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Regulating the tragedy of commons: nonlinear feedback solutions of a differential game with a dual interpretation*

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Abstract

A well established dynamic model describing the impact of oligopolistic interaction on a renewable resource is revisited here to illustrate its dual interpretation as a waste removal differential game. The regulatory implications are illustrated by assuming that the public agency may control market price and possibly also access to the commons. Two different formulations of the managerial or CSR objective are envisaged, based on a combination of profits and either output or the individual share of the waste stock. It is shown that if the representative firm's objective includes the residual waste stock, there exists a unique regulated price driving to zero the steady state stock itself. Hence, the present analysis delivers some useful indications concerning an appropriate definition of the CSR objective firms should adopt.

JEL Codes: C73, L13, Q20, Q53

Keywords: waste removal; resource extraction; feedback information; regulation; tragedy of commons; CSR

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1 Introduction

Free access to the commons is the driver of the original formulation of the tragedy in Gordon (1954) and Hardin (1968). This, in terms of oligopoly games, directly translates into the question as to whether there might exist an optimal industry structure, or, an optimal number of firms in the commons. The analysis of this problem can be traced back to Cornes *et al.* (1986), Mason *et al.* (1988) and Mason and Polasky (1997, 2002).

What follows presents in a single model the impact of oligopolistic interaction on a renewable resource and a waste stock via a differential game approach. The idea that originated this paper stems from an elementary analogy between the exploitation of a renewable natural resource and waste removal, provided the dynamics according to which these two magnitudes grow over time can be assumed to be exogenously given and identical.[1] The issue at stake, then, boils down to the following: if the state is a natural resource or species, in line of principle it would be desirable to have the largest possible stock of it left at the steady state, while the opposite holds if the state variable consists of waste. Hence, the policy implications of the ensuing analysis will be opposite in the two cases.[2]

In building up the model, I will pose that firms define their individual objective functions attaching a positive weight to their output levels or harvest rates or, alternatively, to the individual symmetric share of the stock. That is, a firm's objective function is defined as a combination of profits and either the control or the state variable. One way or the other, this approach, in the light of the typical interpretation deriving from an established view in the theory of industrial organization, amounts to saying that firms have separated ownership from control via delegation contracts to managers *à la*

[1] I will leave the recycling issue outside the model. The dynamic analysis of the interplay between waste removal and recycling can be found in Highfill and McAsey (2001), Lavee *et al.* (2009), Di Corato and Montinari (2014) and Gambella *et al.* (2019), *inter alia*.

[2] A parallel literature discusses the efficiency of waste removal and the appropriate policies which can be designed to facilitate it by subministering waste disposal incentives to households in general equilibrium models often featuring recycling (see Dinan, 1993; Palmer and Walls, 1997; Fullerton and Wu, 1998; Eichner and Pethig, 2001; Gaudet *et al.*, 2001; Walls and Palmer, 2001; and Wagner, 2011, *inter alia*).

Vickers (1985). However, also in this respect one can spot a dual nature of this additional feature, whereby if the common pool is a stock of waste then maximising a combination of profits and output reveals the adoption of a CSR stance by the same firms.[3]

For the sake of simplicity, in the remainder I will quite freely refer to the state variable as a renewable resource or a waste stock, and specify the relevant interpretation of the state when it comes to evaluating the consequences of firms' behaviour, and therefore also the design of an appropriate regulation.

In particular, if the state measures a stock of waste, the ensuing analysis shows that including the state in the maximand is definitely preferable to the alternative based on a combination of profits and individual output (or waste removal). This is because under this specification of the model the regulator avails of a unique regulated price which drives to zero the residual stock associated to any stable equilibria arising under feedback information. It is also worth mentioning that a possible interpretation of the ensuing models is that the resource or waste stock consists of compressed wood or biomass pellets used as a renewable energy source to feed heating appliances in residential and/or industrial spaces, with the regulator being in a position to drive industry behaviour in the desired direction, which in this case implies the minimisation of the residual biomass.

The structure of the paper is the following. The basic setup is laid out in section 2. The first version of the game, where the CSR or managerial objective features the output level, is fully characterised in section 3, including the unregulated open-loop, linear and nonlinear feedback solutions as well as the regulated feedback game. Section 4 accounts for the linear and nonlinear feedback solutions of the alternative approach in which the objective function includes the state variable. Plausible extensions and concluding remarks are in section 5.

[3]This analogy between strategic delegation and corporate social responsibility has already beeen highlighted in the literature. See Lambertini (2013), Lambertini and Tampieri (2015) and the references therein. For an updated and exhaustive discussion of the scope of CSR and its interpretation, see Kudłak (2019).

2 The setup

The setup broadly relies on a model used, among others, by Tornell and Velasco (1992), Benchekroun and Long (2002), Fujiwara (2008) and Lambertini and Mantovani (2014, 2016), where a common property productive asset oligopoly with a linear state equation is considered.[4] The original model illustrates a differential oligopoly game of resource extraction unravelling over continuous time $t \in [0, \infty)$. The market is supplied by $n \geq 1$ firms[5] producing a homogeneous good, whose inverse demand function is $p(t) = a - Q(t)$ at any time t, with a constant reservation price $a > 0$ and $Q = \sum_{i=1}^{n} q_i(t)$, in which $q_i(t)$ is the quantity of firm i at a generic instant. Firms share the same technology, characterised by the cost function $C_i = cq_i^2(t)$, parameter $c \in (0, a)$ being constant over time. Firms operate without any fixed costs. During production, each firm exploits a renewable natural resource, whose accumulation is governed by the following dynamics:

$$\dot{S}(t) = F(S(t)) - Q(t) \tag{1}$$

with

$$F(S(t)) = \begin{cases} \delta S(t) \ \forall \, S(t) \in (0, S_y] \\ \\ \delta S_y \left(\dfrac{S_{\max} - S(t)}{S_{\max} - S_y} \right) \ \forall \, S(t) \in (S_y, S_{\max}] \end{cases} \tag{2}$$

where $S(t)$ is the instantaneous resource stock, $\delta > 0$ is its *implicit* growth rate when the stock is at most equal to S_y and δS_y is the maximum sustainable yield. Taken together, (1-2) imply that (i) if the resource stock

[4] A piecewise linear approximation of the logistic growth curve appears instead in Benchekroun (2003, 2008) and Colombo and Labrecciosa (2015). To allow for the dual interpretation of the model, the linear approximation is appropriate. The proper logistic growth model has been extensively investigated in monopoly or perfect competition (see Clark, 1990, for an overview; see also Neubert, 2003; Bressan and Staicu, 2019). It has also been used, although seldom, in oligopolistic models (see Damania and Bulte, 2007, and Lambertini and Leitmann, 2019).

