

Provision of reactive power services by energy communities in MV distribution networks

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ARTICLE INFO

Article history:

Received 28 September 2022
Received in revised form 28 March 2023
Accepted 30 March 2023
Available online 3 April 2023

Dataset link: <https://doi.org/10.17632/47rnmx7hkn.1>

Keywords:

Active distribution network
Ancillary services
Local energy community
Energy management system
Power factor compensation
Reactive power services

ABSTRACT

The paper presents a procedure for the optimal operation of a community of prosumers connected to a medium voltage distribution network equipped with generation and storage units that considers the penalization for low power factor operation, the exploitation of direct exchanges of both active and reactive power between the prosumers and the provision of reactive power services by the community to the local distribution system operator and the transmission system operator. The proposed procedure calculates the maximum and minimum reactive power deviations that each community participant can provide with respect to the reference profile. A deterministic day-ahead scheduling problem is considered assuming the forecast of load and photovoltaic production known without uncertainties. The formulation of the optimization problems and the solution computational requirements are suitable for the inclusion in a stochastic approach. The effectiveness of the approach is supported by numerical simulations of the daily scheduling for different test cases.

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1. Introduction

1.1. Motivations and literature review

Electricity distribution systems are facing innovations and new operation methods are being introduced with the aim of increasing the production from renewable energy resources, to reduce the climate change impact and promote a sustainable development. New regulatory frameworks have been set up to increase final user participation in the electricity market in several parts of the world. The implementation of energy communities and direct electricity trading exchange schemes between neighbors has become a new opportunity in Europe and elsewhere. These communities and the relevant direct energy transactions take fundamental advantage from the deployment of advance metering infrastructures, storage units to compensate the mismatches between consumption and the production by renewable resources, and the adoption of specific optimization algorithms. The regulatory framework on energy communities is in evolution to include different perspectives of the transition to a low carbon society other than the technical aspects of power system operation, such as environmental issues, eradicating energy poverty, sustainable development (e.g., [1] and references therein). Moreover, energy communities are expected to provide services (such as active and reactive power balancing) to the distribution and transmission networks to which they are connected.

Significant examples of these new regulatory frameworks are the EU Directive on common rules for the internal electricity market (2019/944/EU) and the revised Renewable energy directive (2018/2001/EU) that increase the role of renewables self-consumers and renewable energy communities [2].

Emerging regulations are fostering the participation of final users, single or aggregated collectives, in both the energy market and the ancillary services markets. As an example, the Italian Regulatory Authority for Energy, Networks and Environment (ARERA) has issued a call for projects for the provisions of local ancillary services (resolution 352/2021/R/eel), i.e. those useful for the operation of distribution networks, that completes a previous resolution (300/2017/R/eel) relevant to global ancillary services, i.e. those acquired by the transmission system operator, in the framework of the electricity market regulation (322/2019/R/eel).

These services for the distribution network operation are expected to be provided by microgrids and energy communities, with the use of qualified generating and storage units, reactive power compensation devices, and the implementation of demand response techniques. Another case will be represented by communities owning or operating the part of the public network to which the participants are connected (as foreseen, for example, by the ARERA resolution 120/2022/R/eel), therefore acting as distribution system operator (DSO) providing services to all the connected users and to the transmission system operator (TSO).

This paper focuses on provision of reactive power compensation services by a local energy community of prosumers connected to the same medium voltage (MV) distribution network.

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There is a growing literature on this specific topic both for separately managed final users or prosumers and for communities. In these studies, reactive and active compliant and non-compliant zones of operation are often considered, including penalties for non-compliant absorption of reactive power. Indeed, when a photovoltaic (PV) system, installed in a final user site, is operated at unitary power factor, it decreases the local active power demand with a corresponding worsening of the power factor of the site. The presence of penalties for non-compliant absorption of reactive power is indeed an issue. There are several studies relevant to the reactive power control of distributed generators, with specific reference to PV systems, e.g. [3–6] that consider also the relationship with active power curtailments and transformer control.

Among the auxiliary services for distribution network operation, the provision of reactive power flexibility is one of the most important as it can be used to obtain improved voltage profiles, avoiding or postponing the need of expensive voltage control devices by the DSO (such as static var compensators and voltage regulators).

The changes of the user reactive power injections or absorptions need to be coordinated with the control of the transformers equipped with on-load tap changers (OLTC) [7]. Optimization approaches have been developed for the reactive power management in grids with renewables [8], which need to consider both P-Q inverter capability curves and compliant regions (that may also involve the voltage value at the connection bus) defined in various countries [9–11].

The preliminary calculation of the maximum deviations with respect to a reference value [12,13] are useful for DSO and TSO decisions relevant to the provision of ancillary services. For the specific case of communities equipped with energy storage systems and aggregated distributed energy resources, [14] focuses on active load flexibilities and storage capacity sharing and [15] focuses on voltages ancillary services with the use of phasor measurement units for the coordination of the different control means.

1.2. Contributions and paper organization

This paper presents an optimization model for the scheduling of the community resources that considers both active and reactive power direct exchanges among the community participants. Active power exchanges allow to reduce the energy procurement costs of the community with respect to the case in which the users can only transact with an external energy provider; reactive power exchanges are aimed at reducing the noncompliance penalties associated with low power factor operation.

A day-ahead scheduling problem of a community connected to a MV distribution network is considered. It is assumed that network users are members of the same community and have a common provider (identified for simplicity with the utility). The transactions among different users of the low voltage network connected to the same node of the MV network are aggregated without effects on the results. Daily profiles of the price of the energy bought from the utility, price profiles recognized for the energy sold to the utility, and penalizations for energy exchanges with low power factor are predefined. Direct exchanges of both active and reactive power are allowed among the community participants. The proposed optimization procedure of the community calculates the scheduling of these exchanges through the MV network and the prices of the transactions. The objective of the procedure minimizes the energy procurement costs of the community for the next day together with the penalizations for low power factor operation. Moreover, specific optimization procedures are used to assess the maximum up and down reactive

power variations that can be offered as a flexibility service by the community to the utility for the following day, being the flexibility reward tariffs predefined. The utility can use the reactive power flexibility service offered by the community for the online reactive power/voltage control in the network during the day. As these optimization procedures calculate the maximum reactive power increase and decrease at each node and at the substation that connects the distribution network with the transmission network, these flexibility capabilities can be exploited for both the distribution operation (by the DSO) and for the transmission system at the DSO/TSO interface. In any case, the community is rewarded by the variation of the reactive power at the connection points of the participants.

The developed model considers voltage control devices, such as transformers equipped with OLTC, capacitor banks or static var compensators, as well as the reactive power injection or absorption by distributed generators and storage units.

The structure of the paper is the following. Section 2 presents an overview of the procedures. Section 3 presents the details of the optimization model for the day-ahead scheduling of the energy community resources and the calculation of the reference reactive power profiles. Section 4 deals with the calculation of the maximum deviations of reactive power exchanges for each community member with respect to the reference profiles. Section 5 describes the test cases and presents the results. Finally, Section 6 concludes the paper.

2. Structure of the optimization procedures

In this paper we consider a deterministic day-ahead scheduling problem, i.e., the optimization of the energy resources and control means for the 24 h of the next day with 15 min resolution, assuming the forecast of load and photovoltaic production known without uncertainties. The optimization models can be adopted in scenario-based stochastic approaches and intraday rolling-horizon procedure (as described in e.g., [16]) able to cope with uncertainties. The formulation of the optimization problems and the solution computational requirements are suitable for the inclusion in a stochastic approach, although this is beyond the scope of the paper.

The considered scheme includes the following steps.

1. Reference optimization: calculation of the scheduling of both active and reactive power resources that minimize the energy procurement costs of the entire community assuming known tariffs for the active power exchanges with the external provider and considering the penalties for the participants that operate with power factor lower than a predefined limit;
2. Qdown optimization: calculation of the maximum decrease of reactive power absorption or maximum increase of reactive power injection at the terminals of each community participant that minimizes the energy procurement costs considering the revenues (with predefined €/kvarh price) from the provision of a reactive power flexibility service consisting in the decrease with respect to the reactive power profile (assumed positive when power is absorbed) calculated in the reference step, according to DSO/TSO requests;
3. Qup optimization: calculation of the maximum increase of reactive power absorption or maximum decrease of reactive power injection at the terminals of each community participant that minimizes the energy procurement costs considering the revenues from the reactive power increase with respect to the reference profile.

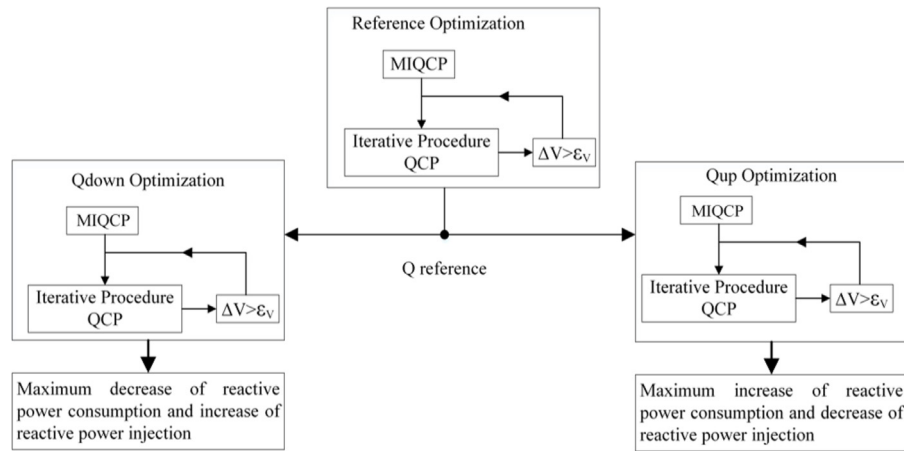


Fig. 1. Scheme of the procedure.

Qdown and Qup optimizations are considered independent due to the lack of constraints relevant to reactive power compensation decisions taken in different times.

Following the typical local energy community scheme, participants are allowed to provide active power to other participants. The role of provider and consumer can vary at each period according to the generation and load levels inside each participant. The proposed optimization model provides the fair price for each transaction as the value of the shadows price of equilibrium constraints.

In the reference optimization, also reactive power exchanges among the participants in the community can be allowed, i.e., a participant that absorbs too much reactive power with the respect to active one (as it operates at low power factor), can reduce the penalty by the help of the reactive power injections of other participants. The model is conceived so that also these internal reactive power exchanges among the community participants are balanced to avoid excessive reactive power exchanges with the grid, both positive and negative. Section 5 will show the results obtained with and without reactive power compensations between community participants for the considered test cases.

The optimization models consider the typical operating constraints of the distribution network: maximum bus voltage deviations with respect to the reference value, maximum current limits in the branches, limitation in the OLTC of transformers and voltage regulators, maximum reactive power of variable capacitor banks.

