SHORT-PAPER

Rule-based Classifier Models

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ICAIL 2025: 20th International Conference on Artificial Intelligence and Law
June 16 - 20, 2025
IL, Chicago, USA

ICAIL '25: Proceedings of the Twentieth International Conference on Artificial Intelligence and Law (June 2025)
https://doi.org/10.1145/3769126.3769243
ISBN: 9798400719394

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Abstract

We extend the formal framework of classifier models used in the legal domain. While the existing classifier framework characterises cases solely through the facts involved, legal reasoning fundamentally relies on both facts and rules, particularly the *ratio decidendi*. This paper presents an initial approach to incorporating sets of rules within a classifier. Our work is built on the work of Canavotto *et al.* (2023), which has developed the rule-based reason model of precedential constraint within a hierarchy of factors. We demonstrate how decisions for new cases can be inferred using this enriched rule-based classifier framework. Additionally, we provide an example of how the time element and the hierarchy of courts can be used in the new classifier framework

CCS Concepts

• **Computing methodologies** → **Knowledge representation and reasoning**; • **Applied computing** → **Law**.

Keywords

Legal Case Based Reasoning, Legal Classifiers, Rule Based Reasoning, Factor Hierarchies

ACM Reference Format:
Cecilia Di Florio, Huimin Dong, and Antonino Rotolo. 2025. Rule-based Classifier Models. In *20th International Conference on Artificial Intelligence and Law (ICAIL 2025), June 16–20, 2025, Chicago, IL, USA.* ACM, New York, NY, USA, 5 pages. https://doi.org/10.1145/3769126.3769243

1 Introduction

Machine learning classifiers in predictive justice perform *case-based reasoning* (CBR), in the sense that they provide the outcomes of new cases on the basis of previous cases. Based on this intuition, [13] investigated a correspondence between existing formal models of legal CBR in [10] and binary input classifier models proposed in [12]. The classifiers framework for legal reasoning was then expanded in [7–9]. In particular in [7], classifier models were enriched to account for two fundamental elements in legal practice: the temporal element and the hierarchical structure of courts within the legal system under consideration. These two elements played a crucial role for refining the notion of constraining/binding precedent and for solving conflicts among binding precedents. We believe the classifier models proposed in [7] can be further refined. In these framework, every case is essentially described in terms of the facts that appear in the case and of the court that assessed the case. Additionally in [7], it is assumed that the decision making process for each new case is a single-step process that, on the basis of the binding precedents, leads directly from the facts of case to its outcome. In legal practice, however, cases are not defined solely in terms of facts, but also by the *ratio decidendi* of the case, which is the reason for the courts decision. The *ratio decidendi* is generally binding on lower courts and later judgments[11]. In some formal models of legal CBR [10], the *ratio decidendi* is represented as a rule linking directly specific facts of the case to the conclusion of the case. However, as suggested in [3, 6], and recalled in [6], the reasoning pattern in judicial opinions is often based on inferences linking portions of the facts of the case to intermediate conclusions supported by those facts. In [3] it is argued that also these intemediate rules, referred to as "precedents constituents," should constrain in later cases. Different approaches have been proposed about how portions of precedents may condition future decisions [1, 2, 5, 6].

The aim of this article is not to argue for or against different approaches to precedential constraint, but to develop a rule-based classifier model that integrates rules into the existing classifier framework. We want to enrich the classifier models so that the states of the classifiers are defined both in terms of facts and of *sets* of rules. Our long-term goal is to show how the framework in [7] can be combined with classifiers that take different rule systems into account. In this work, as a starting point, we build classifier modeled on the CBR framework provided in [6].[6] develops the reason model of precedential constraint [10] in the context of a factor hierarchy, so that the final decision of a case depends on intermediate decisions and on the rules used in those decisions.
The paper is structured as follows. Section 2 briefly introduces the framework in [6]. Section 3 enriches the framework of classifiers model with sets of rules and defines a decision-making process. Section 4 illustrates the benefits of integrating the framework from [7] in the new classifier models. Section 5 concludes. [1]

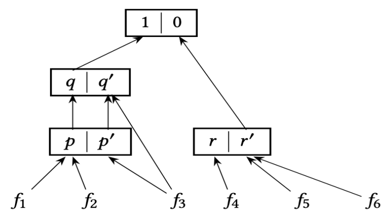
2 Hierarchical Reason Model

In this section we will recall few notions and assumptions introduced in [6], adapting the notation and terminology to our purposes. The work in [6] addresses the problem of precedential constraint with respect to a single top-level issue. With respect to the issue the judge can decide in favor of the plaintiff or in favor of the defendant. We write 0 for a pro-defendant decision and 1 for a pro-plaintiff decision; the set of these two decisions will be denoted as $Dec = \{0, 1\}$. In addressing the case, the judge starts by considering a set of base factors (factual facts), denoted as Atm_0. Given the base factors, it