[5] Under monopoly the delegation to managers would not be operated by stockholders, but CSR could be adopted, so I'm intentionally not ruling out the monopoly case. Another good reason not to do so pops up in section 4.

is sufficiently small the population grows at an exponential rate; and (ii) beyond S_y, the asset grows at a decreasing rate. Moreover, $S_{\max}$ is the *carrying capacity* of the habitat, beyond which the growth rate of the resource is negative, being limited by available amounts of food and space. In the remainder, we will confine our attention to the case in which $F(S(t)) = \delta S(t)$.

Some additional remarks are in order, to clarify the interpretation of this setup when it is used to illustrate waste management. In such a case, the state equation $\dot{S}(t) = \delta S(t) - Q(t)$ portrays a scenario in which a waste volume $\delta S(t)$ is produced at every instant (due either to production or consumption, or both) by a number of other sectors which are left unmodelled, and $Q(t)$ is the collective instantaneous effort exerted by the n firms to remove it from the environment. If the tariff charged by firms is determined by a market mechanism, the above demand tells that the representative individual is willing to pay a tariff decreasing in the volume of waste being collected. The presence of a quadratic cost in firms' profit functions indicates that waste removal takes place at decreasing returns to scale, for any $c > 0$. Having defined the basic elements of the setting, henceforth I will omit the time argument for the sake of brevity.

Firms play noncooperatively and choose their respective outputs simultaneously at every instant. At $t = 0$, each firm hires a manager whose contract specifies the instantaneous objective which the manager has to maximise. Delegation contracts are observable. As in Vickers (1985), the delegation contract establishes that the instantaneous objective function of manager i is a linear combination of profits and output:[6]

$$M_i = \pi_i + \theta q_i \tag{3}$$

in which θ determines the relevance of output in the firm's objective.

An alternative approach consists in supposing that the CSR managerial incentive is

$$M_i = \pi_i - \theta \cdot \frac{S}{n} \tag{4}$$

[6]This contract is equivalent to that considered in Fershtman and Judd (1987) and Sklivas (1987), where the maximand is a weighted average of profits and revenues, $M_i = \alpha\pi_i + (1-\alpha) R_i$, $R_i = pq_i$. A proof of the equivalence is in Lambertini and Trombetta (2002).

where θ is a weight attached to the individual symmetric share of the stock. In both cases, the manager will be rewarded with a two-part salary consisting of a fixed component meeting her/his participation constraint and a variable component increasing in M_i. According to (3), the manager receives an explicit incentive to increase firm i's waste removal activity q_i. If instead the delegation contract is based upon (4), managerial remuneration is penalised in proportion to the symmetric share of total waste not collected, if any.

Intuitively, $\theta > 0$ seems appropriate if the state is a stock of waste (while $\theta < 0$ would be obvious if the model were concerned with the preservation of a livestock). In both alternatives, θ is treated as a constant and is symmetric across the population of firms. Although it might be endogenised by modelling the optimal delegation contract, still treating θ as a parameter allows one to conduct some intuitive comparative statics about its effect on steady state magnitudes.

The i-th manager maximises the following discounted payoff flow

$$\Omega_i = \int_0^\infty M_i e^{-\rho t} dt, \tag{5}$$

under the constraint posed by the state equation

$$\dot{S} = \delta S - Q \tag{6}$$

Parameter $\rho > 0$ is the discount rate, common to all managers and constant over time. Obviously, if $\theta = 0$, firms behave as pure profit-seeking entrepreneurial units.

The analysis will be carried out under the following assumption:

Assumption 1 $\delta \geq \max \{\rho [n(n+2c)+1] / [2(1+c)], n\rho + \theta(2n-1)/(np))\}$.

This guarantees the non-negativity of the residual resource stock and the associated equilibrium quantity at the steady state under any feedback rules, in either game. Throughout the ensuing analysis, I will examine the possibility of resource exhaustion or full waste removal, depending on the interpretation of S, in terms of the key parameters $\{n, \delta, \rho, \theta\}$ and, under price regulation, p.

In the remainder of the paper, I will refer to the game relying on (3) as model I, while that using (4) will be model II.

3 Model I

A few words will suffice to capture the essence of the open-loop solution, which, for several reasons, is of very little interest. In the remainder of this section, I will treat θ as a constant and pose $\sigma \equiv a + \theta$ for the sake of simplicity. If firms don't internalise the consequences of their behaviour at any time and play the individual (static) Cournot-Nash output

$$q^{CN} = \frac{\sigma}{n + 1 + 2c} \tag{7}$$

at all times, then the residual amount of the natural resource in steady state is $S^{CN} = n\sigma / [\delta(n + 1 + 2c)] = Q^{CN}/\delta$. As the remainder of the analysis is about to show, it is worth noting that the static solution corresponds to the open-loop steady state one, which in this game is unstable (see below). Let the initial condition be $S(0) = S_0 > 0$. The relevance of the size of S_0 on the final resource stock as well as on the stability of solutions will be discussed in the ensuing analysis.

3.1 The linear feedback solution

The game can be solved under feedback rules conjecturing a linear-quadratic value function with unknown coefficients to be determined solving the resulting system of equations to determine coefficients, or following an alternative but equivalent procedure consisting in solving the relevant first order condition w.r.t. the partial derivative of the value function. For reasons which will become evident below, here I take the latter route. The Hamilton-Jacobi-Bellman (HJB) equation writes as:

$$\rho V_i(S) = \max_{q_i} \left[(\sigma - Q + cq_i) q_i + V_i'(S)(\delta S - Q) \right] \tag{8}$$

where $V_i(S)$ is the firm i's value function; and $V_i'(S) = \partial V_i(S)/\partial S$. The first order condition (FOC) on q_i is

$$\sigma - 2(1 + c) q_i - \sum_{j \neq i} q_j - V_i'(S) = 0 \tag{9}$$

In view of the *ex ante* symmetry across firms, one can impose the symmetry conditions $q_i = q(S)$ and $V_i(S) = V(S)$ for all i and solve FOC (9) to obtain

$$V'(S) = \sigma - [n + 1 + 2c]\, q \tag{10}$$

Substituting this into (8) yields an identity in S. Differentiating both sides with respect to S and rearranging terms, any feedback strategy is implicitly given by the following differential equation:

$$q'(S) = \frac{(\delta - \rho)\,[\sigma - (n + 1 + 2c)\, q(S)]}{\sigma(n-1) + \delta(n + 1 + 2c)S - 2\,[n^2 + c(2n-1)]\, q(S)}, \tag{11}$$

which must hold together with terminal condition $\lim_{t\to\infty} e^{-\rho t} V(s) = 0$. Examining expression (11) reveals that

$$q'(S) = 0 \Leftrightarrow q_0(S) = \frac{\sigma}{n + 1 + 2c} \tag{12}$$

$$q'(S) \to \pm\infty \Leftrightarrow q_\infty(S) = \frac{\sigma(n-1) + \delta(n + 1 + 2c)S}{2\,[n^2 + c(2n-1)]} \tag{13}$$

