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Characterizing the physical properties of gelatin, a classic analog for the brittle elastic crust, insight from numerical modeling

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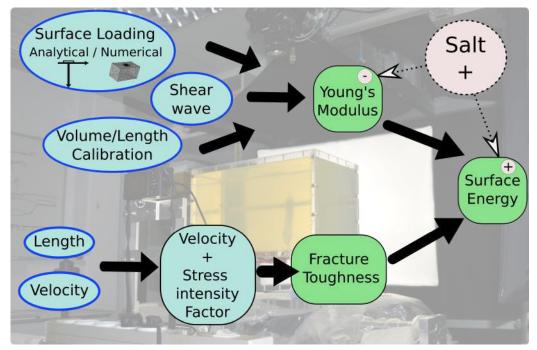
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## Graphical Abstract

Characterizing the physical properties of gelatin, a classic analog for the brittle elastic crust, insight from numerical modeling

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### Highlights

### Characterizing the physical properties of gelatin, a classic analog for the brittle elastic crust, insight from numerical modeling

Smittarello, D., Pinel, V., Maccaferri, F., Furst, S., Rivalta, E., Cayol, V.

- Numerical models improve the gelatin Young's modulus estimation by surface loading.
- Gelatin Young's modulus may be overestimated by 5 % when using an analytical solution.
- The gelatin Young's modulus can be derived from the length for an air-filled crack.
- Addition of salt to gelatin decreases its rigidity and increases its surface energy.

## Characterizing the physical properties of gelatin, a classic analog for the brittle elastic crust, insight from numerical modeling

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### Abstract

Precise characterization of the mechanical properties of gelatin, a classic analog of the elastic crust, is necessary for scaling the mechanical models of the Earth's crust behavior in laboratory experiments. Here we reassess how to accurately calculate the Young modulus (E) of gelatin contained in experimental tanks. By means of dedicated analog experiments and finite element simulations, we estimate the bias introduced by using equations appropriate for a half-space to interpret the subsidence due to a cylindrical surface load applied on the gelatin. In the case of a standard experimental setup with gelatin adhering to the tank wall, we find E is overestimated by at least 5 %for a box with lateral size smaller than 20 times the cylinder diameter. In addition, we deduce a correction factor to be applied when using an analytical formula. We confirm that measuring the shear velocity leads to accurate estimates for the rigidity of gelatin. We also propose a new method for in situ Young's modulus estimation, relying on the length of air-filled propagating crack. Indeed, for a given injected volume, this length depends only on the density contrast between air and gelatin and on the Young's modulus

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of the gelatin. The fracture toughness of the gelatin is estimated independently. Direct comparison between fracture toughness and Young's modulus shows that for a given Young's modulus, salted gelatin has a higher fracture toughness than unsalted gelatin.

*Keywords:* Analog modeling, Gelatin, Young's modulus, Fracture toughness, Crack propagation

### 1 1. Introduction

Gelatin, a transparent animal-derived biopolymer in its sensu stricto ap-2 pellation [1], has been used as an analog of the crust and lithosphere, in a 3 wide range of laboratory experiments [2]. It has proven useful to study the 4 seismic cycle in subduction zones [e.g. 3], the deformation of the upper crust 5 around magma storage zones [e.g. 4, 5] or magma transport through either 6 open conduits [e.g. 6] or magma-filled cracks [e.g. 7, 8, 4, 9, 10, 11, 12, 13]. A physical understanding of the Earth can be gained from analog experiments 8 provided that they are geometrically, kinetically and dynamically scaled [14]. 9 As a consequence the physical and rheological characterization of the gelatin 10 has been addressed by numerous studies in Earth Sciences [e.g. 1, 15, 16] as 11 a complement to work done in the food industry [e.g. 17]. 12

Gelatin has been shown to behave in its gel-like state as a visco-elastic 13 medium, which makes it particularly appropriate for tectonic studies [1]. The 14 balance between the viscous and elastic behavior has been investigated by 15 measuring the storage (G') and the loss (G'') moduli over a broad range of 16 deformation rates. This balance depends mainly on gelatin concentration, 17 temperature and aging [1, 16, 15]. If prepared with a concentration between 18 2 and 5 wt %, gelatin behaves elastically at low temperatures (6-14°C) for 19 time scales up to a few hours [1, 16, 15]. The main focus of the present study 20 is the characterization of gelatin elastic behavior during fluid-filled crack 21 propagation experiments, with application to the study of magma transport 22 through the crust. 23

Elastic behavior of gelatin can be characterized by its Poisson's ratio  $\nu$ and its Young's modulus *E*. Poisson's ratio of gelatin is generally assumed to be 0.5, which means that gelatin is incompressible [18, 19]. Slightly smaller Poisson's ratio with values around 0.45 for gelatin concentration greater than 3 wt % were measured by van Otterloo and Cruden [15] but the values they obtained for smaller concentration were unrealistic [15]. Pansino and Taisne [20] inferred a value larger than 0.47 with a 2.7 wt % concentration. Because
these values are very close to 0.5, we will further consider gelatin as an
incompressible medium. In contrast, crustal rocks generally have a Poisson's
ratio around 0.25.

The Young's modulus can be derived from the limit of the storage mod-34 ulus G' when the frequency tends to infinity and thus measured on small 35 samples of gelatin with a rheometer [1]. However this method is destructive 36 and limited to a reduced size sample. The gelatin Young's modulus increases 37 with the gelatin concentration and decreases with the temperature. It also 38 depends on the gelatin composition, on the preparation protocol and on the 39 cooling history [16]. It might sometimes be useful to add salt to the gelatin 40 in order to slightly increase its density and improve the scaling in specific 41 conditions [21, 22, 23]. In particular, when attempting to model viscous ef-42 fects on magma propagation, the use of salted gelatin is required in order to 43 guarantee a sufficient buoyancy of the oils injected inside the gelatin. Brizzi 44 et al. [24] have shown that the addition of salt dramatically affects not only 45 material behavior, but also gel structure stability. Adding salt induces a 46 decrease of both the Young's modulus and the viscosity, an increase of the 47 time required for cooling down to a stable state and it tends to promote the 48 elastic behavior compared to the viscous one. They also noted that the trans-49 parency might be reduced by salt addition and that the mechanical properties 50 become highly sensitive to the preparation protocol such that it is more dif-51 ficult to control the reproducibility of the experiments. Due to the complex 52 behavior of the gelatin and the variability of experimental conditions, it is 53 recommended to quantify the Young's modulus associated with each gelatin 54 tank, which requires the use of non destructive and *in situ* measurements. 55

The Young's modulus of crustal rocks can be measured in the laboratory 56 by means of uniaxial strain-stress experiments [25]. However, interpolating 57 laboratory measurements on small rock samples to infer the Young's modulus 58 value at depth and the crustal behavior is not trivial. The main issues in 59 laboratory experiments are related to the scaling and use of non-fractured 60 samples at low pressure and temperature that may not reflect the "in situ" 61 conditions at depth in the crust. There are basically two ways of quantifying 62 the in-situ crustal Young's modulus: either using the surface displacement 63 induced by surface loading or unloading events [26, 27, 28], often referred to 64 as a "static" estimation, or using seismic wave velocities derived from local 65 tomography surveys [29, 30], often referred to as "dynamic" estimation. A 66 systematic discrepancy has been evidenced between the dynamic and static 67

<sup>68</sup> Young's moduli. For rocks, the dynamic Young's modulus is always larger <sup>69</sup> by a factor which depends on the porosity [31, 32].

For gelatin in a tank, the most commonly used in-situ and non-destructive 70 method is based on surface loading. It consists of measuring the vertical dis-71 placement induced by a circular load applied at the surface and inverting for 72 the Young's modulus using an analytic formula for a circular rigid load ap-73 plied on an infinite half-space [e.g. 33, 11, 16, 34, 35]. Due to the assumption 74 of an infinite medium, the diameter of the load should be smaller by a factor 75 of ten than the smaller dimension of the tank (either vertically or laterally) 76 [16].77

However, gelatin tanks used for experiments are usually of limited size 78 mainly to ensure a volume which is manageable to produce. It follows that 79 instrumentalists always have to be cautious of boundary effects. One way to 80 account for the rigid boundaries at the box walls consists in using numerical 81 models to infer the actual state of stress within the gelatin [e.g. 11]. For 82 instance, Corbi et al. [36] calculated the state of stress inside their gelatin 83 tank using a 2D-axisymmetric Finite Element Model (FEM) in order to in-84 terpret the crack path observed in experiments. Pinel et al. [37] improved 85 the estimate of the stress field acting within the gelatin tank previously used 86 by Watanabe et al. [38] to quantify the influence of the external stress field 87 on the fluid-filled crack path. This was done by computing the stress field in-88 duced by a surface load using a 3D numerical simulation, taking into account 80 the geometry of the tank and the geometry of the load, and by comparing 90 it to the analytical solution previously used. In the same way, Maccaferri 91 et al. [13] computed both the local stress field and the surface displacement 92 induced by a load of given size, accounting for the rigid boundaries of the 93 box and used this information to obtain an accurate value of the Young's 94 modulus for gelatin tanks. 95

More recently, Pansino and Taisne [39, 20] proposed to derive the Young's 96 modulus from the propagation velocity of shear waves. They took advantage 97 of gelatin being a birefringent photo-elastic material. Its refractive index 98 varies with the stress applied such that, using a pair of polarizing filters 99 enables to visualize the deviatoric stress and track the propagation of shear 100 waves within the gelatin. This method can be thought of as the equivalent 101 of crustal Young's modulus estimations from seismic waves. The results 102 were compared with estimations made using the static loading method and 103 showed to be in good agreement [20]. In addition, Pansino and Taisne [20] 104 proposed that the shear wave method could potentially be used to quantify 105

<sup>106</sup> any variations of the Young's modulus inside the tank.