[1] An extended version is available on arXiv at: http://arxiv.org/abs/2505.00474

ACM ISBN 979-8-4007-1939-4/25/06
https://doi.org/10.1145/3769126.3769243

Figure 1: An Abstract Example.



may be decided whether some intermediate factors apply in the case within a multi-step procedure. The set of intermediate factors is denoted as Int. We define $X \subseteq Atm_0 \cup Int$ a factual situation. We assume that if p is an intermediate factor in Int, then its contrary, noted p', is also in Int. On the basis of the intermediate factors, some further intermediate factors could be inferred. This step-by-step procedure is performed until a decision for the top issue (0 or 1) is reached. Base factors and intermediate factors may favor the decision for an intermediate factor p or its contrary; or they may favor the decision for 0 or 1. We use t, u, v for variables ranging over elements in $Atm_0 \cup Int \cup Dec$. Given $t \in Int \cup Dec$, we denote $\bar{t}$ its opposite. If $t = p \in Int$, $\bar{t} = p'$, if $t = p'$ then $\bar{t} = p$. Also, if $t = 0 \in Dec$, $\bar{t} = 1$, it $t = 1 \in Dec$ then $\bar{t} = 0$. Finally, $\bar{\bar{t}} = t$.

A hierarchy of factors is used to specify which intermediate factor/final decision is favored by each base factor/intermediate factors. Specifically, given $t \in Atm_0 \cup Int$ and $u \in Int \cup Dec$, "a *factor link* is a statement of the form $t \longrightarrow u$ indicating that the presence of the factor t in some situation directly favors a decision that t holds as well, or simply that t directly favors u. A *factor hierarchy* is a set H of factor links" [6].

An example of hierarchy is represented in Figure 1. Here, $Atm_0 = \{f_1, f_2, f_3, f_4, f_5, f_6\}$, $Int = \{p, p', r, r', q, q'\}$, and $Dec = \{0, 1\}$.

For $t \in Atm_0 \cup Int$, we say that t favors u, if in H there is a direct edge $t \longrightarrow u$ or there is a direct edge $\bar{t} \longrightarrow \bar{u}$. We define $Facts^u$ the set of factors favoring u. Given $t \in Int$ an intermediate factor, we use notation $t/\bar{t}$ to indicate an intermediate concern. Intuitively, the concern represented by $t/\bar{t}$ is whether the intermediate factor t or its opposite $\bar{t}$ should hold in the situation under consideration. Similarly, given $t \in Dec$ top-level decision, we define $t/\bar{t}$ as a top-level concern. A top-level concern is also called top-level issue. The set of all the concerns (intermediate and top-level) is $Concern$.

The idea in [6], is that the concerns should be solved sequentially, following their position in the hierarchy: concerns with lower position should be solved before and, on the basis of their solutions, concerns with higher position can be solved. Namely, concerns should be solved following the order provided by their *degree*. The definition of $degree(t/\bar{t})$ is detailed in extended version and [6].

In assessing each concern raised by a fact situation the court will determine not only the outcome of the decision for the concern but also the *reason* for that decision. In this sense in each decision on a concern, a rule is established: the antecedent of the rule is the *reason* for the decision, and the conclusion of the rule is the outcome related to the decision. The rules used in a decision must be admissible wrt the hierarchy: a rule cannot introduce links among factors that cannot be extracted from the hierarchy. More precisely, a rule admissible in H is a statement of the form $r : U \longrightarrow t$, where $t \subseteq Int \cup Dec$ and $U \subseteq Facts^t$. In addition, $Antecedent(r) = U$ is the antecedent of the rule and $Conclusion(r) = t$ its conclusion. $Rules$ is the set of all admissible rules in H. Given $t/\bar{t} \in Concern$, a rule $r \in Rules$ is *related to concern* $t/\bar{t}$ iff $Conclusion(r) \in \{t, \bar{t}\}$.

