ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

ARCHIVIO ISTITUZIONALE
DELLA RICERCA

Alma Mater Studiorum Università di Bologna
Archivio istituzionale della ricerca

Logical pluralism, indeterminacy and the normativity of logic

This is the final peer-reviewed author's accepted manuscript (postprint) of the following publication:

Published Version:
Ferrari, F., Moruzzi, S. (2020). Logical pluralism, indeterminacy and the normativity of logic. INQUIRY, 63(3-4), 323-346 [10.1080/0020174X.2017.1393198].

Availability:
This version is available at: https://hdl.handle.net/11585/746785 since: 2022-11-16

Published:
DOI: http://doi.org/10.1080/0020174X.2017.1393198

Terms of use:

Some rights reserved. The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

This item was downloaded from IRIS Università di Bologna (https://cris.unibo.it/).
When citing, please refer to the published version.

(Article begins on next page)

12 October 2024

This is the final peer-reviewed accepted manuscript of:

Filippo Ferrari & Sebastiano Moruzzi, *Logical pluralism, indeterminacy and the normativity of logic*, «Inquiry», 63:3-4, 323-346.

The final published version is available online at:

https://doi.org/10.1080/0020174X.2017.1393198

Terms of use:

Some rights reserved. The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

This item was downloaded from IRIS Università di Bologna (https://cris.unibo.it/)

When citing, please refer to the published version.

Logical Pluralism, Indeterminacy and the Normativity of Logic

Filippo Ferrari (Universität Bonn), fferrari@uni-bonn.de

Sebastiano Moruzzi (Università di Bologna), sebastiano.moruzzi@unibo.it

Abstract

According to the form of logical pluralism elaborated by Beall and Restall there is more than one relation of logical consequence. Since they take the relation of logical consequence to reside at the very heart of a logical system, different relations of logical consequence yield different logics. In this paper, we are especially interested in understanding what are the consequences of endorsing Beall and Restall's version of logical pluralism vis-à-vis the normative guidance that logic is taken to provide to reasoners. In particular, the aim of this paper is threefold. First, in sections 2 and 3, we offer an exegesis of Beall and Restall's logical pluralism as a thesis of semantic indeterminacy of our concept of logical consequence—i.e. understood as indeterminacy logical pluralism. Second, in sections 4 and 5, we elaborate and critically scrutinise three models of semantic indeterminacy that we think are fit to capture Beall and Restall's indeterminacy logical pluralism. Third, in section 6, following Beall and Restall's assumption that the notion of logical consequence has normative significance for deductive reasoning, we raise a series of normative problems for indeterminacy logical pluralism. The overall conclusion that we aim to establish is that Beall and Restall's indeterminate logical pluralism cannot offer an adequate account of the normative guidance that logic is taken to provide us with in ordinary contexts of reasoning.

1. *Beall and Restall's Logical Pluralism*

According to the form of pluralism discussed by Beall and Restall (henceforth *B&R*) in their book *Logical Pluralism*,[1] there is more than one relation of logical consequence. Since B&R take the relation

ACKNOWLEDGEMENTS

We would like to dedicate this publication to Eva Picardi. Eva has been an extremely inspiring teacher for us, and a good friend. She has accompanied us through the various intricate steps of our academic careers with unfading intellectual generosity and enthusiasm. She will be greatly missed, but we hope to keep her memory alive by continuing pursuing what she truly loved—philosophical research.

of logical consequence to reside at the very heart of a logical system, different relations of logical consequence yield different logics. The basic idea is that the concept of logical consequence (and its linguistic counterpart, namely the meaning of "follows from", cf. B&R 2006: 29) has some core features that allow for different relations to count as logical consequence.[2] The first of these core features is a generalization of Tarski's analysis of logical consequence. According to Tarski, logical consequence should be understood in terms of necessary truth-preservation (Tarski 1956: 411), which, in turn, can be sharpened model-theoretically as follows: a sentence *p* follows logically from a set of sentences *S* just in case every model of *S* is a model of *p* (Tarski 1965: 417). Roughly, Tarski defined a model of a set of sentences *S* as a way of interpreting the sentences that would make them come out as true, by assigning semantic values of the appropriate kind to each type of non-logical expressions. Crucially, Tarski took these models to yield classical logic (Tarski 1956: 197; see also B&R 2006: 39-40). According to B&R, Tarski's conception can be generalized in order to allow for notions other than Tarskian models and thus for different logics than classical logic:

GENERALIZED TARSKI'S THESIS (GTT): an argument is valid_x if and only if, in every case_x in which the premises are true, so is the conclusion (B&R 2006: 29)

The notion of a *case* is intended to include among its instances not only Tarskian models, which yield classical logic, but also other notions such as constructions and situations that, respectively, yield intuitionistic logic and relevant logic. A plurality of consequence relations results from the variety of ways of understanding the notion of a case over which GTT quantifies. The resulting consequence relations are all admissible because they satisfy the three core features of logical consequence that function as constraints on the admissibility of instances of GTT: *necessity*, *formality* and *normativity*. According to *necessity* a valid argument necessitates the truth of its conclusion (B&R 2006: 14). *Formality* states that an argument is valid purely in virtue of its form (B&R 2006: 18). Finally, *normativity*, says

In writing this paper we have enormously benefited from various discussions with Elke Brendel, Massimiliano Carrara, Nathan Kellen, Vittorio Morato, Eugenio Orlandelli, Nikolaj Pedersen, Andrea Sereni, Maria Paola Fogliani Sforza, Elena Tassoni, Giorgio Volpe, Dag Westerståhl, Jeremy Wyatt, Luca Zanetti. We are particularly indebted to Matti Eklund and Erik Stei for their very generous written comments on earlier drafts of this paper. Thanks also to an anonymous referee for various precious comments which helped to improve the final version of the paper. One of the authors, Filippo Ferrari, would like to acknowledge the generous support of the Deutsche Forschungsgemeinschaft (DFG—BR 1978/3–1) for sponsoring his postdoctoral fellowship at the University of Bonn, within the project "Disagreement in Philosophy" directed by Elke Brendel and Thomas Grundmann.

[1] B&R (2006).

[2] Following B&R we will treat the expressions "the concept of logical consequence" and "the meaning (in English) of *follows from*" as interchangeable.

that it is wrong to accept the premises of a valid argument while rejecting its conclusion (B&R 2006: 16). GTT, together with these three constraints, gives us the settled core of the concept of logical consequence.

Although B&R are not fully explicit about this issue, throughout the paper we will assume that the kind of philosophical project they are engaging with is mainly descriptive, namely it is aimed to analyse the ordinary concept of logical consequence. According to this project the ordinary concept of logical consequence is imprecise since it allows for different precisifications. However, no revision of the ordinary concept is enforced by the possibility of different precisifications.

That said, the aim of this paper is threefold. First, in sections 2 and 3, we offer an exegesis of B&R's logical pluralism as a thesis of semantic indeterminacy of our concept of logical consequence—i.e. as a form of indeterminacy logical pluralism. Second, in sections 4 and 5, we elaborate and critically scrutinise three models of semantic indeterminacy that we think might be fit to capture B&R's indeterminacy logical pluralism. Third, in section 6, following B&R's assumption that the notion of logical consequence has normative significance for deductive reasoning, we raise a series of normative problems for indeterminacy logical pluralism. The overall conclusion that we aim to establish is that B&R's logical pluralism, understood as indeterminacy logical pluralism, cannot offer an adequate account of the normative guidance—as predicted by the normativity constraint—that logic is expected to provide us with in ordinary contexts of reasoning.

2. *B&R's Pluralism* versus *Carnapian Conventionalism*

According to B&R, logical pluralism is true because there are at least three admissible instances of GTT that yield three different logics. But what exactly is the nature of this pluralism? For our purposes, it is important to stress that it is not a version of Carnapian conventionalism.[3] This point is significant because it allows us to legitimately investigate into the semantic status of validity judgements—i.e. judgements about what follows from what—and about the normative profile of such judgements—i.e. the rational requirements imposed by a validity judgement (see section 6 below).[4] To see why

[3] For an elaboration of the comparison between Beall and Restall's logical pluralism and Carnapian conventionalism, see Restall (2002).

[4] We take judgements to be cognitive mental acts (Shah & Velleman 2005). This notion might be thought equivalent, for our purposes, to the notion of endorsing a belief by means of a process of rational deliberation (which is, of course, only one of the canonical ways to form beliefs—so no claim of exhaustivity with respect to the normativity of cognitive, mental categories is intended here).

