**skan	**kctōons**k	**kctons**ilont
6	**k'alt**	**k'alt**	**k'alt**	**k'ald**al	**k'aolt**skan	**k'alt**k	**k'ald**elont
7	**t'epqalt**	**t'epqalt**	**t'epqalt**	**t'epqald**al	**t'epqalt**skan	**t'epqalt**k	**t'epqald**elont
8	guandalt	**yuktalt**	**yuktalt**	**yuktlead**al	**ek'tlaed**skan	**yuktalt**k	**yuktald**elont
9	**kctemac**	**kctemac**	**kctemac**	**kctemac**al	**kctemae**tskan	**kctemac**k	**kctemas**ilont
10	**gy'ap**	**gy'ap**	**kp**ēel	**kp**al	**kp**ēetskan	**gy'ap**sk	**kp**eont

[a] These data were collected more than a century ago and already not long after the publication of Conant's book, a different transcription of Tsimshian numerals was provided in Boas [1911, 396]. Still, their use is justified for two reasons: the first is that these are the forms cited by Schmandt-Besserat [1984]; the second is that a more recent compilation of data from one of the dialects of Tsimshian dialect confirms the validity of the earlier material. Thus, for Coast Tsimshian Dunn [1979, 38-40] registers e.g. *txaalpx* 'four', *txalpxdool* 'four (human beings)', *txa'apsxn* 'four (long objects)', *txaalpxsk* 'four (canoes and vehicles)', *txaalpdaat* "four (human beings aboard canoes and vehicles)", *txaalpxl'on* "four (volume or general measure units)".

Tsimshian, spoken in northwestern British Columbia. In Nivkh, the word for 'two' is *mex* when referring to spears or oars, whereas *mik* is used of projectiles, berries, etc., *meqr* for landscape features, houses, etc., *merax* for two eyes, hands, buckets and footprints, and so on. A recent study of numeral systems in Nivkh reports that there are up to 33 classes of numerical expressions for counting specific groups of things [Gruzdeva, 2004]. As regards the number words of Tsimshian, Schmandt-Besserat cites the data presented in Conant [1896, 87; see also Chrisomalis, 2009b, 501-502], as shown in Table 1.

An aspect not explicitly mentioned by Schmandt-Besserat is that the Nivkh and Tsimshian numerical words she mentions involve something called *numeral classifiers* [e.g. Gil, 2013]. Although English does not contain such classifiers, their function can be illustrated with English phrases such as *one pair of boots* or *one cup of coffee*: here, *one* is the numeral, *boots* and *coffee* are the nouns counted or measured, and *pair* and *cup* are the intervening elements which, much like classifiers in classifier languages, assist counting or measuring.

As just noted, not all languages have numeral classifiers: they can be absent (as in English), optional (as in Hungarian), or compulsory, the latter being the case for Nivkh and Tsimshian [Gil, 2013].[3] This aspect is crucial, because (to state the obvious) numeral classifiers classify *numerals*. In other words, numerals exist as morphemes (words or parts of words) and concepts in the heads of speakers independently from the classifiers. Accordingly, Nivkh *meqr* (or *me-qr*) does not signify 'two boats'. It means strictly "two (said of vessels and other objects)" and must be used as part of larger phrases such as *mu-me-qr-* "two boats", which literally translates as 'boat-two-CLASSIFIER' (*-qr-* is a generic classifier) [Gruzdeva, 2004, 307, 319, 325].

It is also clear from Schmandt-Besserat's other examples that the various words for counting two things in Nivkh all contain the morpheme *m*(V)- 'two': *mex* "two (said of spears, oars and other inanimate objects)", *mik* "two (of projectiles, berries, teeth and fists)", *merax* "two (of body parts, parts of plants, textiles, etc.)", and so forth. Other Nivkh numerals behave similarly [Gruzdeva, 2004]. Hence it has been suggested that numerals and numerical classifiers were independent, unbound morphemes at a certain stage of lan-

[3] Though they also occur more isolated in other areas of the globe, languages with numeral classifiers are highly concentrated in South and East Asia and other regions in and around the Pacific, such as the Indonesian and Polynesian archipelagos, and North and Central America. They include Burmese, Chinese, Korean, Japanese, Kiribati, the Mayan languages, among others.

guage evolution, before classifiers became embedded grammatically. As shown in Table 1, this is also supported by the use of one set of Tsimshian numerical lexemes (*gyak* "one", *t'epqat* "two", *guant* "three", *tqalpq* "four", and so on) for general counting, but the same morpheme integrates words that count specific sorts of things: thus, *tqalpq* "four" is contained etymologically in *tqalpq-dal* "four (said of men)", *tqaap-skan* "four (said of long objects)", *tqalpq-alont* "four (said of measures)", etc. This general set of numbers was referred to as "abstract" by Boas [1911, 396].

The linguistic data of Nivkh and Tsimshian does more than confirm that number words dissociated and directly associated with the names of things counted can be synchronic. It indirectly gives evidence that independent number words existed in previous historical stages of these languages, contradicting the evolutionist view that "concrete" *always* predates "abstract". Numeral classifiers can be derived or are assumed to derive from other categories of words, especially nouns [Allan, 1977, 293; Aikhenvald, 2000, 120-121, 353ff; Gruzdeva, 2004, 302]. Linguists refrain from asserting the universality of this historical development, but at least in the case of Nivkh there is evidence to reconstruct it: for example, the Nivkh classifier for paired objects, *-vasq*/*-vysq*/*-vsq*/*-fasq*, derives from the noun *pasq* "one of a pair", whereas *-m*, the classifier for vessels, derives from *mu* "boat" [Gruzdeva, 2004, 317]. Likewise, Boas [1911, 397] observed that the Tsimshian classifying suffix for long objects, *-skan*, was likely derived from the noun *sgan* "stick".