In Qdown and Qup optimizations, the voltage control and reactive power compensation means of the distribution network (i.e., OLTCs and variable capacitor banks) can be operated to maximize the rewards (this can describe the case of an energy community that also acts as the operator for the relevant part of the distribution system) or can be operated to maintain the voltage profile close to the rated value (as in the typical case of a separate DSO from the community). Section 5 compares the results obtained for the two different ways of OLTCs and capacitor banks operation.

For the power flow representation, we have chosen the convex relaxation approach described in e.g., [17,18], based on the DistFlow method [19], considering an equivalent single-phase representation of the three-phase network assumed as balanced.

Although some downsides (analyzed in e.g., [20,21]) and the need of a careful model formulation to guarantee that the solution will meet the equality of the relaxed constraints, this approach appears suitable for a first implementation and comparison of novel objectives and strategies, to explore the new scenario characterized by the presence of communities, direct

energy transaction among prosumers, and their participation to local and global ancillary service markets.

The relationship between load modeling and volt-var optimization can be significant, as shown in e.g., [22] and references therein. Since the study is focused on reactive power provision services, the proposed model includes the voltage dependence of active and reactive power loads represented by the ZIP model (i.e., combination of constant impedance, constant current, and constant power loads), other than transformers equipped with OLTCs, capacitor banks, and the representation of the charging current of the branch lines (i.e., the line shunt capacitance).

The linear representation of the ZIP model presented in [23] has been suitably adapted to be included in the DistFlow method. Moreover, to increase the accuracy of the solution and to avoid any link between loads and branch currents, the optimization is included in an iterative procedure in which, at the first calculation, the ZIP models are evaluated at the voltage of the secondary side of the feeding OLTC transformers, and in the following iterations, loads are represented as constant power calculated at the bus voltage value of the previous iteration. The optimization model of the following iterations is simpler than that of the first iteration, as described in Section 3.9, so to significantly speed up the calculation. The iterative procedure stops when the maximum difference between the voltage values in consecutive iterations is smaller than a predefined tolerance.

The structure of the entire procedure is illustrated in Fig. 1. The figure indicates the type of the implemented optimization problems: MIQCP refers to mixed-integer quadratically constrained programming and QCP refers to quadratically constrained programming (without binary variables).

The next two sections describe the optimization model adopted in this paper for the scheduling of the community resources together with the calculation of the reference reactive power profiles and the models for the calculation of the maximum reactive power deviations, respectively.

3. Reference optimization model of the distribution network with the presence of a local energy community

The T equivalent circuit is adopted as line model, which may be advantageous with respect to the Π line model, adopted in other DistFlow-based optimization models as in, e.g. [24], for the easier calculation of the currents at the line ends.

The network is assumed to be radial. The number of buses (excluding the connection to the transmission network) and branches is the same. The set of the bus and branches is denoted by Ω . For simplicity, a reference direction is assumed for the

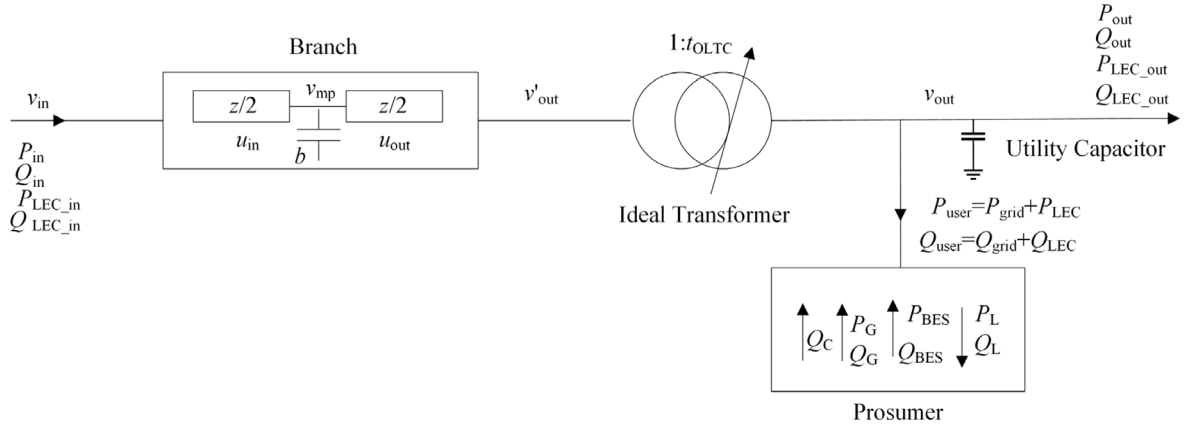


Fig. 2. Scheme of the model of a generic branch with the connection point of a user.

power flow along the lines from the external grid to the terminal buses. Each branch is denoted by the index i of the sending bus. Each time interval Δt of the considered optimization horizon T is equal to 15 min. According to the DistFlow method, each generic branch of the network is represented by the line model connected to the input terminal or receiving bus, a transformer, a load and a shunt capacitance connected to the output terminal or receiving bus, as shown in Fig. 2. v_{in} , v_{out} denote the squared rms values of the voltages at the receiving and sending bus, respectively, while v_{mp} and v'_{out} refer to the internal and the sending bus of the T-model. u_{in} , u_{out} are the squared rms values of the line currents in the two terminals of the T-model. $z = r + jx$ is the line series impedance and jb is the line shunt admittance.

The transformer can be present or not. If present, a transformer ratio t_{OLTC} different than 1 is considered while the short circuit impedance and the magnetizing inductance are modeled by using the T equivalent circuit.

The community participants can transact with the external energy provider and among themselves, at prices taken equal to the marginal costs calculated as shadow prices of specific equality constraints (described in Section 3.7), under the assumption that the participants of the community are not in competition. Extending the approach presented in [25], the exchanges between participant i and any other participant in time t are represented by variables $P_{LEC\ i,t}$, $Q_{LEC\ i,t}$, for active and reactive power, respectively. Analogously, the exchanges with the external provider, which for simplicity we identify with the utility, are described by variables $P_{grid\ i,t}$, $Q_{grid\ i,t}$. The local active and reactive power $P_{user\ i,t}$, $Q_{user\ i,t}$, measured by the meter at the participant connection, should be equal to the sum of the transaction with the grid with the community. For each participant, the signs of the two exchanges with the grid and with the other participants are constrained to be the same, to avoid reselling. Variables $P_{LEC\ in\ i,t}$, $Q_{LEC\ in\ i,t}$ and $P_{LEC\ out\ i,t}$, $Q_{LEC\ out\ i,t}$ allow to represent the power flows associated with power generated and consumed in the same t inside the community.

At its point of connection, each community participant absorbs active and reactive power (nonnegative $P_L\ i,t$, $Q_L\ i,t$), injects active and reactive power by local generator ($P_G\ i,t$, $Q_G\ i,t$), BES unit ($P_{BES\ i,t}$, $Q_{BES\ i,t}$), and capacitor bank (nonnegative $Q_C\ i,t$).

3.1. Objective function

The considered objective minimizes the objective function OF that includes for both total community costs/revenues due to the transactions (positive or negative) with the utility grid $C_{grid\ i,t}$, the penalization (with price μ_{PF}) for the power factor noncompliance $Q_{PF\ i,t}$, with the predefined power factor limit pf_{min} for simplicity

considered the same for all the users, the penalization of the joule power loss in the branches $\ell_{i,t}$ and of the losses of battery charging and discharging processes $\ell_{BES\ i,t}$:

$$OF = \sum_{i \in \Omega} \sum_{t \in T} \left(C_{grid\ i,t} + \mu_{PF} Q_{PF\ i,t} + \mu_{loss\ i,t} \ell_{i,t} + \mu_{BES\ i,t} \ell_{BES\ i,t} + \mu_{LEC} \hat{P}_{LEC\ i,t} \right) \Delta t \quad (1)$$

where μ_{PF} , μ_{loss} , μ_{BES} , μ_{LEC} are the penalization coefficients of the noncompliance of the minimum power factor, of the branch power loss, of charge and discharge BES losses, of the internal power exchanges to avoid the reselling of the power bought from the grid to the other participants and vice versa. The value of μ_{PF} is assumed known, i.e. fixed by the regulatory authority or the utility. The values of μ_{loss} , μ_{BES} and μ_{LEC} are selected quite small so that their contribution to the objective function is negligible with respect to $C_{grid} + \mu_{PF} Q_{PF}$. In case the obtained solution is not feasible, the optimization is automatically repeated with two additional penalization terms that become null when a feasible solution is obtained, as explained in detail in Section 3.8.

As the number and characteristics of the installed components are fixed in the considered day-ahead scheduling problem, as well as the community composition, only the costs that depend on the decision variables (i.e., the active and reactive power outputs of the controllable energy resources that are already available in the system, the transactions among the community participants, and the OLTC positions) are included.

The objective function does not include generation costs since we assume here that all the local generation is provided by PV systems. If generation costs vary with production, they affect the optimal prices of the transactions among the community participants as shown in [26] where the presence of biogas units is considered.

As the price $\pi_{buy,t}$ for buying energy from the utility is higher than the price $\pi_{sell,t}$ recognized when the community participants sell energy to the utility at each time t , the feasible region of cost $C_{grid\ i,t}$ is defined by the minimization of following convex epigraph

$$C_{grid\ i,t} \geq \pi_{buy,t} P_{grid\ i,t} \quad \text{and} \quad C_{grid\ i,t} \geq \pi_{sell,t} P_{grid\ i,t} \quad \pi_{buy,t} \geq \pi_{sell,t} \quad (2)$$

where $P_{grid\ i,t}$ is positive when the participant buys from the utility.

The area that complies with pf_{min} is illustrated in Fig. 3, where $\hat{Q}_{i,t} = Q_G\ i,t + Q_C\ i,t + Q_{BES\ i,t} + Q_{LEC\ i,t}$ is the sum of reactive power decision variables, i.e. controllable reactive power resources ($Q_G\ i,t$, $Q_C\ i,t$, $Q_{BES\ i,t}$) and reactive power exchanges

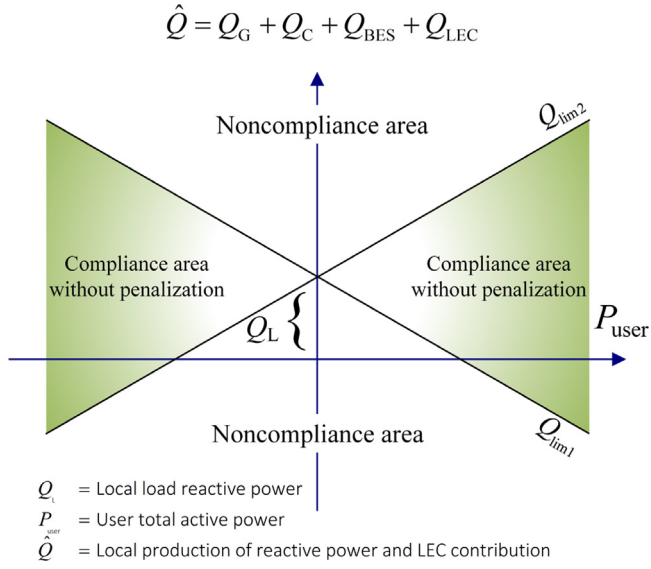


Fig. 3. Operating region that complies with the minimum power factor.

among the users $Q_{LEC\ i,t}$. According to (1), when the operating point is outside the compliance area, a penalty is applied proportional to the noncompliance amount $Q_{PF\ i,t}$. In some regulatory framework, for small active power consumption or production a fixed reactive power exchange is allowed without penalization, as illustrated in [7,11] with reference to Swiss and Belgian regulation. For simplicity, these peculiarities are not included in the implementation of the model presented in this paper, as customer penalizations for bus voltage violations.

