Efficiency of bilateral delegation in a mixed Cournot duopoly

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Abstract: We consider a bilateral delegation mixed duopoly with quantity setting, where the objective function of the public managers is based on strategic manipulation of a Generalized Welfare Function. We show that such manipulation, coupled with strategic delegation by the private firm, enables the government to enforce an efficient outcome at equilibrium. When the manipulation/delegation choices and their sequence are endogenized, public manipulation and private delegation are supported at equilibrium, with the government, as first mover, setting the weights of the Generalized Welfare Function at the most efficient level consistent with private firm retaining a manager. This ensures maximum welfare, as compared with all other organizational structures.

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1 Introduction

The basic issue addressed by the literature on mixed markets is the indirect regulatory role of public firms – to what extent their interaction with private firms can lead to market outcomes which are more efficient *vis à vis* the equilibria obtaining when the market is populated by private firms only (Lambertini, 2017). This paper deals with this efficiency issue in managerial mixed markets, i.e. under the assumption that market decisions of both the public and the private firms are possibly taken by managers, whose objective functions are strategically defined by their respective firms' owners. In particular, we identify the structure of the delegation contract offered to public managers which, coupled with a standard delegation contract on the private side, ensures the best welfare performance.

The analysis of bilateral delegation in mixed markets dates back to Barros (1995) and White (2001). Their crucial assumption is that there exists a single market for managers, serving both the private shareholders and the government. This implies that the structure of the delegation contract is the same for both private and public managers - their assigned objective function is of either the Fershtman and Judd (1987) or the Vickers (1985) type, combining the firm's profits with the revenues or the sold quantities. In this framework what distinguishes a public from a private manager is the relative weight of the components of their objective function, defined according to the ultimate objectives (welfare or profits) of the firms' owners. From an efficiency perspective, the key results of this approach is that for the public firm hiring a manager is not necessarily welfare enhancing: under price competition, bilateral delegation is indeed the equilibrium outcome, but it entails lower welfare as compared with a standard mixed duopoly (Bárcena-Ruiz, 2009); under quantity competition, the equilibrium asymmetric organizational structure is characterized by delegation only on the private side (Chirco *et al*, 2014).[1]

An alternative, less investigated, approach builds on the idea that public and private owners, having different objectives, are likely to offer their managers different types of contracts. This is the case, e.g., with Heywood and Ye

[1]The assumption that the structure of the delegation contract is the same for private and public managers is adopted in many mixed market models. Under this assumption, the issue of endogenous timing is addressed by Nakamura and Inoue (2007, 2009), and Bárcena-Ruiz (2013); further extensions to the content of the delegation contracts are in Nakamura (2019a).

(2009) – where the public managers are instructed to maximize a linear combination of welfare and profits (i.e. to follow a partial privatization scheme), while the private managers are a offered a sales-based contract of the Fershtman and Judd type. Also in this case no univocal indication emerges as to the welfare gains of hiring a public manager: in the same vein as Delbono and De Fraja (1989) it is the number of private competitors which plays a key role – while in a mixed duopoly a welfare gain is observed, in oligopoly this may or may not occur depending on the technology properties.[2]

In this paper we follow the approach which allows for different contract structures for private and public managers. However, we move from the previous literature by assuming that the government – the owner of the public firm – does not commit to any predetermined contract structure, but rather chooses a managerial contract of the Generalized Welfare type: managers are instructed to behave according to an objective function, where each welfare component (consumer's surplus, public profits and private profits) enters separately and is strategically weighted. This form of strategic delegation has been originally investigated by White (2002), who interpreted it in terms of manipulation of the objectives of the public firms. More recently, Benassi *et al* (2014) analyze the optimal manipulation in a unilateral delegation framework, i.e. when market decisions are taken by public managers with Generalized Welfare objectives, and by profit maximizing private firms. In this framework, the key result is that a Generalized Welfare delegation encompasses all forms of delegation based on welfare component: strategic choices like partial privatization, relative-profit-based delegation, consumers' oriented management, are specific implementations of an optimal manipulation rule, and are obtained by imposing on the latter appropriate additional constraints on the weights of the welfare components.

In what follows we extend the analysis of the effects of a Generalized Welfare delegation by coupling it with a standard Vickers type delegation by the private firm. We prove that in a quantity-setting mixed duopoly this structure of the delegation contracts ensures full efficiency of the market outcome. Given a managerial organizational structure for both the public and private firms, the adoption of the optimal Generalized Welfare delegation ends up assigning a full regulatory role to the presence of a public firm. This sharp

[2] Other examples of mixed market models with different delegation contracts for public and private managers are in Nakamura (2015, 2019b) – where a sales-based contract on the private side is coupled with a contract for public managers combining welfare and the excess of the consumers' surplus over profits.

result calls for an assessment of the incentives perceived by the private side of the market to strategically rely on managers. We verify that when the choice to hire a manager is endogenized, bilateral delegation is observed only when the decisions on the organizational structure are sequential, and the government commits to limit its aggressiveness. This configuration is shown to have the properties of a subgame-perfect Nash equilibrium in timing, organizational structure and market interaction, and, though not fully efficient, is anyhow the most efficient outcome as compared to any form of unilateral delegation or bilateral delegation with alternative (more restrictive) structures of managerial incentives for the public managers.

Our discussion is organized as follows. In Section 2 we develop our quantity-setting mixed duopoly model with bilateral delegation, and we prove the efficiency property of the equilibrium of the multistage game in the delegation/manipulation parameters and quantities. In Section 3 we endogenize the choices whether to hire a manager or not, and the timing of the above decisions. Some conclusions are gathered in Section 4.