The idea that bound numeral classifiers, as seen in Nivkh and Tsimshian, were originally separate words (which later became embedded grammatical features) is also supported by languages where classifiers (or words with similar functions) are still independent. Take the vestigial English expression *three head of cattle*, or Japanese *san wa no tori* "three birds", literally 'three CLASSIFIER LINK bird' [Crump, 1990, 37]. The Japanese classifier *wa* means "wing" so the original sense of this last construction is "three wings of bird". In turn, this is reminiscent of phrases like *one cup of coffee*, and suggests that classifiers originate as words which either help measure mass nouns (e.g. *coffee*) or assist in counting nouns which are ambiguous in their number (e.g. collective *cattle*), but then become generalized and extend to other (discrete) nouns that are more easily individuated (e.g. *birds*). Again, in these languages the thing counted is not represented by the classifier bound to the numeral, but by a separate noun.

In conclusion, a close examination of the data from Nivkh and Tsimshian used by Schmandt-Besserat to justify the "tokenist" theory does not support the view that the speakers of these languages fused number and entity counted. At least linguistically, the separation between the two is evident. And while this piece of counterevidence may be well known to many scholars, especially linguists, it may have escaped the notice of archaeologists, epigraphists, textual critics, and historians of mathematics working on Mesopotamian contexts, or on early writing in general.

The case of Mangarevan [Beller and Bender, 2008; also Bender, 2013], a language spoken in the Pacific island of Mangareva that belongs to the Oceanic subgroup of the Austronesian linguistic family, is relevant here. Beller and Bender report that speakers of Mangarevan used both an abstract decimal numeration system, inherited from Proto-Oceanic, and systems for counting specific items which derived from it, as shown in Table 2.

This example again shows that "concrete" counting must not be taken as a sign of inability to conceive numbers as relational properties independent from concrete objects. Beller and Bender [2008, 214] also make the suggestion that systems of specific counting—in this case spoken, not material—can be a response to the social need to deal with large numbers in the absence of numerical notation.

Also, no compelling example has been provided of a language whose abstract number words etymologically derive from concrete number words, which would support the proposed unidirectional development. Schmandt-Besserat [1984, 51] cites Niuean, another Polynesian language, as having abstract number words "that meant literally 'one fruit, two fruits, three fruits'", but numbers in this language are part of a decimal system which, like Mangarevan numbers, was inherited from Proto-Oceanic. One needs only to compare Niuean *taha* "one", *ua* "two", *tolu* "three", *fā* "four", *lima* "five", *ono* "six", *fitu* "seven", *valu* "eight", *hiva*

Table 2
Mangarevan counting systems (adapted from Beller and Bender, 2008, 214, Table 2; the parenthetical analyses are ours).[a]

Objects/ Numerals	General (inherited from Proto-Oceanic)	Group 1 (tools, sugar cane, breadfruit, pandanus)	Group 2 (ripe breadfruits and octopus)	Group 3 (first breadfruits and octopus of a season)
1	*tahi*	–	–	–
2	***rua***	*tauga*	–	–
4	***hā***	***rua*** *tauga* (= 2 *tauga* = 2 × 2)	*tauga*	
8	***varu***	***hā*** *tauga* (= 4 *tauga* = 4 × 2)	***rua*** *tauga* (= 2 *tauga* = 2 × 4)	*tauga*
10	*rogo'uru*	–	–	–
20	*takau*	*paua*	–	–
40		*tataua*	*paua*	
80		***varu*** (= 8 → 80)	*tataua*	*paua*
160		***rua varu*** (= 2 × 8 → 2 × 80)	***varu*** (= 8 → 160)	*tataua*
200				
320		***hā varu*** (= 4 × 8 → 4 × 80)	***rua varu*** (= 2 × 8 → 2 × 160)	***varu*** (= 8 → 320)
640		***varu varu*** (= 8 × 8 → 8 × 80)	***hā varu*** (= 4 × 8 → 4 × 160)	***rua varu*** (= 2 × 8 → 2 × 320)

[a] The data in Beller and Bender [2008] is cited from a grammar and dictionary compiled by the French missionaries of the Congrégation des Sacrés-Cœurs de Picpus [Janeau, 1908, 18-21, 101] at the beginning of the 20th century. Bender [2013] adds data from a slightly different account [Buck, 1938] and mentions that the Mangarevan counting systems may have changed over time, due to European influence.

"nine", *hogofulu* "ten" [Sperlich, 1997] with some of the Mangarevan numerals shown here in Table 2: *tahi* "one", *rua* "two", *hā*, "four", *varu* "eight" and *rogo'uru* "ten". Even a cursory look suggests that these numerical lexemes are so varied in their phonological shape that it is highly unlikely they once shared a morpheme signifying "fruit".