Then, assuming that the extraction strategy is a linear function of the stock at any time, I assume $q(S) = \alpha + \beta S$, whereby (11) is satisfied by any pair (α, β) solving the following system:

$$\begin{gathered}\sigma\,[\beta(n-1) - \delta + \rho] + \alpha\left[(n + 1 + 2c)(\delta - \rho) - 2\left(n^2 + c(2n-1)\right)\beta\right] = 0 \\ \beta\left[(n + 1 + 2c)(2\delta - \rho) - 2\left(n^2 + c(2n-1)\right)\beta\right] = 0\end{gathered} \tag{14}$$

System (14) is solved by the pairs

$$\alpha_1 = -\frac{\sigma\,[2\delta(1+c) - \rho(n(n+2c)+1)]}{2\delta(n + 1 + 2c)\,[n^2 + c(2n-1)]}\,;\; \beta_1 = \frac{(n + 1 + 2c)(2\delta - \rho)}{n^2\,[n^2 + c(2n-1)]} \tag{15}$$

$$\alpha_2 = \frac{\sigma}{n + 1 + 2c}\,;\; \beta_2 = 0 \tag{16}$$

so that the individual equilibrium output is

$$q^{LF}(S) = \alpha_1 + \beta_1 S \tag{17}$$

$$q^{OL} = \alpha_2 = \frac{\sigma}{n + 1 + 2c} = q_0(S) \tag{18}$$

where superscripts LF and OL stand for *linear feedback* and *open-loop*, respectively. That is, since the game is a linear state one by construction, one of the linear feedback strategies delivered by the solution of the HJB equation degenerates in the open-loop one, coinciding with the static Cournot-Nash solution.[7] The expression on the r.h.s. of (17) belongs to $[0, \sigma/(n+1+2c)]$ for all

$$S \in \left[\frac{\sigma [n(n+2c)+1]}{\delta (n+1+2c)^2}, \frac{\sigma [2(1+c)\delta - \rho (n(n+2c)+1)]}{(n+1+2c)^2 (2\delta - \rho)\delta} \right] \tag{19}$$

If $q = q^{LF}(S)$, the steady state level of the natural resource stock is

$$S^{LF} = \frac{n\sigma [2(1+c)\delta - \rho (n(n+2c)+1)]}{\delta (n+1+2c)[2c(\delta - n\rho) + n(2\delta - \rho (n+1))]} \geq 0 \tag{20}$$

for all values of δ satisfying the first threshold appearing in Assumption 1. It is evident that $\partial S^{LF}/\partial\theta > 0$ since $\partial S^{LF}/\partial\sigma > 0$. That is,

Lemma 1 *At the linear feedback equilibrium, any increase in the extent of delegation increases the residual stock of resources in steady state.*

If instead $q = q^{OL}$, the steady state level of the natural resource stock associated with open-loop strategies is

$$S^{OL} = \frac{n\sigma}{\delta (n+1+2c)} > 0 \tag{21}$$

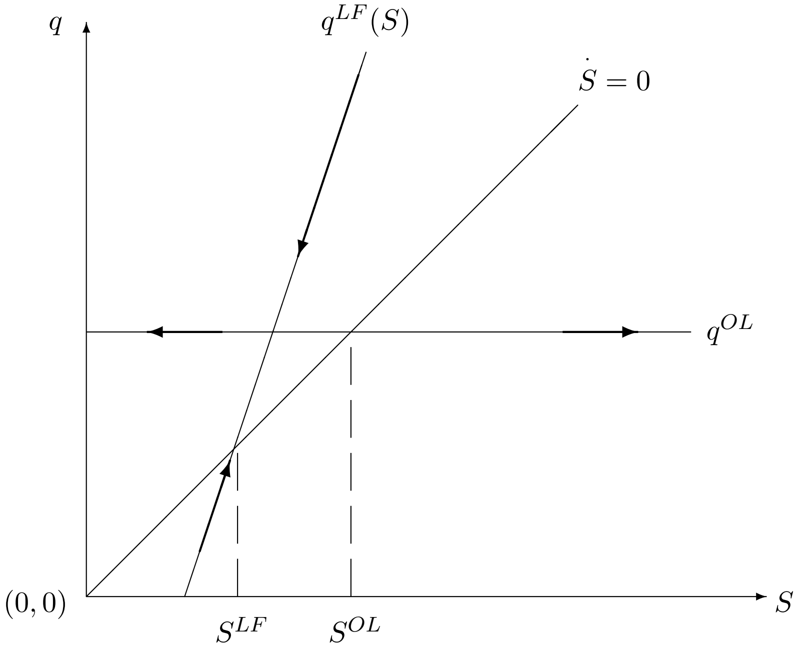
everywhere, but the dynamic properties of the state-control system imply that the industry is unable to reach point (S^{OL}, q^{OL}).

The linear solutions q^{OL} and $q^{LF}(S)$, together with the locus $\dot{S} = 0$, are represented in the space (q, S) in Figure 1, where indeed the arrows illustrate the dynamics of variables and the stability of $q^{LF}(S)$, as opposed to the instability of the open-loop solution q^{OL}. If firms adopt this strategy, the resource stock is bound to shrink to zero for all $S_0 < S^{OL}$. Otherwise, for

[7] For more on classes of differential games in which the open-loop solution is subgame perfect (or strongly time consistent), see Dockner *et al.* (1985), Fershtman (1987), Mehlmann (1988), Dockner *et al.* (2000), Cellini *et al.* (2005) and Lambertini (2018).

all $S_0 > S^{OL}$, the stock will grow beyond S_y, not represented along the horizontal axis of Figure 1. Hence, under open-loop rules, the ultimate destiny of the natural resource (or the waste stock) depends on initial conditions. It is also worth stressing that $\partial S^{OL}/\partial\theta > 0$, which implies that the interval of initial conditions leading to resource extinction or full waste removal under open-loop (or quasi-static) strategies expands in the extent of managerial delegation.