For the definitions of decision, complete resolution, opinion, case base (CB) and (in)cosistency refer to the extended version or [6]

3 Generalizing Classifier Models: From Single to Many Decisions

In this section, we show how the hierarchical models presented in [6] can be imported into classifier models introduced in [12] and adapted to legal case-based reasoning in [7–9, 13]. In the classifier models, there are essentially two components: a set of states and a classification function that associates each state with a possible outcome. The states are described in terms of input features/atoms. In the classifier models for the legal domain, the input features can be the facts/factors that may appear in a case, while the outcome can be 0 (pro-defendant deicision), 1 (pro-plaintiff) or ? (absence of decision). We aim to extend the classifier framework to include rules alongside factors in case descriptions. More precisely, a state of the classification model will be a set of facts together with a set of rules, used to describe the case. In this paper we focus on the specific sets of rules; we call these sets solutions.

Given a hierarchy H and $X \subseteq Atm_0$, a *solution* in H *applicable to* X is any $sol \subseteq Rules$ s.t.: 1) $\forall r \in sol$: if $p = Antecedent(r) \in Atm_0$ then $p \in X$; 2) for all $t/\bar{t} \in Concern_H(Conclusion(r')) \cup Concern_H(X)$ with $r' \in sol$, there is a unique $r \in sol$ related to this concern; 3) for all rules $r \in sol$, it is the case that $Antecedent(r) \subseteq X \cup \bigcup_{r' \in sol} Conclusion(r')$. Condition 2) states that for each concern raised by the conclusion of the rules in the solution or by the factual situation in X, there is a rule related to the concern. Condition 3) states that in a solution there are not rules whose antecedent is not in X or that cannot be derived from other rules in the solution. Given $X \subseteq Atm_0$, we let Sol_X be the set of all applicable solutions to X and $Sol =_{def} \{Sol_X \mid X \subseteq Atm_0\}$ be the set of all solutions in H. From the previous definition, it follows that, in a solution sol, there cannot be two conflicting rules: namely, there cannot be two rules $r, r' \in Sol$ such that $Conclusion(r) = t$ and $Conclusion(r') = \bar{t}$. Given $sol \in Sol$, for each concern $t/\bar{t} \in Concern$, we denote $rule_{sol}(t/\bar{t})$ as the unique $r \in sol$ related to the concern $t/\bar{t}$, if it exists (it is unique due second property in solution definition).

We introduce now the framework of rule-based classifer models.

Definition 1 (Rule-based Classifier Model). *A rule-based classifier model (RCM) is a triple* $C = (S, H, f)$ *where: a)* $S = \{(X, sol) \mid X \subseteq Atm_0, sol \in Sol_X\} \subseteq 2^{Atm_0} \times Sol$*; b)* H *is a hierarchy; c)* $f : S \longrightarrow Val$ *is a decision function, where* $Val = \{0, 1, ?\}$.

In a RCM, each state of the model $s = (X, sol) \in S$ contains a factual situation X with an applicable solution sol. The classification function maps each state into a possible value, namely $\{0, 1, ?\}$. So, for each $s \in S$, we can have either that $f(s) = o$ with $o \in \{0, 1\}$ and so s represents a precedent case; or we can have that $f(s) =$?, and so s is an unassessed/new case. We impose the following constraint on the rule-based classifier models: 1)$\forall s = (X, sol) \in S : f(s) = ? \Rightarrow sol = \emptyset$; 2) $\forall s = (X, sol) \in S : f(s) \neq ? \Rightarrow$

$f(s) = Conclusion(rule_{sol}(0/1))$ With first condition we require that there is no solution for the undecided cases. With second condition, we require that the classification of the state is consistent with a solution in the state.

A *RCM* can be translated into a case base, mimicking Def. 5 in [6]. We write CB_C for the case base obtained from $C = (S, f)$.

3.1 Constraining New Cases

In this section, we show how the output for new cases provided by a classifier should be constrained by relevant cases.

3.1.1 Relevance Relation Modelling Precedential Constraint. For $s = (X, sol) \in S$, we define $\tilde{s}$ as the set containing the situation X of s and all the conclusions of the rules in the solution sol of s, except the rules pertaining the issue $0/1$. Hence, $\tilde{s}$ contains both base factors of s and intermediate factors that apply to s. Given $t \in Int \cup Dec$, we define $Facts^t(s) =_{def} \tilde{s} \cap Facts^t$. So, $Facts^t(s)$ is the set of factors in s favoring side t wrt the concern $t/\bar{t}$, while $Facts^{\bar{t}}(s)$ is the set of factors in s favoring side $\bar{t}$.