Q_G , Q_{BES} are constrained by the minimum power factor of the local generator and BES. Q_C is fixed or limited by the maximum output of the switchable capacitor bank (for simplicity, discrete switching is not represented).

The noncompliance $Q_{PF\ i,t}$ is a nonnegative variable under the assumption that there is no reward for operating inside the compliance area. As the compliance area shown in Fig. 3 is nonconvex, the representation of $Q_{PF\ i,t}$ includes a condition on the sign of $P_{user\ i,t}$ (dealt with the inclusion of binary variables):

$$Q_{PF\ i,t} \geq \begin{cases} Q_{lim\ 1\ i,t} - \hat{Q}_{i,t} & \text{if } P_{user\ i,t} \geq 0 \\ -Q_{lim\ 2\ i,t} + \hat{Q}_{i,t} & \text{if } P_{user\ i,t} < 0 \end{cases} \quad \text{or} \quad \begin{cases} -Q_{lim\ 1\ i,t} + \hat{Q}_{i,t} & \text{if } P_{user\ i,t} < 0 \\ Q_{lim\ 2\ i,t} - \hat{Q}_{i,t} & \text{if } P_{user\ i,t} \geq 0 \end{cases} \quad (3)$$

where

$$Q_{lim\ 1\ i,t} = Q_{L\ i,t} - \tan[\arccos(pf_{min})]P_{user\ i,t} \quad (4)$$

$$Q_{lim\ 2\ i,t} = Q_{L\ i,t} + \tan[\arccos(pf_{min})]P_{user\ i,t}$$

Joule power loss $\ell_{i,t}$ in each branch is

$$\ell_{i,t} = 0.5r_i u_{in\ i,t} + 0.5r_i u_{out\ i,t} \quad (5)$$

Power losses associated with BES discharging and charging (corresponding to P_{BES} positive and negative, respectively) are

$$\ell_{BES\ i,t} \geq \begin{cases} (1 - \eta_{discharge})P_{BES\ i,t} \\ (1 - 1/\eta_{charge})P_{BES\ i,t} \end{cases} \quad (6)$$

where $\eta_{discharge}$ and η_{charge} are efficiency factors lower than one. P_{BES} is constrained by the maximum power limit of the battery.

Nonnegative variable \hat{P}_{LEC} is defined by

$$\hat{P}_{LEC\ i,t} \geq \begin{cases} P_{LEC\ i,t} \\ -P_{LEC\ i,t} \end{cases}, \quad (7)$$

3.2. Coupling constraints

Values of $v_{out\ i,t}$, $P_{out\ i,t}$, $Q_{out\ i,t}$, $P_{LEC_out\ i,t}$, $Q_{LEC_out\ i,t}$ should be equal to the values of $v_{in\ i+1,t}$, $P_{in\ i+1,t}$, $Q_{in\ i+1,t}$, $P_{LEC_in\ i+1,t}$, $Q_{LEC_in\ i+1,t}$ considering i as the upstream branch and $i+1$ the downstream one. Generalizing to the case of multiple branches terminating and originating from the same bus:

$$v_{out\ i \in \Omega_j^f,t} = v_{in\ i \in \Omega_j^s,t} = v_{j,t} \quad (8)$$

$$\sum_{i \in \Omega_j^f} P_{out\ i,t} = \sum_{i \in \Omega_j^s} P_{in\ i,t} \quad (9)$$

$$\sum_{i \in \Omega_j^f} P_{LEC_out\ i,t} = \sum_{i \in \Omega_j^s} P_{LEC_in\ i,t} \quad (10)$$

$$\sum_{i \in \Omega_j^f} Q_{out\ i,t} = \sum_{i \in \Omega_j^s} Q_{in\ i,t} \quad (11)$$

$$\sum_{i \in \Omega_j^f} Q_{LEC_out\ i,t} = \sum_{i \in \Omega_j^s} Q_{LEC_in\ i,t} \quad (12)$$

where $v_{j,t}$ is the squared voltage of bus j and Ω_j^f , Ω_j^s denote the sets of branches connected to bus j as the sending and receiving end, respectively. The squared voltage V_0^2 at the connection point to the transmission network (slack bus 0) is assumed to be known and, for simplicity, is here assumed constant during the day.

Transactions between the participants of the community do not cause any power flow exchange with the utility, i.e.,

$$\sum_{i \in \Omega_0} P_{LEC_in\ i,t} = 0 \quad (13)$$

$$\sum_{i \in \Omega_0} Q_{LEC_in\ i,t} = 0 \quad (14)$$

3.3. Branch constraints

According to the DistFlow method, for each branch i and time interval t , the relationships between the voltages at the terminals and the power flows are given by the following relationships¹:

$$\begin{aligned} v_{mp\ i,t} &= v_{in\ i,t} - r_i P_{in\ i,t} - x_i Q_{in\ i,t} + 0.25(r_i^2 + x_i^2)u_{in\ i,t} \\ v_{in\ i,t} - v'_{out\ i,t} &= 2r_i P_{in\ i,t} + 2x_i Q_{in\ i,t} - 0.75(r_i^2 + x_i^2)u_{in\ i,t} \\ &\quad - 0.25(r_i^2 + x_i^2)u_{out\ i,t} + x_i b_i v_{mp\ i,t} \end{aligned} \quad (15)$$

where

$$\begin{aligned} P_{in\ i,t} &= P'_{out\ i,t} + 0.5r_i u_{in\ i,t} + 0.5r_i u_{out\ i,t} \\ Q_{in\ i,t} &= Q'_{out\ i,t} - b_i v_{mp\ i,t} + 0.5x_i u_{in\ i,t} + 0.5x_i u_{out\ i,t} \end{aligned} \quad (16)$$

$$P'_{out\ i,t} = P_{out\ i,t} + P_{user\ i,t}$$

$$Q'_{out\ i,t} = Q_{out\ i,t} + Q_{user\ i,t} - Q_{cap\ i,t}$$

being $Q_{cap\ i,t}$ the reactive power injection of the utility capacitor bank connected at bus i (as shown in Fig. 2) if present.

Nonnegative variable $u_{in\ i,t}$, $u_{out\ i,t}$ are constrained to be lower than the square of the maximum branch current limit ($I_{max\ i}^2$) and

¹ As complex power is equal to voltage and conjugate current product, the sum of the squares of the real and imaginary parts gives $v_{in\ i,t} - v_{mp\ i,t} = r_i P_{in\ i,t} + x_i Q_{in\ i,t} - 0.25(r_i^2 + x_i^2)u_{in\ i,t}$ at node in and $v_{mp\ i,t} - v'_{out\ i,t} = r_i P_{mp\ i,t} + x_i Q_{mp\ i,t} - 0.25(r_i^2 + x_i^2)u_{out\ i,t}$ at node mp of Fig. 2, where $P_{mp\ i,t} = P_{in\ i,t} - 0.5r_i u_{in\ i,t}$ and $Q_{mp\ i,t} = Q_{in\ i,t} + b_i v_{mp\ i,t} - 0.5x_i u_{in\ i,t}$.

nonnegative variables $v_{in\ i,t}$, $v_{out\ i,t}$ are constrained between the square of the minimum and maximum bus voltage limits ($V_{\min i}^2$, $V_{\max i}^2$).

3.4. Cone constraints

As usually done to represent the DistFlow model as a quadratically constrained problem, the apparent power equalities are relaxed as

$$\begin{aligned} P_{in\ i,t}^2 + Q_{in\ i,t}^2 &\leq v_{in\ i,t} u_{in\ i,t} \\ P_{mp\ i,t}^2 + Q_{mp\ i,t}^2 &\leq v_{mp\ i,t} u_{in\ i,t} \\ P_{mp\ i,t}^2 + Q_{mp\ i,t}^2 &\leq v_{mp\ i,t} u_{out\ i,t} \\ P_{out\ i,t}^2 + Q_{out\ i,t}^2 &\leq v'_{out\ i,t} u_{out\ i,t} \end{aligned} \quad (17)$$

where

$$\begin{aligned} P_{mp\ i,t} &= P'_{out\ i,t} + 0.5r_i u_{out\ i,t} \\ Q_{mp\ i,t} &= Q'_{out\ i,t} + 0.5x_i u_{out\ i,t} \\ Q'_{mp\ i,t} &= Q_{mp\ i,t} - b_i v_{mp\ i,t} \end{aligned} \quad (18)$$

In a feasible solution, all (17) should be verified as equalities, as well as one of (6). A specific check is automatically performed in the procedure and if the mismatch is greater than a predefined small tolerance the optimization is repeated as described in Section 3.8.

3.5. OLTC and capacitor constraints

For the branches relevant to OLTC transformers, the constraints are

$$t_{\min}^2 v'_{out\ i,t} \leq v_{out\ i,t} \leq t_{\max}^2 v'_{out\ i,t} \quad (19)$$

where t_{\max} and t_{\min} are the upper and lower bounds of t_{OLTC} (for the branches that describes a line $t_{\max} = t_{\min} = 1$). This formulation, for simplicity, does not explicitly represent discrete steps. A refined result that considers the discrete steps is obtained by the iterative optimization procedure described in Section 3.9.

We consider two ways to operate the OLTC:

- the OLTC ratio is optimized, together with the other decision variables, to minimize the objective function or
- the OLTC tap is chosen to control the voltage by two linearized constraints (by neglecting $\Delta V_{i,t}^2$ in the first constraint and assuming v_{out} close to 1 pu in the second one)

$$\begin{aligned} v_{out\ i,t} &= V_r^2 + 2V_{ri} \Delta V_{i,t} \\ \Delta V_{i,t} &= s_R P'_{out\ i,t} + s_X Q'_{out\ i,t} \end{aligned} \quad (20)$$

where V_r is the rated voltage at the secondary side of the transformer, s_R and s_X are positive parameters that represent the regulator compensation settings. To consider t_{\max} and t_{\min} , (20) is conditioned by (19) with the inclusion of three binary variables (each corresponding to $v_{out\ i,t} = t_{\max}^2 v'_{out\ i,t}$, $v_{out\ i,t} = t_{\min}^2 v'_{out\ i,t}$, and $v_{out\ i,t}$ between $t_{\min}^2 v'_{out\ i,t}$ and $t_{\max}^2 v'_{out\ i,t}$) whose sum must be 1.