B&R's pluralism is different from Carnapian conventionalism, let us consider Carnap's idea of tolerance famously expressed in the following passage:

> In logic there are no morals. Everyone is at liberty to build his own logic, i.e. his own language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (Carnap 1937: §17)

According to Carnap, one is free to stipulate new frameworks which involve different rules for the logical connectives thus yielding different logics. There is no question of which logical framework is correct since the choice of a framework is based on practical considerations.[5]

In contrast with Carnapian conventionalism, B&R take their kind of pluralism to arise even within a language (B&R 2006: 78-9; Restall 2002). For B&R the concept of logical consequence, although it admits of different relations of logical consequence, has a unity enforced by GTT together with the three constraints of normativity, necessity and formality. In other words, the unified concept of logical consequence (the intension) allows for a variety of logical consequence relations (extensions):

> We take it that the notion of logical consequence is irreducibly plural in its application. That is, we take it that there are at least two distinct relations of logical consequence and not simply two distinct relations in intension, but two distinct relations in extension. (Restall 2002: 426)

Thus, even within the same language, we make judgements concerning the validity of arguments that have the following features: i) they are meaningful, and ii) they allow for different and conflicting assessments.[6] This is because there are different relations of logical consequence which are admissible extensions of the concept of logical consequence and these extensions give rise to conflicting assessments of some arguments. To illustrate, consider the following English sentence expressing one instance of the explosion principle (i.e. anything follows from a contradiction):

---

[5] See Carnap (1950), "Let us be cautious in making assertions and critical in examining them, but tolerant in permitting linguistic forms". The idea of tolerance does not itself imply conventionalism—i.e. that logic is determined by the conventions of a linguistic frameworks. Carnap held both tolerance and conventionalism, but for our purposes it is important the conventionalist thesis only—on this, see Shapiro (2014: ch.1).

[6] We can distinguish between a linguistic and a mental level by referring to mental acts (judgements) and linguistics acts (assertions). The normative problems we raise in this paper concern judgements, however we think that analogous considerations pose a normative problem for assertions.

(MOON) "The moon is made of cheese" follows from "2 is even" and "2 is not even"

B&R point out that there are admissible instances of GTT according to which MOON is true, namely those instances that take cases either as Tarskian models or as constructions. However, they also point out that there is an admissible instance of GTT according to which MOON is false, namely the instance that takes cases as situations.

One might now wonder whether or not MOON is true—i.e. whether it is in fact the case that "The moon is made of cheese" follows from "2 is even" and "2 is not even". In other words, one might wonder how to answer to the following question:

(QUESTION) Does "The moon is made of cheese" follow from "2 is even" and "2 is not even"?

Notoriously, Carnap thought that there are two readings of QUESTION: one internal to a framework and the other external to it (Carnap 1950). According to Carnap, there is always a fact of the matter as to whether the internal reading of QUESTION receives a determinate answer. This is because every linguistic framework determines a logic. If, however, QUESTION is read as an external question, it is meaningless since there is no notion of validity detached from a particular framework.

Contrary to Carnap, B&R take it that there is no fact of the matter, even within a language, as to which is the correct answer to QUESTION:

> As a pluralist about logical consequence, I take it that there is no further fact of the matter as to whether explosion or disjunctive syllogism are *really valid*. For me, that question makes no more sense than to ask if a function on the real line is really smooth, without saying more about the notion of smoothness. A function might be smooth in the sense of continuity without being smooth in the sense of differentiability. The same goes with logical consequence. I take it that this is a pretheoretic notion which may be made precise in a number of different ways (Restall 2002: 426-27; italics added).

If we take "really valid" to express the thought that there is a fact of the matter concerning the truth of a validity judgement, what Restall is saying in the above passage is that, unless more is said to make the informal notion of *valid* precise, there are validity judgements for which there is no fact of the matter about the truth of the proposition involved. Let's apply this to MOON. MOON is a meaningful sentence of English, though there is no fact of the matter as to whether it is true. This is because different precisifications of the concept of logical consequence allow for different assessments of the

truth value of MOON. The question we will investigate in the following section is thus: what is the semantic status of sentences used to express validity judgements? Since we will use MOON as the working example for assessing this question, our question amounts to asking what is the semantic status of MOON.