2 A mixed duopoly with managerial firms: private delegation and public manipulation

We consider a duopolistic market for a differentiated product, populated by a continuum of identical consumers (normalized to 1) whose preferences are described by the following semi-linear utility function:

$$V = U(q_0, q_1) + y = q_0 + q_1 - \frac{1}{2}\left(q_0^2 + q_1^2 + 2\gamma q_0 q_1\right) + y \tag{1}$$

where q_i, $i = 0, 1$, denotes the consumption of the two varieties, y is the *numéraire* commodity traded in a perfectly competitive market, and $\gamma \in (0, 1)$ measures the degree of product substitutability.

On the supply side of this market a public firm (indexed by 0), and a private firm (indexed by 1) strategically compete with respect to quantities. The two firms produce under the same constant returns to scale technology, with a constant average and marginal cost $c \in [0, 1)$. We assume that both firms are managerial. For the private firm, consistently with the standard delegation literature, this means that while its owner has an ultimate objective in terms of her own profit

$$\pi_1 = (p_1 - c)\, q_1 \tag{2}$$

actual market decisions are delegated to a manager whose objective function combines profits and output (Vickers, 1985):

$$M_1 = (p_1 - c)\, q_1 + \theta q_1 \tag{3}$$

In (3) the delegation parameter θ is strategically set by the owner in a profit maximization perspective.

As far as the public firm is concerned, its characterization as managerial is not straightforward. As is well known, its owner, the government, has an ultimate objective in terms of social welfare, i.e. the sum of the consumer's surplus and the firms' profits, $W = CS + \pi_0 + \pi_1$. When we consider a public managerial firm, we depart from the most common approach which attributes to public managers the same objective function as private managers. Rather, we assume that the government is free to instruct public managers to behave on the market according to a so-called Generalized Welfare (GW) objective, i.e. a linear combination of the welfare components (White, 2002; Benassi *et al,* 2014):

$$GW = \alpha CS + \beta \pi_0 + \delta \pi_1$$

where the weights α, β, and δ, are strategically set by the government in a welfare maximizing perspective. This manipulation of the weights of the welfare components can be seen as the distinctive feature of the delegation contract offered to public managers; accordingly, these weights play the same strategic role as delegation parameters. Without any loss of generality, in the sequel we shall set $\beta = 1$, thus interpreting α and δ as the *relative* weights public managers are instructed to assign to the consumer's surplus and the private firm's profits, in terms of the weight assigned to the public firm profits.

As we shall see below, the behaviour of this market is described by a three-stage game, which is solved by backward induction. At the last (third) stage of the game, the market stage, the managers of both firms decide the optimal quantities according to their objective functions, given the delegation and manipulation parameters set by the respective owners. These parameters are set in two previous stages. The reason why two stages are required is the following. As we shall see in detail below, when the public and private owners have to determine the manipulation-delegation parameters a key asymmetry arises: while profit maximization with respect to the delegation parameter allows to identify univocally the optimal response of the private owner to the relative weights set by the government, welfare maximization with respect to the manipulation parameters does not identify specific values of the relative weights in response to the private delegation parameter, but it determines a linear rule that the optimal relative weights must satisfy. The existence of this degree of freedom in the government's choice is well known in the literature (White 2002, Benassi et al 2014). In our framework, it implies that the full solution of the game requires (a) a second stage at which the private delegation parameter, and only one of the two relative weights, are actually determined, both contigent on the value of the other relative weight, and (b) a first stage at which the remaining relative weight is set by the government. In particular, in the sequel we shall assume that at the first stage the government sets the relative weight of the consumer's surplus; at the second stage the private firm and the government set respectively the delegation parameter and the relative weight of private profits in the welfare function; at the third stage private and public managers strategically compete with respect to quantities.

2.1 The market stage

Since preferences are described by (1), at the market stage the two firms face the following inverse demand functions:

$$p_0 = 1 - q_0 - \gamma q_1 \tag{4a}$$
$$p_1 = 1 - q_1 - \gamma q_0 \tag{4b}$$

Consider now the public firm. Given (4a) and (4b), the social welfare can be written as:

$$\begin{aligned} W = {} & \tfrac{1}{2}\left((1-\gamma)\left(q_0^2+q_1^2\right)+\gamma\left(q_0+q_1\right)^2\right)+ \\ & +\left(1-q_0-\gamma q_1-c\right)q_0+\left(1-q_1-\gamma q_0-c\right)q_1 \end{aligned} \tag{5}$$

so that the objective function assigned to the public manager at the market stage is the following Generalized Welfare function:

$$\begin{aligned} GW = {} & \tfrac{\alpha}{2}\left((1-\gamma)\left(q_0^2+q_1^2\right)+\gamma\left(q_0+q_1\right)^2\right)+ \\ & +\left(1-q_0-\gamma q_1-c\right)q_0+\delta\left(1-q_1-\gamma q_0-c\right)q_1 \end{aligned} \tag{6}$$

By maximizing (6) with respect to q_0 under the above normalization we obtain the following reaction function of the public firm's manager:

$$q_0 = q_0\left(q_{1.}\right)=\frac{1}{2-\alpha}\left((1-c)-(1-\alpha+\delta)\gamma q_1\right) \quad \text{if } q_0\left(q_1\right)>0 \tag{7a}$$
$$q_0 = 0 \quad \text{if } q_0\left(q_1\right)\leq 0$$

the second order conditions being satisfied for $\alpha < 2$.[3]