Figure 1 Open-loop and linear feedback solutions in the (S, q) space



Should we interpret the state variable as recyclable waste to be used, e.g., in the production of renewable energy, then the open-loop strategy q^{OL} would look even less reliable or appealing than it already does on the basis of the formal argument about its instability. Indeed, its adoption would cause the exhaustion of the stock in finite time for any initial condition $S_0 < S^{OL}$, putting an end to energy supply and calling for an alternative source. In terms of the extraction of a biological resource, the obvious interpretation of the open-loop solution is that myopic firms adopting a quasi-static strategy may cause the extinction of the resource population through a tragedy of commons. Entirely analogous considerations can be extended to any of the open-loop strategies arising in the ensuing reformulations of the differential game.

3.2 Nonlinear feedback equilibria

The present game produces infinitely many nonlinear feedback solutions whose continuum can be fully characterised using the same procedure as in Lambertini (2016a) and Lambertini and Mantovani (2016), which in turn relies on Rowat's (2007).[8] Without replicating the entire analysis of the nonlinear case, here it suffices to characterise the degenerate nonlinear solution identified by the tangency between the highest isocline of the representative firm and the steady state locus $\dot{S} = 0$ in the state-control space.

To do so, one has to go back to (11) and note that the slope of the steady state locus $\dot{S} = 0$ is

$$\left.\frac{\partial q(S)}{\partial S}\right|_{\dot{S}=0} = \frac{\delta}{n} \tag{22}$$

which must coincide with $q'(S)$ when $q(S) = \delta S/n$, in such a way that (11) becomes:

$$\frac{\delta}{n} = \frac{(\delta - \rho)[n\sigma - \delta(n + 1 + 2c)S]}{(n-1)[n\sigma - \delta(n + 2c)S]} \tag{23}$$

whose unique solution w.r.t. the state variable is

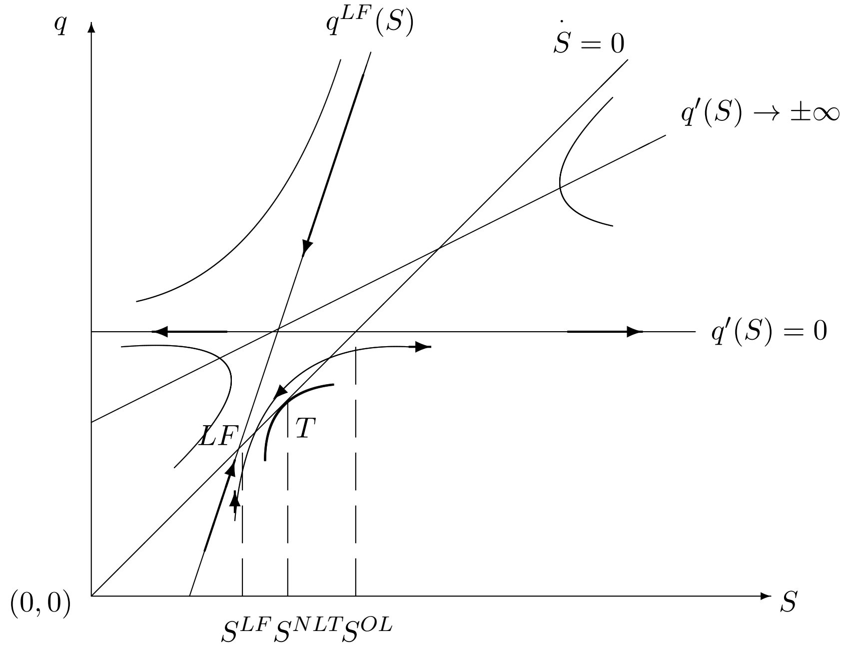
$$S^{NLT} = \frac{n\sigma(\delta - n\rho)}{\delta[2c(\delta - n\rho) + n(2\delta - \rho(n+1))]} \tag{24}$$

which is positive in the parameter range wherein $S^{LF} > 0$. The associated individual output is $q^{NLT} = \delta S^{NLT}/n$. Superscript NLT mnemonics for *nonlinear tangency solution*.

[8]Nonlinear feedback solutions have been investigated in oligopoly theory, environmental and resource economics and other fields. See Tsutsui and Mino (1990), Shimomura (1991), Dockner and Long (1993), Dockner and Sorger (1996), Itaya and Shimomura (2001), Rubio and Casino (2002) and Colombo and Labrecciosa (2015), *inter alia*.

Figure 2 describes the evolution of state and control variables over time, enabling one to single out the properties of any nonlinear feedback solutions, including the very specific one generated by the tangency point with the locus $\dot{S} = 0$ (point T in the figure). Figure 2 (which is nothing but a more detailed version of Figure 1) also portrays the flat locus $q'(S) = 0$ (along which $q_0(S) = q^{OL}$) and the non-invertibility one, $q'(S) \to \infty$.[9]

Figure 2 Linear and nonlinear feedback solutions in the (S, q) space



The arrows along the isocline tangent to the locus $\dot{S} = 0$ in point T show that the tangency solution is indeed semi-stable. However, there exist infinitely many stable solutions identified by the intersections along the seg-

[9]As in Rowat (2007), there also exist nonlinear feedback strategies outside the cone defined by linear feedback solutions in Figure 2. However, all of these alternative nonlinear strategies can be excluded either because they intersect the non-invertibility locus - and when they do so, they cease to be functions (cf. Rowat, 2007, Lemma 3, p. 3192) - or because they never intersect the steady state locus.

ment delimited by points T and LF. This set of *stable nonlinear solutions*, which can be labelled as $SNLS$, is sensitive to the extent of delegation θ, which affects the loci $\dot{S} = 0$ and $q^{LF}(S)$, and therefore also the position of the tangency point T. The set $SNLS$ has the size of such a segment:

$$SNLS = \sqrt{\left(S^{NLT} - S^{LF}\right)^2 + \left(q^{NLT} - q^{LF}\right)^2} \tag{25}$$

This yields a measure of the continuum of stable equilibria delivered by the unregulated version of Model I. This set, although bounded on both sides, is of course infinitely dense, but the problem connected with the multiplicity of equilibria - which is commonplace in uncountably many game-theoretical models - is worsened by any increase in the level of the managerial incentive θ. The impact of delegation can be appreciated through the following procedure. Using the corresponding expressions for the steady state values of state and control, one obtains $SNLS = \sigma\sqrt{\Phi(n, \delta, \rho)}$, with $\Phi(\cdot) > 0$. Consequently,

$$\frac{\partial SNLS}{\partial \theta} = \sqrt{\Phi(n, \delta, \rho)} > 0 \tag{26}$$

by the definition of σ. This boils down to the following:

Proposition 2 *The separation between ownership and control via delegation contracts based on output expansion enlarges the set of stable nonlinear feedback solutions.*

In particular, since $\partial S^{NLT}/\partial\theta > 0$, the above proposition is accompanied by a relevant corollary:

Corollary 3 *The adoption of managerial incentives based on output expansion increases the upper bound of the $SNLS$ set.*

That is, under this type of delegation both S^{NLT} and S^{LF} increase linearly in θ, the former more than the latter, thereby reinforcing the issue posed by the multiplicity of equilibria, and implying the arising of progressively higher residual stocks in correspondence of stable equilibria, as the extent of delegation increases.