In (a) the community also operates as a DSO for the network to which the participants are connected, in (b) DSO and community are separated so the DSO operates the transformers to control the voltage near to the rated value.

Analogously, also the variable capacitor bank can be considered belonging to the community participant connected to the same bus or to the utility network. In both cases, the nonnegative variable of reactive power injection $Q_{Ci,t}$ is limited by maximum

value $Q_{C\max i}$. If the capacitor bank belongs to the community participant, the capacitor reactive power injection is included in the evaluation of Q_{user} as shown in Fig. 2.

3.6. User plant constraints

The net power for each user is given by:

$$\begin{aligned} P_{user\ i,t} &= P_{L\ i,t} - P_{Gi,t} - P_{BES\ i,t} + \ell_{BES\ i,t} \\ Q_{user\ i,t} &= Q_{L\ i,t} - Q_{Gi,t} - Q_{BES\ i,t} - Q_{Ci,t} \end{aligned} \quad (21)$$

The adopted simple model of the BES unit is represented by:

$$\begin{aligned} E_{i,t} &= E_{i,t-1} - P_{BES\ i,t} \Delta t \quad \text{for } 1 < t < 96 \\ E_{i,1} &= E_{\max i} - P_{BES\ i,t} \Delta t \quad \text{and} \quad E_{i,96} = E_{\max i} \end{aligned} \quad (22)$$

where $E_{i,t}$ is the energy content constrained by the minimum and maximum energy levels $E_{\min i}$, $E_{\max i}$. In the numerical tests, $E_{i,t}$ is assumed equal to $E_{\max i}$ at beginning and the end of the optimization horizon ($t = 1$ and $t = 96$, respectively).

The linearized ZIP model of the load (written in pu) is

$$\begin{aligned} P_{L\ i,t} &= P_Z\ i,t v_{out\ i,t} + (P_{i\ i,t} + \Delta P_{i\ i,t}) + P_{Pi,t} \\ Q_{L\ i,t} &= Q_Z\ i,t v_{out\ i,t} + (Q_{i\ i,t} + \Delta Q_{i\ i,t}) + Q_{Pi,t} \end{aligned} \quad (23)$$

where P_Z and Q_Z represent the consumption at the rated voltage of the constant impedance component, P_i and Q_i represent the consumption at the rated voltage of the constant current component, P_p and Q_p represent the consumption of the constant power component, and $\Delta P_{i\ i,t}$, $\Delta Q_{i\ i,t}$ (different from zero only when $P_{i\ i,t}$ and $Q_{i\ i,t}$ are not null) represent the linearized voltage dependence of the constant current component consumption²:

$$\begin{aligned} 2(\Delta P_{i\ i,t} P_{i\ i,t} + \Delta Q_{i\ i,t} Q_{i\ i,t}) &= (v_{out\ i,t} - 1)(P_{i\ i,t}^2 + Q_{i\ i,t}^2) \\ Q_{i\ i,t} \Delta Q_{i\ i,t} - P_{i\ i,t} \Delta P_{i\ i,t} &= 0 \end{aligned} \quad (24)$$

3.7. Active and reactive power exchanges among the community participants

As mentioned, each community participant can exchange active and reactive power with other participants ($P_{LEC\ i,t}$, $Q_{LEC\ i,t}$) and with the utility ($P_{grid\ i,t}$, $Q_{grid\ i,t}$). The balance is

$$P_{user\ i,t} = P_{grid\ i,t} + P_{LEC\ i,t} \quad (25)$$

$$\begin{aligned} Q_{user\ i,t} &= Q_{grid\ i,t} + Q_{LEC\ i,t} \quad \text{if } Q_{LEC\ i,t} \geq 0 \\ Q_{user\ i,t} &= Q_{LEC\ i,t} \quad \text{if } Q_{LEC\ i,t} < 0 \end{aligned} \quad (26)$$

The representation of Q_{LEC} exchanges includes a condition on the sign of $Q_{LEC\ i,t}$ (dealt with the inclusion of binary variables), in order to avoid that one participant may absorb reactive power from the utility to provide Q_{LEC} to other participants who need it to reduce the noncompliance penalty Q_{pf} .

Since it is in general avoided to inject reactive power to the grid, if not requested, $Q_{grid\ i,t}$ is constrained by

$$Q_{grid\ i,t} \geq 0 \quad (27)$$

The $P_{LEC\ i,t}$ and $Q_{LEC\ i,t}$ flows in the network are represented by

$$P_{LEC\ i,t} = P_{LEC\ in\ i,t} - P_{LEC\ out\ i,t} \quad (28)$$

$$Q_{LEC\ i,t} = Q_{LEC\ in\ i,t} - Q_{LEC\ out\ i,t} \quad (29)$$

² The first of (24) comes from $(P_{i\ i,t} + \Delta P_{i\ i,t})^2 + (Q_{i\ i,t} + \Delta Q_{i\ i,t})^2 = v_{out\ i,t} (P_{i\ i,t}^2 + Q_{i\ i,t}^2)$ by neglecting $\Delta P_{i\ i,t}^2$ and $\Delta Q_{i\ i,t}^2$ and assuming the square of the voltage reference value equal to 1 pu.

The shadow prices associated with the active power constraints (28) are used to define the prices of the transactions among the participants of the community.

In summary, the objective function of the MIQCP reference problem is given by (1) with constraints (2)–(29) and the lower and upper limits of the variables.

3.8. Repeated optimization to achieve a feasible solution

The model described in the previous subsections includes the two following relaxations:

- (a) the convex representation of the losses in the battery (6)
- (b) the conic model of power flows represented by (17)

Relaxation (a) is valid when the solution reaches an equality conditions for at least one constraint of (6). This condition is facilitated by the minimization of the summation of $\ell_{BES\ i,t}$, explicitly considered in the objective function. However, since the compliance reactive power limits Q_{im1} and Q_{im2} depends on the active power consumption P_{user} , as shown by (4), and due to the relationship between P_{user} and ℓ_{BES} shown by (21), in some cases, the lowest value of the objective function is achieved without reaching the minimum BES losses condition, if $\mu_{BES\ i,t}$ are not increased so much that the BES power loss minimization term becomes prevalent. To avoid this issue, in constraint (4) the calculation of P_{user} does not include ℓ_{BES} .

Relaxation (b) is valid when the solution reaches the equality conditions for all constraints in (17). The achievement of this solution is facilitated by the minimization of the branch power losses explicitly considered in the objective function. However, due to the voltage dependence model of the loads (23), (24) and the relationship between bus voltages and branch currents (15), in some cases, the lowest value of the objective function is achieved without reaching the minimum power losses condition, if $\mu_{loss\ i,t}$ are not increased so much that the power loss minimization term becomes prevalent. In order to overcome this condition, in the first optimization (MIQCP), the load consumption is made independent from the branch currents: in (23) and (24) $v_{out\ i,t}$ is replaced by variable $v_{load\ i,t}$ that is equal to the voltage at the secondary side of the feeding OLTC transformer or to V_0^2 if there are no OLTC transformers between bus i and slack bus 0. Analogously, to make the voltage at the secondary side of OLTC transformers independent from branch currents, in (19) $v'_{out\ i,t}$ is replaced by V_0^2 or, if there is another upstreaming OLTC, by its secondary side voltage v_{out} .

If at the end of an optimization neither of the two constraints (6) is satisfied as equality for some of the BES units, despite the described countermeasure, the optimization is repeated with the inclusion of an additional nonnegative penalization in (1), greater than the difference between ℓ_{BES} and the maximum of the right side terms of (6) calculated by using the $P_{BES\ i,t}$ values provided by the previous solution.

Analogously if at the end of an optimization, constraints (17) are not satisfied as equality for some branches, the optimization is repeated by adding a penalization in (1), greater than the difference between u_{in} , u_{out} and the maximum values of $(P_{in}^2 + Q_{in}^2)/v_{in}$, $(P_{mp}^2 + Q_{mp}^2)/v_{mp}$ and $(P_{out}^2 + Q_{out}^2)/v_{out}$, respectively, evaluated according to the previous solution.

3.9. Iterative procedure to obtain a refined solution

The iterative procedure mentioned in Fig. 1 improves the accuracy of the results. The model (1)–(29) is iteratively solved again with these changes:

- (a) the voltage at the secondary side of OLTC transformers are fixed in agreement of the step nearest to the previously calculated value,
- (b) the sign of P_{user} and the sign of Q_{LEC} at each bus are fixed as previously calculated,
- (c) the total power of each load is recalculated by using the bus voltage value obtained in the previous iteration and all load types are transformed in constant P.

The iterative procedure ends when the difference between the bus voltage values in two subsequent iterations becomes lower than a predefined tolerance. As the binary variable are fixed the model (1)–(29) becomes a quadratically constrained problem (QCP).

At the end of this procedure, the reference profile $Q_{ref\ i,t}$ for each participant is defined equal to the calculated $Q_{user\ i,t}$. The same reference profile is used in both the Qdown and Qup procedures described in the next sections.

4. Calculation of the maximum and minimum reactive power deviations

To exploit the community willingness to provide a variation of the reactive power consumptions or injections with respect to the reference profile, DSO and TSO needs to know the maximum amount of the reactive power flexibility.

These flexibility limits are calculated by two distinct optimization models, one (Qdown) provides the maximum value of reactive power consumption decrease or of reactive power injection increase for each t and i ; the other (Qup) provides the maximum value of reactive power consumption increase or of reactive power injection decrease for each t and i .

In both optimizations, the objective functions the revenues for the provision of the flexibilities replace the penalizations for the noncompliance of the minimum power factor.

4.1. Maximum increase of reactive power injection or decrease of reactive power absorption

The considered objective function of the Qdown problem is

$$OF_{Qdown} = \sum_{i \in \Omega} \sum_{t \in T} (C_{grid\ i,t} - \pi_{down} Q_{down\ i,t} + \mu_{loss\ i,t} \ell_{i,t} + \mu_{BES\ i,t} \ell_{BES\ i,t}) \Delta t \quad (30)$$

where π_{down} is the amount of money that DSO/TSO gives to the community for each kvarh of consumption decrease or injection increase, Q_{down} is the variation of the reactive power at each t and i with respect to the reference value Q_{ref} calculated by the reference optimization procedure, i.e.:

$$Q_{down\ i,t} = Q_{ref\ i,t} - Q_{user\ i,t} \quad (31)$$

Function (30) is conceived under two assumptions:

- (a) reactive power decisions in one period do not have significant relationship with the reactive power decisions taken in previous periods,
- (b) the inclusion of the reward term does not significantly affect costs C_{grid} (i.e., active power decisions).