As far as the private firm is concerned, substituting (4b) into (3) and maximizing M_1 with respect to q_1, yields the reaction function of the private firm's manager:[4]

$$\begin{aligned} q_1 &= q_1(q_0) = \frac{1}{2}(1 - c + \theta - \gamma q_0) \qquad \text{if } q_1(q_0) > 0 \qquad (7b) \\ q_1 &= 0 \qquad \text{if } q_1(q_0) \leq 0 \end{aligned}$$

The solution of (7a) and (7b) gives the following quantities:

$$q_0(\alpha, \delta, \theta) = \frac{(\gamma(1 - \alpha + \delta) - 2)(1 - c) + \gamma(1 - \alpha + \delta)\theta}{(1 - \alpha + \delta)\gamma^2 + 2(\alpha - 2)} \qquad (8a)$$

$$q_1(\alpha, \delta, \theta) = \frac{(\alpha - (2 - \gamma))(1 - c) - (2 - \alpha)\theta}{(1 - \alpha + \delta)\gamma^2 + 2(\alpha - 2)} \qquad (8b)$$

which, if positive, are the equilibrium quantities at the market stage,

2.2 Optimal delegation and optimal manipulation

Equations (8a) and (8b) describe the outcome at the market stage of the interaction between the public and the private manager, in terms of the manipulation and the delegation parameters set by the government and the private firm's owner. It is by anticipating this outcome that the government and the owner of the private firm define their optimal manipulation and delegation strategies, according to their ultimate objectives.

Again, we consider first the public side. If the government assigns to the public firm's manager a GW-type objective, it has to set, according to a welfare maximizing criterion, two manipulation parameters, α and δ, the relative weight of the consumer's surplus and the private profit respectively. This kind of manipulation, however, exhibits a key property (White, 2002; Benassi *et al*, 2014): the optimality (first order) conditions with respect to the above parameters are not linearly independent. In our case, when (8a)

[3]Should the weight assigned to the consumer surplus be greater than the double of the weight assigned to the own profits, the convex component of the *GW* function would dominate the concave one, and the overall shape of *GW* would turn turn out not to be concave in q_0.

[4]For the private firm, the second order conditions at the market stage are always satisfied.

and (8b) are substituted into the welfare function (5), maximization with respect to α or δ yields for any θ the same optimal relation between α and δ, i.e. the following optimal manipulation rule:[5]

$$\delta = \frac{\theta\gamma^3-(2-\gamma)^2(1-c)}{\gamma(2(1-\gamma)(1-c)+(2-\gamma^2)\theta)} + \frac{\gamma\left(1-\gamma^2\right)\theta+\left(4-2\gamma^2-\gamma\right)(1-c)}{\gamma(2(1-\gamma)(1-c)+(2-\gamma^2)\theta)}\alpha \tag{9}$$

which is consistent with positive quantities of the private firm at the market stage (equation 8b) for

$$\theta > \frac{2\left(\gamma-1\right)\left(1-c\right)}{\left(2-\gamma^2\right)} \tag{10a}$$

and with positive quantities of the public firm (equation 8a) for

$$\theta < \frac{\left(4-3\gamma\right)\left(1-c\right)}{\gamma} \tag{10b}$$

Equation (9) has a twofold interpretation. The first stresses its being indeed a rule. As in the case of unilateral manipulation by the public firm, the government, by instructing its manager to assign to the welfare components *any* pair of relative weights consistent with the rule, achieves at the market stage the same outcome as a Stackelberg leader along the private firm's reaction function. Clearly, once delegation by the private firm is allowed for, the structure of the optimal rule is affected by the private delegation parameter θ, since the latter acts as a shift parameter of the private firm reaction function at the market stage.[6] The second interpretation sees it as a reaction function. Once one of the two parameters under the government control is given, equation (9) defines the optimal response of the other parameter to the delegation choice of the owner of the private firm.

The delegation problem is easily stated. By using the inverse demand function (4b) and equations (8a) and (8b) into (2), profit maximization with respect to θ gives the following optimal reply of the private firm to the choice of the manipulation parameters by the public one:

$$\theta = \frac{1}{2}\gamma^2\left(1-c\right)\frac{\left(1-\alpha+\delta\right)\left(2-\alpha-\gamma\right)}{\left(2-\alpha\right)\left(2-\alpha-\gamma^2\left(1-\alpha+\delta\right)\right)} \tag{11}$$

[5]Indeed, the second order condition $\partial^2 W/\partial\delta^2 < 0$, or $\partial^2 W/\partial\alpha^2 < 0$, is verified if (9) holds.

[6]Indeed, it is straightforward to check that (9) collapses to the optimal rule in Benassi *et al* (2014), once θ is set to zero, and our normalization $\beta = 1$ is taken into account.

The second order condition requires:

$$\delta < \frac{\left(2-\alpha-\gamma^2\left(1-\alpha\right)\right)}{\gamma^2} \tag{12a}$$

given the second order condition for GW maximization at the quantity stage, $\alpha < 2$. Given these restrictions, equation (11) is consistent with positive quantities of the private firm for:

$$\alpha < 2-\gamma \tag{12b}$$

and with positive quantities of the public firm for:

$$\delta < \frac{\gamma\left(\alpha-1\right)\left(2+\gamma-\alpha\right)+2\left(2-\alpha\right)}{\gamma\left(2+\gamma-\alpha\right)} \tag{12c}$$

It can be checked that for any γ, if (12b) and (12c) are satisfied, then the second order condition (12a) is also satisfied.