Experiments of fluid-filled crack propagation also require the characterization of the brittle behavior of the gelatin. A key parameter is the fracture toughness. Crack propagation will only occur once the stress intensity factor at the tip, which depends on the applied stress and the shape of the crack, exceeds the fracture toughness of the surrounding medium. The fracture toughness  $K_c$  is linked to the Young's modulus by the following equation expressed by Griffith [40]:

$$K_c = \sqrt{2\gamma_s E},\tag{1}$$

with  $\gamma_s$  the surface energy of the solid, which for gelatin is estimated around 114  $1.0 \pm 0.2$  J.m<sup>-2</sup> [33, 16]. This relationship is often used to derive the fracture 115 toughness but it should be kept in mind that the value of  $\gamma_s$  is expected to 116 depend on the gelatin composition and might be different when salt is added 117 [22]. Alternatively, the fracture toughness can be quantified by measuring 118 the pressure required to propagate a pre-existing fluid-filled crack [33, 16]. 119 Another method consists in retrieving the stress intensity factor of propa-120 gating cracks for various velocities of propagation: the fracture toughness is 121 then given by the limit of the stress intensity factor when the propagation 122 velocity tends to zero [41]. 123

In the current study, we further detail how numerical simulations may 124 improve the determination of gelatin Young's modulus by surface loading. 125 In particular, we provide an accurate estimate of the error resulting from 126 using the analytical formula when deriving the Young's modulus by surface 127 loading. We also compare Young's modulus obtained by surface loading to 128 values derived by the shear wave velocity method. Then, Young's modulus 129 estimates are used to derive a calibration of the relation between the injected 130 volume and the crack length in case of air-filled cracks. We thus provide a 131 new method for Young's modulus estimation. Finally, we characterize the 132 fracture toughness of the gelatin to quantify the effects of adding salt to 133 gelatin on its brittle behavior. 134

### 135 2. Methods

### 136 2.1. Laboratory technique

${ m K}_{ m SD}^{ m 3D}$ Pa.m $^{1/2}$	54	56	64	53	60	53	58	48				73		47	45				62	62	65			82			19						33	57	40	42	37
$\mathrm{K}_{\mathrm{c}}^{\mathrm{2D}}$ Pa.m $^{1/2}$	49	51	58	48	54	48	52	44				99		42	41				56	57	59			74			17						30	51	36	38	33
: with exact $E^{shw\pm} std$ (Pa)	(5-1)																					$903 \pm 11.52$	$1269 \pm 40.16$	$1462 \pm 28.27$	$1408 \pm 19.43$	$3232 \pm 77.15$	$100 \pm 2.618$	$405 \pm 4.78$	$408 \pm 6.99$	$172 \pm 3.25$	$733 \pm 5.62$	$1177 \pm 9.59$	$212 \pm 4.09$				
tanks. $E_{an}^{load}$ is calculated only when a circular load was applied to the surface with the Finite Element model with the rigid load condition taking into account the exact s is for the number of injections performed and used in this study. Salt $\rho_{gel}^{end} \pm T_{gel}^{el}$ Duration Injections $E_{and}^{end} \pm \operatorname{std}$ Shear velocity $\pm \operatorname{std}$ $E^{\operatorname{shw}_{\pm}}$ (Pa)	(a /																					$54.32 \pm 3.54$	$61.46 \pm 10.89$	$65.97 \pm 7.14$	$64.73 \pm 5.00$	$98.07 \pm 13.11$	$17.25 \pm 2.53$	$34.71 \pm 2.30$	$34.86 \pm 3.34$	$22.64 \pm 2.39$	$46.72 \pm 2.00$	++	$25.14\pm2.71$				
culated only when a circular load was applied model with the rigid load condition taking int of injections performed and used in this study. Duration Injections $E_{inn}^{land\pm} \pm std = E_{inn}^{lond\pm} \pm std = SheaDuration Injections (Pa)$	$1637 \pm 27$	$2277 \pm 136$	$2315 \pm 118$	$2312 \pm 105$	$2408 \pm 354$	$2286 \pm 142$	$2342 \pm 276$	$2117 \pm 130$	1308	$2001 \pm 174$	630	$2201 \pm 352$	$834 \pm 128$	$966 \pm 121$	$845 \pm 257$	$909 \pm 98$	$800 \pm 101$	$938 \pm 100$	2201	1937	2085	1885	1995	$1443 \pm 77$	$1290 \pm 40$	$2892 \pm 236$	$306 \pm 61$	$846 \pm 263$	$441 \pm 32$	$597 \pm 47$	$762 \pm 76$	$950 \pm 168$	$595 \pm 19$	$1847 \pm 20$	$1005 \pm 20$	$1113 \pm 16$	$666 \pm 16$
circular lo 1 load conc ed and use E <sup>load</sup> ± std (Pa)																								$1769 \pm 94$	$1783 \pm 55$	$3649 \pm 297$		$1532 \pm 477$	$700 \pm 50$		$1382 \pm 137$	$1500 \pm 265$					
y when a a the rigic s perform [0]	11	6	7	9	×	7	7	9	J.	4	9	7	9	7	7	4	×	4	7	7	6	7	Q	IJ	1	1	×	1	ç	ъ	1	°°	12	7	10	6	6
ilated only nodel with f injection Duration	1.6	1.7	1.1	1.8	2.1	1.6	1.6	1.3	1.8	3.3	2.9	4.4	3.8	4.0	5.2	3.5	4.0	2.5	0.4	0.2	1.1	0.8	1.6	2.1	0.5	1.7	2.9	0.4	1.7	2.1	0.8	0.7	3.5	1.5	1	1	1
$E_{ann}^{load}$ is calculate Element 1 the number of $\rho_{\text{gel}}^{\rho_{\text{gel}}} = \mathbf{T}_{e^{\text{gel}}}^{\rho_{\text{gel}}}$	6																		10	?-12	?-12					10	14	×	8-11	14	8-12	8-14	14	5.2 - 7.2	10.2 - 13.3	8.2-13.3	
s. $E_{an}^{loai}$ Finite E or the n $\rho_{gel}^{\rho_{gel}}$	1020	1020	1020	1020	1020	1020	1020	1020	1080	1060	1120	1120	1120	1120	1120	1120	1120	1120	1020	1020	1020	1020	1120	1120	1120	1120	1120	1120	1120	1120	1120	1120	1120	1020	1020	1020	1120
tank the 1 the 1 the 1 the 2 salt (%)	0	0	0	0	0	0	0	0	11	10	15	15	15	15	15	15	15	15	0	0	0	0	15	15	15	15	15	15	15	15	15	15	15	0	0	0	15
the 37 d with jection (%)	2	2	2	2	2	2	2	0	2.5	3	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	2	2	2	2	3.5	3.5	2.5	3.5	2	1.5	1.5	2	2	0	7	1.8	1.6	1.5	2
es for t lculated ied. Inj Volume (L)	16.00	16.00	16.00	16.00	16.00	16.00	16.00	16.00	7.70	17.60	18.12	18.17	17.45	18.40	17.84	17.54	17.86	17.57	9.65	15.01	15.05	15.35	12.70	10.44	4.36	4.20	17.54	4.46	4.12	17.10	4.16	4.63	17.22	18.00	18.00	18.00	18.00
Gelatin properties for the 37 tanks. Trans $E_{num}^{load}$ is calculated with the Fi of the load applied. Injections is for Tank Height Volume Gel. Salt reconstry (cm) (J) (%) (%) k	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	9.62	22.00	22.65	22.71	21.81	23.00	22.30	21.92	22.32	21.96	12.06	18.76	18.81	19.19	15.88	13.05	22.56	26.18	21.92	23.09	25.66	21.37	21.53	28.81	21.53	22.50	22.50	22.50	22.50
Table 1: Gelatin properties for the 37 tanks. Eq. 5 whereas $E_{num}^{load}$ is calculated with the Fi geometry of the load applied. Injections is for Tank Tank Height Volume Gel. Salt mumber geometry (cm) (T) (%) (%) 1	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	big	small	cylinder	big	small	cylinder	big	$\operatorname{small}$	$\operatorname{cylinder}$	big	big	big	big	big
Table 1: Eq. 5 whe geometry Tank	1701	1702	1703	1704	1705	1706	1707	1708	1801	1802	1803	1804	1805	1806	1807	1808	1809	1810	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910	1911	1913	1914	1915	1916	2002	2003	2004	2005

### 137 2.1.1. Gelatin preparation

We used type A pig-skin gelatin (Bloom number 280g) purchased from Italgelatine S.p.A., in the form of solid granules. Gelatin with different concentrations were prepared and salt was added to some preparations in order to increase the gelatin density. In total we made 37 tanks of gelatin, 15 at 1.5 to 2 wt % without adding salt and 22 varying the gelatin concentration from 1.5 wt % to 3.5 wt % and the salt concentration from 10 to 15 wt % (Tab. 1).

To prepare the unsalted gelatin, we completely dissolved the granules of 145 gelatin with hot water ( $\sim 60^{\circ}$ C). In order to get a transparent solid, a perfect 146 dissolution of the gelating ranules is required. We proceeded in several steps, 147 dissolving only a fraction of the total mass needed to prepare one tank in 148 a 2 L beaker and holding the solution on warm plate at 60°C. When all 149 the granules were dissolved we diluted with warm water to reach the desired 150 volume and concentration. We let it cool down to room temperature for a few 151 hours. When the preparation had reached  $\sim 32^{\circ}$ C, we mixed it to homogenize 152 the gelatin and we transferred the liquid gelatin into the tank, before placing 153 it into the refrigerator at  $\sim 8^{\circ}$  C for 20 hours. According to Kavanagh et al. 154 [16] and van Otterloo and Cruden [15] liquid gelatin solidifies, behaving as a 155 visco-plastic solid below 25°C and as an elastic solid below 15°C. Note that 156 during the preparation, the temperature of the gelatin should never exceed 157  $70^{\circ}$ C or drop below 4°C to avoid the denaturation of the peptide chains by 158 heating or freezing, respectively [1]. Following Brizzi et al. [24]'s protocol, in 159 order to add salt to the recipe, we first dissolved the salt in hot water, then we 160 used the salty water to dissolve the gelatin granules, as previously described. 161 It is important to first dissolve the salt and then the gelating ranules to avoid 162 the precipitation of the peptide chains and to preserve the transparency of 163 the solid. 164

To prevent the formation of a tough "skin" on the gelatin surface caused by water evaporation, a thin layer of vegetable oil was poured on top of the gelatin (salted and unsalted) before placing it into the fridge. The oil was removed before starting the experiments.