3. *Indeterminacy Pluralism*

Given this setting, we submit that the proper way of understanding B&R's form of pluralism is by means of an indeterminacy claim about what "follows from" means in English:

INDETERMINACY: the concept of logical consequence is indeterminate.[7]

3.1. *Inadequate Models of Indeterminacy Pluralism*

There are different ways in which the notion of semantic indeterminacy can be modelled. One simple model for semantic indeterminacy is given by an ambiguity thesis:

AMBIGUITY: there are several concepts of logical consequence with no common feature except for the fact that they are all expressed by a univocal linguistic expression (e.g. "follows from" in English)

However, AMBIGUITY is inconsistent with the idea that there is conceptual common core given by GTT together with the three constraints (normativity, formality and necessity).[8]

Another model for semantic indeterminacy is given by partial definitions:

PARTIALITY: the concept of logical consequence is incomplete, in the sense that some arguments are neither valid nor invalid.

---

[7] See footnote 2 above. See Eklund (2017) for a similar interpretation of B&R's logical pluralism. For a contextualist interpretation of logical pluralism see Caret (2016). Although the contextualist reading is certainly a possible interpretation of B&R's central thesis, we think that, all things considered, the textual evidence that can be found in the book strongly favours an indeterminacy interpretation over a contextualist interpretation. In fact, in the book B&R make an explicit analogy between vagueness and the indeterminacy of GTT, and they use the language of indeterminacy with words such as "precisification" "unsettledness" (see especially B&R 2006: 27-29). Moreover, B&R (2006: 88) seems to reject a contextualist and relativist interpretation of their main thesis. Lastly, we think that the contextualist interpretation is in tension with the requirement of formality—on a standard indexical form of contextualism, "follows from" would express different contents in different contexts, and there would be no overarching concept of logical consequence valid in all contexts.

[8] See Keefe (2014: 1381) for a discussion of the inadequacy of the ambiguity interpretation of B&R's pluralism.

PARTIALITY doesn't seem the proper way of capturing the kind of indeterminacy at the core of B&R's pluralism either. To see why consider the following analogy: suppose it is stipulated that an argument is glorious if it is sound and convincing and that it is not glorious when it is not sound. What about arguments that are sound but not convincing? The stipulation is silent about such cases: if there is no other reason—beside the stipulation—for classifying the argument as glorious or not glorious, why should it be permissible to read off a plurality of candidates for being a glorious argument from such a stipulation? It seems just implausible to extrapolate a multiplicity of readings from the partial definition given by the stipulation. If we carry this analogy over to B&R's pluralism, indeterminacy pluralism cannot be grounded on PARTIALITY. In fact, PARTIALITY gives rise to a form of indeterminacy that does not plausibly motivate any plurality of candidates for logical consequence.

The indeterminacy claim hospitable to B&R's pluralism requires a conception of indeterminacy that allows for the possibility of different admissible ways for an argument to be valid. This multiple admissibility arises from the constitutive features of logical consequence—i.e. its settled core and role. Following a suggestion discussed by B&R themselves (B&R 2006: 27), the proper model for capturing INDETERMINACY is along the lines of a popular semantic model for vagueness according to which the indeterminacy of a vague expression is due to the existence of a multiplicity of equally admissible precisifications of it. Let us consider the following example: when I say that Aldobrando is bald, where Aldobrando is a borderline case of baldness, what I say is semantically indeterminate in that there are different equally admissible precisifications of "bald"—i.e. different ways of drawing the line between bald and non-bald people. If we keep in mind this model of indeterminacy, then INDETERMINACY can be supplemented as follows:

MULTIPLICITY There are different but equally admissible ways of precisifying the concept of logical consequence.

The multiplicity interpretation of B&R's pluralism predicts that the thoughts and practices of the speakers of the language, together with the non-linguistic facts, do not determine whether arguments such as MOON are valid.[9] In this respect, B&R's pluralism has the consequence that any claim of the form "argument X is valid" that would not hold in all of the admissible precisifications is indeterminate. As a further consequence, any reasoning involving these types of argument turns out

[9] See McGee and McLaughlin (1995: 214).

to be indeterminately valid. We argue that this causes problem for B&R's logical pluralism especially in connection with the normativity requirement they endorse (see section 6). However, before addressing this normative issue, we need to clarify the conception of semantic indeterminacy underlying MULTIPLICITY. Doing so will allow us to assess what the semantic status of validity judgements is. Mirroring the debate on vagueness, we first give two ways of conceptualizing MULTIPLICITY (section 4), we then articulate three semantic models of indeterminacy grounded in MULTIPLICITY (section 5), and finally we consider their normative consequences (section 6).