The system of optimal replies (9) and (11) is clearly underdetermined. This reflects the fundamental asymmetry between the two firms: while the private owner controls one delegation parameter, a government delegating according to a generalized welfare criterion enjoys the freedom to choose among infinite pairs of optimal relative weights ensuring the same welfare outcome. In other words, the system of equations (9) and (11) being underdetermined is the consequence of the degree of freedom characterizing equation (9).

The existence of a degree of freedom for the government implies that a full solution for the delegation-manipulation equilibrium requires two stages: a second stage of the game in which consistency between the optimal public manipulation rule and the private delegation reply is ensured, and a first stage in which the government exploits its degree of freedom in order to achieve the best implementation of its optimal rule.

2.2.1 The second stage

We now solve equations (9) and (11) for the delegation parameter θ and one of the two manipulation parameters set by the public firm. In what follows we solve the system in terms of θ and δ, for given α. This amounts to assuming that in the sequence of government's decisions, the first parameter set by the government itself, i.e. the parameter set at the first stage, is the relative weight of the consumer's surplus. [7]

The following Proposition can now be established:

Proposition 1 *Let* $\underline{\alpha}(\gamma) = 0$ *for* $\gamma \in (0, \overline{\gamma}]$ *– with* $\overline{\gamma} \approx 0.793$ *– and* $\underline{\alpha}(\gamma) = (4 - \gamma^3 + 2\gamma^2 - 6\gamma) / (2 + \gamma^2 - 4\gamma)$ *for* $\gamma \in (\overline{\gamma}, 1)$*; then for any* $\alpha \in (\underline{\alpha}(\gamma), 2 - \gamma)$ *there exists a unique subgame equilibrium in the delegation-manipulation game.*

Proof. Consider $\alpha < 2 - \gamma$, so that (12b) is satisfied. By solving equations (9) and (11) for θ and δ, we obtain:

$$\theta(\alpha) = \frac{1}{4}\gamma \frac{((4-3\gamma)\alpha - (4 - 3\gamma(2-\gamma)))(1-c)}{(1-\gamma^2)(2-\alpha)} \tag{13a}$$

$$\delta(\alpha) = \frac{(4-\gamma)(1-\gamma^2)\alpha^2 + (\gamma^4 - 3\gamma^3 + 7\gamma^2 + 6\gamma - 12)\alpha + (\gamma^3(2-\gamma) + 8(1-\gamma))}{\gamma((\gamma^2 - 4\gamma + 2)\alpha + (\gamma^3 - 2\gamma^2 + 6\gamma - 4))} \tag{13b}$$

These are the solution of the second stage, if they ensure positive quantities at the market stage, i.e. if they are consistent with (10a), (10b) and (12c). By substituting $\theta(\alpha)$ and $\delta(\alpha)$ in (8a), the quantity produced at the market stage by the public firm can be written as:

$$q_0(\alpha) = \frac{1}{4} \frac{(3\alpha\gamma - 4\alpha - \gamma^2 - 6\gamma + 8)(1-c)}{(2-\alpha)(1-\gamma)(\gamma+1)} \tag{14a}$$

which is positive for $\alpha < (8 - \gamma^2 - 6\gamma)/(4 - 3\gamma)$. Since the latter is greater than $2 - \gamma$, $q_0(\alpha) > 0$ for all $\alpha < 2 - \gamma$. Substitution of $\theta(\alpha)$ and $\delta(\alpha)$ in (8b) restates the quantity produced by the private firm at the market stage as:

$$q_1(\alpha) = \frac{1}{4} \frac{(\alpha(\gamma^2 - 4\gamma + 2) + \gamma^3 - 2\gamma^2 + 6\gamma - 4)(1-c)}{(\alpha - 2)(1-\gamma)(\gamma+1)} \tag{14b}$$

For (14b) to be positive it is necessary that $\alpha(2 + \gamma^2 - 4\gamma) < 4 - \gamma^3 + 2\gamma^2 - 6\gamma$. This inequality holds for any $\alpha \in (0, 2-\gamma)$ when $\gamma \in (0, \overline{\gamma}]$, where $\overline{\gamma} \approx 0.793$

[7] While in principle the alternative sequence can be considered, where the relative weight of the private firm's profit is set at the first stage, having a first-stage choice involving the consumer's surplus – thus making all other decisions conditonal on it – is arguably more attuned with the specific role played by public firms in mixed markets. Solving the model with this sequence has also an analytical justification which will be touched upon later in this Section (see f.note 9).

is the real root of $(4-\gamma^3+2\gamma^2-6\gamma)$. For $\gamma > \overline{\gamma}$ the above inequality holds for $\alpha > \underline{\alpha}(\gamma) = (4-\gamma^3+2\gamma^2-6\gamma) / (2+\gamma^2-4\gamma)$. Clearly, when the conditions for positive quantities are met, $\delta(\alpha)$ and $\theta(\alpha)$ satisfy the set of inequalities (10a), (10b) and (12c). ■

Proposition 1 establishes that (13a) and (13b) are the unique subgame equilibrium of the delegation-manipulation game for a range of values of α, strictly lower than $2-\gamma$. It may be useful to recall that for any given α, (13a) determines the position of the private firm reaction function at the market stage, as optimally chosen at equilibrium by the private owner, while (13b) ensures that the government achieves the Stackelberg outcome on that reaction function. Therefore, these equations highlight very clearly the key property of our model: through its degree of freedom – i.e. through the availability of the additional strategic instrument α – the public firm has the possibility not only to choose a specific point on its rival's reaction function at the market stage, but also to affect the position of that reaction function.

Proposition 2 deals with the limit case $\alpha = 2-\gamma$.