The gelatin density was computed as the ratio between its mass and the volume. Using a graduated cylinder, we measured a volume of  $50\pm1$  mL of liquid gelatin and weight it with a gram-accurate scale. Density of unsalted and salted (15 wt %) gelatin were estimated to  $1020\pm40$  kg.m<sup>-3</sup> and  $1120\pm40$  kg.m<sup>-3</sup>, respectively. Although, we have a quite large formal absolute error due to the poor scale resolution, we obtain very good reproducibility in our measurements from one tank to another. We checked that density
changes due to cooling and solidification were negligible by repeating density
measurements at different temperatures, and checking that the gelatin level
in the tank did not change after solidification.

### 179 2.1.2. Experimental setup and recordings

We used three different plexiglas tanks: a cylindrical one with a diameter 180 of 14.3 cm and a height of 30 cm and two cuboids with dimensions  $L \times l \times H$ 181 of  $13.9 \text{ cm} \times 13.9 \text{ cm} \times 24 \text{ cm}$  for the smaller one and  $40 \text{ cm} \times 20 \text{ cm} \times 25 \text{ cm}$  for 182 the larger one. The larger tank corresponds to the one used for experiments 183 described by Maccaferri et al. [13] (see Fig. 1). The height of gelatin was usu-184 ally close to 20 cm (see Tab. 1). For experiments 2002, 2003 and 2004, both 185 the room and the gelatin temperature were continuously recorded during the 186 whole duration of the experiment by thermocouples, whereas for some other 187 experiments the gelatin temperature was measured by an infrared thermome-188 ter. However, we do not have temperature records for all the experiments: 189 when available the gelatin temperature, or temperature values for the begin-190 ning and the end of the experiment are given in Tab. 1 together with the 191 duration of the whole experiment. 192

In order to measure the gelatin Young's modulus by static deformation, 193 the gelatin surface was loaded with rectangular  $(6 \times 14 \text{ cm})$  or circular shapes 194 (with diameters ranging from 2 to 4 cm) and masses ranging from 3 g to 331 g. 195 With the corresponding pressure range, we induced surface displacement 196 large enough to be accurately measured, without damaging the gelatin. We 197 used a digital caliper, whose accuracy is  $10^{-2}$  mm, to measure the subsidence. 198 We fixed the caliper to a rigid support on the top of the tank (Fig. 1c). We 199 measured the distance to the gelatin surface without load  $d_1$  and the distance 200 to the top of the load  $d_2$ . Knowing the load thickness e, we can derive the 201 vertical surface displacement u induced by the load (Eq. 2): 202

$$u = d_2 + e - d_1 \tag{2}$$

The main limitation to the measurement accuracy comes from the ability of experimentalists to use the caliper without deforming the gelatin surface while measuring the subsidence. To reduce the uncertainties, we repeated the measurements for each tank at least three times using the same load until getting 3 values less than 0.2 mm apart, and when possible, we used several loads with increasing mass.

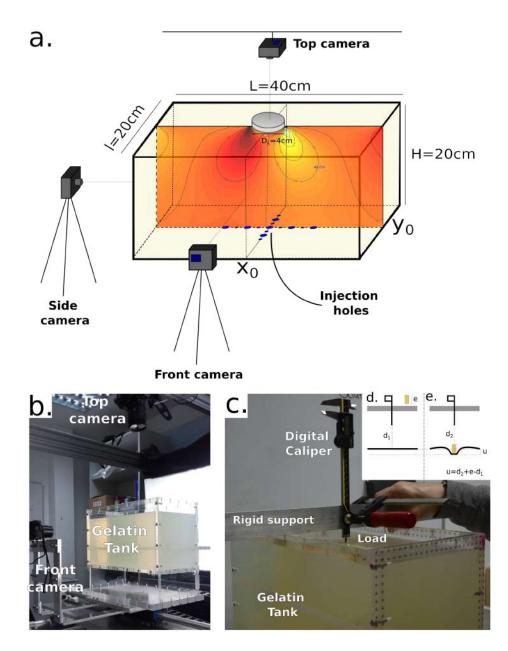


Figure 1: Experimental set-up. (a) Sketch of the experimental set-up and location of the three cameras. Section along plane y = 0 shows shear stress induced by the loading (grey cylinder). The 15 injection holes (2 sizes) are marked by blue circles on the underside of the tank. (b) Photography of the experimental set-up. Two lamps illuminate the tank from the back and right sides through the white screens. (c) Photography of the measurement of the surface vertical downward displacement induced by a load applied at the surface. Inset diagrams schematize the two steps of a measurement (d) measuring the reference distance  $d_1$  before putting the load and (e) measuring the distance  $d_2$  to the top of the load of known thickness e.

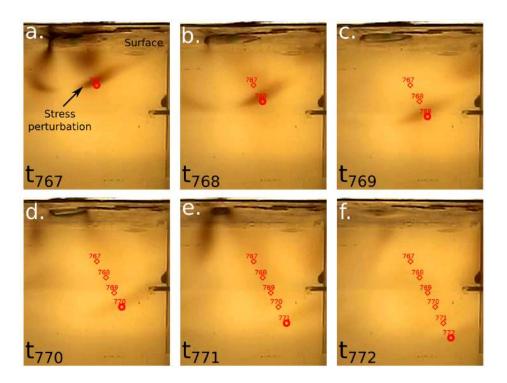


Figure 2: Shear wave velocity measurements with TRACKER. Panels (a to f) display screenshot of six successive images  $(t_{767}$  to  $t_{772})$  from the video records of tank 1910 showing shear wave propagation front after exciting the surface. Red dots indicate the manual picking of the propagation front position to compute the velocity. Note that the shear wave attenuates quickly and it becomes increasingly difficult to track it manually.

In order to estimate the gelatin Young's modulus by measuring the shear-209 waves propagation velocity, we added polarizing filters on both the front and 210 back sides of the tank. Following Pansino and Taisne [39, 20], we excited the 211 gelatin surface with a spoon, and we recorded the shear wave propagation 212 with the front camera. Using the open source software TRACKER [12], we 213 measured the propagation velocity (Fig. 2). For each tank, we performed 214 velocity measurements on several wave trains by manually picking the prop-215 agation front on at least 4 to 5 successive images. Then we computed the 216 mean velocity and the standard deviation associated to each tank. 217

In order to study the fluid-filled crack propagation, we used a syringe to inject a finite volume of air from some holes at the bottom of the tank. The fracture orientation was controlled by carefully orienting the needle used for the injection. No slit was needed, we let the air create its own fracture. On

the bottom of the larger rectangular tank, 15 holes with 2 cm in between 222 allow to perform several injections into the same gelatin. Three perpendic-223 ular cameras recorded the fluid-filled crack shape and path. Two spotlights 224 illuminated the tank from the back and right sides (Fig. 1). Videos subsam-225 pling was done with the video editing and open source software Shotcut [42]. 226 The software TRACKER was used to measure the length L' of the air-filled 227 cracks and to extract the path and the velocity of the crack. When all in-228 jections were completed in a tank, we took several pictures of a ruler at the 229 location of the cracks in order to measure the calibration factor F needed to 230 scale the videos. The crack length L was computed as  $L = F \times L'$ . Thus, 231 the statistical error on the crack length is given by: 232

$$\frac{\sigma_L}{L} = \sqrt{\left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_{L'}}{L'}\right)^2} \tag{3}$$

Where  $\sigma$  is the standard deviation. Given that  $\frac{\sigma_F}{F} = 0.024$  and  $\frac{\sigma_{L'}}{L'} = 0.026$ , we obtain  $\frac{\sigma_L}{L} = 0.035$ .

### 235 2.2. Numerical simulations

In order to take into account the finite size of the tank and the exact 236 shape and size of the circular load, we use a 3D FEM to compute the sur-237 face displacement induced by the applied load. Numerical simulations are 238 performed with the commercial software COMSOL [43] applying a zero dis-239 placement condition to the lateral and bottom boundaries of the gelatin to 240 reproduce the adherence of the gelatin to the tank walls. We use a mesh 241 made of about 330,000 tetrahedral units, refined in a vertical plane centered 242 below the load as well as on the upper surface around the load (minimum 243 size of the mesh was set to 2 mm). The upper surface is considered as a free 244 surface except where the load is applied. To simulate the loading, the easiest 245 boundary condition to be considered would be a constant pressure 246

$$P_{Load} = \frac{4m_L g}{\pi D_L} \tag{4}$$

with g the standard acceleration due to gravity and  $m_L$  the load mass applied on the circular surface of diameter  $D_L$  (Fig. 3a). We will refer to this condition, as the "uniform pressure condition". However, this boundary condition is not fully satisfactory as rigid loads are applied to the surface inducing a uniform vertical displacement below the load. To better reproduce this