Indeed, as shown in the test cases, in general, the replacement of noncompliance penalization with the flexibility reward does not significantly modify the value of C_{grid} . In cases this is not true (i.e., when reactive power compensation or voltage control do affect active power outputs or consumptions), it might be appropriate to include the expected probability ϕ_t that DSO/TSO will require the activation of the reactive power flexibility at

period t of the next day in the objective function. With this change, the flexibility reward is weighted by ϕ_t and the objective function also includes the noncompliance penalization weighted by $(1 - \phi_t)$. For simplicity, the consideration of this aspect is not addressed in the paper.

Problem Q_{down} includes constraints (2), (5)–(11), (13), (15)–(25), (28), (31). If the OLTC transformers are operated according to (20), the model is still MIQCP although binary variables are limited to those relevant to such a constraint. If (20) is not included (i.e., OLTCs are optimized to minimize the objective function), the model does not include binary variables and it is classified as QCP.

The same repeated solutions and iterative procedures describe in Sections 3.8 and 3.9 are also applied in the Q_{down} procedure, to achieve a feasible and accurate solution.

4.2. Maximum decrease of reactive power injection or increase of reactive power absorption

Analogously, the objective function of Q_{up} procedure is

$$OF_{Q_{\text{up}}} = \sum_{i \in \Omega} \sum_{t \in T} (C_{\text{grid } i,t} - \pi_{\text{up } Q_{\text{up } i,t}} + \mu_{\text{loss } i,t} \ell_{i,t} + \mu_{\text{BES } i,t} \ell_{\text{BES } i,t}) \Delta t \quad (32)$$

where π_{up} is the amount of money that DSO/TSO gives to the community for each kvarh of consumption increase or injection decrease, Q_{up} is the variation of the reactive power at each t and i with respect to the reference value Q_{ref} calculated by the reference optimization procedure, i.e.:

$$Q_{\text{up } i,t} = Q_{\text{user } i,t} - Q_{\text{ref } i,t} \quad (33)$$

Problem Q_{up} includes constraints (2), (5)–(11), (13), (15)–(25), (28), (33). It is a MIQCP or QCP model if (20) is included or not. The repeated solutions and iterative procedures of Sections 3.8 and 3.9 are applied to achieve a feasible and accurate solution.

5. Model implementation, test cases description and results

The complete procedure of Fig. 1, which illustrates all the described optimization models, has been implemented in AIMMS Developer modeling environment [27], using Gurobi 9.5 solvers (MIQCP for the first reference optimization and QCP for Q_{down} , Q_{up} and iterative solutions respectively). The results have been obtained by using a computer equipped with an Intel-i7 and 32 GB of RAM, running 64-bit Windows 10.

The numerical tests included in this paper consider three different test cases. The complete set of data of the 3 test cases is included in the Excel file available at <https://doi.org/10.17632/47rnm7hkn.1>. The file contains also the schemes of the networks, the 96 period per unit load profiles used in all the test cases obtained by the CREST tool [28] using different numbers of dwellings, the daily profiles of π_{buy} and π_{sell} , and the daily profile of the ratio between power output and panel surface, assumed the same for all PV units.

Each prosumer may be equipped with a PV system, a load, and a BES unit. All the prosumers belong to the same energy community. All the calculations refer to a time window of one day, divided into 96 periods of 15 min each. Noncompliance penalty tariff μ_{PF} is equal to 5 €/kvarh, flexibility reward tariffs π_{down} and π_{up} are equal to 3 €/kvarh, in agreement with [11]. The minimum power factor value that complies with the requirements is assumed equal to 0.9. The bus voltages values are constrained to be inside of the 0.9 pu, 1.1 pu interval.

Regarding the iterations needed to obtain feasible (Section 3.8) and accurate (Section 3.9) solutions, a predefined tolerance of

1% is adopted for both the branch maximum current limit and the difference between the bus voltage values in two subsequent iterations. A 1% value is also set for the mixed integer relative optimality tolerance of the global optimum gap in the MIQCP solver.

5.1. Test case A

The test system, adapted from [29], is a 14-bus network, in which three feeders are connected to the same substation bus.

All the BES units can inject or absorb reactive power as determined by the optimization procedure. The minimum and maximum reactive power limits are $\pm 48.43\%$ (0.9 power factor) of the rated value. All PV units operate at unitary power factor.

The forecasted total energy demand during the day is 349.4 MWh, the PV energy generation is 52.4 MWh (15% of the load), the total storage capacity installed is equal to 5.5 MWh (10.5% of the daily PV generation).

For the provision of reactive power, All the BES units can inject or absorb reactive power as determined by the optimization procedure. The minimum and maximum reactive power limits are $\pm 48.43\%$ (0.9 power factor) of the rated value. All PV units operate at unitary power factor.

Assuming that all variable capacitor banks belong to community participants and reactive power exchanges between participants are allowed, Fig. 4 shows the profiles of P_{LEC} and Q_{LEC} during the day.

Fig. 5 shows the bus voltages, Fig. 6 compares the profile of the average price of the internal transactions π_{LEC} with π_{buy} and π_{sell} , i.e. the prices of the transaction with the external energy provider. Fig. 6 also shows the profile $P_{\text{grid tot}}$, i.e., the sum of $P_{\text{grid } i}$. As expected, the prices of the internal transactions are close to π_{buy} as $P_{\text{grid tot}}$ always positive during the day. It has been verified that the both sum of all $P_{\text{LEC } i}$ and $Q_{\text{LEC } i}$ in each period are null and that the voltage profiles (and the relevant power flows in the network) corresponds to those provided by Matpower [30]. The same tests have been carried out for all the other test cases.

The calculations are repeated for 4 scenarios that differentiate for the type of operation of the capacitor banks and whether Q_{LEC} exchanges are allowed (scenario 0 is without community, i.e., without P_{LEC} ; in all the other scenarios P_{LEC} transactions are allowed):

- scenario 0 - all capacitors belong to the utility without community;
- scenario 1 - all capacitors belong to the prosumers without Q_{LEC} exchanges;
- scenario 2 - all capacitors belong to prosumers and Q_{LEC} exchanges are allowed;
- scenario 3 - all capacitors belong to the utility and Q_{LEC} exchanges are allowed;
- scenario 4 - all capacitors belong to the utility without Q_{LEC} exchanges.

Table 1, provides the values of the objective function, of the total daily costs of the exchanges with the energy provider and the daily value of the power factor noncompliance penalty obtained by the first (MIQCP) and the final of the iterative solutions (QCP), for the 4 scenarios.

Tables 2 and 3 provide the solution results for the Q_{down} and Q_{up} procedures where the rewards corresponding to the provision of reactive power change with respect to the reference value replace the noncompliance penalties.

For all the calculations, the computer time is indicated. The final solutions (denoted in Tables 2 and 3 as final iter.) are achieved with a single iteration.

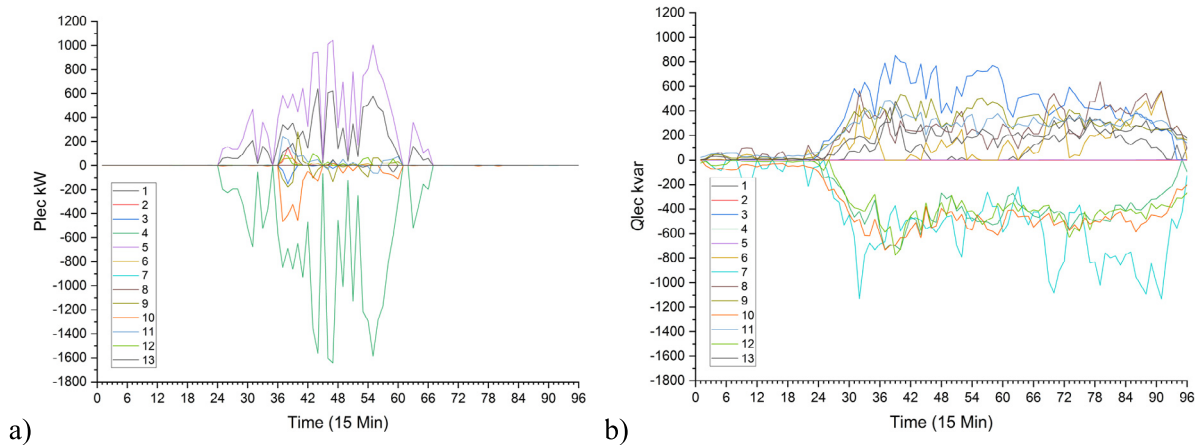


Fig. 4. Profiles of the (a) active and (b) reactive power exchanges between community participants. Test case A.

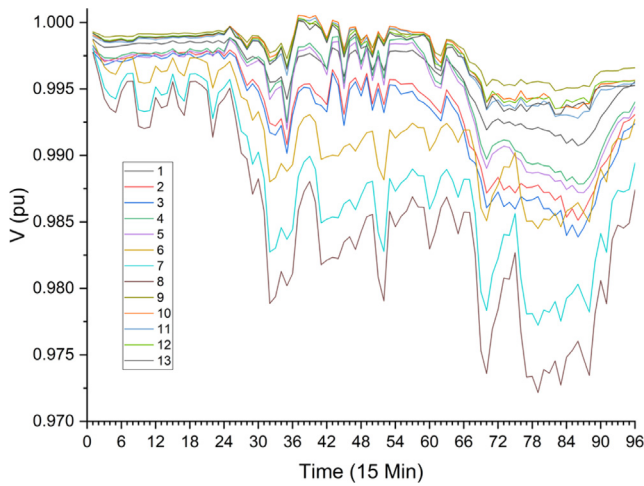


Fig. 5. Bus voltage profiles. Test case A.

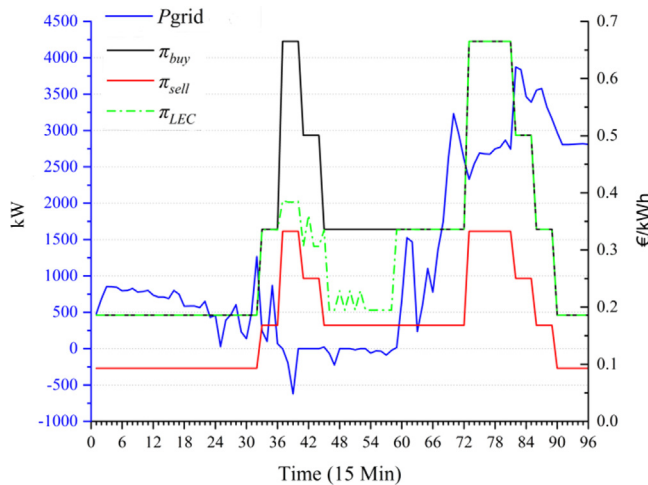


Fig. 6. Prices of the internal transactions, prices of the transactions with the external energy provider, cumulative value of the power exchanged with the energy provider. Test case A.