4 Two conceptions of MULTIPLICITY: Underspecification and Overspecification

There are at least two ways of conceptualizing MULTIPLICITY—either in terms of the notion of underspecification, or by means of its dual notion, namely that of overspecification.

According to the *underspecification* understanding of MULTIPLICITY as applied to the concept of logical consequence, we have that the thoughts and practices of the speakers of the language, together with the non-linguistic facts, underdetermine whether an argument is valid (Lewis 1986: 213). Applying this conception to MULTIPLICITY we obtain:

UNDERSPECIFICATION: the concept of logical consequence underspecifies the way in which it can be admissibly precisified. The validity of an inference is determinate just in case *every* admissible precisification of the concept of logical consequence validates that inference.

The dual of UNDERSPECIFICATION is the *overspecification* understanding of MULTIPLICITY—i.e. the idea that the thoughts and practices of the speakers of the language, together with the non-linguistic facts, overderdetermine whether an argument is valid. The idea of overdetermination is intended to capture a claim of semantic hyper-decision. Applying this conception to MULTIPLICITY we obtain:

OVERSPECIFICATION: the concept of logical consequence overspecifies the way in which it can be admissibly precisified. The validity of an inference is determined just in case *at least one* admissible precisification of the concept of logical consequence validates that inference.

Let us say that a piece of reasoning is *multiplicity-indeterminate* if its validity is indeterminate because of the truth of MULTIPLICITY. If your reasoning involves multiplicity-indeterminate sentences, it is indeterminate whether it exemplifies a valid argument. This might happen either because no specific

consequence relation is selected—UNDERSPECIFICATION—or because many consequence relations are selected—OVERSPECIFICATION. With this in hand, the urgent question for B&R is: what is the semantic status of validity judgements involving sentences that are multiplicity-indeterminate?

5. *Three Semantic Models of* INDETERMINACY

In this section, we consider three ways of semantically modelling INDETERMINACY given the two conceptions of MULTIPLICITY listed above. They correspond to what is known as *standard supervaluationism*, *subvaluationism* and *non-standard supervaluationism* (or *plurivaluationism*) in the vagueness literature.

Before discussing these models in some detail, we need to make the connection between a *validity judgement* and a *validity sentence* of a natural language L explicit. A *validity judgement* is a mental act presenting as true the proposition semantically expressed by a sentence—i.e. a validity sentence—of L stating that a certain conclusion follows from certain sentences (we are bracketing here limitations of expressibility of natural languages). To exemplify: the validity judgement that "The MOON is made of cheese" follows from "2 is even" and "2 is not even" involves the proposition semantically expressed by MOON in English. Given this assumption, in the following we will use the expression *J*-MOON as a shorthand for "the validity judgement directed towards what is semantically expressed in English by the validity sentence MOON".

We will also assume the following connection between sentential truth and propositional truth: a sentence S of language L is *true simpliciter* if and only if the proposition semantically expressed by S in L is true simpliciter—we are here bracketing issues related to context dependence and we are taking truth simpliciter of a proposition as truth of the proposition in the actual world.

5.1. *Gappy Underspecificationism*

UNDERSPECIFICATIONISM is naturally associated to standard supervaluationist semantics (Lewis 1970, 1993, Fine 1975, Keefe 2000). Truth is defined as truth under all the specifications, and falsity is defined as falsity under all the specifications. An underspecified sentence is true under some specifications and false under others, hence according to supervaluationist semantics the sentence is neither true nor false. Underspecification-driven indeterminacy gives rise to truth-value gaps. Let's consider an example: according to supervaluationism, if Aldobrando is a borderline case of baldness, there are specifications of "bald" under which Aldobrando belongs to the extension of "bald" and others under which he does not. Thus, if the sentence "Aldobrando is bald" says that Aldobrando is

bald, then "Aldobrando is bald" is neither true nor false.[10] The supervaluationist semantic machinery allows for determinate compound sentences with indeterminate components: if every specification is classically bivalent, then "Either Aldobrando is bald or Aldobrando is not bald" will be true in every specification and thus true *simpliciter* even though its disjuncts are neither true nor false. Let's call such a position *gappy underspecificationism*. Let's now apply gappy underspecificationism to B&R's pluralism. Consider again the following sentence:

(MOON) "The moon is made of cheese" follows from "2 is even" and "2 is not even"