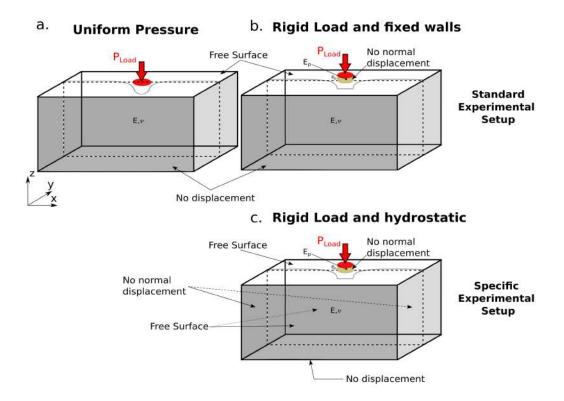


Figure 3: Boundary conditions applied in the numerical model and modeled surface displacements. The gray line shows the shape of the vertical component of the surface displacement induced by the boundary condition applied to the surface: (a) uniform pressure with gelatin adhering to the tank walls, (b) rigid load with gelatin adhering to the tank walls and (c) rigid load with gelatin in contact with water on two of the lateral sides of the gelatin block.

condition we simulate a thin (thickness  $e_p$ ) rigid plate characterized by a 252 large Young's modulus  $E_p$ . On the upper surface of this plate we apply the 253 pressure  $P_{Load}$ . We also apply a condition of zero horizontal displacement to 254 the lateral edge of this rigid plate (Fig. 3b). We will refer to this condition 255 as the "rigid load condition". We set the values of  $E_p$  and  $e_p$  to 10<sup>9</sup> Pa 256 and 4 mm respectively, to ensure a uniform vertical displacement below the 257 applied load. As a theoretical value of 0.5 for the Poisson's ratio cannot be 258 handled numerically, we set it to 0.49. 259

We compare the simulations using constant pressure and rigid load in similar conditions. We also run a set of simulations for a given load, increasing the box size. Additionally, several studies [34, 44], have presented an experimental setup where gelatin is in contact with water on two lateral sides and detached from the other two sides of the tank. In order to discuss the influence of such a specific setup, we also applied another set of lateral conditions with a free surface on the two proximal lateral sides and a roller condition (zero displacement in the direction perpendicular to the tank wall) on the distal lateral sides (Fig. 3c).

### <sup>270</sup> 3. Young's modulus determination by surface loading

#### 271 3.1. Validity domain of the analytical solution

The relationship between the vertical surface displacement  $(U_z)$  beneath a rigid load of mass  $m_L$  applied to the surface of a half-space is given by the analytical formula (see Timoshenko et al. [45]):

$$U_z = \frac{m_L g(1 - \nu^2)}{D_L E},$$
(5)

where  $D_L$  is the load diameter. This method has been followed by most ana-275 log modelers to estimate the Young's Modulus of gelatin [e.g. 33, 11, 16, 34, 276 35]. However, assuming an infinite half space, such formula may introduce 277 an error. This error was reported to remain small providing that the tank 278 minimum size is ten times larger that the load diameter [16]. This estimation 279 was derived from a correlation study between the Young's modulus estimate 280 and the ratio of the load versus tank dimensions [16]. As the tank size is 281 usually limited for practical reasons, this condition might be difficult to ful-282 fill, in particular, because the load applied should be large enough to ensure 283 significant displacement and it should be applied over an area large enough 284 to avoid damaging the gelatin. In our case, even with our largest cuboid 285 tank, the tank minimum dimension is indeed only five times larger than the 286 load we applied. In order to quantify the error performed when using Eq. 5 287 as a function of the tank size, we compare the surface displacement calcu-288 lated from the numerical simulation with the one predicted by the analytical 289 formula. 290

Numerically, we apply a circular load (mass m = 8 g and diameter  $D_L =$ 40.04 mm) to the center of our largest rectangular tank and we progressively multiply the tank size by a factor f up to 80. Calculations are done both for the uniform pressure condition (Fig. 3a) and the rigid load condition (Fig. 3b)

with gelatin adhering to the tank walls. The gelatin Young's modulus is set 295 to 1000 Pa. Displacement profiles are shown in Fig. 4. As expected, the 296 uniform pressure solution (dashed lines in Fig. 4) results in a maximum 297 displacement below the load center while the rigid load condition produces 298 a smaller and uniform displacement below the load which better reproduces 299 our experimental conditions. We also represent the analytical solution for a 300 uniform pressure load, which can be derived from Sneddon [46] as proposed 301 by Sigmundsson and Einarsson [47] and Pinel et al. [28]: 302

$$U_z = \frac{4}{\pi} \frac{m_L g(1 - \nu^2)}{D_L E}$$
(6)

We find that using a uniform pressure (Eq. 6) instead of a rigid load (Eq. 5)303 as boundary condition for the loading, would produce an overestimate of the 304 Young's modulus E by a factor  $4/\pi \sim 1.27$  for an infinite half-space. More-305 over, both analytical formulas (Eq. 5 and 6) predict more vertical displace-306 ment than the corresponding numerical solution. Therefore, if such formula 307 is applied to our largest cuboid tank (Tab. 2), E gets overestimated by 15%308 and 21% for the uniform pressure and the rigid load conditions, respectively. 309 In Tab. 2, displacements at the center of the load are also estimated 310 using analytical formulas and a Poisson's ratio value of 0.49 (instead of 0.5) 311 similarly to numerical simulations. Using a Poisson's ratio of 0.49 for the rigid 312 load condition (Eq. 5) produces an overestimate of the vertical displacement 313 by 1% (0.02mm). Such an overestimate is negligible in comparison to the 314 error made by ignoring the boundary effect. 315

Both Fig. 4 and Tab. 2 show that when the tank size increases, numerical solutions tend to the value given by the analytical ones, which confirms the validity of the numerical model.

We compare the estimation of the Young's modulus when using the ana-319 lytical formula (Eq. 5) to the numerical solution for the rigid load conditions 320 (Tab. 2) as a function of the relative size of the load and the tank (Fig. 5). 321 The caliper we used to measure the subsidence has an accuracy of 0.01 mm 322 which corresponds to an error of 5% on the Young modulus estimate. Ac-323 cording to Fig. 5, the ratio between the shortest dimension of the tank and 324 the load diameter must be at least 21 for the error being less than 5% in 325 the case of an experimental setup with gelatin adhering to the tank walls. 326 This ratio is twice larger than the ratio previously recommended based on 327 correlation studies [16]. For ratios below twenty, side effects due to the rigid 328 walls of the tank cannot be neglected and the analytical formula significantly 329

overestimates the displacement induced by a given load thus producing an 330 overestimation of the Young's modulus of the gelatin (see also Tab. 1). Fig. 5 331 also shows that, when a specific experimental setup that ensures hydrostatic 332 conditions on the proximal lateral sides of the gelatin block is used, the use 333 of the analytical solution is less problematic. Then the analytical solution 334 can be used provided that the shortest dimension of the tank remains 7 times 335 larger than the load diameter. However even in the few studies that consider 336 these specific lateral conditions, the Young's modulus was measured before 337 the gelatin sides were melted and replaced by water meaning that the gelatin 338 adhered to wall during the Young's modulus measurement. While most ex-339 perimenters indicate the size of the tank they are using, the size of the load 340 applied during the Young's modulus measurement is never provided except 341 by Kavanagh et al. [16]. In the literature, the minimum size of the tank 342 ranges from 8.6 cm [16] to 80 cm [6], which means that the maximum size 343 of the applied load should be less than 0.4 cm for the smallest tanks and 344 less than 3.8 cm for the largest. We provide a solution to this problem as 345 from our results, it is possible to derive a correction factor to be applied to 346 the value derived using the analytical formula in order to take into account 347 the actual size of the tank (Fig. 5) and thus obtain a reliable value for the 348 Young's modulus. 349

#### 350 3.2. Young's modulus estimations by surface loading

Based on the results presented in the previous paragraph, the best method 351 to estimate the Young's modulus by surface loading is to use the numerical 352 simulation with a rigid load condition. We thus follow this strategy. Using 353 the FEM simulation, with a geometry corresponding to each tank and each 354 load considered, we compute the surface displacement  $U_z^{FEM}$  induced by 355 the load with a Young's modulus value of  $E^{FEM} = 1000$  Pa, Poisson's ratio 356  $\nu = 0.49$  and gravity  $g = 9.81 \text{ m.s}^{-2}$ . For each measurement of surface 357 subsidence in the analog setup  $U_z^{mes}$ , we can determine a Young's modulus 358 value  $E^{load}$  using the following equation: 359

$$E^{load} = \frac{E^{FEM}U_z^{FEM}}{U_z^{mes}} \tag{7}$$

The Young's modulus is estimated for the 37 gelatin blocks (15 tanks filled with unsalted gelatin at 1.5 to 2 wt % and 22 tanks filled with salted gelatin at 1.5 to 3.5 wt % of gelatin and 10 to 15 wt % of salt). We estimate a value

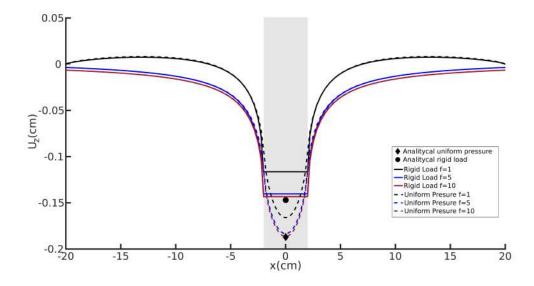


Figure 4: Profile of vertical surface displacement along x-axis (y=0, profile at the load center) for a circular load (diameter  $D_L = 40.04$  mm, mass  $m_L = 8$  g) applied to the surface of a rectangular tank (dimensions L×l×H of 40 cm×20 cm×20 cm multiplied by a factor f, the value f = 1 corresponding to the largest rectangular tank we used in our experiments). The Young's modulus is set to 1000 Pa. Black, blue and red solid lines are profiles calculated with the FEM for, respectively f = 1, f = 5 and f = 10. Dashed lines are for solutions derived with the uniform pressure condition, whereas plain lines are for solutions derived with the rigid load condition (in the case of the experimental setup with gelatin adhering the tank walls). Black diamond and circle represent the vertical displacement from analytical formulas (in x=0 at the load center) considering, the uniform pressure and the rigid load condition, respectively.