Conversely, the etymologies of small numbers worldwide point to sources that have nothing to do with specific items counted, but rather with how counting was performed (often through gestures by means of the hands or other body parts). Thus, "two" frequently means "next one, (the) following" or similarly, "five" is often etymologically related to "hand" or "fist", and so on [Dehaene, 2011, 80; Everett, 2017, 104-105]: cf. e.g. Yup'ik Eskimo (spoken in Alaska) *mahl'uk* "two", from the phrase "one that follows" [Koo, 1980, 282] and Latin *secundus* "second", literally "following" [De Vaan, 2008, 556]; Aghu (spoken in Papua New Guinea) *bidikimu* ∼ *bidikuma* "five", literally "hand" [De Vries, 1994, 556], and the already mentioned Niuean *lima* "hand-arm, five"; as well as the Portuguese phrase *mão cheia* (lexicalized by some writers as *mancheia*) and its English equivalent *handful*, synonymous with "five".

This excursus helped to show the situational nature of the emergence of numbers in a less rigid frame that takes into account their inherent rapport with things counted. While the evolution cannot be taken to be unidirectional and fixed, it is important to expand the context in which numbers emerged in the gradual development of their earliest representations, as offered by the extant evidence from Mesopotamia.

3. Context-dependent numbers in late 4th-millennium Mesopotamia

Linguistic evidence from Sumerian—or any another language—is missing for the pre- and proto-literate phases in Mesopotamia (i.e. before 3000 BCE), meaning that lexical numbers are not attested for the period that interests us. Thus, numeracy can be assessed exclusively on tokens, numerical tablets, and so forth. Yet,

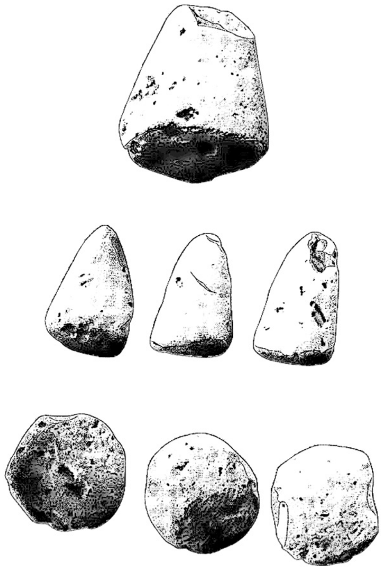


Figure 1. Conical and discoid tokens found inside, and impressed on the outside of a clay envelope Sb 1927 from Susa (drawn after Englund [1998, 48, Fig. 12]; not to scale).

in what follows we show that specialized or context-dependent numbers already subsume abstraction, and not just in spoken counting systems. All this contradicts the *concretum pro abstracto* theory. Material, and in particular notational, arithmetic practices of late 4th-millennium Mesopotamia lead to similar conclusions.

Schmandt-Besserat maintained that the tokens merged the notion of quality and two concepts of quantity. In addition to complex tokens, which she proposed denoted specific commodities by resemblance or iconicity (such as ovoids allegedly standing for amounts of oil jars), she also interpreted plain tokens as representative of grain measures. In this view, the cones would count small measures of grain and spheres would stand for large measures of grain. These grain tokens would in turn provide the basis for the later numerical notations of proto-cuneiform, which includes conical numerical symbols, "because grain was the commodity most widely exchanged in the ancient Middle East" [Schmandt-Besserat, 1984, 57]. This is the development claimed to mark the shift from a concrete to an abstract concept of number, coinciding with the beginnings of writing.

Yet the functionality of tokens was interpreted in terms of what their shapes were thought to represent, and by making inferences based on later technologies (such as clay bullae, and numerical and numero-ideographic tablets), which Damerow [1996, 213] admits leads to a "high degree of uncertainty". This functional interpretation of tokens finds no independent confirmation. As noted already by several scholars, archaeological evidence either does not corroborate them, at least not all of the time.

Three such criticisms are worth reiterating here. First, certain tokens claimed to be counters have been found in archaeological contexts that are difficult to reconcile with accounting practices, including child burials [Michalowski, 1993, 997; Englund, 1998, 46-47]. Second, the "qualitative" identification of certain objects attempted by Schmandt-Besserat is contradicted by their distribution: for example, her token shape for "sheep", which would have been an important commodity in 4th-millennium Mesopotamia, is scarcely attested [Zimansky, 1993; Friberg, 1994]. Third, it has been emphasized that the tokens compiled by Schmandt-Besserat come from much-varied geographical and chronological settings, and cannot correspond to a single practice. In conclusion: methodologically, the meaning of tokens, and whether they (or some of them) functioned as a system at all, would need to be interpreted on contextual evidence, rather than mere resemblances. Which tokens appear together, and in which settings? If certain tokens occur together in the same contexts, how do they interact? Are there meaningful repetitions or complementary distribu-

Table 3
Sexagesimal (S) system of proto-cuneiform numerals [after Englund, 2011, 39, Fig. 2.4].

Value	1/2 or 1/10	1	10	60	600	3,600	36,000
Sign							
Sign No.	N_8	N_1	N_{14}	N_{34}	N_{48}	N_{45}	N_{50}

Table 4
GAN_2 (G) system of proto-cuneiform numerals, used for area measurements [after Nissen et al., 1993, 28].

Value		10x(?)	6x	3x	10x	6x
Sign						
Sign No.	N_8	N_1	N_{22}	N_{14}	N_{50}	N_{45}

tions of tokens in token groups? Discovering such patterns would provide a basis to make inferences about their meanings.