Given $s = (X, sol) \in S$, $u/\overline{u} \in Concern$, and $t \in \{u, \overline{u}\}$, we define: 1) $dec^{u/\overline{u}}(s) = Conclusion(r)$ if exists $r \in Rules$ such that $r = rule_{sol}(u/\overline{u})$, otherwise $dec^{u/\overline{u}}(s) =?$; 2) $Reas^t(s) = Antecedent(r)$ if exists $r = rule_{sol}(u/\overline{u})$ such that $t = Conclusion(r)$, otherwise $Reas^t(s) = \emptyset$. Intuitively, $Reas^t(s)$ is the reason for $dec^{u/\overline{u}}(s) = t$, which is the decision in s wrt the concern $u/\overline{u}$ (if it exists).

Definition 2 (Concern-based Relevance). *Given a classification function f and $u/\overline{u} \in Concerns$, we define $\mathcal{R}^{u/\overline{u}} \subseteq 2^{Atm_0 \cup Int} \times 2^{Atm_0 \cup Int} \times S \times S$ as a relevance relation, when it satisfies: $(D, G, s, s') \in \mathcal{R}^{u/\overline{u}}$ iff $Reas^t(s) \subseteq Reas^t(s') \cup D^t$ and $Reas^{\bar{t}}(s') \cup G^{\bar{t}} \subseteq Facts^{\bar{t}}(s)$, where $dec^{u/\overline{u}}(s) = t$ and $f(s) \neq ?$.*

Example 1. The definition of relevance provided above offers a computational method to verify precedential constraints between two states [6, 10]. In other words, it helps determine whether one state should be (or should have been) assessed based on the decision made for the other state, following a form of *a fortiori* reasoning. This verification can be done by fixing specific D and G, which in turn depends on the decisions associated with the states. Consider two states $s_1, s_2 \in S$. We focus on the concern p/p'. Assume s_1 and s_2 were decided in opposite ways: $dec^{p/p'}(s_1) = p$, while $dec^{p/p'}(s_2) = p'$. Suppose $(Facts^p(s_2), \emptyset, s_1, s_2) \in \mathcal{R}^{p/p'}$, that is: (a) $Reas^p(s_1) \subseteq Facts^p(s_2)$ and (b) $Reas^{p'}(s_2) \subseteq Facts^{p'}(s_1)$.

Statement (a) tells us that $Reas^p(s_1)$, the reason for p in s_1, is contained in the factors favoring p, in s_2. Hence, it could have been used in s_2. On the other hand, according to (b), the reason for p' in s_2, $Reas^{p'}(s_2)$, is contained in the factors favoring p', in s_1; so it could have been used in s_1. But s_1, was decided for p. Thus, in deciding s_1 the reason $Reas^p(s_1)$ for p was considered stronger than the reason $Reas^{p'}(s_2)$ for p'. So, — supposing that s_2 was decided later than s_1 — s_2, should have been decided as s_1, *a fortiori*. What has just been discussed is nothing new: if we see it from the perspective of case base models [6], we have simply said that s_2 has violated the precedential relation established by s_1 — that is, the decision for s_2 is inconsistent with the decision for s_1. Indeed, we have the following result.

Proposition 2. *Let $u/\overline{u} \in Concern$. Given $s = (X, sol), s' = (X', sol') \in S$, where $f(s) \neq ?$ and $f(s') \neq ?$, we can define $CB = \{c, c'\}$ with $c = (X, op(sol), f(s))$ and $c' = (X', op(sol'), f(s'))$. Then (1) and (2) are equivalent: (1) $(Facts^t(s'), \emptyset, s, s') \in \mathcal{R}^{u/\overline{u}}$, with $dec^{u/\overline{u}}(s) = t$ and $dec^{u/\overline{u}}(s') = \bar{t}$; (2) CB is inconsistent wrt $u/\overline{u}$.*

3.1.2 Classifications by Relevance: From Precedents to New Cases. From the previous section we know that we can express the precedential constraint between decided states (in the opposite direction) on the basis of the relevance relation. We would like to do the same for undecided states, i.e. we would like to express via the relevance relation, when a new state s^*, is to be decided in a certain direction with respect to a concern, based on the precedential constraint. To do this, however, we must consider some aspects. Firstly, in a new case we only know the base factors, and not the intermediate factors. And solely on the basis of the base factors, we cannot always apply the precedential constraint to decide on concerns of degree greater than 1 (e.g. top issue $0/1$). What we can do is to apply a precedential constraint 'conditioned' by the presence of certain intermediate factors in a set F, i.e. we can say that if certain intermediate factors in a set F applied in s^*, then s^* would be constrained to be decided in a certain way on the basis of some previous cases. Can F be any set of facts? No, F should consists of factors that can be obtained from the factual situation in s^*, i.e. for which there exists a solution sol applicable to the factual situation in s^* and such that the rules in sol support the factors in F. In this sense, F is obtainable from s^*. In particular, for $F \subseteq Int$ and $X \subseteq Atm_0$, we say that F *is obtainable from* X if there is a solution $sol \in Sol$, such that: a) sol is applicable to X; b) $\forall p \in F$: $\exists r \in sol$ such that $Conclusion(r) = p$.