Table 2: Comparison of modeled vertical displacement induced by a circular load (diameter  $D_L = 40.04 \text{ mm}$ , mass  $m_L = 8 \text{ g}$ ) applied at the surface of a rectangular tank (dimensions  $L \times l \times H$  of 40 cm  $\times 20 \text{ cm} \times 20 \text{ cm}$  multiplied by a factor f, the value f = 1 corresponding to the largest rectangular tank we used in our experiments). The Young's modulus is set to 1000 Pa. Two analytical solutions are tested also with three numerical solutions ("uniform pressure condition" with gelatin adhering to the walls of the tank, "rigid load condition" with gelatin in contact with water on two of the lateral sides of the gelatin block) for several values of the tank size. In numerical simulations, the Poisson's ratio value is set to 0.49. FEM: Finite Element Model ; P: uniform pressure ; rgdL: rigid load; Swall: gelatin adhering to the tank walls; Fwall: gelatin in contact with water on two of the gelatin block.

Model		Tank size l (dm)	$f Ratio l/D_L$	$egin{array}{c} \mathbf{U_z} \ (\mathbf{mm}) \end{array}$
Analytic rgdL	$\nu = 0.5$	$\infty$	$\infty$	1.4695
Analytic rgdL	$\nu = 0.49$	$\infty$	$\infty$	1.4889
Analytic P	$\nu = 0.5$	$\infty$	$\infty$	1.8711
Analytic P	$\nu = 0.49$	$\infty$	$\infty$	1.8958
FEM P-Swall	f=1	2	5	1.5919
FEM P-Swall	f=10	20	50	1.8648
FEM rgdL-Swall	f=1	2	5	1.1640
FEM rgdL-Swall	f = 1.5	3	7.5	1.2633
FEM rgdL-Swall	f=2	4	10	1.3146
FEM rgdL-Swall	f=3	6	15	1.3660
FEM rgdL-Swall	f=4	8	20	1.3915
FEM rgdL-Swall	f=5	10	25	1.4064
FEM rgdL-Swall	f=6	12	30	1.4162
FEM rgdL-Swall	f=7	14	35	1.4237
FEM rgdL-Swall	f=8	16	40	1.4286
FEM rgdL-Swall	f=9	18	45	1.4323
FEM rgdL-Swall	f=10	20	50	1.4381
FEM rgdL-Swall	f=20	40	100	1.4521
FEM rgdL-Swall	f = 50	100	250	1.4594
FEM rgdL-Swall	f = 60	120	300	1.4623
FEM rgdL-Swall	f=80	160	400	1.4634
FEM rgdL-Fwall	f = 0.5	1	2.5	1.2944
FEM rgdL-Fwall	f=1	2	5	1.3785
FEM rgdL-Fwall	f = 1.5	3	7.5	1.4056
FEM rgdL-Fwall	f=2	4	10	1.4217
FEM rgdL-Fwall	f=3	6	15	1.4375
FEM rgdL-Fwall	f=4	8	20	1.4453
FEM rgdL-Fwall	f=6	$17 \ 12$	30	1.4521
FEM rgdL-Fwall	f=8	16	40	1.4555
FEM rgdL-Fwall	f=10	20	50	1.4597
FEM rgdL-Fwall	f=20	40	100	1.4628

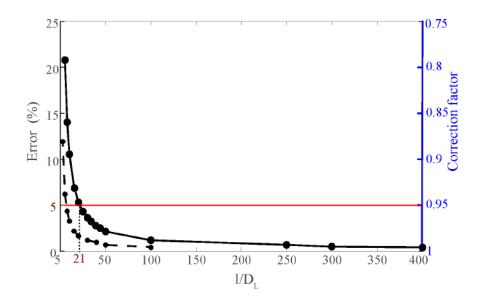


Figure 5: Estimation of the error resulting from the use of the analytical solution for a rigid circular load applied on a half-space (Eq. 5) when estimating the surface displacement induced by a circular load as a function of relative size of the load applied and the tank. The right axis gives the corresponding correction factor to be applied to the value derived analytically in order to obtain the actual value of the Young's modulus inside the tank. The plain and dashed lines are, respectively, for the experimental setup with gelatin adhering to the tank walls and for the case where gelatin is in contact with water on the two proximal lateral sides. Circles indicate the numerical simulations we performed. The red line is for a 5 % error.

of the Young's modulus for each measurement of the vertical displacement. 363 Then for each tank we compute the mean and standard deviation values from 364 all available measurements (Fig. 6 and Tab. 1). For unsalted gelatin at 2 wt 365 %, we estimate  $E = 2150 \pm 230$  Pa, with a good reproducibility. Only the first 366 tank (1701) is found to have a lower value E = 1640 Pa. However, because 367 neither the length, nor the shape nor the velocity of the cracks in this gelatin 368 block is significantly different from the others, we believe that this low value 369 is due to an error on the measurement of the vertical displacement induced 370 by the load. 371

Varying gelatin and salt concentration, Young's modulus of salted gelatin 372 range between 300 and 2900 Pa (Tab. 1). As found by Brizzi et al. [24], 2 wt 373 % gelatin with 15 wt % salt has lower Young's modulus than unsalted gelatin 374 at 2 wt %. By increasing the gelatin concentration to 3.5 wt %, we prepared 375 salted gelatin with Young's modulus similar to unsalted gelatin at 2 wt %. 376 We made 3 tanks at 3.5 wt % and 15% salt and we obtain  $E = 2065 \pm 120$  Pa. 377 a value close to the Young's modulus estimated for unsalted gelatin at 2 wt 378 %. 379

## 4. Comparison of Young's modulus estimates either by shear-wave velocity or surface loading

An alternative method for non-destructive and in-situ measurements of Young's modulus has been proposed recently by Pansino and Taisne [39, 20]. It takes advantage of the birefringent photo-elastic property of the gelatin. This property allows the measurement of the shear wave velocity  $\nu_s$  and the determination of Young's modulus  $E^{shw}$  with the following equation:

$$E^{shw} = 2(1+\nu)\rho_g \nu_s^2 \tag{8}$$

where  $\rho_g$  is the gelatin density and  $\nu$  is its Poisson's ratio, here, assumed to be 0.5.

Shear wave velocities measurements are performed for 12 gelatin blocks (1 unsalted gelatin and 11 salted gelatin) enabling for a direct comparison with the previous estimates based on surface loading. We compute the Young's modulus  $E^{shw}$  for each measured value of shear wave velocity, then we use the mean value and the standard deviation obtained for each gelatin block (Fig. 6 and Tab. 3). Young's modulus estimates using both the surface loading method and the shear wave method have the same order of magnitude.

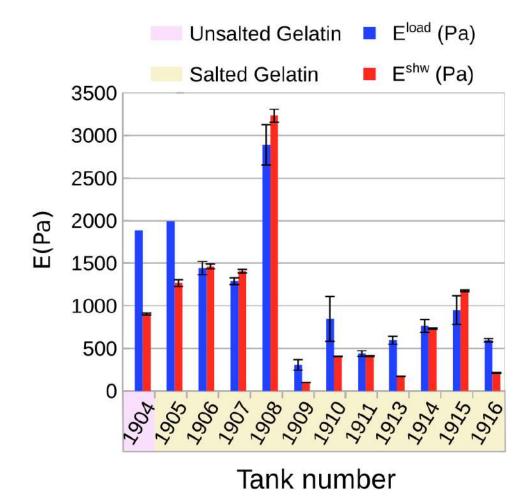


Figure 6: Comparison of Young's Modulus estimated for 12 gelatin tanks and two methods. The average value obtained from several measurements is represented, either derived by the surface loading method ( $E^{load}$  in blue), or from shear waves velocity measurements ( $E^{shw}$  in red). Tank numbers of unsalted gelatin and salted gelatin are highlighted in purple and orange respectively. Error bars represent the standard deviation.

Tank	E <sup>load</sup> (Pa)	$E^{shw}$ (Pa)	$\Delta E$ (Pa)	$rac{\Delta \mathrm{E}}{\mathrm{E}^{\mathrm{load}}}$ (%)	$rac{\mathbf{E}^{\mathbf{load}}}{\mathbf{E}^{\mathbf{shw}}}$
1904	1885	905	980	52~%	2.08
1905	1995	1270	725	36~%	1.57
1906	1445	1460	-15	-1 %	1.00
1907	1290	1410	-120	-9 %	0.91
1908	2890	3230	-340	-12 %	0.89
1909	305	100	205	67~%	3.05
1910	845	405	440	52~%	2.09
1911	440	410	30	6~%	1.07
1913	600	170	430	72~%	3.53
1914	760	735	25	3~%	1.03
1915	950	1170	-220	-23 %	0.81
1916	595	210	385	64~%	2.83

Table 3: Comparison of Young's modulus estimation by surface loading  $E^{load}$  and shearwave velocity measurement  $E^{shw}$  for 12 tanks.  $\Delta E$  is the absolute difference in Pa,  $\frac{\Delta E}{E^{load}}$  is the percentage of difference between both measurements.