Around 3500 BCE a new administrative technology emerges: the clay bullae or (spherical) envelopes [Schmandt-Besserat, 1980; Englund, 1998, 48-49, and 2004, 122]. Geometric or "plain" tokens made of clay, including spheres, cones and disks, were enclosed inside these envelopes, which were then impressed with cylinder seals. The outer surfaces of the envelopes were also impressed with counters in a one-to-one correspondence with the tokens contained on the inside. Amiet [1966, 20-21] interpreted the enclosed objects as "calculi" denoting small and large quantities of various commodities. He published a clay envelope from Susa (inv. no. Sb 1927), containing one big cone, three small cones and three disks inside [Figure 1], and the same kind and amount of impressed marks on the outside [Amiet, 1972, pl. 168, no. 539; see also Carter et al., 1992, 56 and Englund, 1998, 48, Fig. 12]. Their resemblance to proto-cuneiform numerical signs is impressive.

Indeed, Lambert (1966) made the suggestion that the first numerical signs impressed on flat and rounded clay tablets (ca. 3400-3350 BCE, cf. Englund [2004, 123]) were actually made with counters or styli and imitated the forms of these plain tokens. These numerical signs, precursors of the later proto-cuneiform numbers (ca. 3350-3200 BCE) [Table 3], were evidently dots or circles and conical or bullet-like shapes. According to the evidence surveyed and reported by Schmandt-Besserat [1980, 367-370, Fig. 6; 1992, 127-128], the token shapes found inside clay envelopes are limited to spheres, disks, cones, cylinders, tetrahedrons, parabolae and ovoids.[4] Of these, only the ovoids—which she interprets as representative of jars—are obvious candidates for "iconic" tokens.

A still unproven inference is that some of the plain tokens operated like the early numerical notations of Mesopotamia, as their forerunners, and had comparable values. Conversely, "iconic" or "complex" tokens could *at best* be proposed as, but were not necessarily, the precursors of (some of) the pictorial ideograms of proto-cuneiform (i.e. signs depicting vessels, animals, etc.) [Friberg, 1994, 484].

If some of the plain tokens are related to the first numerical notations of Mesopotamia, what can the latter tell us about numeracy in their context? In ca. 3200-3000 BCE, numerical signs impressed on clay tablets incorporated different systems, where their value was dependent on the items being counted or measured. Nissen et al., [1993, 25-29] showed that proto-cuneiform contained as many as fifteen different numerical systems, even though only five of them were particularly common: the sexagesimal system or S [Table 3], the bisexagesimal system, the ŠE system, the GAN_2 system [Table 4], and the EN system.

[4] Unfortunately, there is yet no comprehensive list of bullae opened or examined by means of tomographies, or of the typology of the tokens that they contained.

These systems were of two different basic types. The bisexagesimal and sexagesimal system were used for counting discrete items and the latter was used in almost half the texts from Uruk. The ŠE system was used to measure grain, the GAN_2 system served to measure field areas, and the purpose of the EN system remains obscure.

Another system, called U_4, measured time and combined numerical and non-numerical signs to denote specific calendrical units [see also Damerow, 1996, 292 and Englund, 1998, 120].[5] Importantly, while the same sign shapes were used in many of the systems, their value often varied from one to another. For example, a dot or disk (•) represented the equivalent of ten conical impressions in some systems, but six in others [Table 4]. Damerow [1996, 292-293] interpreted this fact as an indication that the signs had different arithmetical values in particular contexts. This supposed polyvalence and its context dependence led him to see the proto-cuneiform numerical systems as reflecting a concrete concept of number.

However, as Overmann [2018, 10] points out, "specification" may have been a change from earlier "non-specifying" counting technologies—again, with concrete counting practices emerging anew. Actually, the problem of variability in proto-cuneiform numbers echoes the way in which Mangarevans counted things orally until recent times. As we have seen, *varu* was the generic Mangarevan word for "eight", but when counting breadfruit, sugar canes, etc., *varu* meant "80" or, if we prefer, "eight (tens)", as these commodities were counted in groups of multiple items [Table 2]. Quite simply, in these and other specific counting systems, the reference units differed, yet evidently Mangarevans still had free-standing numbers.

The situation with *spoken* Mangarevan numerals may have been true also of proto-cuneiform *inscribed* numbers: that is, certain things (grain, field areas, etc.) were counted with specific units. As Michalowski [1993, 998] notes, a wedge (or, in the case of proto-cuneiform, a small horizontal cone) "preceding a pictograph was not '1', but 'one unit of x'". In this scenario, it is unnecessary to question the existence of an abstract concept of number to account for the different progressions found in different proto-cuneiform numerical systems. For example, in the sexagesimal system a small horizontal cone ( ) stands for "1" and a large dot or disk (●) is the equivalent of 3,600 small horizontal cones. However, the same large dot corresponds to only 1,080 similar cones (i.e. one-third fewer) in the GAN_2 system, which measured field areas [Table 4].

Put differently, the large dot means "3,600" in the regular sexagesimal system, but may represent something like "3,600 thirds" in the field-measuring notation. Compare this with the use of *varu* "eight" as "eight tens" i.e. "80" in the breadfruit and sugarcane system of spoken Mangarevan. That a certain field measure of early Mesopotamia might have been conceived as "one-third" of another unit is not as far-fetched as it might seem. Measure systems commonly involve fractional units worldwide and this may have been the case with certain proto-cuneiform metrological notations too. One may compare also the old Spanish and Portuguese weight unit *arroba*, whose name derives from Arabic *rub'* "one-fourth" and which corresponded to a quarter of another weight measure called *quintal* [Lopes, 2003]. In Mesopotamia, thus, the quality or kind of thing counted was expressed by an iconic ideogram, but the value of numerical signs, albeit dependent on context, was strictly quantitative.