MOON is true under some admissible specifications of "follows from": those specifications where GTT is instantiated in relation to cases that are either Tarskian models or constructions. However, there is at least one admissible precisification—i.e. the one that reads cases as situations—in which MOON is false since *explosion* fails. Thus, according to gappy underspecificationism, MOON is neither true nor false. Hence, given that MOON is gappy and assuming that it expresses a unique proposition, J-MOON is directed towards a proposition that is neither true nor false. Let us call *controversial validity sentences* those validity sentences that come out true in at least one, but not every, precisification of the concept of logical consequence. With this notion in hand, the general consequence of gappy underspecificationism is that all controversial validity sentences are neither true nor false, and the associated validity judgements are directed towards gappy propositions.[11]

5.2. *Classical Underspecificationism*

Another way of modelling UNDERSPECIFICATION does not commit us to truth-value gaps. This modelling is usually known as "non-standard" supervaluationist semantics or as "plurivaluationism" (Smith 2008, McGee & MacLaughlin 1994, Iacona 2010, Eklund 2010). According to non-standard supervaluationism, vague expressions are indeterminate in content. There are different precisifications of "bald" that capture equally well the meaning of "bald". Since it is indeterminate what "bald" means, the truth-value of "Aldobrando is bald"—where Aldobrando is a borderline case of baldness—is unsettled. Following Eklund (2010), the indeterminacy in question should not be understood in terms of the sentence possessing a third semantic value, beside truth and falsity (first-level indeterminacy). Rather, such indeterminacy should be understood in terms of the existence of a multiplicity of

[10] We are here bracketing indeterminacy arising from the context of use of the sentence.
[11] See also Keefe (2014: 1380 and footnote 7).

contents attached to an expression for which there is no fact of the matter which is the one that is semantically expressed (second-level indeterminacy). So, instead of being determinate that the sentence "Aldobrando is bald" is neither true nor false,[12] it is not determinate that the sentence is true and it is not determinate that the sentence is false: although the sentence has one of these truth-values it is indeterminate which one it has. The supervaluationist machinery allows for compound sentences with indeterminate components to receive a determinate semantic status: "Either Aldobrando is bald or Aldobrando is not bald" will be true in every specification and thus determinately true even though each disjunct is indeterminately true and indeterminately false. Although this option is consistent with the abandonment of classical semantics, we will call it *classical underspecificationism* since it is usually adopted with a classical semantics. The central idea of classical underspecificationism is thus that an expression is indeterminate because of the indeterminacy concerning what content it semantically expresses. According to the classical underspecificationism interpretation of B&R's pluralism, MOON indeterminately expresses a variety of propositions: a true proposition under the (admissible) classical specifications of "follows from", a false proposition under the (admissible) relevant precisification of "follows from". Since it is indeterminate which of the two propositions MOON semantically expresses, MOON is indeterminately true and indeterminately false (while being determinately either true or false).

Connecting second-level indeterminacy to judgements, the resulting picture would then be the following: logical consequence is an unspecified concept whose specification gives rise to a cluster of specified concepts. When controversial validity sentences are considered, this lack of specificity gives rise to acts which are related to a cluster of precisified propositions with different truth-values (e.g. *J*-MOON). Furthermore, there is no fact of the matter as to which of these propositions is the act directed towards. Call *MOON_x* a specified proposition related to an admissible precisification of logical consequence ("MOON_x" is a shorthand for the specified proposition <"The moon is made of cheese" follows_x from "2 is even" and "2 is not even">, where "follows_x" is an admissible specification of validity). Second-level indeterminacy gives then rise to the following indeterminacy theses:

(Judgement Indeterminacy MOON) It is indeterminate whether J-MOON is directed towards MOON_x.

More generally, according to classical underspecificationism all controversial validity sentences are indeterminately true and indeterminately false and determinately either true or false (but not both true

[12] We are here ignoring issues connected to higher-order vagueness.

and false). Moreover, the related validity judgements are indeterminately directed to a cluster of propositions.