In this case, we say that sol is *compatible* with F and X. We denote $Sol_{F,X}$ the set of all $sol \in Sol$ compatible with F and X.

However, it may happen that a solution compatible with F may have rules that support more intermediate factors than those in F. Given a specific concern $u/\overline{u}$, we will require that the solution compatible with F is minimal with respect to $u/\overline{u}$, i.e. no more intermediate factors favoring $u/\overline{u}$ than those in F and in the factual situation of s^* are introduced by the solution. In particular, given $sol \in Sol_{F,X}$ a compatible solution with F and X, we say that sol is *$u/\overline{u}$-minimal wrt F and X* iff, for any $r = rule_{sol}(u/\overline{u})$, it is the case that $Antecedent(r) \subseteq (Facts^u \cup Facts^{\overline{u}}) \cap (X \cup F)$.

Here is where the relevance relation comes into play. In general, when we write $(D, G, s, s^*) \in \mathcal{R}^{u/\overline{u}}$, we mean that s is relevant to s^*, given the presence of D and G. Also, notice that, given $dec^{u/\overline{u}}(s) = t$ and $f(s^*) =?$ (which leads to $Reas^t(s^*) = \emptyset$), by Definition 2, the following two statements are equivalent: 1) $(D, G, s, s^*) \in \mathcal{R}^{u/\overline{u}}$; 2) both $Reas^t(s) \subseteq D^t$ and $G^t \subseteq Facts^{\bar{t}}(s)$.

Which sets D and G are we interested in? Consider F a set obtainable from s^*. So D and G under consideration are now respectively, $F^t \cup Facts^t(s^*)$ (the factors in favor of t in F and s^*) and $F^t \cup Facts^{\bar{t}}(s^*)$ (the factors in favor of $\bar{t}$ in F and s^*). Suppose $(F^t \cup Facts^t(s^*), F^{\bar{t}} \cup Facts^{\bar{t}}(s^*), s, s^*) \in \mathcal{R}^{u/\overline{u}}$, with $dec^{u/\overline{u}}(s) = t$. So we know that $Reas^t(s) \subseteq F^t \cup Facts^t(s^*)$ and $F^{\bar{t}} \cup Facts^t(s^*) \subseteq Facts^{\bar{t}}(s)$. These two conditions are crucial. Why? Take any compatible and minimal solution sol' wrt F and the factual situation X^* of s^*, we can define a new state $s' = (X^*, sol')$. Now, $Reas^{\bar{t}}(s') \subseteq F^{\bar{t}} \cup Facts^t(s^*)$, whatever the solution wrt $t/\bar{t}$ for s' is. Indeed, we have two possibilities. Solution sol' for s' satisfies $dec^{u/\overline{u}}(s') = t$ (that is, s' is decided as t given sol'). So $Reas^{\overline{t}}(s') = \emptyset$ and trivially $Reas^{\overline{t}}(s') \subseteq F^{\overline{t}} \cup Facts^{t}(s^*)$. If, instead, solution sol' for s' satisfies $dec^{u/\overline{u}}(s') = \overline{t}$, the minimality of sol' will ensure $Reas^{\overline{t}}(s') \subseteq F^{\overline{t}} \cup Facts^{t}(s^*)$ — because sol' cannot introduce more factors favoring $\overline{t}$ than the ones in F and s^*. On the other hand, the compatibility of solution ensures that $F^t \cup Facts^t(s^*) \subseteq Facts^t(s')$ — s' and s^* have the same factual situation X^* and, by the compatibility, every element in F is the conclusion of some rule in sol'. Combining the conditions obtained so far, we can have that $Reas^t(s) \subseteq Facts^t(s')$ and $Reas^{\overline{t}}(s') \subseteq Facts^{\overline{t}}(s)$, which leads to the main result: $(Facts^t(s'), \emptyset, s, s') \in \mathcal{R}^{u/\overline{u}}$.