Damerow [1996, 289] signaled as evidence for concrete numeracy the fact that non-numerical (e.g. an animal ideogram) and numerical proto-cuneiform signs could be combined in a ligature to indicate, for

[5] Also at the end of the 4th millennium BCE, the archaeological record of western Persia, and particularly Susa, shows a trajectory analogous to that of Uruk-influenced Mesopotamia. The development of proto-writing, the use of seals, and the so-called tokens are the most significant indicators of this common path. In the same region, the proto-Elamite "script" presents many shared features with proto-cuneiform, including the same system of numerical signs of proto-cuneiform. This system comprised a sexagesimal (S) set comparable to the proto-cuneiform one, and there is little doubt that proto-Elamite and the latter were related. There is, however, a significant divergence: the existence of a derivative proto-Elamite decimal sub-set of signs without parallel in southern Mesopotamia, which was used for counting discrete animate objects, in particular domesticated animals and human workers [Englund, 2004, 107, 120, 124].

example, an animal of a certain age. However, users of these signs may well have been able to conceive an abstract number (in this case referring to age) independently from the animal. Again, this would be the same as saying that a person using the English word "twins" does not separate the abstract concept of "two(ness)" from that of "person". Rather, devices such as these graphic symbols are context-specific.

We should also note that clay documents containing only numerical signs predated, or were contemporary with, other clay documents containing non-numerical signs expressing commodities alongside numbers. Hence, it is possible to turn Damerow's theory around, and posit that non-numerical ideograms—a crucial step towards writing—were invented to register the things that were being counted and their contexts, which the numerical signs alone could not express with clarity. In other words, the "abstractness" of numerical notations might have facilitated the process that led to ideograms representing specific things.

Finally, the possibility that the plain tokens, encased in, and represented on, clay envelopes, are the immediate forerunners of impressed numerical signs, warns us that these tokens might also have been used to express numbers independently from the things counted. Ifrah [2000, 125-126] collects ethnographic examples from pre-industrial, state and non-state societies which used similar objects (of low iconicity), including a system of counting using pebbles, with a decimal base, among the Malagasy of Madagascar; other peoples used sticks, pearls, shells, pellets, buttons, etc. sometimes pierced and slid into threads or sticks (cf. the abacus), or placed on boards or in compartments. Friberg [1994, 485] notes that likely "the plain tokens got their distinctive geometric shapes just because it is particularly easy to fabricate small plain spheres, cones, etc." out of clay.

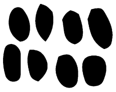
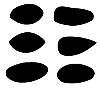
4. Numbers and first writing: from abstract to power-based

We have seen that Schmandt-Besserat connected the invention of writing in southern Mesopotamia at the end of the 4th millennium BCE with the introduction, for the first time, of the so-called "abstract" numbers. In the previous sections, we noted that no evidence rules out the possibility that, before then, the Mesopotamians conceived abstract mathematical thought. All numbers are inherently conceptual and imply a degree of abstraction, even if they rely concretely or materially on specific objects.

We suggest that emphasis should be transferred from the "concrete" vs. "abstract" dichotomy to the emergence of signs for powers of one or more bases as a defining feature of numerical notations. Archaeological evidence shows that the *structured* numerical notations seen in proto-cuneiform were a recent invention, and therefore chronologically close to proto-writing (at pre-phonetic stage). Clay or gypsum tablets bearing only numerical signs similar to the proto-cuneiform ones have been found at Syrian, Mesopotamian and western Iranian sites, which immediately predate (3500-3350 BCE) or overlap with the beginnings of proto-cuneiform, in the so-called Uruk IV stage (3350-3200 BCE) [Nissen et al., 1993, 127-130; Englund, 1998, 50-51; Ross, 2014, 298-299]. In proto-cuneiform a sign for a higher power tends to be used once signs for lesser powers come into place [Damerow, 1996, 280]. However, this rule does not apply systematically, which implies that no single conventionalized system of numerical signs is comparable to that of fully fledged proto-cuneiform.

For example, numerical tablet JA 104, from the Syrian site of Jebel Aruda, features 22 dots [van Driel, 1982, *apud* Nissen et al., 1993, 130, Fig. 114 and Englund, 1998]. This implies that the standardization of the systems seen from the Uruk IV period onwards was still ongoing. Thus Nissen et al., [1993, 130] deduced "that there was a close connection between the invention of writing and the formation of standardized systems of measures and numbers". In other words, whereas the historical development from "concrete" to "abstract" concept and representations of number is not supported by empirical data, it does seem that the advent of writing in Mesopotamia more or less coincides with the crystallization of a system for notating numbers using powers grounded on numerical bases. Indeed, numerical notations cannot be taken for granted before the introduction of powers [Chrisomalis, 2010, 4, 362]. These are key in providing

Table 5
Numerical signs attested in the corpus of inscribed tags from the U-j Tomb (redrawn after Dreyer et al., 1998, Figs. 74-75).

structure, and this is tantamount to systematized notations; without structure, these would merely represent "tally marks".

The Mesopotamian data is serendipitous because, both in quantity and in time frame, it allows us to reconstruct the evolution of numeracy, and literacy, with a modicum of continuity in terms of material evidence and the types of support for notations. In regions of the world where the inception of numeracy is not so extensively documented, the situation, even though defined as enigmatic [Schmandt-Besserat, 1996, 1], can yield evidence that needs further probing.