5.3. *Glutty Overspecificationism*

Let's now consider overspecified sentences. Subvaluationist semantics is the machinery generally used to model overspecification. Such a machinery gives rise to truth-value gluts—i.e. to sentences that are both true and false.[13] Truth is defined as truth under at least one specification, and falsity is defined as falsity under at least one specification. An overspecified sentence is true under some specifications and false under others. Hence, by means of the subvaluationist machinery it is both true and false. Subvaluationism is a paraconsistent theory in the sense that a sentence might both be true and false without however generating triviality. According to subvaluationism, if Aldobrando is a borderline case of baldness there are specifications of "bald" under which Aldobrando belongs to the extension of "bald" and others under which he does not. Thus, if the sentence "Aldobrando is bald" says that Aldobrando is bald, then "Aldobrando is bald" is both true and false. However, according to the subvaluationist semantics a conjunction in which both conjuncts are glutty—i.e. both truth and false—does not inherit the glutty status—i.e. it is not itself both true and false but false only. Thus, if every specification is classically bivalent, then "Aldobrando is bald and Aldobrando is not bald" will be false *only* in every specification and thus false simpliciter even though its conjuncts are both true and false. Let's call such position *glutty overspecificationism*. According to the glutty overspecificationism interpretation of B&R's pluralism, given that MOON is true under some admissible specifications of the concept of logical consequence and false under others, MOON is both true and false. Hence, given that MOON expresses a unique glutty content, J-MOON is directed towards a proposition that is both true and false. More generally, all controversial validity sentences are both true and false.

We would like to conclude this section by highlighting what the connection between classical underspecificationism and glutty overspecificationism is. Although they both are forms of semantic indeterminacy, there is an important difference between the two. According to classical underspecificationism, controversial validity sentences turn out to have an unsettled semantic status—these sentences are indeterminately true and indeterminately false. According to glutty overspecificationism, controversial validity sentences have a settled semantic status—i.e. they are both true and false. Moreover, the logic in the background of these options is different: whereas classical

[13] Subvaluationism has been advanced as a theory of vagueness by Hyde (1997). Hyde (2010) explores in details issues related to the duality between subvaluationism and supervaluationism.

underspecificationism is consistent with classical logic, glutty overspecificationism requires paraconsistency.

6. *Varieties of Indeterminacy Pluralism and the Normativity of Logic*

In this section, we will lay out some normative principles governing judgement in relation to validity sentences, and we will discuss the consequences that adopting any of the three conceptions of indeterminacy outlined in the previous section have on the normative status of validity sentences. In section 6.1 we articulate the truth norm and the so-called bridge principles that are meant to capture the normativity of logic. In section 6.2 we argue that this normative setting gives rise to two problems for the gappy underspecificationist interpretation of B&R's pluralism: (i) endorsing this interpretation seems to preclude a commitment to a genuine pluralistic stance towards the notion of logical consequence (we call this the *Permissibility Problem*); (ii) within this model it is hard to make sense of the normative guidance provided by the bridge principles (we call this the *Absence of Guidance Problem*). In section 6.3 we argue that this guidance problem affects also Classical Underspecificationism and that moreover this view is either hostage to the Permissibility Problem or to a normative version of the so-called Collapse Problem for logical pluralism. Finally, in section 6.4 we critically assess the glutty overspecificationist interpretation of B&R's pluralism. We argue that this interpretation is vulnerable to both the Collapse Problem and a specific problem related to the normative guidance of logic that we call the *Normative Conflict Problem*.

6.1. *Normative Principles*

We outline here a normative setting for judgement and deductive inference. If we call *J-P* the act of judging that P, we take on board the following truth-norm:

(TN) J-P is permissible if and only if <P> is true.[14]

Logic is normative, or so it is often claimed.[15] In particular, logic is taken to provide thinkers with rules of thought and thus to guide them in reasoning. Although this understanding of the normative role

[14] Where "<P>" names the proposition that P.

[15] The idea that logic is normative has a long tradition in analytic philosophy. Frege conceived of logic as laying down the laws of thought, not as a theory for describing our psychology of reasoning but as laying down the correct ways of

of logic has been contested by some philosophers (e.g. Harman 1984, 1986), it is generally considered the standard interpretation of the sense in which logic is normative.[16] In this paper, we assume the standard interpretation. B&R seem to agree with the thesis that logic is normative since they include among the core features of logical consequence its normative role (B&R 2006: 16-18). But in what exactly does the normativity of logic consist in? Unfortunately, they do not say much about this important issue—all they have to offer in terms of a positive specification of what the normativity of logic consists in is the following principle: "it is a mistake to assert the premises of a valid argument while denying the conclusion" (B&R 2006: 18). However, recent literature on the normativity of logic (MacFarlane 2004; Steinberger 2015, 2016; Field 2009) has helped to shed light on this intricate issue. Following the lead of MacFarlane (2004) and Steinberger (2016: 16-17) we can state (at least) two dimensions which are relevant to assess the normative status of judgements involving validity sentences. The first dimension has to do with the scope of the normative operator (we only focus here on the deontic formulation cashed out in terms of 'ought'), which can be either narrow or wide. The second dimension has to do with whether some doxastic restrictions are imposed or not—i.e. with whether the principles should somehow reflect the doxastic state of the subject (subjective reading) or not (objective reading). Accordingly, we have the following four formulations:

(N1) If B follows from A1, … , An, then (if S judges all the Ai*s*, S ought to judge that B).