Now we summarize. From $(F^t \cup Facts^t(s^*), F^{\overline{t}} \cup Facts^{\overline{t}}(s^*), s, s^*) \in \mathcal{R}^{u/\overline{u}}$, we can retrieve $(Facts^t(s'), \emptyset, s, s') \in \mathcal{R}^{u/\overline{u}}$ for any new state s' obtained by enriching s^* with a minimal compatible solution. So, by Proposition 2 we have that, if s' is decided differently than s wrt $u/\overline{u}$ then s' is inconsistent wrt s; and viceversa, if s' is inconsistent wrt s then s' is decided differently than s wrt $u/\overline{u}$. Differently stated, $(F^t \cup Facts^t(s^*), F^{t'} \cup Facts^{t'}(s^*), s, s^*) \in \mathcal{R}^{u/\overline{u}}$ tell us that if we choose a solution for s^* that does not add further elements wrt F and X^* pertaining $u/\overline{u}$ (i.e. it is minimal) we introduce incosistency wrt s, pertaining $u/\overline{u}$, only if the solution goes against s wrt the decision for $u/\overline{u}$. This insights are behind the following proposition.

Proposition 3. *Given $u/\overline{u} \in Concern$, we define $s = (X, sol) \in S$ such that $f(s) \neq ?$ and $dec^{u/\overline{u}}(s) = t \in \{u, \overline{u}\}$ and define $s^* = (X^*, sol^*) \in S$ such that $f(s^*) = ?$ and $sol^* = \emptyset$. Given $F \subseteq Int$, we also define $F^t = F \cap Facts^t$ and $F^{\overline{t}} = F \cap Facts^{\overline{t}}$. Now, given $sol' \in Sol_{F,X^*}$ such that sol' is $u/\overline{u}$-minimal wrt F and X^*, we define $s' = (X^*, sol')$. Let $CB = \{c, c'\}$ be with $c = (X, op(sol), f(s))$ and $c' = (X', op(sol'), f(s'))$. Assume $(F^t \cup Facts^t(s^*), F^{\overline{t}} \cup Facts^{\overline{t}}(s^*), s, s^*) \in \mathcal{R}^{u/\overline{u}}$. Then: $dec^{u/\overline{u}}(s') = \overline{t} \Leftrightarrow CB$ is incosistent wrt $u/\overline{u}$.*

Given $s^* \in S$ a new case we want to define a way to infer a decision for the new case. In this sense, we want to infer a decision making process for assessing the case. In particular, we define a decision making process for the top issue $0/1$, that is iteratively constructed on decision making processes for intermediate concerns $u/\overline{u}$. These processes are defined on the basis of the relevance relation we extracted from the precedential contraint in [6].

Definition 3 (Decision-making Process). *Assume that $s^* = (X^*, \emptyset) \in S$ with $f(s^*) = ?$. Given $m = degree(0/1)$, we iteratively define a decision making process for s^*, based on the relevant states, $h^* : S \longrightarrow 2^{\{0,1\}}$, as follows. Define $F_0 = X^*$.*

Let $n = 1, ..., m$. Define $u/\overline{u} \in Concern$ with $degree(u/\overline{u}) = n$.

- *For $t \in \{u, \overline{u}\}$, define $cite^t(s^*) =_{def} \{s \mid (F^t_{n-1}, F^{\overline{t}}_{n-1}, s, s^*) \in \mathcal{R}^{u/\overline{u}}\}$; that is, $cite^t(s^*)$ is the set of relevant states for s^* that can be cited to constrain a decision for s^* as t.*
- *Define the decision making process related to $u/\overline{u}$ as a function $h^*_{u/\overline{u}} : S \longrightarrow 2^{\{u,\overline{u}\}}$ such that: for all $s \in S$,*

$$h^*_{u/\overline{u}}(s) =_{def} \begin{cases} \{dec^{u/\overline{u}}(s')\} & \text{if } s \neq s^*; \\ \{t \mid cite^t(s^*) \neq \emptyset\} & \text{otherwise.} \end{cases}$$

- *Define $F_n =_{def} F_{n-1} \cup \bigcup_{u/\overline{u}:degree(u/\overline{u})=n} h^*_{u/\overline{u}}(s^*)$. For $t \in Int \cup Dec$, define $F^t_n =_{def} F^t \cap F_n$.*

Notice that at each step n, the set F_n collects all base factor in X^* (since they are in F_0) along with all conclusions from the decision making process, pertaining to concerns up to degree n. We define the set of base and intermediate factors and final decisions determined for s^* by the decision making process as $F_{s^*} =_{def} F_m$.