This result prompts for the analysis of the so-called *voracity effect* (Lane and Tornell, 1996; Tornell and Lane, 1999), which can be briefly summarised as follows. In line of principle, one would expect that the higher the resource growth rate is, the higher should be the volume of that resource surviving in steady state. However, this may not hold true as firms respond to any increase in the growth rate by hastening resource extraction, whereby one observes that $\partial S/\partial\delta < 0$ in steady state, at least for sufficiently high levels of δ. The arising of such voracity effect has been highlighted, with pure profit-seeking units, in Benchekroun (2008) and Lambertini and Mantovani (2014). As in Lambertini and Mantovani (2014, p. 121), also here it can be easily shown that under linear and nonlinear feedback information the voracity effect operates.

Take the weighted average of S^{LF} and S^{NLT} :

$$\overline{S} = \phi S^{LF} + (1-\phi) S^{NLT} \tag{27}$$

with $\phi \in [0,1]$. There emerges that $\partial\overline{S}/\partial\delta < 0$ for sufficiently high levels of the growth rate δ, for any $\phi \in [0,1]$, thereby including the extremes of the relevant interval of resource stock volumes in steady state. However, this property, combined with Lemma 1, Proposition 2 and Corollary 3, entails

Proposition 4 *Managerial incentives allowing for output expansion soften the voracity effect over the entire interval of nonlinear feedback solutions* SNS.

It would be tempting to interpret this conclusion as implying a beneficial effect of managerialization on resource preservation (or, an undesirable effect upon waste removal, in which case voracity is most welcome for intuitive reasons). However, this would be hazardous as the same issue should indeed be reassessed in presence of alternative incentive schemes, based for instance on market shares (Jansen *et al.*, 2007; Ritz, 2008) or comparative performance evaluation (Salas Fumas, 1992; Miller and Pazgal, 2001). Yet, the possibility that delegating control to agents interested in expanding production or extraction might ultimately mitigate the pressure on the resource is a striking and unexpected feature of the present model. This fact finds its explanation in the multiplicative effect of this form of delegation on equilibrium outputs and the resource stock, as the delegation parameter θ appears in market size σ and makes it larger as seen from the managers' standpoint. Since σ is at the same time a measure of profitability or demand level, this type of delegation (i) increases the maximum mark-up from $a-c$ to $a-c+\theta$ or equivalently (ii) shifts the demand upwards by θ. Consequently, the managerial inclination to expanding output is routed in the direction of affecting the mark-up level and this mechanism operates as a partial remedy to voracity, in the range where the latter takes place. Therefore, albeit with some caution, this design of delegation contracts - admittedly, far from being general - is of public interest because it couples the usual elements connected with consumer surplus and profits with additional motives (perhaps more far-reaching) dealing with the impact of the separation between ownership and control on resource (and species) preservation.

Moreover, there remains the open question as to how a public agency could regulate access to the commons, in presence of a single stable linear feedback equilibrium and infinitely many stable nonlinear feedback equilibria. A plausible solution is proposed in the next section.

3.3 The regulated case

The model remains the same as for the resource dynamics (6) and firms' technology. Instead, here the price p is exogenously given, being a policy instrument in the hands of a public authority in charge of regulating access to the common resource pool or waste stock.

Accordingly, firm i's instantaneous maximand writes

$$M_i(t) = (p - cq_i + \theta)\, q_i. \tag{28}$$

The problem is formally defined as above, as firm i's HJB equation is

$$\rho V_i(S) = \max_{q_i} \left\{ M_i + \frac{\partial V_i(S)}{\partial S} \cdot (\delta S - Q) \right\} \tag{29}$$

Solving the game on the basis of the same procedure (or equivalently using the method of the undetermined parameters), one obtains the following pair of strategies:

$$q_p^{OL} = \frac{\sigma_p}{2c}\,;\; q_p^{LF} = \frac{2c\delta\,(2\delta-\rho)\,S - (\delta - n\rho)\,\sigma_p}{2c\delta\,(2n-1)} \tag{30}$$

where (i) $\sigma_p \equiv p + \theta$; (ii) superscripts have the same meaning as above; and (iii) subscript p indicates that the price of the final good is being regulated. While $q_p^{OL} > 0$ over the entire parameter space, $q_p^{LF} > 0$ for all[10]

$$S > \frac{(\delta - n\rho)\,\sigma_p}{2c\delta\,(2\delta-\rho)} > 0 \tag{31}$$

In this range, $\partial q_p^{LF}/\partial n < 0$ for all admissible levels of the stock: this entails that q_p^{LF} becomes steeper as the number of firms decreases, which is bound to come to bear while assessing the impact of industry structure on the uncollected stock at the steady state.

The interesting implication of price regulation is that, irrespective of the information structure underpinning firms' strategies, the residual steady state resource stock is exactly the same:

$$S_p = \frac{n q_p^{LF,OL}}{\delta} = \frac{n\sigma_p}{2c\delta} \tag{32}$$

which amounts to the following:

Proposition 5 *Regulating price eliminates the multiplicity of stable feedback equilibria, with the single linear feedback one surviving.*

Moreover, (32) has two relevant implications that should equally attract the attention of the authority:

- since $\partial q_p^{LF}/\partial n < 0$, S_p monotonically increases in n: hence, the minimum residual stock obtains in correspondence of $n = 1$. Recalling the dual interpretation of the nature of S, this fact has completely opposite implications concerning the socially efficient access to the commons.