Proposition 2 *For $\alpha = 2-\gamma$, there exists a unique equilibrium of the delegation-manipulation game, where $\theta = (1-c)/(1+\gamma)$ and $\delta = (1+\gamma-\gamma^2)/\gamma$.*

Proof. According to Proposition 1, $\theta(\alpha)$ and $\delta(\alpha)$ are defined for $\alpha \in (0, 2-\gamma)$ if $\gamma \leq \overline{\gamma} \approx 0.793$, and for $\alpha \in (\underline{\alpha}, 2-\gamma)$ if $\gamma > \overline{\gamma}$, where $\underline{\alpha} = (4-\gamma^3+2\gamma^2-6\gamma) / (2+\gamma^2-4\gamma)$. Consider now the following pair (θ, δ):

$$\lim_{\alpha \to (2-\gamma)^-} \theta(\alpha) = \frac{1-c}{1+\gamma}$$

$$\lim_{\alpha \to (2-\gamma)^-} \delta(\alpha) = \frac{1+\gamma-\gamma^2}{\gamma}$$

When $\alpha = 2-\gamma$, the pair $\theta = (1-c)/(1+\gamma)$, $\delta = (1+\gamma-\gamma^2)/\gamma$ lies on the government's reaction function at the delegation-manipulation stage. However, if $\alpha = 2-\gamma$, for $\delta = (1+\gamma-\gamma^2)/\gamma$ the reaction function of the private owner cannot be defined since the second order condition (12a) is not satisfied: the profits of the private firm are equal to zero for all values of θ, and therefore the choice of θ is indifferent for the owner. Hence, the pair $\theta = (1-c)/(1+\gamma)$, $\delta = (1+\gamma-\gamma^2)/\gamma$ is a Nash equilibrium, to which equations (8a)–(8b) associate the following positive quantities: $q_0 =$

$q_1 = (1-c)/(1+\gamma)$. The latter also coincide with $\lim_{\alpha\to(2-\gamma)^-} q_0(\alpha)$ and $\lim_{\alpha\to(2-\gamma)^-} q_1(\alpha)$ and therefore, given the continuity of the $q_0(\alpha)$ and $q_1(\alpha)$ functions around $\alpha = 2-\gamma$, with $q_0(2-\gamma)$ and $q_1(2-\gamma)$.[8] The above equilibrium is also unique: given $\alpha = 2-\gamma$, there cannot be an equilibrium with two active firms if $\delta \neq (1+\gamma-\gamma^2)/\gamma$. Indeed, for $\delta > (1+\gamma-\gamma^2)/\gamma$ the private firm profit function is convex in θ; for $\delta < (1+\gamma-\gamma^2)/\gamma$, the profit function has a maximum in $\theta = 0$, which, coupled with $\alpha = 2-\gamma$, implies $q_1 = 0$. ■

Propositions 1 and 2 allow to identify the range of values of the weight of the consumer's surplus α, $\alpha \in (\underline{\alpha}(\gamma), 2-\gamma]$, which ensures that a subgame equilibrium exists with positive quantities at the market and delegation stage. We now turn to the optimal choice of α, made by the government at the first stage of the game.

2.2.2 The first stage

By using (14a) and (14b) into (5) and (2) (using 4b) for $\alpha \in (\underline{\alpha}(\gamma), 2-\gamma]$, we obtain the following expressions for welfare and profits in terms of α only (given the structural parameters γ and c):

$$W(\alpha) = \frac{1}{32}(1-c)^2 \frac{\left(\left(3\gamma^2-32\gamma+28\right)\alpha^2+\left(6\gamma^3-12\gamma^2+120\gamma-112\right)\alpha+3\gamma^4-12\gamma^3+8\gamma^2-112\gamma+112\right)}{(2-\alpha)^2(1-\gamma^2)} \tag{15a}$$

$$\pi_1(\alpha) = \frac{1}{8}(1-c)^2 \frac{\left(\left(2+\gamma^2-4\gamma\right)\alpha+\gamma^3-2\gamma^2+6\gamma-4\right)(\alpha-2+\gamma)}{(2-\alpha)^2(1-\gamma^2)} \tag{15b}$$

We can now establish the following Proposition.

Proposition 3 *In a mixed duopoly with manipulation of the public managers objectives and delegation by the private owner, there exists a unique subgame-perfect equilibrium and this equilibrium is efficient.*

Proof. Consider equation (15a). It can be checked that $W'(\alpha) > 0$ for $\alpha \in (\underline{\alpha}(\gamma), 2-\gamma)$ and that $W'_{-}(2-\gamma) = 0$, so that the value of α which maximizes (15a) is:

$$\alpha^* = 2-\gamma$$

[8]Since for $\alpha = 2-\gamma$ the Nash equilibrium values of θ and δ equal the limit of the corresponding continuous functions $\theta(\alpha)$ and $\delta(\alpha)$, the functions $q_0(\alpha)$ and $q_1(\alpha)$ are defined over the interval $(\underline{\alpha}, 2-\gamma]$.