Discrepancies over 500 Pa are found only for 2 tanks (1904, 1905) and discrep-396 ancies over 60% are found for 3 other tanks (1909, 1913 and 1916) having the 397 smallest values of Young's modulus. In Tab. 3 we report the ratio between 398  $E^{load}$  and  $E^{shw}$ , which is smaller in our case (between 0.81 and 3.53) than in 390 Pansino and Taisne [20] (between 0.79 and 5.29). In both studies, a better 400 agreement between estimation by surface loading and by shear waves velocity 401 is observed for more rigid gelatins. Pansino and Taisne [20] attributed the 402 largest values given by the surface loading method to the non homogeneous 403 cooling of the gelatin. Whereas the shear wave method allows to quantify 404 the strength of the interior region of the tank, the surface loading method 405 quantifies the strength of the upper layer, whose hardening by cooling is 406 much quicker than the interior of the tank. This conclusion was supported 407 by larger discrepancy between  $E^{load}$  and  $E^{shw}$  being observed after a shorter 408 duration of cooling. However, because precise dimension of the load are not 409 provided for each tank of Pansino and Taisne [20]'s study, an overestimation 410 of the Young's modulus by surface loading due to the use of the analytical 411 formula cannot be completely excluded. 412

### <sup>413</sup> 5. Young's modulus estimation by crack length measurement

### 414 5.1. Principles and theoretical background

The length of propagating air-filled cracks in a gelatin block depends on the injected air volume and on the physical properties of the gelatin [10] such that cracks shape might be used to estimate the Young's modulus [8]. Whereas the critical fluid volume required to ensure a buoyant crack propagation is a function of the fracture toughness [48], the relationship between the injected volume and the crack length only depends on the Young's modulus and buoyancy.

In the framework of the Weertman's theory, a static crack of length L = 2a, filled with an incompressible fluid, is characterized by the following halfopening profile w along z direction [49, 50]:

$$w(z) = \frac{1 - \nu^2}{E} \Delta \rho g \sqrt{a^2 - z^2} (a + z); -a \le z \le a,$$
(9)

where *a* is the half-length of the crack and  $\Delta \rho = \rho_{solid} - \rho_{fluid}$  is the density contrast between the fluid and the host rock. Integrating the opening profile (Eq. 9) over the crack's length gives the area *A* of the crack's cross section:

$$A = \frac{\pi (1 - \nu^2)}{8} \frac{\Delta \rho g}{E} L^3 \tag{10}$$

The influence of fluid compressibility on Eq. 10 can be shown by using a 2D numerical, boundary-element model (see Fig. 7). In 3D, one could expect for the volume V of the crack :

$$V = \alpha (1 - \nu^2) \frac{\Delta \rho g}{E} L^4, \qquad (11)$$

431 with  $\alpha$  a constant.

<sup>432</sup> Considering that in side view a rising crack has a shape close to an ellip-<sup>433</sup> soid in its upper part and close to a rectangular in its lower part (Fig. 8a), <sup>434</sup> and that the crack half breadth or lateral dimension r is comprised between <sup>435</sup> 3/4a and a [51], it follows that  $\alpha$  is expected to range between 0.22 and 0.30.

### 436 5.2. Calibration of the Volume-Length relation in 3D

Volumes ranging between 0.4-20 mL were injected with syringes of different sizes  $(2\pm0.2, 10\pm1 \text{ or } 20\pm2 \text{ mL})$ . Measurements of the crack length from

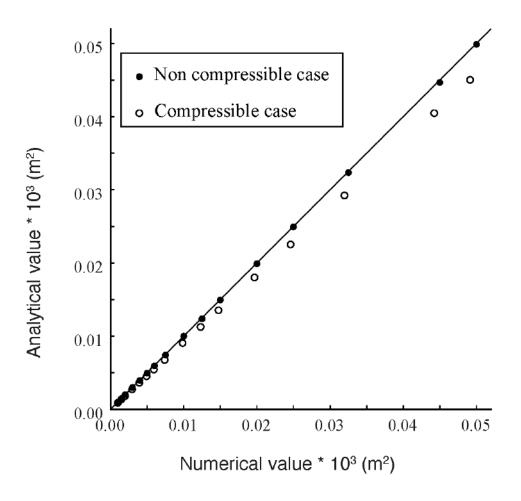


Figure 7: Comparison of the analytical value and the numerical value for the 2D crosssectional area of a finite length static crack in an infinite elastic medium. Analytical value is given by Eq. 10, the numerical value is calculated with the Boundary Element model described in Maccaferri et al. [52], considering increasing crack lengths. For the compressible case, the bulk modulus of the fluid is set to 100 Pa whereas for the non compressible case it is set to  $10^6$  Pa.

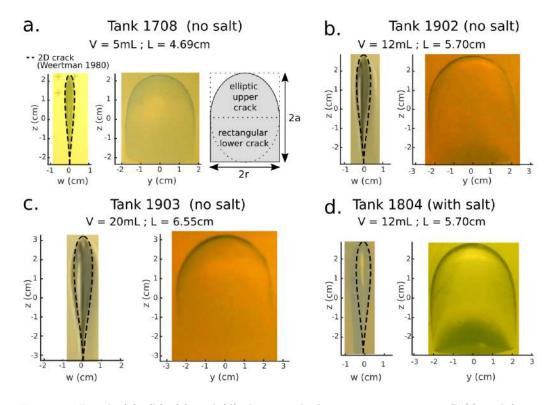


Figure 8: Panels (a), (b), (c) and (d) show crack shapes in cross section (left) and front (right) view of four experiments. Profile opening along cross sections is computed with Eq. 9 (dashed line). Panel (a) also shows a scheme of the theoretical shape of front view as a combination of an elliptic upper part which radius are r (half-width) and a (half-length) and a rectangular lower part which sides are 2r and a.

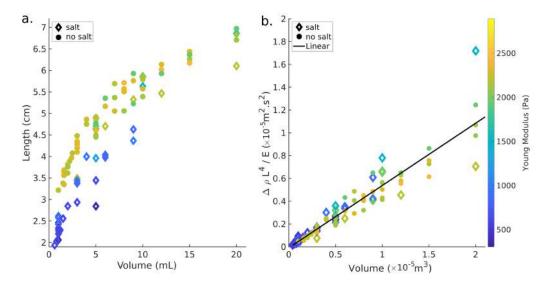


Figure 9: Relationship between crack length (L) and injected volume (V) for air-filled cracks (for all injections performed in tanks 1701 to 1916 listed in Tab. 1). Circles and diamonds represent injections inside unsalted gelatin and salted gelatin, respectively. Colors represent the Young's modulus  $(E = E^{load})$  of the tank estimated by surface loading. (a) Crack length (L) as a function of injected volume of air (V). (b)  $\frac{\Delta \rho L^4}{E}$  as a function of the injected volume of air (V), the linear tendency gives  $\alpha = 0.25$  for Eq. 11.

<sup>439</sup> both cameras are consistent and, for most of the injections, the crack length<sup>440</sup> remains constant during crack propagation.

Fig. 8 shows a selection of cross-section and front views of cracks compared to the opening profile predicted by the Weertman's theory (Eq. 9). As shown by experiments performed injecting 12 mL in tank 1902 (unsalty) and in tank 1804 (salty) which have similar Young's modulus, the crack length is not affected by the addition of salt into the gelatin.

Fig. 9a shows the crack length (L) as a function of the injected volume (V) for each air injection. For a given injected volume, shorter cracks form in gelatin with lower Young's modulus. In order to test the validity of Eq. 11, Fig. 9b represents  $\frac{\Delta \rho L^4}{E}$  as a function of the injected volume V. We use our numerically estimated Young's modulus for the first 33 tanks listed in Tab. 1 to calibrate Eq. 11 by determining the proportionality coefficient  $\alpha$ . The linear tendency allows to estimate  $\alpha_{air}$  to 0.25.

The crack length versus injected volume relationship (Eq. 11 with  $\alpha_{air}$ set to 0.25) can thus be used to infer the Young's modulus inside any gelatin tank by measuring the crack length when injecting a known volume of air or <sup>456</sup> of any non-viscous buoyant fluid.

In order to further validate this method, we derive the Young's modulus, from Eq. 11 ( $E_{\alpha}$ ) with  $\alpha$ =0.25 considering several injected volumes in four tanks (tanks 2002, 2003, 2004 and 2005 as listed in Tab. 1), which were not used to derive  $\alpha$ . We then compute the relative differences between  $E_{\alpha}$  and  $E^{load}$  (Eq. 12) for several injections (Fig. 10):

$$\frac{\Delta E}{E}(\%) = \frac{|E^{load} - E_{\alpha}|}{E^{load} + E_{\alpha}} \times 200$$
(12)

We get a mean error and a standard deviation of  $17 \pm 9$  % which reflects the dispersion of our data due to measurements uncertainties on length, volume and  $\Delta \rho$ . Whereas the absolute accuracy of this method does not seem better than 15%, it enables us to evidence potential changes in the Young's modulus along the crack path as discussed below.

# 467 5.3. Evidence of a Young's modulus vertical gradient in some experimental 468 tanks

If the Young's modulus of the gelatin tank is homogeneous, the length of 469 the crack does not vary, except in the close vicinity of the upper free surface 470 where it is expected to decrease [53]. Otherwise, a progressive change in the 471 crack length of an ascending crack may reflect a gradient in the rigidity of 472 the gelatin. In two tanks (1806 and 1807), we injected the air a few hours 473 after taking the gelatin out of the fridge. In those cases, cracks are getting 474 shorter and thicker during their ascent consistently with a decrease of the 475 Young's modulus value at shallower depths. In tank 1806, we measure the 476 crack length at several depths for injections 1834 and 1835. Using Eq. 11, 477 we evidence a vertical gradient of rigidity of 35  $Pa.cm^{-1}$  and 43  $Pa.cm^{-1}$ . 478 respectively (Fig. 11). 479

Those tanks are filled with salted gelatin but the observed gradient can-480 not originate from a gradient in salt concentration. If it had been the case 481 larger salt concentration would have been expected in the lower part of the 482 gelatin tank resulting in smaller values of Young's modulus at the bottom 483 of the tank. An effect of a gradient in gelatin concentration cannot be ex-484 cluded, however it is unlikely as such a gradient was never observed in other 485 tanks obtained following the same protocol. Such a decrease of the Young's 486 modulus with decreasing distance to the gelatin surface cannot be due to 487 an ongoing cooling either, as it would result in the reverse gradient. One 488 explanation would be a progressive re-heating of the gelatin tank due to the 489

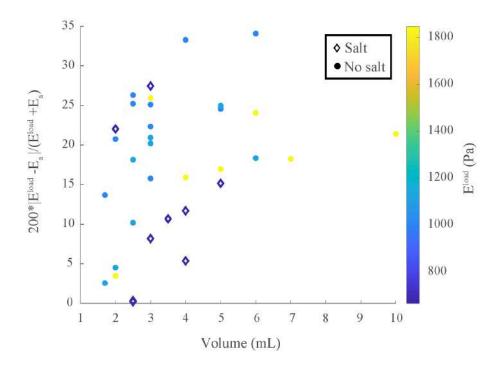


Figure 10: Relative difference between the Young's modulus estimated by air-filled crack length  $(E_{\alpha})$  and by surface loading  $(E^{load})$  for four tanks (2002, 2003, 2004 and 2005 as listed in Tab 1). Circles and diamonds represent injections inside unsalted gelatin and salted gelatin, respectively. Colors represent the Young's modulus  $(E = E^{load})$  of the tank estimated by surface loading.  $E_{\alpha}$  is computed with  $\alpha = 0.25$  in Eq. 11.