Based on the cardinality of the decision outcomes related to a concern, we determine if the decision making process in unambiguous wrt the concern. Namely, the decision making function $h^*_{u/\overline{u}}$ is *unambiguous* for s^* iff $|h^*_{u/\overline{u}}(s^*)| = 1$, i.e. when s^* can be decided in a unique way; it is *ambiguous* iff $h^*_{u/\overline{u}}(s^*) = \{u, \overline{u}\}$, i.e. when s^* can be decided both as u and as u'. We can prove that if the starting case base is consistent, then the decision making process is unambiguous for top-level issue and for all intermediate concerns.

The viceversa does not hold: we can have a decision making process that is unambiguous with respect to the top-level issue, even though the case base is inconsistent with respect to some concern (and so, inconsistent).This may happen when we have negligible concerns with respect to a decision: $u/\overline{u}$ is *negligible* with respect to the decision for s^*, if, although the decision about $u/\overline{u}$ is ambiguous, the decision for $0/1$ can be made unambiguously. More precisely, $u/\overline{u}$ is negligible wrt decision for s^*, pertaining the top issue $0/1$, iff $|h^*_{0,1}(s^*)| = 1$ and $h^*_{u,\overline{u}}$ is ambiguous for s^*.

We have a decision making process to work out which intermediate factor and which top issue should be chosen for the new state s^*, on the basis of relevant states. If the decision making process leads to a non-ambiguous solution for the top-level issue (i.e. $|h^*_{0,1}(s^*)| = 1$), which solution can be chosen for s^*? Intuitively, any solution that is compatible with the set of base factors and intermediate factors in F_{s^*} and is $u/\overline{u}$-minimal, for all concerns raised by F_{s^*}, could be chosen. The issue is that for negligible concern $u/\overline{u}$ both u and $\overline{u}$ are in F_{s^*}. In this situation, no solution that is compatible with F_{s^*} can be found (in a solution there can be at most one rule for each concern). We must therefore partition F_{s^*} into subsets containing, for each concern $u/\overline{u}$, either u or $\overline{u}$. Namely, we define the *conflict-free partition* $\mathcal{P}$ of F_{s^*} as the set $\{P \subseteq F^*_{s^*} \mid |P \cap \{u, \overline{u}\}| = 1$ for all $u/\overline{u} \in Concern$ such that $\{u, \overline{u}\} \cap F_{s^*} \neq \emptyset\}$.

Based on partitions, we define a decision process providing admissible solutions for the case under consideration. We want this decision making process to be in line, as much as possible, with the precedential constraint. Specifically, we seek to avoid introducing new inconsistencies beyond those already present in the classifier model (i.e., in the derived case base). To do this, we must ensure that the rules contained in the solution for s^* are compatible with the rules already present in the solutions of the relevant states for s^*. The condition $\star$ in the following definition is for this purpose: in the new solution, we enter a rule pertaining a concern, only if the antecedent of that rule is stronger than (i.e. contains) the reason for the decision pertaining to that concern of a case relevant to s^*.

Definition 4 (Decision making process for a solution). *Let $s^* = (X^*, \emptyset) \in S$ be with $|h^*_{0,1}(s^*)| = 1$. The* decision making process/function for a solution for s^*, based on relevants states*, is the function $g^* : S \longrightarrow 2^{Sol}$ defined as follows:*

$$g^*(s) = \begin{cases} \{sol\} & \text{if } s = (X, sol) \neq s^*; \\ \{sol_P \mid P \in \mathcal{P}\} & \text{otherwise.} \end{cases}$$

where $sol_P \in Sol_{P \cap Int, X^*}$ must satisfy this condition: for all $t \in P$

$$r : Antecedent(r) \rightarrow t \in sol_P \textit{ iff}$$
$$Reas^t(s) \subseteq Antecedent(r) \subseteq P^t, \textit{for some } s \in cite^t(s^*) \qquad (\star)$$

We can prove that if the starting case base is consistent, then the decision making process provides at least a solution, i.e. $g^*(s^*) \neq \emptyset$. Also, the case base with the new decided case is consistent.