When the government at the first stage instructs its managers to assign this weight to the consumer's surplus, then at the second stage the equilibrium choices are $\delta^* = (1+\gamma-\gamma^2)/\gamma$ from the public side, and $\theta^* = (1-c)/(1+\gamma)$ from the private owner (Proposition 2).The corresponding quantities at the market stage are $q_0^* = q_1^* = (1-c)/(1+\gamma)$. These quantities are efficient, in that they are the solution of the unconstrained welfare maximization problem, and they are therefore associated with marginal cost pricing and zero profits.[9] ∎

Why is it that the outcome of the interaction between a delegating private firm and a GW-type managerial public firm delivers an efficient outcome, while this is not the case in the framework of unilateral manipulation? When public manipulation is unilateral, the market decisions of public managers are constrained by the profit maximizing reaction function of the private firm. This however prevents achieving full efficiency: indeed, though manipulation ensures the maximum welfare consistent with that reaction function, the latter does not include the efficient outcome, which accordingly lies outside the set of viable policy options. In this case, the advantage of the degree(s) of freedom granted by the GW-type manipulation boils down to the (political) possibility of implementing the optimal rule in different guises, all of which nonetheless deliver the same (not fully efficient) equilibrium welfare level.

When instead the private firm engages in a strategic competition at the delegation stage, the set of options available to the public firm widens. It is still true that for any *given* delegation choice of the private firm, the maximum welfare consistent with the private reaction function can be achieved by

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[9] One might wonder whether this result is robust to (a) alternative formulations of the private delegation contract, and (b) a reversal in the order of the government's decisions.

As to point (a), if the Fershtman and Judd (1987) paradigm is adopted, the manager objective function is a linear combination of profits and revenues, $M_1 = \psi\pi_1 + (1-\psi)\,p_1 q$; solving the model, the equilibrium relative weights of the GW function are again $\alpha = 2-\gamma$ and $\delta = \left(1+\gamma-\gamma^2\right)/\gamma$, while the equilibrium value of the delegation parameter is $\psi = (c\,(2+\gamma)-1)\,/c\,(1+\gamma)$. These manipulation and delegation parameters ensure that the efficient quantities $q_0^* = q_1^* = (1-c)/(1+\gamma)$ are produced.

As to point (b), it amounts to solving the second stage (eqts (9) and (11)) for α and θ given δ, and then for δ at the first stage. Calculations are in this case more cumbersome and numerical simulations are called for. These show that: (i) the same efficient solution emerges straightforwardly for $\gamma \geq 1/2$; (ii) for $\gamma < 1/2$ there are two equilibria at the second stage, but the efficient solution is still the unique subgame perfect equilibrium.

We thank an anonymous referee for drawing our attention to both these points, the detailed calculations for which are available upon request.

any combination of parameters satisfying the optimal rule; however, under the GW manipulation the optimal private firm delegation depends on both parameters controlled by the government: as a result, the latter can exploit its degree if freedom to force the private firm delegation choice in such a way that its reaction function actually includes the efficient outcome. Accordingly, in this framework, the two weights become independent instruments, through which the government is able to (a) affect the strategic interaction in such a way that the efficient outcome becomes a feasible option, and (b) achieve that outcome through the actual implementation of the optimal rule.

3 The endogenous choice of the organizational structure

The analysis of previous section leads to a very sharp result: in the presence of a private managerial firm, the government is able to achieve an efficient outcome through an appropriate manipulation of the weights of the components of its objective function. This immediately raises the question of whether both firms being managerial is actually an equilibrium configuration of a mixed duopoly, when the GW-type manipulation is allowed for. The issue of the endogenous choice of the firms' organizational structure is discussed in this Section under two alternative hypotheses on the timing of this decision. Given the peculiar characteristics of the GW manipulation, in this context the crucial distinction is not whether decisions are taken simultaneously or sequentially. Rather, the key element is whether, in case of public manipulation, the private owner may take the hiring decision after a commitment has been taken by the government on the relative weight assigned to the consumer's surplus α – which in a way could compensate the advantage of the public side of unilaterally choosing that value.

Therefore, in the sequel we shall consider two cases. In the first, the decisions whether to manipulate/delegate are taken in a pre-stage, simultaneously or sequentially, before any other decision. In particular, the private owner has to decide the organizational structure before any government's decision on the weights assigned to the welfare components. In the second case, which is sequential by nature, the private owner, in the presence of public manipulation, decides whether to hire a manager once the weight of the consumer's surplus α has been set by the government. The outcome of these two cases will also allow us to endogenize the timing of the above decisions.

3.1 The pre-stage case

The implicit hypothesis of the model developed in Section 2 is that before entering the strategic interaction on the delegation/manipulation parameters and the market decisions, both the government and the private owner have taken a binding commitment to hire managers. When the latter decision is endogenized, and is assumed to be simultaneously taken at a pre-stage of the delegation and market game, the reduced form at the pre-stage is synthesized by the payoffs matrix of Table 1, where M and NM denote manipulation and non-manipulation by the government, and D and ND denote delegation and non delegation by the private owner:

Table 1
The payoffs matrix at the pre-stage

	Private D	Private ND
Gvt M	$W(M,D)=\frac{(1-c)^2}{\gamma+1}$ $\pi_1(M,D)=0$	$W(M,ND)=\frac{(1-c)^2(7-6\gamma)}{2(4-3\gamma^2)}$ $\pi_1(M,ND)=4\frac{(1-c)^2(1-\gamma)^2}{(4-3\gamma^2)^2}$
Gvt NM	$W(NM,D)=\frac{(1-c)^2(7+\gamma)}{8(1+\gamma)}$ $\pi_1(NM,D)=\frac{(1-c)^2(1-\gamma)}{4(1+\gamma)}$	$W(NM,ND)=\frac{(1-c)^2\left(7-2\gamma^2-6\gamma+2\gamma^3\right)}{2(2-\gamma^2)^2}$ $\pi_1(NM,ND)=\frac{(1-c)^2(1-\gamma)^2}{(2-\gamma^2)^2}$

In Table 1, $W(M,D)$ and $\pi_1(M,D)$ are the outcome of the model of Section 2, $W(M,ND)$ and $\pi_1(M,ND)$ are the outcome in the case of unilateral manipulation by the public firm (Benassi *et al*, 2014), $W(NM,D)$ and $\pi_1(NM,D)$ are the firm's payoffs in the case of unilateral private delegation (Chirco *et al*, 2014), and $W(NM,ND)$ and $\pi_1(NM,ND)$ are the payoffs of the standard mixed Cournot duopoly with imperfect substitutability (Fujiwara, 2007). By comparing the payoffs in Table 1, we can establish the following Proposition.