To evaluate the hypothesis of a similar correlation between the introduction of power-based numerical representations and iconic signs, we should include other regional cases of numerals at the earliest possible stage of their development [Chrisomalis, 2010] *vis-à-vis* examples of newly invented systems of writing. Here we consider data from the Maya setting, the Chinese, the Egyptian and the Aegean, all cases whose original development can be assumed with a degree of probability. Other cases may also be added (Nahuatl, Anatolian Hieroglyphic), but for reasons of space they will not be part of this examination.

4.1. Egypt

The earliest attestation of Egyptian numerical symbols is on some of the ca. 200 inscribed ivory and bone tags from the tomb U-j at Abydos (Naqada IIIa, ca. 3200 BCE). Alongside more than a hundred ceramic jars painted with individual or paired symbols, these inscriptions constitute also the oldest evidence of proto- or full writing in Egypt [Görsdorf et al., 1998; Baines, 2004, 153-157]. Their meaning is far from certain, but Wengrow [2009, 1025] notes similarities with larger tags from 1st Dynasty elite burials, which carry hieroglyphic inscriptions of quantities of valuable products, alongside royal names and complex kingly scenes. Whatever the case, it is remarkable that these early numerical symbols never mix with the pictorial signs that also occur on the tags, and which are the precursors of Egyptian hieroglyphs.

Crucially, the numerical symbols on the U-j tags do not reflect the standard Egyptian number system known from later phases. Only three types of signs are attested: vertical strokes, horizontal strokes, and a spiral shape, probably depicting a curled rope [Table 5] [Dreyer et al., 1998, 113-118 *apud* Baines, 2004, 157]. Respectively, these signs may stand for units, tens, and hundreds. Moreover, the tags never combine different numerical signs. In fact, we infer that they are numerical signs only because we find reiterations of strokes. Some tags show strokes in groups of more than 10. Baines stresses that this is not necessarily an indication that the number notation of U-j used a non-decimal system. Rather, the decimal system we find after the cementing of Egyptian writing around the time of Narmer's reign was possibly still under development, or not yet standardized, at this point. This makes for a possible analogy with the trajectory of numerical signs and writing in Mesopotamia, as described above.

Similar tags dating to the Naqada III and Early Dynastic periods were found in other cemeteries at Abydos and Naqada. They show groupings of three types of numerical symbols in the same tag. One sign shaped like a curved rope (𓍢) is followed by groups of bow-shaped (∩) and vertical strokes in a number that does not exceed 9. Along with their shapes, this arrangement in a clear decimal system [Table 6] indicates that the notation known from king Narmer's time onwards was now in place [Imhausen, 2016, 22-25].

Table 6
Egyptian hieroglyphic numerical notation (after Gardiner [1957, 191-192]).

	1	10	100	1,000	10,000	100,000	1,000,000
Hieroglyph	𓏺	𓎆	𓍢	𓆼	𓂭	𓆐	𓁨
Phonetic reading	*wʿ*	*mḏ*	*št*	*ḫꜣ*	*dbʿ*	*ḥfn*	*ḥḥ*

The Egyptian sign for units is a simple vertical stroke, perhaps originating in a tally system [Imhausen, 2016, 18]. The arch-shaped symbol for 10 (𓎆), however, lends itself to other kinds of interpretation. For Gardiner [1957, 524] this was a bar-less version of another hieroglyph, the 'hobble for cattle' 𓎅, which is the determinative for the Egyptian word *mḏt* 'stable; cattle stall'. Because the Egyptian word for 'ten' was *mḏ*(*w*), Chrisomalis [2010, 36-37, following Sethe, 1916, 2] asserts that the hobble sign might have become numerical via rebus. But we could turn the question around: perhaps the Egyptian word for 'ten' *mḏ* is etymologically connected to *mḏt* 'stable' (*'place where animals are tied'?) and alludes to the representation of ten counted things by tying a knot on a cord (or similar) among prehistoric Egyptians. Of course, this cannot be proved, but it may not be a coincidence that the word for the next power, *št* 'one hundred', was written with a hieroglyph in the shape of a coiled length of rope (𓍢). In fact, Imhausen [2016, 18, n. 4] gathers evidence that the latter may have represented a rope employed in the measurement of field areas, corresponding to a length of 100 cubits. The finger hieroglyph (𓂭), signifying 'ten thousand', also represented a subdivision unit of the cubit, and had a strong association with the act of measuring, suggested by its use in the Egyptian script as a determinative signifying "accurate" and "precise" [Gardiner, 1957, 456]. Yet this can hardly be the direct rationale for its use as a numeral, and Imhausen suggests that the finger stood for something like a "bundle (of thousands)". The other numerical signs, all pictorial, are less likely to be rooted in material counting practices than metaphorical or metonymic allusions. For instance, the use of a tadpole hieroglyph for "100,000" could be reminiscent of the massive reproduction of frogs.

In conclusion, Egyptian numerical signs through to the hundreds are relatively schematic, perhaps rooted in a tally system involving simple marks, a chord-based counting system, or both. As in Mesopotamia, these signs are non-pictorial, perhaps reflecting the transposition of material counting practices into graphic form. And as in Mesopotamia, the first evidence for a numerical notation based on powers as explained above is roughly contemporaneous with the first evidence of writing.