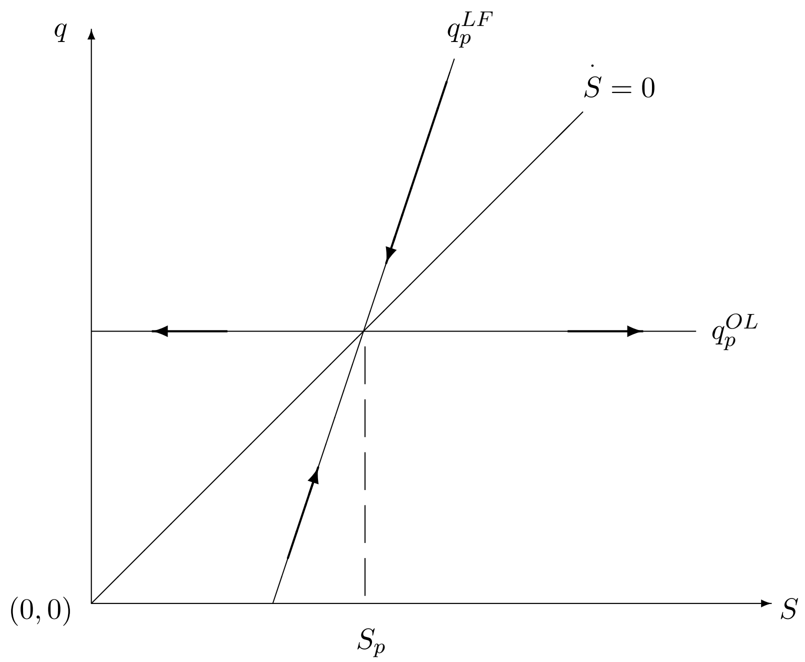
[10]The demonstration that indeed

$$\frac{(\delta - n\rho)\,\sigma}{2c\delta\,(2\delta-\rho)} > 0$$

derives from the solution of the model in which firms are pure-profit-seeking agents (i.e., $\theta = 0$) and price is endogenously determined via the linear demand function instant by instant (see Fujiwara, 2008; and Lambertini and Mantovani, 2014).

- S_p monotonically increases in σ and therefore also in the extent of delegation, θ: this reveals that including the individual instantaneous harvest rates in the delegation contracts (or, adopting a CSR stance) might or might not mean good news from the regulator's standpoint, again in view of the dual interpretation of the model as for the nature of the state variable.

Figure 3 The regulated case



Be that as it may, the picture looks as in Figure 3, where again the arrows indicate the dynamics of the state S and illustrate that q_p^{OL} is unstable while q_p^{LF} is stable. Therefore, although they seem to yield the same steady state, open-loop and feedback information structures are not equivalent at all. In particular, the outcome engendered by q_p^{OL} can either drop to $S = 0$ or exceed S_y, depending on the initial stock,[11] while the volume of the long-run

[11] A peculiar and somewhat paradoxical feature of the case of waste removal is that if

equilibrium state variable generated by q_p^{LF} is surely $S_p = n\sigma_p / (2c\delta)$.

To close the discussion carried out in this section, let's focus our attention onto the case in which S is a stock of waste. If so, then monopoly is the socially efficient structure, with

$$S^{NLT}\big|_{n=1} = S^{LF}\big|_{n=1} \tag{33}$$

for obvious reasons, and

$$S^{LF}\big|_{n=1} - S_p\big|_{n=1} = \frac{ac - \theta - p(1+c)}{2\delta c(1+c)} > 0 \tag{34}$$

for all

$$p < \min\left\{0, \frac{ac-\theta}{1+c}\right\} \tag{35}$$

which entails that waste removal in monopoly should be subsidised if $\theta > ac$.

4 Model II

Here, the contract based on (4) says that the firm attaches a negative weight to the residual individual share of waste at the symmetric equilibrium. I will briefly illustrate the case in which price is endogenously determined along the demand function, and then focus on the regulated case, as the continuum of stable feedback equilibria arising with an unregulated price survives the regime change, unlike what happens in model I.

4.1 The unregulated game

If price is not being regulated, the HJB equation of firm i is

$$\rho V_i(S) = \max_{q_i}\left[\left(a - (1+c)q_i - \sum_{j\neq i} q_j\right)q_i - \frac{\theta S}{n} + V_i'(S)(\delta S - Q)\right] \tag{36}$$

---

the initial stock is sufficiently low, firms might involuntarily drive to zero the residual stock under myopic open-loop rules. Of course it is also true that if the inital stock is large then the adoption of open-loop strategies might cause the waste stock to shoot up to plus infinity.

and, imposing symmetry across quantities and solving the FOC, one obtains $V'(S) = a - (n + 1 + 2c)\, q_i$. Proceeding as in subsections 3.1 and 3.2, the equation of the continuum of feedback strategies writes as follows:

$$q'(S) = \frac{a(\delta - \rho) + \theta - n(n + 1 + 2c)(\delta - \rho)\, q(S)}{n\left[2\left(n^2 + c(2n - 1)\right) q(S) - a(n - 1) - S(n + 1 + 2c)\,\delta\right]} \tag{37}$$

Hence, the open-loop strategy solves

$$q'(S) = 0 \Leftrightarrow q_0(S) = \frac{a(\delta - \rho) + \theta}{n(n + 1 + 2c)(\delta - \rho)} \tag{38}$$

and the non-invertibility threshold of output is

$$q'(S) \to \pm\infty \Leftrightarrow q_\infty(S) = \frac{a(n - 1) + S(n + 1 + 2c)\,\delta}{2\left[n^2 + c(2n - 1)\right]} \tag{39}$$

Then, posing $q(S) = \alpha + \beta S$, one may solve (37) to characterise the linear strategies, the first of which indeed coincides with $q_0(S)$:

$$q^{OL} = \frac{a(\delta - \rho) + \theta}{n(n + 1 + 2c)(\delta - \rho)}$$

$$q^{LF} = \frac{an\left[(1 + n(n + 2c))\,\rho - 2(1 + c)\,\delta\right] + 2\theta\left[n^2 + c(2n - 1)\right] - n(n + 1 + 2c)^2(\rho - 2\delta)\,\delta S}{2n(n + 1 + 2c)\left[n^2 + c(2n - 1)\right]\delta} \tag{40}$$

The corresponding steady state stocks are $S^{OL} = nq^{OL}/\delta$ and

$$S^{LF} = \frac{an\left[2(1 + c)\,\delta - (1 + n(2c + n))\,\rho\right] - 2\left[n^2 + c(2n - 1)\right]\theta}{(n + 1 + 2c)\left[2c\delta - n(\rho(n + 1 + 2c) - 2\delta)\right]\delta} \tag{41}$$