Proposition 4 *In a mixed duopoly where the government has the option to manipulate the public managers objectives according to a GW criterion, and the private firm has the option to delegate, if the decision on manipulation/delegation are taken simultaneously at a pre-stage, the subgame-perfect equilibrium is characterized by unilateral public manipulation.*

Proof. Since $W(M,D) > W(NM,D)$ and $W(M,ND) > W(NM,ND)$, manipulation is a dominant strategy for he public firm. Moreover, $\pi_1(M,ND) > 0 = \pi_1(M,D)$. Therefore, the subgame-perfect equilibrium is characterized by unilateral manipulation by the public firm, with the private firm choosing to be profit maximizer at the market stage. ■

It's worth stressing that in the situation discussed above, the choice of a GW-type manipulation on the public side leads to an asymmetric organizational structure with a public managerial firm, which is the opposite of the asymmetric structures in White (2001) and in Chirco *et al* (2014), where it is the private firm to be managerial at equilibrium.

It can be checked that the same result as the above simultaneous game is obtained if the decisions on the organizational structure are sequential, the private firm being the first mover. On the contrary, an asymmetric structure with unilateral delegation of the private firm arises if the game is sequential, with the government as first mover. The government anticipates the decisions of the private owner, i.e. to choose an entrepreneurial structure in case of public manipulation, and a managerial structure in case of pure welfare maximization. Since $W(NM,D) > W(M,ND)$, the sub-game perfect equilibrium involves a non-manipulating public firm and private delegation.

When the timing of decisions on the organizational structure is itself endogenized through a standard delay game (Hamilton and Slutsky, 1990), multiple equilibria arise. Indeed. since unilateral public manipulation is the outcome of both the simultaneous and the private first-mover game, while unilateral private delegation is the outcome of the public first-mover game, we obtain the following solution: a) the sequential structure with the private firm as first mover is an equilibrium for all values of γ; b) the simultaneous structure is an equilibrium as long as $\pi_1(M,ND) > \pi_1(NM,D)$, which occurs for $\gamma \leq 2\sqrt{2}/3$, while c) the sequential structure with the government as first-mover is an equilibrium for $\gamma \geq 2\sqrt{2}/3 \approx 0.943$.

The key result of the above analysis is that the bilateral manipulation-delegation setup discussed in the previous Section is not observed as an equilibrium in the extended game involving the choice of the organizational structure. The ability of the government to guide the interaction between the managerial public and private firms to an efficient outcome is in a sense self-defeating: the private firm perceives an incentive to deviate from any situation in which it commits to hiring a manager if this implies a zero profits perspective. But the latter is the inevitable outcome if the government fully exploits the advantage of the additional degree of freedom of a GW-type manipulation. Bilateral delegation can be an equilibrium organizational structure only if the government commits to a less aggressive behaviour, by foregoing the advantages of that degree of freedom through a binding pre-announcement of the relative weight assigned to the consumer's surplus α. This alternative setup is investigated in the next subsection.

3.2 The precommitment case

Consider now the following alternative sequence of decisions: the government decides whether to manipulate or not its managers' objective function, and, in the case manipulation is chosen, it commits to assign a specific value to the relative weight of consumer's surplus α; then the private owner, knowing α, decides whether to hire a manager or not. The following Proposition can now be established.

Proposition 5 *Consider a mixed duopoly where the government has the option to manipulate the public managers objectives according to a GW criterion, and the private firm has the option to delegate. If (a) the decisions on manipulation/delegation are sequential, the government being the first mover, and (b) in case of manipulation the government commits to a specific weight of the consumer's surplus α before any delegation decision by the private firm, then the subgame-perfect equilibrium is characterized by both firms being managerial, with* $\alpha = \widetilde{\alpha} = (8 - 3\gamma^4 + 6\gamma^3 - 6\gamma^2 - 4\gamma) / (8 + 3\gamma^3 - 8\gamma^2 - 2\gamma) < 2 - \gamma$.

Proof. Assume the government decides not to manipulate. Then the payoffs are $W(NM, ND)$ and $\pi_1(NM, ND)$ (see Table 1) if the private firm hires no managers, while the payoffs are $W(NM, D)$ and $\pi_1(NM, D)$ if the private firm delegates. Therefore, since $\pi_1(NM, D) > \pi_1(NM, ND)$, if the government decides not to manipulate, the final outcome is that the private firm delegates and the payoffs are $W(NM, D)$ and $\pi_1(NM, D)$. Assume now that the government decides to manipulate, in which case it has to commit to a value of α. When deciding upon α, the government anticipates that the private firm will compare the profits associated to bilateral delegation, $\pi_1(\alpha)$ in equation (15b), with the profits associated to public unilateral manipulation, $\pi_1(M, ND)$. If $\pi_1(\alpha) < \pi_1(M, ND)$, then the private owner does not hire a manager and the government, in the subsequent stage, sets the value of δ