Fig. 7 compares the profiles of the sum of the $Q_{user\ i}$ values calculated by the reference, Q_{down} , and Q_{up} procedures for the

considered scenarios, showing the margins for each period that can be used as provision of the reactive power flexibility service by the community.

In Table 1, the comparison between scenarios 1 and 2 and between scenarios 3 and 4 show the capability of QLEC exchanges to significantly reduce the noncompliance penalties.

In this case, the optimization of the available capacitor banks by the prosumers (scenarios 1 and 2) provides a significant advantage only for the provision of the Qdown reserve, whilst the penalty in the reference case is not reduced, as shown by the comparison between scenarios 1 and 4.

In all the scenarios, the energy procurement costs due to the transactions with the external energy provider are similar, being higher for scenario 0 (without community) especially for the reference case in which the voltages are kept high to increase the active power consumption by the voltage dependent loads and reduce the noncompliance penalties.

The comparison between scenarios 0 and 4 for the reference case shows that, in this test case, the decrease of energy procurement costs by the participation in the community leads to a slightly increase of the noncompliance penalties.

For this test case A, as well as for test case B and C, it has been verified that participating in the community does not disadvantage any of the prosumers.

In the case the variable capacitor banks belong to the utility, if QLEC exchanges are allowed, all the capability of providing reactive power by the participants is already used in the reference optimization to reduce noncompliance penalties. Therefore, the margin allowed for the provision of Qdown services is almost zero for large part of the day as shown in Fig. 7(c) and the low value of the corresponding reward in Table 2.

In order to show the sensitivity of the results for different values of noncompliance penalty and flexibility reward tariffs, Table 4 reports the summaries for scenario 3 with $\mu_{PF} = 0.1$ €/kvarh and $\mu_{PF} = 2.5$ €/kvarh (other than 5 €/kvarh as in the previous results), $\pi_{down} = \pi_{up} = 1.5$ €/kvarh (other than 3 €/kvarh as in the previous results). The results show that, as expected, decreasing the value of μ_{PF} decreases the noncompliance penalty, whereas lowering $\pi_{down/up}$ reduces the flexibility rewards. No effect is observed on the overall cost of exchanges with the energy provider.

5.2. Test case B

Test case B is based on 13-bus IEEE feeder [31]. All the branches are considered symmetrical by averaging the non-zero values of the diagonal and off diagonal elements of the impedance

Table 1
Summary of the results for the reference optimization of case study A.

		Objective function	Cost of exchanges with the energy provider (k€)	Noncompliance penalty (k€)	CPU time (s)
Scenario 0	1st solution	230.5 10 ³	114.8	115.6	9.0
	Final iter.	228.9 10 ³	114.1	114.7	4.9
Scenario 1	1st solution	229.5 10 ³	113.0	116.4	3.2
	Final iter.	227.5 10 ³	112.2	115.2	2.6
Scenario 2	1st solution	112.4 10 ³	112.2	0	3.2
	Final iter.	111.6 10 ³	111.5	0	2.4
Scenario 3	1st solution	159.2 10 ³	113.1	45.9	14.7
	Final iter.	157.7 10 ³	112.5	45.1	2.2
Scenario 4	1st solution	229.4 10 ³	113.0	116.3	3.0
	Final iter.	227.8 10 ³	112.3	115.4	3.0

Table 2
Summary of the results for the Qdown optimization of case study A.

		Objective function	Cost of exchanges with the energy provider (k€)	Reward (k€)	CPU time (s)
Scenario 0	1st solution	22.9	113.7	90.9	3.4
	Final iter.	19.4	113.0	93.8	3.7
Scenario 1	1st solution	-278.7 10 ³	112.2	391.0	3.2
	Final iter.	-281.7 10 ³	111.6	393.5	1.9
Scenario 2	1st solution	-175.3 10 ³	112.2	287.7	3.2
	Final iter.	-178.3 10 ³	111.6	290.1	1.9
Scenario 3	1st solution	89.6 10 ³	112.2	22.8	1.4
	Final iter.	86.1 10 ³	111.5	25.6	1.8
Scenario 4	1st solution	21.9 10 ³	112.2	90.4	1.4
	Final iter.	18.4 10 ³	111.5	93.3	1.9

Table 3
Summary of the results for the Qup optimization of case study A.

		Objective function	Cost of exchanges with the energy provider (k€)	Reward (k€)	CPU time (s)
Scenario 0	1st solution	-178.8 10 ³	113.7	292.6	3.4
	Final iter.	-176.1 10 ³	112.9	289.1	3.9
Scenario 1	1st solution	-211.4 10 ³	112.2	323.8	5.9
	Final iter.	-207.9 10 ³	111.2	319.3	2.1
Scenario 2	1st solution	-314.7 10 ³	112.2	427.1	6.0
	Final iter.	-311.3 10 ³	111.2	422.7	2.1
Scenario 3	1st solution	-248.5 10 ³	112.2	360.8	6.6
	Final iter.	-245.8 10 ³	111.4	357.3	1.9
Scenario 4	1st solution	-180.8 10 ³	112.2	293.2	6.7
	Final iter.	-178.1 10 ³	111.4	289.6	2.0

Table 4
Summary of the results for case study A Scenario 3 with different values of reactive power penalty and flexibility reward (in parenthesis, percentage variations with respect to Tables 1–3).

μ_{PF} , $\pi_{down/up}$ (€/kvarh)	Reference		Qdown		Qup	
	Cost of exchanges with the energy provider (k€)	Noncompliance penalty (k€)	Cost of exchanges with the energy provider (k€)	Reward (k€)	Cost of exchanges with the energy provider (k€)	Reward (k€)
0.1, 3	111.6 (-0.8)	1.1 (-97.6)	111.5 (0)	26.3 (2.7)	111.4 (0)	356.6 (-0.2)
2.5, 3	112.5 (0)	22.7 (-49.7)	111.5 (0)	25.1 (-2.0)	111.4 (0)	357.7 (0.1)
0.1, 1.5	111.6 (-0.8)	1.1 (-97.6)	111.5 (0)	13.1 (-48.8)	111.4 (0)	178.3 (-50.1)
2.5, 1.5	112.5 (0)	22.7 (-49.7)	111.5 (0)	12.6 (-50.8)	111.4 (0)	178.9 (-49.9)
5, 1.5	112.5 (0)	45.1 (0)	111.5 (0)	12.8 (-50.0)	111.4 (0)	178.7 (-50.0)

and shunt admittance matrices and using the positive sequence values. The loads are assumed balanced too, increasing the original load values indicated in [31].

All the BES units can operate can inject or absorb reactive power with minimum and maximum limits equal to $\pm 48.43\%$ of the rated value. All PV units operates at unitary power factor.

The forecasted total energy demand during the day is 52.5 MWh, the PV energy generation is 24.5 MWh (46.7% of the load),

the total storage capacity installed is equal to 1.7 MWh (6.9% of the daily PV generation).

Both transformers at the substation and the one feeding a low voltage bus are considered equipped with OLTCs, between 0.9 pu and 1.1 pu.

Assuming that OLTCs and variable capacitor banks as operated by the community with Q_{LEC} exchanges allowed, Fig. 8 compares the profile of the average price profile of the internal transactions

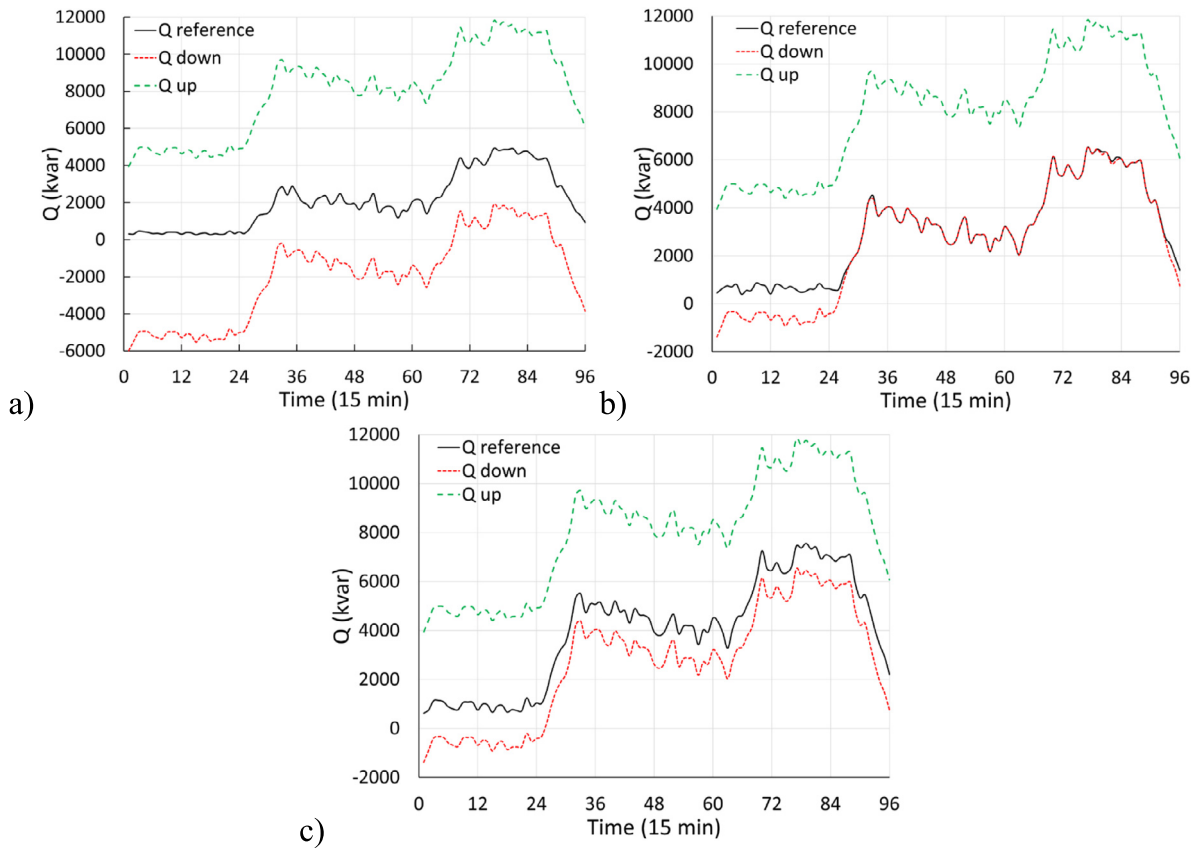


Fig. 7. Profiles of the cumulative value of the community reactive power calculated by the reference, Qup and Qdown procedures for scenarios: (a) 2 (like 1 not shown), (b) 3, (c) 4 (like 0 not shown). Test case A.