4.2. China

Not unlike Egypt, the first known numerical notation in China appears alongside the first phonetic writing, in the final phase of the Shang period (ca. 1200-1050 BCE). These first written records, found in the last Shang capital, Anyang, were made on turtle shell and cattle and ovicaprid scapulae used in the context of divination at the royal court. Yet inscribed bronze vessels, jade *ge* (halberds), and pottery are also known, if in fewer numbers [Bagley, 2004]. The act of divining in Shang China consisted in heating the bones or shells with fire till cracks were produced, which, according to unknown criteria, would then reveal the answer of royal ancestors to specific questions. While this practice predates any notations [Keightley, 1978a, 3], numerical signs came to be used to number the cracks, and oracular texts in general recorded the results of consultations, which often had to do with funerary offerings, as well as agricultural and military affairs of concern to the king and the State.

Describing these records, Keightley [1978b, 215] writes that their logic was "itself frequently bureaucratic" and that the success of the recorded offerings "depended upon correct fulfillment of (. . .) the right number of cattle, to the right ancestor, on the right day". A jade halberd found in a tomb dated to ca. 1200 BCE explicitly registers the sending of five such ceremonial weapons [Bagley, 2004, 215], again illustrating the initial uses of numerical records in China.

Table 7
Shang Chinese numerals [after Chrisomalis, 2010, 260-261].

1	2	3	4	5	6	7	8	9	10	100	1,000	10,000
–	=	≡	≣			+	)(		|			

Table 8
Early Mesoamerican numerical notation or 'bar-and-dot' system (signs redrawn after Macri and Looper [2003]).

1	5	20 (Maya)	0 (Maya)
•	—		

The Shang numerical notation was well embedded in the writing system, whose signs have as direct forerunners the pictorial symbols found on somewhat earlier bronze vessels (ca. 1500 BCE), and which are interpreted as "clan-name insignia" [Boltz, 1994, 45-50; Demattè, 2010, 212]. However, the shapes of some of its numerical signs give us clues about their origins. In the Shang numerical system, numbers 1 through 4 are notated with additive horizontal bars, whereas 10 is represented with a vertical bar [Table 7] [Chrisomalis, 2010, 260-261]. This betrays their origin from a simple system of markings that became structured on a decimal base: i.e. originally, after having counted 9 units through horizontal strokes, the passage to the power of 10 may have been marked by shifting to vertical ones. The Shang characters for 5 through 9 are perhaps logographs that came to signify number words phonetically via rebus. This is likely at least in the case of , which represented logographically the Old Chinese word **kruk* 'dwelling', but through near-homophony it came to transcribe **krung* 'six' as well [Liu, 1993]. Rebus may well have applied in the genesis of the sign for 1,000 () as well. The latter is similar to the character for 'man' () [Djamouri, 1994, 15], and the reconstructed Old Chinese words **s.n̥ˤi(ŋ)* 'thousand' and **n̥i(ŋ)* 'man, person' [Baxter and Sagart, 2014] also show some phonic resemblance.

Thus, the early Chinese numerical signs for lower magnitudes, in the shape of strokes, and the decimal base of the system may be based on previous tallying marks predating the invention of writing, with some of the numerals between 5 and 9. In this aspect too, the trajectory of China is comparable to that of Mesopotamia and Egypt. And like Egypt, some Chinese numerical signs for higher magnitudes are of obscure origin, but others are pictorial logographs that represent number words through rebus.

4.3. Mesoamerica

In the Maya script, for instance, attested towards the end of the 1st millennium BCE, zero is represented by means of a shell-looking (or lobed flower?) glyph, and units are dots, and 5 is a bar [Table 8]. The dots and bar had evident roots in a tally system of schematic signs [Justeson, 1986, 440-441]. These signs are combined progressively to notate numbers from 1 to 19 [Chrisomalis, 2010, 284-287]. This seems to be the case also from the beginning of literacy, in stages that are for the most part still very obscure, like the Epi-Olmec (or Isthmian) and Zapotec inscriptions. One of the earliest examples of a combined bar-and-dot phrase is found on Stela 12, from Monte Albán, a site in the Oaxaca Valley. It coincides with the earliest secure evidence for writing, which comes from the first occupational phase of this site (ca. 500-200 BCE).

During this period, the inhabitants of the site also raised more than 300 carved stone monuments, known as *danzantes* [Marcus, 1980, 52-53]. These are stone slabs with depictions of persons in distorted and convoluted positions, often accompanied by short columns of glyphs. The most accepted interpretation nowadays is that they represent slain and mutilated captives or defeated enemies [Scott, 1978, 21-30; Mar-

Table 9
Cretan Hieroglyphic signs for whole numbers [Olivier and Godart, 1996, 17, 434-444].

Value	1		10	100	1,000
Sign					

Table 10
Linear A signs for whole numbers [based on Godart and Olivier, 1976-1985].

Value	1	10		100	1,000
Sign					

cus and Flannery, 2003], and the glyphs may have served to name them. For instance, *danzante* D-40 presents a glyph comprising a dot and a bar, hence ‘6’. Mesoamerican documents of a later date show the long-standing tradition, common to different societies in the region, of naming people with day names calqued from the local calendar (e.g. ‘1 Earthquake’, ‘3 Deer’, and so forth), and it has been proposed that this was also the case earlier, at Monte Albán.

This evidence, however scanty or related to stages of literacy that may (or may not) be the very earliest in absolute terms, reflects numerical notations that arose from three-dimensional (but still symbolic, abstract) counters, such as pebbles and maybe shells and sticks. Materiality is present in graphic form as much as abstraction. From basic tallies, logography was then used to extend the numerical system of notation in the Maya script: thus, the glyph for ‘20’ [Table 8] depicted the moon and also signified both this celestial body and the lunar month [Chrisomalis, 2010, 285]. The similarity of this trajectory with that of the regions appraised above is again striking.