At the tangency point, $q(S) = \delta S/n$ and $\partial q(S)/\partial S = \delta/n$, so that

$$S^{NLT} = \frac{n\left[a(\delta - n\rho) - \theta\right]}{\left[2c\delta - n(\rho(n + 1 + 2c) - 2\delta)\right]\delta}\,;\ q^{NLT} = \frac{\delta S^{NLT}}{n} > q^{LF} \tag{42}$$

Using again $\overline{S} = \phi S^{LF} + (1 - \phi)\, S^{NLT}$, one may verify that $\overline{S} = 0$ for all[12]

$$\theta \geq \theta_0 \equiv \frac{an\left[(n + 1 + 2c)(\delta - n\rho) - \phi(n - 1)(\delta - \rho)\right]}{n(n + 1 + 2c) + \phi(n - 1)(n + 2c)} \tag{43}$$

with $\partial\theta_0/\partial\phi < 0$ everywhere. This yields

[12] The threshold on the r.h.s. of (43) is positive for all $\phi < (n + 1 + 2c)(\delta - n\rho)/\left[(n - 1)(\delta - \rho)\right]$, which in turn is lower than one for all $\delta > \rho\left[n(n + 2c) + 1\right]/\left[2(1 + c)\right]$, the latter condition being included in Assumption 1.

Proposition 6 *In the unregulated game in which the CSR stance takes the form of a negative weight attached to the residual stock, the extent of delegation ensuring full waste removal is unique for any ϕ in the unit interval, and it is minimal in correspondence of the linear feedback solution.*

The interpretation is intuitive: since $q^{NLT} > q^{LF}$ and consequently $S^{NLT} > S^{LF}$, the intensity of the CSR incentive required to ensure the collection of the whole stock of waste at the steady state diminishes monotonically as firms move from the tangency point to the intersection between the linear feedback strategy and the steady state locus. And yet, in this scenario full removal strictly depends on a crucial detail written in the delegation contract and therefore also in the firms' objective functions. This prompts for the analysis of the regulator's role.

4.2 The regulated game

Here, p is an instrument in the regulator's hands. Additionally, for reasons which will become apparent below, I will confine myself to interpreting the game as one in which firms collect a waste stock. The HJB equation of firm i is

$$\rho V_i(S) = \max_{q_i} \left[(p - cq_i) q_i - \frac{\theta S}{n} + V_i'(S)(\delta S - Q) \right] \tag{44}$$

and the FOC yields $V_i'(S) = p - 2cq_i$. This expression can be substituted into (44), and then, manipulating the latter following the same procedure as in section 3.3, one obtains the following equation, describing the continuum of feedback strategies:

$$q'(S) = \frac{n(\delta - \rho)[p - 2cq(S)] - \theta}{n[p(n-1) + 2c(\delta S - (2n-1)q(S))]} \tag{45}$$

Therefore,

$$\begin{aligned} q'(S) = 0 \Leftrightarrow q_0(S) &= \frac{np(\delta - \rho) - \theta}{2cn(\delta - \rho)} \\ q'(S) \to \pm\infty \Leftrightarrow q_\infty(S) &= \frac{p(n-1) + 2c\delta S}{2c(2n-1)} \end{aligned} \tag{46}$$

Hence, (45) reveals that, in model II, the shape of the managerial objective function yields infinitely many subgame perfect strategies even under price regulation, precisely because delegation concerns the state variable instead of the individual volume of waste removal. Indeed, this fact becomes explicit by noting that the two linear feedback strategies that can be identified plugging $q(S) = \alpha + \beta S$ and $q'(S) = \beta$ in (45),

$$q_p^{OL} = \frac{np(\delta - \rho) - \theta}{2cn(\delta - \rho)} = q_0(S) \; ; \; q_p^{LF} = \frac{n\left[np\rho + 2\left(2c\delta^2 S + \theta\right) - \delta(p + 2c\rho S)\right] - \theta}{2cn\delta(2n-1)} \tag{47}$$

do not cross the steady state locus at the same point and thus produce two different values of the residual waste stock in steady state:

$$S_p^{OL} = \frac{np(\delta - \rho) - \theta}{2c\delta(\delta - \rho)} \; ; \; S_p^{LF} = \frac{np(\delta - n\rho) - \theta(2n-1)}{2c\delta(\delta - n\rho)} \tag{48}$$

with

$$S_p^{OL}, S_p^{LF} > 0 \,\forall\, \delta \in \left(n\rho + \frac{\theta(2n-1)}{np}, \infty\right) \wedge \theta \geq 0 \tag{49}$$

The lower bound of the above interval is the second critical threshold of δ appearing in Assumption 1. In the same range of values of δ, in steady state we also have $q_p^{OL} > 0$ (which is obvious, as the open-loop solution independent of the stock), and

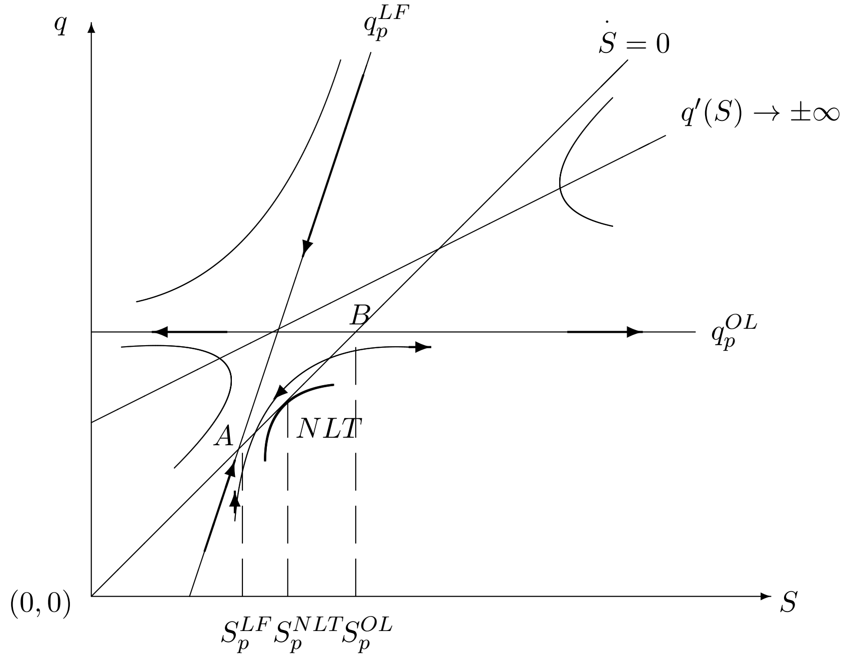
$$q_p^{LF}\left(S_p^{LF}\right) = \frac{\delta S_p^{LF}}{n} = \frac{np(\delta - n\rho) - \theta(2n-1)}{2cn(\delta - \rho)} > 0 \tag{50}$$

The tangency point is identified by noting that, the steady state locus and its slope being, respectively, $q(S) = \delta S/n$ and $\partial q(S)/\partial S = \delta/n$, equation (45) is solved by

$$S_p^{NLT} = \frac{n\left[p(\delta - n\rho) - \theta\right]}{2c\delta(\delta - n\rho)} \in \left(S_p^{LF}, S_p^{OL}\right) \tag{51}$$

The resulting graph replicates the picture appearing in Figure 1, with analogous properties. In particular, once again the first (open-loop) solution is unstable, while the second is stable. Of course, there are infinitely many nonlinear solutions, a subset of which is stable. This is portrayed in Figure 4, which, except for labels, looks like Figure 2.