consistent with the optimal unilateral manipulation rule, given α. The payoffs in this case are $W(M, ND)$ and $\pi_1(M, ND)$. If $\pi_1(\alpha) \geq \pi_1(M, ND)$, which occurs for

$$0 < \frac{4 + 3\gamma^2 - 6\gamma}{4 - 3\gamma} \leq \alpha \leq \frac{8 - 3\gamma^4 + 6\gamma^3 - 6\gamma^2 - 4\gamma}{8 + 3\gamma^3 - 8\gamma^2 - 2\gamma} < 2 - \gamma$$

the private owner hires a manager, and the payoffs are $W(\alpha)$ and $\pi_1(\alpha)$ as in equations (15a) and (15b). Given that $W(\alpha)$ is increasing in α for $\alpha < 2-\gamma$, the welfare maximizing value of α consistent with the private firm hiring a manager is

$$\widetilde{\alpha} = \frac{8 - 3\gamma^4 + 6\gamma^3 - 6\gamma^2 - 4\gamma}{8 + 3\gamma^3 - 8\gamma^2 - 2\gamma}$$

Since $W(\widetilde{\alpha}) > W(M, ND)$, if the government decides to manipulate, it sets $\alpha = \widetilde{\alpha}$, so that the private firm delegates and the payoffs are $W(\widetilde{\alpha})$ and $\pi_1(\widetilde{\alpha})$. Finally, since $W(\widetilde{\alpha}) > W(NM, D)$ the subgame-perfect equilibrium is characterized by public manipulation, $\alpha = \widetilde{\alpha}$, private delegation, $W = W(\widetilde{\alpha})$ and $\pi_1 = \pi_1(\widetilde{\alpha}) > 0$.[10] ■

If the private firm has the opportunity to decide on its being managerial once α is known, then bilateral manipulation/delegation is observed at equilibrium. The question is left whether the sequence of decisions assumed in Proposition 5 has a sound justification.

If the sequence of decisions is endogenized, with the government as first-mover being associated with the pre-commitment on the weight α, this latter case turns out to be an equilibrium configuration. Indeed, all sequences (simultaneous or sequential) are deviation-proof, but the public firm being first-mover Pareto dominates all the others, which are characterized by unilateral public manipulation – $W(\widetilde{\alpha}) > W(M, ND)$.

Finally, it is worth noticing that the solution described in Proposition 5 identifies a set of relative weights in the GW function significantly different from that offered in Proposition 3. Depending on γ, the value of $\widetilde{\alpha}$ ranges from about 0.94 to about 1.02, while the correspondent value of δ from equation (13b) ranges from 1 to about 1.13. This in turn implies that the solution of Proposition 5 is clearly not efficient: it is straightforward to check that the quantity produced by the public firm is higher than the efficient quantity, while the production by the private firm is significantly lower. When the

[10]Notice that $\widetilde{\alpha} > \underline{\alpha}$, so that in this case Proposition 1 holds.

decision of the private owner to hire a manager is endogenized, the efficient solution cannot be achieved – indeed, the private firm will never commit to hire a manager if the government is allowed to set the efficient set of weights. However, for all values of γ, by setting $\widetilde{\alpha}$ and inducing the private firm to delegate, the government achieves the highest level of social welfare as compared to all possible alternatives: standard mixed duopoly, unilateral public manipulation, unilateral private delegation. Moreover, since the GW manipulation nests all forms of manipulation based on the welfare components, and is more welfare enhancing than other forms of public delegation based on quantities and revenues, the solution envisaged in Proposition 5 describes the most efficient achievable outcome of a quantity-setting mixed duopoly.

4 Conclusions

In this paper we have extended the analysis of the effects of a GW-type strategic manipulation of the objective function of public managers to situations in which - in a mixed quantity-setting duopoly - also the private firm is characterized by a managerial structure. A GW-type manipulation allows the government to freely instruct its managers to assign relative weights to the welfare components, without committing to any predetermined structure of these weights. Two basic findings emerge from our analysis. The first is that the GW-type manipulation coupled with strategic delegation by the private firm leads to an efficient outcome. This is a direct consequence of the asymmetric structure of the strategic setting, which allows the government to exploit an additional instrument, as compared to its private rival. The second finding is related to the equilibrium organizational structure of a mixed duopoly. By endogenizing the public and private decisions on manipulation/delegation, and the sequence of these decisions, the equilibrium configuration is shown to exhibit the following properties: the government manipulates and the private owner actually delegates, with a sequential structure of decision where the government, as first mover, refrains from setting the efficient set of weights, but rather it sets the weights at the most efficient level consistent with the private firm finding it profitable to hire a manager. This Nash equilibrium solution ensures the maximum welfare as compared with all other organizational structures of a quantity-setting mixed duopoly.

We believe that this model conveys two noteworthy messages in a policy perspective. The first is that GW manipulation creates a scope for the government to pursue its welfare objective *via* an additional policy instrument, *viz* by conditioning the private firm's organizational structure, thus indirectly affecting the private manager's choices. In other words, the aggressiveness of a public firm can show up not only at the market stage through larger quantities, but also by inducing an aggressive attitude of its private rival, as a reaction to its organizational choices. The second is that for this strategy to be successful, however, the incentive to aggressiveness must be consistent with the profit objectives of the private firm, so as to converge to a bilateral managerial structure. Hence the limits of such a strategy: it should not be pushed too far – too strong an aggressive manipulation policy turns out to be counterproductive, as it leads to a misalignment of the private firm's incentives, by destroying its convenience to adopt a managerial organizational structure.

The robustness of these results to a price-setting framework and the analysis of the strategic choice of the mode of competition is left to future research.

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