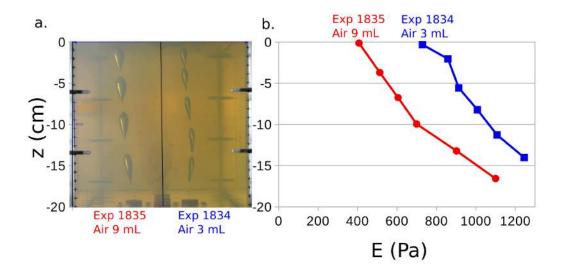


Figure 11: A rigidity gradient affects the tank 1806. (a) Screenshots showing the evolution of the shape of the cracks 1834 (3 mL) and 1835 (9 mL) during propagation. (b) Evolution of E with depth, blue and red curves represent experiments 1834 and 1835 respectively.

lighting. This is consistent with the fact that both experiments were run after a long stay (more than 4 hours) at room temperature. The slightly larger value obtained for the Young's modulus derived from injection 1834, performed before injection 1835, is consistent with this explanation. But the difference between both estimations might also reveal lateral variations in the tank. In particular, the crack in 1835 is injected in the backward side of the tank which is closer to the light source.

### <sup>497</sup> 6. Fracture toughness characterization

Magma transport through the upper crust occurs mainly by dike propa-498 gation. Magma flows inside a planar fracture such that the velocity is partly 499 controlled by fracturing, at least in the tip area, and depends on the crustal 500 fracture toughness [41]. Fracture toughness is thus a key physical parameter 501 of the gelatin for fluid-filled crack propagation experiments. This property 502 is linked to the Young's modulus through the surface energy  $\gamma_s$  (see Eq. 1) 503 [40], which is usually poorly known. Kavanagh et al. [16] estimated the 504 value of surface energy for unsalted gelatin with concentration ranging from 505 5 to 8 % to be around  $1.0 \pm 0.2$  Jm<sup>-2</sup>. It was done measuring indepen-506 dently the Young's modulus by surface loading and the fracture toughness 507

by quantifying the pressure required to propagate a pre-existing crack, using 508 a two-dimensional approximation. Here we followed the strategy proposed 509 by Heimpel and Olson [41] to estimate the fracture toughness. We estimated 510 the velocity for several finite size air-filled cracks of various volumes injected 511 inside the same tank of gelatin. We computed the stress intensity factor 512 based on the crack length. Following Secor and Pollard [49], in 2D, the 513 stress intensity factor in mode I  $K_I^{2D}$ , for a buoyant crack, can be expressed 514 as: 515

$$K_I^{2D} = \Delta \rho g a \sqrt{\pi a} \tag{13}$$

Dahm [51] better characterized the 3D shape of buoyancy-driven propa-516 gating fractures with an approximately circular and straight line boundary 517 at the upper and lower ends, respectively. He showed that fractures are self 518 similar with the lateral extent (half breadth r) linked to the vertical extent 519 (half length a) by the relationship: r = (3/4)a. Using this approximation 520 and the expression for the stress intensity factor in 3D proposed by Heimpel 521 and Olson [41], we can derive a scaling factor between 3D  $(K_I^{3D})$  and 2D 522  $(K_I^{2D})$  stress intensity factors: 523

$$K_{I}^{3D} = \frac{2\sqrt{3}}{\pi} K_{I}^{2D} = \frac{2\sqrt{3}}{\sqrt{\pi}} \Delta \rho g a \sqrt{a} \approx 1.103 K_{I}^{2D}$$
(14)

Fig. 12 shows the evolution of  $K_I^{3D}$  as a function of the crack velocity in three different tanks. The critical value for the stress intensity factor is the minimum which allows for crack propagation and equals the fracture toughness of the host medium. Therefore, a linear regression is used to quantify the fracture toughness of the gelatin in each tank, which corresponds to the vertical intercept.

Derived fracture toughness values are listed in Tab. 1 for each tank where 530 this property was estimated. We use our estimates of  $K_L^{3D}$ , in combination 531 with estimates of the Young's modulus  $E^{load}$ , to compute the surface energy 532  $\gamma_s$  for our gelatins (Eq. 1). Our results (Fig. 13) are mostly consistent with 533 the previous estimate of  $\gamma_s$  [33, 16]. For unsalted gelatin, we derive a value for 534  $\gamma_s$  equal to 0.77 Jm<sup>-2</sup>, which is slightly below the range derived by Kavanagh 535 et al. [16] with more rigid gelatin. Importantly, our results clearly show that, 536 for the same Young's modulus, a salted gelatin is characterized by a higher 537 fracture toughness than an unsalted one. Using all available salted gelatin, 538 for  $\gamma_s$  we obtain a value of 1.32 Jm<sup>-2</sup>. When omitting tank 1906, which could 539 be considered as an outlier in Fig. 13, we obtain for  $\gamma_s$  a value of 1.10 Jm<sup>-2</sup> 540

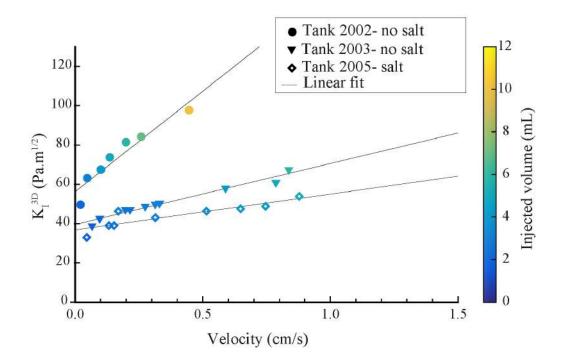


Figure 12: Stress intensity factor  $K_I^{3D}$  as a function of vertical velocity. 26 air injections for 3 different tanks (2002 and 2003 unsalted and 2005 salted) are represented. The stress intensity factor is estimated from the 3D theory (Eq. 14). For each tank, the limit of  $K_I$  when the vertical velocities tends to zero corresponds to the fracture toughness of the gelatin.

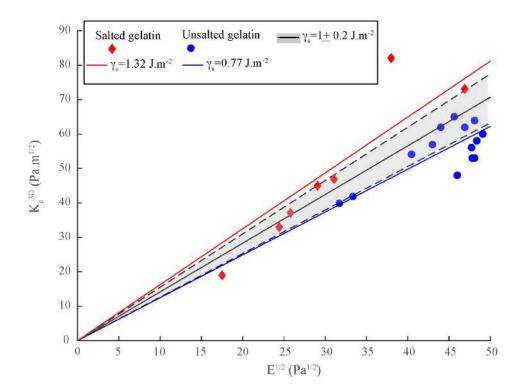


Figure 13: Relationship between the fracture toughness and the Young's modulus. Fracture toughness is estimated using the 3D approximation (Eq. 14). Linear fit, with uncertainties, corresponding to  $\gamma_s$  equal to  $1\pm 0.2 \text{ Jm}^{-2}$  [33, 16] is represented by the black line, whereas fits obtained from the salted and unsalted tanks are represented, respectively, by red and blue lines.

for the salted gelatin, which is significantly larger than the value obtained for
the unsalted one. Further experiments varying the salt concentration would
be useful to better characterize the influence of salt on the surface energy of
gelatin.