4.4. Aegean

The evidence for numerical notations and the earliest stages of writing from Crete date to the beginning of the second millennium BCE. There is no consensus as to whether the concept was borrowed from the Egyptian, but the graphic notations point to an independent creation in terms of the system used, which appears newly invented [Chrisomalis, 2010, 62, with references], with the establishment of the Cretan Hieroglyphic script as early as *ca*. 1850 BCE. The numeral system of Cretan Hieroglyphic is closely linked to that which will later appear in the Linear A syllabary, which continues till the Linear B system of the end of the Bronze Age. While the numerical tradition can be traced and reconstructed across the three Aegean scripts, its source and origin are still steeped in uncertainty.

At its early stage, the Cretan Hieroglyphic number system included signs for units, tens, hundreds, thousands, and fractions, which are geometric and schematic, like vertical and oblique strokes, dots, circles, etc. [Table 9] [Evans, 1909, 256-259; Olivier and Godart, 1996, 17]. Its decimal structure and the fact that it also comprises a subset of signs representing fractions of whole numbers have led to the hypothesis that the Cretan system might have originated from or, at least, been influenced by Egypt [Bennett, 1950, 206, n. 20; Chrisomalis, 2010, 57, with references]. Nevertheless, as shown in the figures [Tables 9 and 10], neither hieroglyphic nor hieratic signs for numbers are truly comparable to the Cretan Hieroglyphic or Linear A ones [Bennett, 1999, 164-175]. The only exceptions might be the vertical stroke, used both in the Aegean and Egypt for units. They cannot be considered diagnostic, though, since the vertical stroke is a common marker for units [Chrisomalis, 2010, 57]. Therefore, the only evidence put forward so far is the decimal power-base typology, which is notably common, and the rare use of special signs for fractions for whole numbers instead of measures, although at least in the case of Egypt we know that the first attested

Figure 2. Example of clay roundel (PYR Wc 4, Agios Nikolaos Museum 12567) inscribed with Linear A and bearing repeated seal impressions on the sides.

fractions were associated with the measuring of the height reached by the flooding of the Nile [Austin and Guillemot, 2017, 54]. The origin of the Aegean numerals, especially with regard to the Cretan Hieroglyphic script, remains in need of further investigation.

Interpretations of writing and numeracy as fully embedded in the administration of Cretan palaces have suggested that a one-to-one correspondence between impressions on seals and commodity units on clay roundels may be observable. The seal impressions on the edges of the clay object [Figure 2] would account for the counting strategy, and would be correlated to the commodity logograms inscribed on the flat face of the roundel. The link of both elements has led to the suggestion that some Cretans may have progressed as far as stage 1 in the alleged evolution of numeracy, namely a one-to-one counting strategy [Weingarten, 2018]. Numerical notations were, however, added on other clay documents used in the administrative machine, and these were embedded in a fully structured decimal system and thus, by definition, "abstract" or independent, even though they seem to be uniquely relegated to the administrative sphere. On this basis, no evidence for a "concrete" conceptualization can be assumed.

5. Foreground

We have looked at numeracy as a phenomenon at large, across its regional and global development, from the vantage point of its end result. Yet, as in any reconstruction, we must also take into consideration the relative, contextual settings in which language expresses itself. A macro-context approach should be balanced by the analysis of the micro-context as well. Linguistic expressions and social practices, including all material and symbolic representations, do not evolve independently, nor do they exist in a vacuum. Rather, they are in constant interaction and fluid dynamics. This makes pinning down specific categories and concepts particularly difficult. Also, global viewpoints, which may build up a new, reassessed theory of how numeracy came about, in each and every regional development, and counterbalanced by some more comparative evidence, lest they fall prey to sweeping statements, must be supported by firm diagnostic data. This is what the foundation of any theory should be based on.

On this basis, at least as far as the Mesopotamian case is concerned, we must conclude that numerical notations and non-numerical proto-cuneiform "iconic" signs represent two distinct but converging developments. This means that a "concrete-to-abstract" step cannot be postulated as an evolutionary or deterministic trigger for the invention of writing, unless we are willing to fall into the pit of absent data. For Mesopotamia and beyond, we should conclude that whatever ended up being defined as "abstract" in regard to numbers was already present before any tangible writing can be observed. Yet, abstractions are not any less situational, since they emerge from specific settings of experiences of counting. Therefore, abstract numbers are not context-free, as much as they do not follow concrete ones.

But we can take this further. We claim that writing emerged out of a converging path that involved numeracy and iconic signs [Justeson, 1986, 439]. But numeracy requires a more precise definition in this case. The crucial aspect is not so much the idea of "abstract" numbers as defined, but rather their representation through a system hinging on powers of specific bases.

In conclusion, at least as far as the Near East is concerned, proto-writing emerged when early numerical notations, representing numbers independently of the things counted, were combined on clay tablets with iconic forms associated with modes of non-verbal expression, such as Late Uruk seal imagery [Pittman, 1994; Ross, 2014]. If this is true, the dichotomy between "abstract" and "concrete" is abolished, and any deterministic or linear evolution rebuked. Not only were numbers (by definition always abstract) alive from the beginning, but their historical development intersects with the invention of writing in a more specific and defined way than previously assumed: their convergence pivots on the appearance of power-based systems of number notation.

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