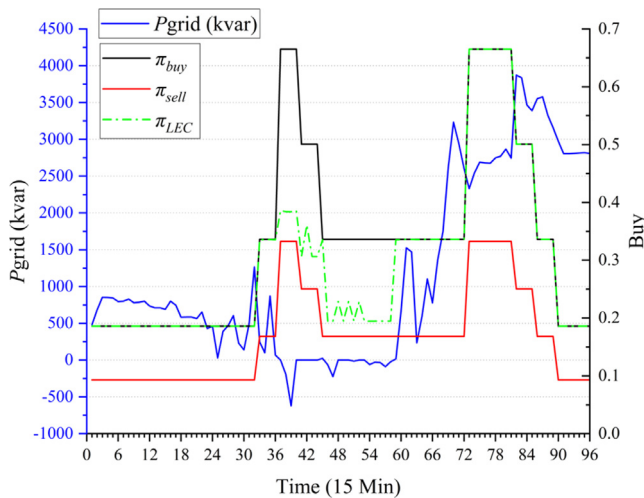


Fig. 8. Comparison between the prices of the internal transactions and the prices of the transactions with the external energy provider. Cumulative value of the power exchanged with the energy provider. Test case B.

π_{LEC} with the profile of $P_{grid\ tot}$. As expected, the prices of the internal transactions are bounded between π_{buy} and π_{sell} , close to π_{buy} when $P_{grid\ tot}$ is positive and close to π_{sell} when $P_{grid\ tot}$ is negative.

The calculations are repeated for 9 scenarios that differentiate for the type of operation of the OLTC transformers, of the capacitor banks, and whether Q_{LEC} exchanges are allowed (scenario 0 is without community, all the other scenarios are with P_{LEC}):

- scenario 0 - OLTCs operated by the utility, all capacitors belong to the utility without community;
- scenario 1 - OLTCs operated by the utility, all capacitors belong to the utility without Q_{LEC} ;
- scenario 2 - OLTCs operated by the community, all capacitors belong to the utility without Q_{LEC} ;
- scenario 3 - OLTCs operated by the utility, all capacitors belong to the prosumers with Q_{LEC} ;
- scenario 4 - OLTCs operated by the community, all capacitors belong to the prosumers with Q_{LEC} ;
- scenario 5 - OLTCs operated by the community, all capacitors belong to the utility with Q_{LEC} ;
- scenario 6 - OLTCs operated by the community, all capacitors belong to prosumers without Q_{LEC} ;
- scenario 7 - OLTCs operated by the utility, all capacitors belong to the utility with Q_{LEC} ;
- scenario 8 - OLTCs operated by the utility, all capacitors belong to prosumers without Q_{LEC} .

Table 5 shows the summary of the values of energy procurement cost from the external provider, noncompliance penalty or reactive power service reward obtained at the last iteration of the reference, Qdown, and Qup procedures, respectively. The average (maximum) computational times in s are: 20.5 (44.4) for reference optimization, 3.3 (5.6) for Qdown, 3.1 (5.4) for Qup. The final solutions are achieved with 1 or 2 iterations.

Fig. 9 compares the profiles of the sum of the $Q_{user\ i}$ values calculated by the reference, Qdown, and Qup procedures for the considered scenarios.

The results of Table 5 show that minimum values of the noncompliance penalties are achieved when Q_{LEC} transactions are allowed (scenarios 3, 4, 5, and 7), reaching the complete

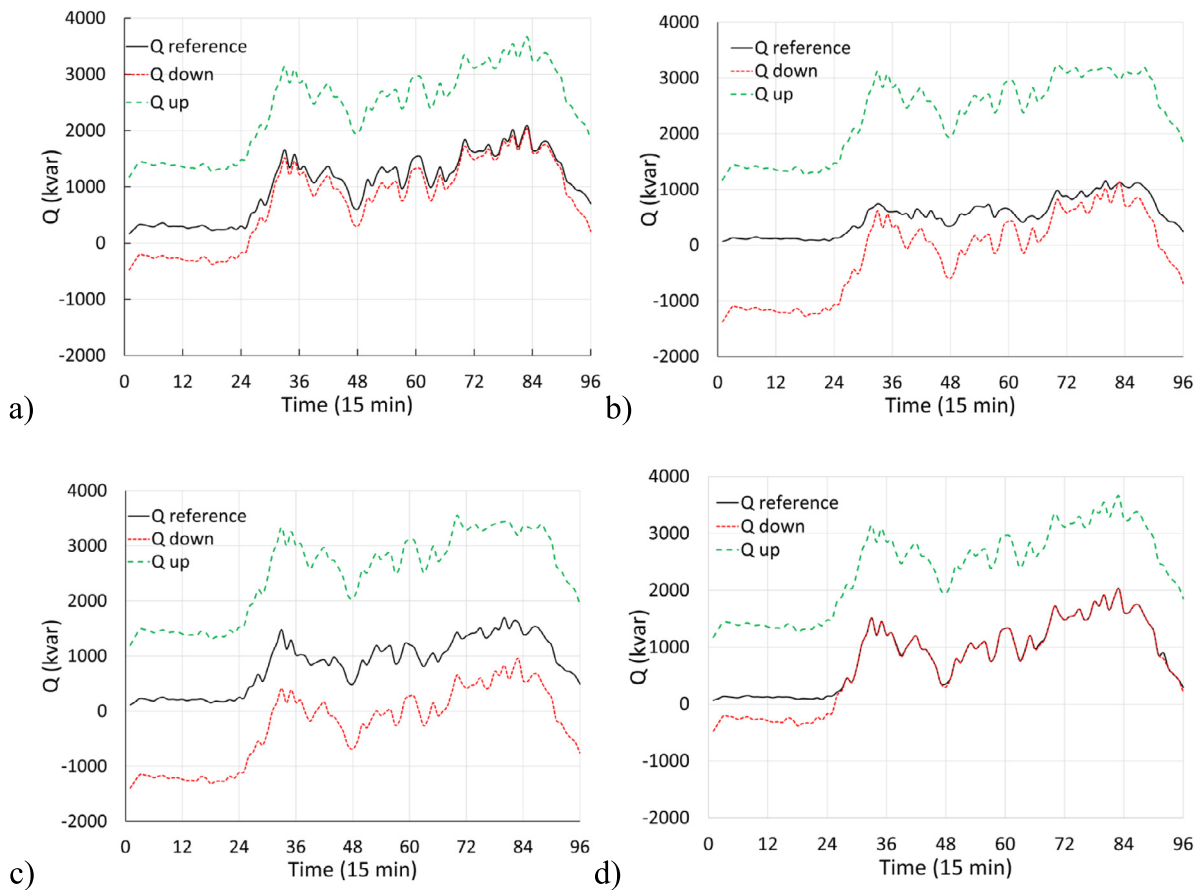


Fig. 9. Profiles of the cumulative value of the reactive powers of the community participants calculated by the reference, Qup and Qdown procedures for scenarios: (a) 1 (like 0 and 2 not shown), (b) 4 (like 3 not shown), (c) 6 (like 8 not shown), (d) 7 (like 5 not shown). Test case B.

Table 5

Summary of the results for the reference, Qup and Qdown optimizations of case study B. C_{EP} indicates the cost of the exchanges with the energy provider, P_{NC} indicates the noncompliance penalty, R indicates the reward.

	Reference		Qdown		Qup	
	C_{EP} (k€)	P_{NC} (k€)	C_{EP} (k€)	R (k€)	C_{EP} (k€)	R (k€)
Scenario 0	12.4	45.5	12.0	20.0	11.9	98.0
Scenario 1	10.8	45.5	10.6	20.5	10.6	97.4
Scenario 2	9.5	40.0	9.4	21.0	12.1	115.1
Scenario 3	10.6	0	10.7	48.0	10.5	131.4
Scenario 4	9.4	0	9.3	35.0	12.0	163.0
Scenario 5	9.5	22.1	9.4	8.7	12.1	127.5
Scenario 6	9.6	39.9	9.4	80.1	12.0	118.0
Scenario 7	10.8	28.0	10.6	7.7	10.5	110.3
Scenario 8	10.7	45.1	10.7	78.1	10.5	101.3

compensation when the capacitor banks belong to the participants (scenarios 3 and 4).

The effects of the different way of operation of the OLTC transformers are quite negligible for both the reference and the Qdown optimizations.

Higher rewards in the Qup optimization are obtained when the OLTC transformers are operated by the community (scenario 2 compared to 1, scenario 4 compared to 3, scenario 5 compared to 7, and scenario 6 compared to 8). In the Qup optimization of these scenarios however, there is also a slight increase of the costs C_{EP} relevant to the transactions with the external energy provider the Qup optimization (from around 10.5 in scenarios 1,3,7,8 to around 12 in scenarios 2,4,5,6).

When Q_{LEC} transactions are allowed and the capacitor banks belong to the utility (scenarios 5 and 7), the reduction margin available is negligible, resulting in very low rewards in the Qdown solution, as also illustrated by Fig. 9d).

As expected, other conditions equal, the highest value of the energy procurement costs from the external provider are those without community (scenario 0).

5.3. Test case C

Test case C is based on 123-bus IEEE feeder [31]. All the lines are considered balanced with positive sequence parameters obtained by averaging self and mutual impedances and admittances given in [31]. The loads are assumed balanced too, by averaging the single-phase loads. 49 PV units are added at load buses, with peak power taken equal to the load power multiplied by a randomly generated factor with a uniform distribution between 0 and 2, provided the production/consumption ratio is greater than 0.9, otherwise taken as zero. The apparent rated power of the PV inverters is increased by 10% respect to the PV rated powers. The BES units operate at unitary power factor, while the PV units may exchange the reactive power determined by the optimization procedure. With reference to the rated power of the inverter, the minimum and maximum reactive power limits are $\pm 48.43\%$ if the produced active power is larger than 10%, $\pm 4.84\%$ otherwise.

The forecasted total energy demand during the day is 13.0 MWh, the PV energy generation is 7.3 MWh (56.2% of the load), the total storage capacity installed is equal to 68 kWh (0.9% of the daily PV generation).

Table 6

Summary of the results for the reference, Q_{up} and Q_{down} optimizations of case study B. C_{EP} indicates the cost of the exchanges with the energy provider, P_{NC} indicates the noncompliance penalty, R indicates the reward, CPU indicates the total computational time.