### 545 7. Discussion

### <sup>546</sup> 7.1. Comparison of the various methods for Young's modulus estimation

<sup>547</sup> Our numerical simulations pointed out that the most common method <sup>548</sup> used by experimentalists to infer Young's modulus is reliable (error <5 %) <sup>549</sup> only if the diameter of the load applied at the surface of the gelatin is 20 times <sup>550</sup> smaller than the minimum dimension of the tank. Otherwise, a numerical

model is required to link the vertical displacement to the Young's modulus of 551 gelatin taking into account the boundary effect of the rigid tank walls. Here, 552 we also provide a correction factor that can be applied to the value derived 553 from the analytical formula. Even considering the improvement brought by 554 the numerical model, this method still suffers some bias. The major bias is 555 that the measurement is done at the surface, which makes it difficult to reveal 556 potential heterogeneities or layering of the gelatin. The same limitation is 557 encountered when deriving a static value of the crustal Young's modulus from 558 surface loading or unloading events. The lateral extent of the load determines 550 the crustal depth over which the Young's modulus is effectively averaged. In 560 particular, one expects that loads applied on a broader area will probe a 561 thicker layer of the underlying medium. In order to characterize this effect, 562 we perform several numerical simulations in 2D axisymmetry using a FEM. 563 Numerical calculations are performed either to match the experiments (with 564 the Poisson's ratio set to 0.49 and a rigid load applied at the surface) or the 565 crustal Earth case (with the Poisson's ratio set to 0.25 and a uniform pressure 566 applied at the surface). The numerical box size is set to 2000 times the size 567 of the load applied at the surface to match the ideal case of a half-space. We 568 set the Young's modulus to be a linear function of depth at shallow level and 569 constant deeper: 570

$$E(z) = E_{surf} + \nabla E \times z \text{ for } z < Z_d$$
  

$$E(z) = E_{surf} + \nabla E \times Z_d \text{ for } z \ge Z_d$$
(15)

where  $E_{surf}$  is the value of the Young's modulus at the surface (in z=0),  $\nabla E$ 571 is its vertical gradient and  $Z_d$  is the depth at which the Young's modulus 572 becomes constant. The numerical simulation is used to calculate the value of 573 the surface displacement induced by the load. From this value, we can esti-574 mate the corresponding Young's modulus  $(E_{eq})$  for a homogeneous medium 575 (using Eq. 5 for the rigid load condition and Eq. 6 for the uniform pressure 576 condition). We then calculate an effective depth  $Z_{eq}$  corresponding to the 577 depth over which the actual Young's modulus inside the medium should be 578 averaged in order to obtain  $E_{eq}$ . It can be expressed by: 579

$$Z_{eq} = 2\frac{E_{eq} - E_{surf}}{\nabla E},\tag{16}$$

providing that  $Z_{eq}$  remains smaller than  $Z_d$ .  $Z_{eq}$  gives an estimate of the penetration depth reached by the surface load for Young's modulus measurements.

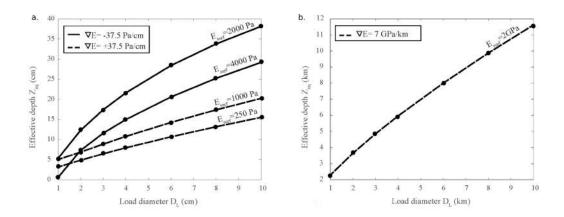


Figure 14: Effective depth over which the Young's modulus is probed as a function of the size of the load applied when there is a vertical gradient. Plain and dashed lines are for a decrease and an increase of the Young's modulus with depth, respectively. Different curves are obtained for various values of the Young's modulus at the surface. Circles are for the numerical simulations performed. a) Cases relevant for the gelatin tank. The depth  $Z_d$  at which the Young's modulus becomes constant is here set to 40 cm, the Poisson's ratio to 0.49 and a rigid load is applied at the surface. b) Case relevant for the Earth's crust. The depth  $Z_d$  at which the Young's modulus becomes constant is here set to 20 km, the Poisson's ratio to 0.25 and a uniform pressure load is applied at the surface.

Fig. 14 shows  $Z_{eq}$  as a function of the size of the load for different Young's 583 modulus profiles. Results are obtained with the depth  $Z_d$  set to 40 cm for 584 the experimental case (Fig 14a) and to 20 km for the crustal case (Fig 14b). 585 In all cases, as expected,  $Z_{eq}$  increases with the diameter of the load applied. 586 Consequently loads of various diameter might be used to evidence a vertical 587 gradient of the Young's modulus in a gelatin block. In case a load of signif-588 icant diameter is required, a numerical solution to interpret the subsidence 589 should then be used. Note that applying the load at various location of the 590 tank surface might also be useful to reveal potential lateral gradients. Once 591 again, the use of the numerical model might be necessary. 592

To infer the gelatin Young's modulus, the alternative method based on 593 the measurement of shear wave velocities as recently proposed by Pansino 594 and Taisne [39, 20] or the new method based on the measurement of the 595 length of a finite volume crack proposed in this study, enable to detect spatial 596 variations of the Young's modulus. They can additionally be used to quantify 597 the potential gradients, which is not possible with surface load measurements. 598 The accuracy of the shear wave velocities method strongly depends on the 599 absolute value of the Young's modulus. Higher rigidity, will produce faster 600

seismic waves, thus reducing the ability to follow a wave train with sufficient
resolution before any reflection occurred on the rigid walls. In contrast, the
method based on the air-filled crack propagation cannot be considered as
strictly non-destructive because once the crack has propagated through it,
the gelatin remains cut along the path followed by the crack.

Going back to the Earth's crust, seismic tomography has been used to 606 infer spatial variations of the Young's modulus at depth, usually showing 607 an increase of the rigidity with depth [54, 29, 30], whereas measurements 608 made by surface loading or unloading only provide a value averaged over 609 a given crustal thickness below the surface. Fig. 14b presents the depth 610 probed by surface loading as a function of the lateral extent of the load for 611 values corresponding to a typical crust. Note that usually when using surface 612 displacements induced by surface loading to infer the crustal rheology, the 613 load size is imposed by the natural phenomenon at play (e.g. lake level 614 change, ice thickness variations, etc.). Also, similarly to what we propose for 615 the gelatin, the length of magma intrusions could potentially be used to bring 616 insight into the crustal Young's modulus. However it might be difficult to 617 have a precise knowledge of the volume of magma involved. Besides, here we 618 derived the relationship between length and volume for a non viscous fluid, 619 which might be not fully appropriate in case of dynamic magma propagation. 620

### <sup>621</sup> 7.2. Critical length for crack propagation

The critical volume required to ensure liquid-filled fracture propagation is 622 key information both in the hydraulic fracturing domain and in volcanology. 623 Using a numerical model and analytical derivation, Davis et al. [48] and Sal-624 imzadeh et al. [55] provided an expression for the critical volume for vertical 625 propagation of a buoyant crack in three dimensions. They underlined that 626 previous estimations for critical "volumes" were only given in terms of crit-627 ical fracture length and based on analyses performed in 2-D. In particular, 628 following Secor and Pollard [49] and using the fact that the stress intensity 629 factor is equal to the fracture toughness at the upper tip and zero at the 630 lower tip of the crack, the critical half-length is given by: 631

$$a_c = \left(\frac{K_c}{\Delta \rho g \sqrt{\pi}}\right)^{2/3},\tag{17}$$

Using again the approximation r = (3/4)a and the expression for the stress intensity factor in 3D proposed by Heimpel and Olson [41], we end up with an expression for the critical length in 3D:

$$a_c = \left(\frac{\sqrt{\pi}}{2\sqrt{3}} \frac{K_c}{\Delta\rho g}\right)^{2/3},\tag{18}$$

Using Eq. 18 and Eq. 11, we obtain an expression for the critical volume and can express it the same way used by Davis et al. [48]. Using  $\alpha$  equal to 0.25 in Eq. 11, we have:

$$V_c = \alpha \frac{2^{13/3}}{9} \frac{1 - \nu}{16\mu} \left(\frac{9\pi^4 K_c^8}{\Delta \rho^5 g^5}\right)^{1/3},\tag{19}$$

$$V_c \approx 0.56 \frac{1 - \nu}{16\mu} \left(\frac{9\pi^4 K_c^8}{\Delta \rho^5 g^5}\right)^{1/3},$$
(20)

with  $\mu$  the shear modulus. Eq. 20 is very close to the expression numerically derived by Davis et al. [48], who gave a coefficient of 0.75 instead of 0.56. This expression of the critical volume can be useful to interpret the volume of magmatic dikes keeping in mind that it was derived neglecting potential viscous effects.

### <sup>643</sup> 7.3. Young's modulus decrease and surface energy increase in presence of salt

We used the independent estimation of Young's modulus and fracture 644 toughness to estimate the surface energy of the gelatin. We evidenced that 645 the surface energy is increased by addition of salt. It follows that for the same 646 value of the Young's modulus, a salted gelatin will have a higher fracture 647 toughness. This is consistent with the roughly six times higher velocity 648 measured in unsalted gelatin, for a similar injected volume and a similar 649 Young's modulus (Fig. 8). In the same way the critical volume for crack 650 propagation is larger for the salted gelatin than for the unsalted one. It is thus 651 important to take into account this influence of the salt on the surface energy 652 of the gelatin when using salted gelatin as a crustal analog. In particular, to 653 enable the injection of viscous fluids like vegetable or silicon oils, the use of 654 salted gelatin might be required in order to guarantee a sufficient buoyancy. 655 In this case, the fracture toughness cannot be simply derived using the surface 656 energy for unsalted gelatin. 657

### 658 8. Conclusion

We illustrated the added value of using numerical simulations to improve 659 the interpretation of analog experiments. In particular, we quantified the 660 errors associated with the use of the analytical formula corresponding to an 661 elastic half-space [45] for a finite medium. An overestimation of 5 % is ex-662 pected when using this analytical formula to derive the Young's modulus if 663 the tank size is not 20 times larger than the load diameter when a standard ex-664 perimental setup is used with gelatin adhering to the tank walls. We showed 665 that using a 3D numerical model removes the constraint of only applying sur-666 face loads of limited diameters to derive the Young's modulus. This enables, 667 for instance, to check for potential heterogeneous elastic properties inside a 668 gelatin tank. Two others methods are suitable to quantify the Young's mod-669 ulus and can alternatively reveal its variations inside a tank. One consists 670 of measuring shear wave velocities, which is fully non-destructive. The other 671 is based on the calibration performed in this study and requires the quan-672 tification of the length of cracks filled with a known volume of a non-viscous 673 buoyant fluid, which can be done with limited alteration of the gelatin. In 674 addition we highlighted the influence of salt on gelatin physical properties. 675 While salt was known to decrease the Young's modulus value of the gelatin. 676 we showed that it also increases its surface energy. More generally, the infor-677 mation provided by numerical models regarding the depth probed by surface 678 loading, might prove to be useful when interpreting Young's modulus values 679 for the Earth's crust derived by static loading/unloading events. Based on 680 our numerical model results in the case of a linear increase of the crustal 681 Young's modulus with depth, we confirm that the lateral size of the surface 682 loading (or unloading) considered should be the same order of magnitude as 683 the crustal thickness to be probed. 684

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