4 Courts and Time for Conflict Resolution

The decision making process on the basis of relevance is not always unambiguous. This also happens because the model seen so far is a flat model *with respect to cases*: all cases are equally important. This is not always the case in the legal domain: some cases are more important than others, because they have been decided, for instance, by a higher court or later in time. Furthermore, not all cases are to be considered binding: binding cases for a case at hand are the relevant cases that were decided earlier and by a court that has binding power on the court deciding the case at hand. Lastly, some binding cases are subject to exceptions: either because they have been overruled (i.e. there is a relevant case decided later in a different manner, by a court with the power to do so) or because they have been decided *per incuriam* (i.e. ignoring a binding precedent themselves). These insights are formalised in [7], where a single-step relevance relation is take into account. Without going into technical details, we illustrate how the framework in [7] can help resolve ambiguities in the proposed decision-making process.

Example 4. We refer to hierarchy in Figure 1. Suppose we have a classifier model $C = (S, f)$ s.t. $f(s_1) = f(s_3) = 1$, $f(s_2) = 0$. Suppose $\{p, q\} \in \tilde{s_1}$, $\{p', r', q'\} \in \tilde{s_2}$, $\{p', q', r\} \in \tilde{s_3}$. Let s^* a new case. Suppose we can verify that s_1 and s_2 are both relevant for s^* with respect to p/p', but in different directions. Also, suppose s_2 and s_3 are relevant for s^* with respect to r/r', but in different directions. And the decisions with respect to p/p' and r/r' are both non-negligible for the overall decision. We have a conflict of precedents. Without any consideration of time or courts, we can not make an unambiguous decision. Assume now we have a simple legal system in which we have three courts k_0, k_1, k_2. Court k_0 is a higher court than k_1, which in turn is a higher court than k_2. Let us assume that *vertical stare decisis* applies (higher courts always bind lower courts). Moreover, a higher court can always overrule a lower court (so k_0 can overrule both k_1 and k_2), while a lower court can never overrule a higher court (for instance, k_2 cannot overrule either k_1 or k_0). Suppose that s_1 has been assessed by court k_1, s_2 has been assessed by court k_0, and s_3 has been assessed by court k_2. In addition, s_1 has been assessed before s_2 and s_2 has been assessed before s_3. Suppose s^* is to be decided by the lowest court k_2.Suppose that s_1 was relevant for s_2 wrt concern p/p'. However, s_2 was decided differently than s_1, by a court, k_0, with overruling power wrt the court of s_1, namely k_1. So, s_2 overruled s_1 wrt p/p'. Suppose that s_2 was relevant for s_3 wrt the concern r/r'. We know s_2 was decided before s_3 and binding for $s_3 - k_0$ is higher than k_2. But, s_3 was decided differently than s_2, by the court k_2 without overruling power wrt the court of s_2, namely k_0. Hence, s_3 was decided *per incuriam* wrt r/r'. Now, we consider the binding precedents for s^* without exceptions (not overruled and not *per incuriam*). Recall that the court in s^* is k_2, which is bound by k_0, k_1 and k_2. Notice that, the only binding precedent without exceptions for s^* wrt p/p' is s_2 (s_1 has been overruled). So, s^* should follow s_2 and be decided as p'. And, on the basis of that, s^* should be decided as q'. The only binding precedent without exceptions for s^*, wrt r/r' is s_2 (s_3 is *per incuriam*). So s^* should be decided as r' (as s_2). So we know that p', q', and r' should apply in s^*. Hence, s^* should be decided as 0, on the basis of the binding precedent s_2.

5 Conclusions

In this paper, we provide a first example of how classifier models in the legal domain can be enriched to take also into account sets of rules used to decide cases (potentially both the *ratio decidendi* of the case and the rules established by the precedents constituents). We have provided an initial example of how this can be done, starting from the framework proposed in [6]. Some points need attention, especially similarities with work in [4, 6]. Specifically, our concern-based relevance relation between cases decided in opposite ways returns exactly the definition of conflicting cases introduced in [4], in the context of the flat reason model. This is because, like [4]'s notion of conflicting cases, our case relevance is extracted from precedential constraint. We suppose that our decision making process leads to the same result that would be obtained if the notion of obligatory decision in [4] were applied in the multi-step framework in [6]. The key difference from [4] is our method for computing admissible rules for a new case, which, to our knowledge, is not explicitly defined there.

Acknowledgments

C. Di Florio is partially supported by ISA doctoral Prize, issued by UNIBO ISA DP. H. Dong is partially funded by the WWTF project nr. 10.47379/ICT23-030. A. Rotolo was partially supported by the projects CN1 "National Centre for HPC, Big Data and Quantum Computing" (CUP: J33C22001170001) and PE01 "Future Artificial Intelligence Research" FAIR (CUP: J33C22002830006).

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