	Reference			Qdown			Qup		
	C_{EP} (k€)	P_{NC} (k€)	CPU (s)	C_{EP} (k€)	R (k€)	CPU (s)	C_{EP} (k€)	R (k€)	CPU (s)
Scenario 0	2.9	3.0	135.2	2.9	10.0	43.6	2.9	32.9	43.5
Scenario 1	2.5	3.0	141.3	2.5	9.9	47.4	2.5	33.0	53.2
Scenario 2	2.5	1.7	464.2	2.5	9.5	48.7	2.5	33.5	52.5
Scenario 3	2.4	2.9	153.3	2.4	9.9	54.8	2.6	34.5	49.1
Scenario 4	2.4	1.6	127.7	2.4	8.8	50.8	2.6	35.5	49.2

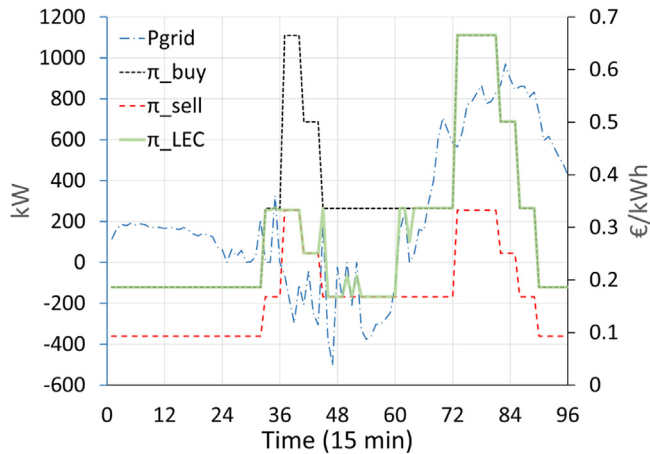


Fig. 10. Comparison between the prices of the internal transactions and the prices of the transactions with the external energy provider. Cumulative value of the power exchanged with the energy provider. Test case C.

The substation transformer and the voltage regulators feeding buses 14, 26, and 67 are considered equipped with OLTCs, between 0.9 pu and 1.1 pu. Variable capacitor banks are connected to buses 83, 88, 90, 92 (with maximum power equal to the average values indicated in [31] for the three phases).

Assuming that OLTCs and variable capacitor banks are operated by the community with Q_{LEC} exchanges allowed, Fig. 10 compares the average price profile of the internal transactions π_{LEC} with the profile of $P_{grid\ tot}$ and, as in previous test cases, the prices of the internal transactions are bounded between π_{buy} and π_{sell} , quite closely following the sign of $P_{grid\ tot}$.

The calculations are repeated for 5 scenarios that differentiate whether Q_{LEC} exchanges are allowed and how the OLTC and capacitor banks are operated (scenario 0 is without community, all the other scenarios are with P_{LEC}):

scenario 0 – OLTCs and capacitors operated by the utility, without community;

scenario 1 – OLTCs and capacitors operated by the utility, without Q_{LEC} ;

scenario 2 – OLTCs and capacitors operated by the utility, with Q_{LEC} ;

scenario 3 – OLTCs and capacitors operated by the community, without Q_{LEC} ;

scenario 4 – OLTCs and capacitors operated by the community, with Q_{LEC} ;

Table 6 shows the summary of the values of energy procurement cost from the external provider, noncompliance penalty or reactive power service reward obtained at the last iteration of the reference, Q_{down} and Q_{up} procedures, respectively, as well as the total computation time for of the three optimizations. Fig. 11 compares the profiles of the sum of the $Q_{user\ i}$ values calculated

by the reference, Q_{down} , and Q_{up} procedures for the considered scenarios. The final solutions are achieved with 4 or 5 iterations.

The results show that in this case considering the capacitor banks as included in the prosumers does not provide significant advantages due to the limited number and size of the banks. The operation as a community reduces the energy procurement costs (C_{EP} values of scenario 0 are the highest). The possibility to exchange reactive power among the community participants significantly reduces the noncompliance penalties (the P_{NC} values of scenarios 2 and 4 are the lowest). The values of the energy procurement cost, associated with the active power exchanges with the external provider, are almost the same in the different reactive power optimizations (there is only a slight increase for the Q_{up} optimization, when OLTCs and capacitor banks are operated by the community, i.e., in scenarios 3 and 4).

6. Conclusions

The paper has presented a procedure for the day-ahead scheduling of an energy community in which direct exchanges of both active and reactive power among the participants are allowed.

Direct transactions of active powers allow to decrease the total costs due to energy procurement from the external provider with respect to the case in which each prosumer can only transact with the energy provider, under the (usual) assumption that the purchase tariffs are higher than sale rates. The procedure calculates the scheduling of the energy resources and the fair prices of the internal transactions among the community participants as the shadow prices of the balance constraints. As these prices are bounded between the purchase and sale rates fixed by the external provider, none of the prosumers suffer an economic disadvantage in participating in the community.

The reactive power exchanges allow to reduce the noncompliance penalties that each prosumer would pay whenever it operates at a power factor lower than the minimum value fixed by the energy authority or the utility. The issue of low power factor operation is of increasing importance with the diffuse installations of PV units that significantly reduce, during the central hours of the day, the active power consumption. The optimization procedure calculates the scheduling of the available reactive power compensation resources, coordinated with the voltage control means of the network. For this purpose, the voltage dependence of the loads is considered.

The procedure is completed by the calculation of the maximum and minimum reactive power deviations that can be provided by the community, following a DSO/TSO request, for each period of the following day. In these calculations, the noncompliance penalties are replaced by the revenues provided by the reactive power flexibility assuming a predefined tariff. The results obtained for three test cases show that the different scheduling of the reactive power compensation resources has a limited impact on the energy procurement costs. This conclusion also applies for lower values of the noncompliance penalties and reactive power remuneration than those assumed in the calculations, as

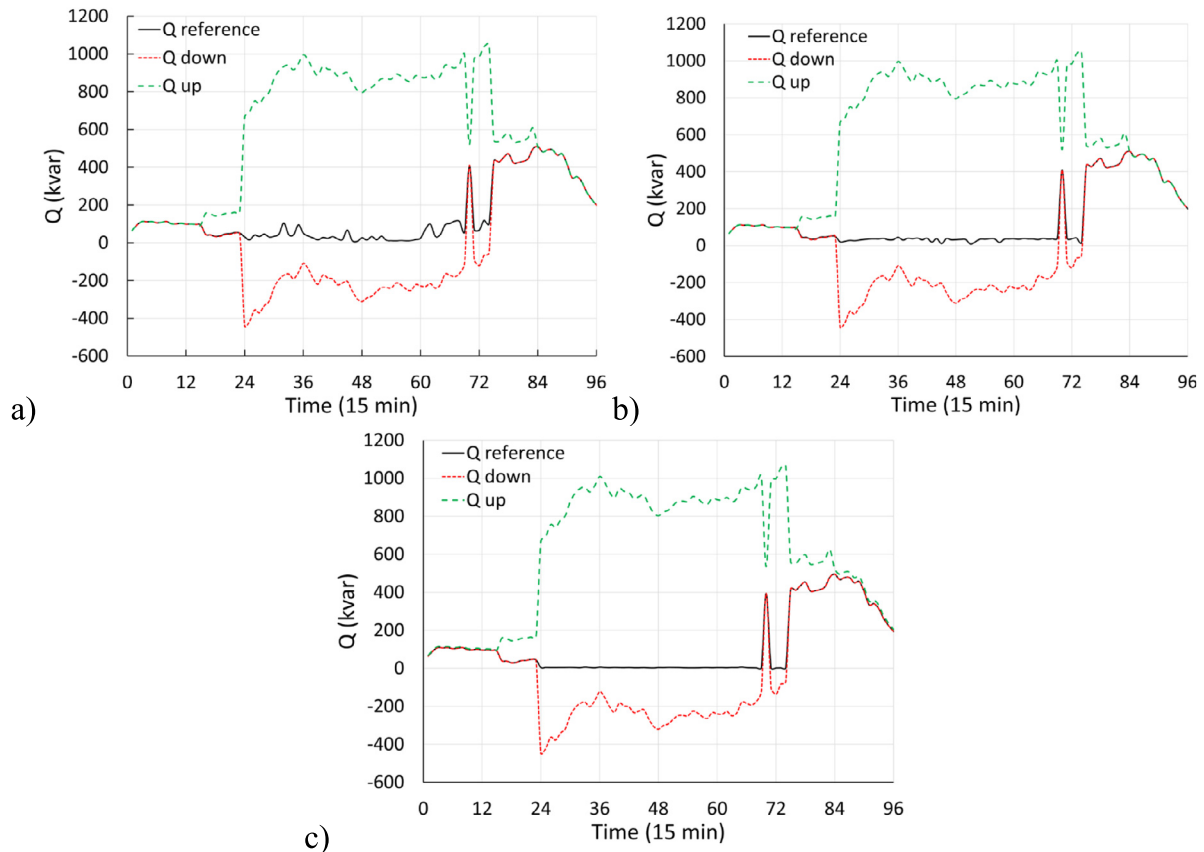


Fig. 11. Profiles of the cumulative value of the reactive powers of the community participants calculated by the reference, Qup and Qdown procedures for scenarios: (a) 1 (like 0 and 3 not shown), (b) 2, (c) 4. Test case B.

they would result in smaller reactive power compensation actions by the community participants. For this reason, and due to the lack of intertemporal coupling constraints in the reactive power decisions, the assumption of neglecting the probability that the flexibility service will be requested during the day appears reasonable.

The computation times are reasonably low for all the calculations. This makes the proposed deterministic models suitable to be included in stochastic procedures that consider the uncertainties related to the PV production and load consumption profiles, other than the already mentioned probability that DSO/TSO can require a reactive power reduction or increase during the day.

In this paper, all the users of the network participate in the same community and share the same energy provider (or at least the same $\pi_{buy,t}$ and $\pi_{sell,t}$ profiles). Although beyond the scope of this paper, the presented modeling approach can be applied for the analysis of systems where the users belong to different communities or do not participate in any community, with the presence of multiple energy providers.

CRedit authorship contribution statement

Tohid Harighi: Conceptualization, Methodology, Software, Investigation, Visualization, Writing – original draft. **Alberto Borghetti:** Conceptualization, Methodology, Software, Investigation, Visualization, Writing – original draft, Supervision. **Fabio Napolitano:** Methodology, Writing – review & editing. **Fabio Tossani:** Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data are available at: <https://doi.org/10.17632/47rnm7hkn.1>.

Acknowledgments

The authors thank Prof. Carlo Alberto Nucci and Ing. Adriano Calzolari for helpful discussions and comments during the development of this work. The research activity has been carried out in the framework of a collaboration agreement between Inrete Distribuzione Energia S.p.A. and University of Bologna, Italy funded by FSE REACT-EU -A.Y. 2021/2022 (PON Project “Research and Innovation”) and by the Italian Ministry for Education, University and Research under the grant PRIN-2017K4JZEE “Planning and flexible operation of micro-grids with generation, storage and demand control as a support to sustainable and efficient electrical power systems: regulatory aspects, modeling and experimental validation”.

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