Figure 4 Non linear solutions in the alternative model



Hence, it is evident that regulation does not deliver uniqueness if the delegation contract (or, the CSR stance) chosen by firms includes the residual stock of waste instead that the individual removal rate as in model I. Yet, at a closer look, this scenario is not as discouraging as it might look at first glance. To grasp the intuition why it is not so, observe first that

$$S_p^{OL} - S_p^{LF} = \frac{(n-1)(2\delta - \rho)\theta}{2c\delta(\delta - \rho)(\delta - n\rho)} > 0 \tag{52}$$

for all $\theta > 0$. This simple result can be formulated as follows:

Lemma 7 *If firms adopt a CSR stance based on a negative weight attached to the individual share of the residual waste stock, the stable feedback solution yields a lower residual stock than the unstable (open-loop) one.*

This fact has several relevant implications: (i) any stable nonlinear solution is more desirable than the open-loop one; (ii) unlike what happens in models dealing with natural resource exploitation, here the voracity effect (Tornell and Velasco, 1992; Lane and Tornell, 1996; Tornell and Lane, 1999) combined with feedback information plays a positive role; and, more importantly, both S_p^{NLT} and S_p^{LF} are monotonically decreasing in θ. Hence, we have the following

Proposition 8 *Intensifying the CSR component in the firm's objective function brings about a decrease in the residual waste stock in any stable steady states reached through feedback strategies.*

It is worth noting that this is the opposite of what happens in the regulated version of the previous model, in which the residual stock is (32).

Last but not least, one may verify from (47-48) that $\partial^2 q_p^{LF}/\partial n \partial p$ and $\partial^2 S_p^{LF}/\partial n \partial p$ are both positive. This indicates that along the stable linear feedback trajectory any increases in price and firms' numerosity behave as complements, and this offers the regulator a way out of the multiplicity issue. To see this, note that, for any $\theta > 0$, there exists a unique level of the regulated price at which $S_p^{LF} = 0$:

$$p\left(S_p^{LF} = 0\right) = \frac{\theta\,(2n-1)}{n\,(\delta - n\rho)} > 0 \tag{53}$$

with

$$\frac{\partial p\left(S_p^{LF} = 0\right)}{\partial n} = \frac{\left[\delta + 2n\,(n-1)\,\rho\right]\theta}{n^2\,(\delta - n\rho)^2} > 0. \tag{54}$$

An analogous result holds in correspondence of any equilibrium generated by nonlinear feedback strategies, whose residual stock is $\widehat{S} = \phi S_p^{LF} + (1-\phi)\,S_p^{NLT}$, $\phi \in (0,1)$, as both S_p^{LF} and S_p^{NLT} are linear in p. Moreover, (52) implies that, in monopoly, $S_p^{OL} = S_p^{LF}$ and therefore the infinitely many nonlinear equilibria vanish. These last findings can be summarised in the following terms:

Corollary 9 *In correspondence of any stable feedback solution there exists a single price driving to zero the residual waste stock at equilibrium. This price takes its minimum value in monopoly, where the stable linear solution is the only one being relevant as the continuum of nonlinear equilibria disappears.*

This suggests that the regulator may indeed rely on a single firm, granting it the lowest price identified by $p\left(S_p^{LF}=0\right)\big|_{n=1}$. This simultaneously solves the problem associated with the multiplicity of equilibria and ensures full removal at the lowest cost for society.

5 Concluding remarks

In a nutshell, the foregoing analysis has shown that the acquired model describing the dynamic exploitation of a common pool renewable resource could be reinterpreted as a game of waste removal, by changing a few labels. Of course, this involves a non trivial change of perspective, in particular when it comes to the need of regulating an oligopoly game generating a continuum of feedback equilibria.

Firms are either managerial or CSR entities - depending on the interpretation being chosen - and their objective functions are defined in two alternative ways. The first formulation stipulates that the relevant objective contains profits and output (or, the instantaneous individual volume of waste removal). In this case, the adoption of feedback information generates a continuum of stable subgame perfect equilibria. The choice of regulating price sweeps away the continuum of equilibria engendered by nonlinear strategies, leaving the regulator with a single stable linear feedback equilibrium whose performance depends on the price level and the number of firms being granted access to the commons. Hence, there appears that, combining appropriately price and entry regulation, the public authority can indeed outperform the most favourable unregulated feedback equilibrium in terms of the residual resource stock at the steady state.

The second formulation assumes that managerial or CSR incentives are based on a combination of profits and the firm's individual share of the residual stock of the state variable. If the latter measures the volume of waste, the model shows that, in correspondence of any stable feedback equilibrium, there exists either a delegation level or a price at which the residual stock is indeed nil. Also in this case, the persistence of the multiplicity of equilibria notwithstanding, the regulated game offers the regulator the possibility of achieving full removal, as the price ensuring this outcome is univocally defined for any industry structure and decreases monotonically in the number of firms. Hence, the regulator may restrict access to a single firm and adopt the lowest of all such prices, thereby attaining the desired goal at the lowest possible tariff.

Needless to say, the foregoing material does not exhaust the analysis of this topic. In addition to the obvious extensions accounting for the aforementioned alternative delegation contracts based upon market shares (Jansen *et al.*, 2007; Ritz, 2008) or comparative performance evaluation (Salas-Fumas, 1992; Miller and Pazgal, 2001), a plausible and promising one is that in which either control variables and/or the stock imply polluting emissions. The first possibility is plausible if the state refers to a natural resource, and the production of a final good based on harvest is polluting the environment; the second is intuitively related to a scenario in which the state variable is a stock of waste. This extension would enrich the currently scant literature modelling the simultaneous presence of resource extraction (or stock removal) and environmental damage or global warming (cf. Lambertini and Leitmann, 2013; and Lambertini, 2016b).

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