(N2) If S recognizes that B follows from A1, … , An, then (if S judges the Ai*s*, S ought to judge that B).

(W1) If B follows from A1, … , An, then S ought to (if S judges all the Ai*s*, then S judges that B).

(W2) If S recognizes that B follows from A1, … , An, then S ought to (if S judges all the Ai*s*, then S judges that B).

(deductive) reasoning (see Mezzadri 2015a, 2015b). For a contrary view see Harman (1984, 1986). See also MacFarlane (2004), Field (2009), and Steinberger (2015, 2016) for some recent discussions of the normative conception of logic.

[16] One might for instance think that logic is indeed normative, and perhaps intrinsically so, but in a much weaker sense than that assumed by the standard interpretation. It might be claimed that the normativity of logic is exhausted by mere criteria of correctness. In this sense logic would provide a tool for distinguishing between correct and incorrect arguments without however giving us any guidance on how we ought to reason. See Ferrari (forthcoming) and Steinberger 2016, 2017 for some useful distinctions in normative functions.

In what follows we will consider only wide-scope formulations of normativity since consensus is gathering in the literature that narrow scope formulations fall prey to what are known as *Harman's challenges* (Harman 1984, 1986).[17] In particular, for the purposes of our discussion we will stick with the simpler principle W1, since it is immaterial what wide-scope formulation is adopted. We now turn to a discussion of how the truth norm and wide scope principles about logic relate to the three models of INDETERMINACY that might underwrite B&R's logical pluralism.

6.2. *Gappy Underspecificationism and Normativity*

Let's consider gappy underspecificationism first. Given that by TN the truth of a proposition is a necessary condition for the permissibility of judging it, it follows that it is impermissible to judge a proposition that comes out as underspecified. Take any controversial validity sentence such as MOON. According to gappy underspecificationism it is not permissible to judge any proposition semantically expressed by such controversial validity sentence. On the contrary, validity sentences that are true in every admissible precisification—i.e., those that are uncontroversial—are permissible to judge. Take conjunction introduction, which is valid in classical, relevant and intuitionistic logic:

(EARTH) "The earth is flat" follows from "The earth is flat and the moon is made of cheese"

If we model B&R's pluralism with gappy underspecificationism, EARTH is true simpliciter since it is true in every admissible precisification of the concept of logical consequence. This is granted by the fact that among the logics that are admissible within their pluralist framework, conjunction elimination is always valid. Hence it is permissible to judge EARTH but impermissible to judge MOON.

We have thus what we think is a first unwelcome consequence for indeterminacy pluralism. We take it that a genuine pluralistic stance towards the notion of logical consequence should involve some kind of acceptance of controversial validity sentences. This is because, according to logical

[17] Note that wide scope principles might require some improvement in order to address one of the aspect of Harman's Challenges—namely, what is generally referred to as the *excessive demand* objection according to which formulations like (W1) and (W2) entail that a subject is under the normative requirement to judge all the logical consequences of her current judgements. There are various ways in which these wide-scope bridge principles can be amended to take care of this issue (see, for instance, MacFarlane 2004 and Steinberger 2016). A rather simple way to do so is to reformulate the wide-scope principle as follows:

(W*) If [S recognizes that] B follows from A1, … , An, S ought to (if S believes all the Ai, then S *does not disbelieve* B)

For the purpose of this paper we won't take a stand on this matter and in order to avoid making things unnecessarily complicated, we will ignore the potential problem generated by the excessive demand objection. Thanks to Erik Stei for pointing this out to us.

pluralism the various admissible notions of logical consequence are all equally good and equally normatively relevant to deductive reasoning. A genuine pluralist stance towards the notion of logical consequence is one that not only admits of a plurality of notions of logical consequence, but it also committed to maintain that none of these notions is overall better than any of the other admissible notions. This commitment is in tension with the prediction given by the gappy underspecificationist interpretation of B&R's pluralism according to which it is impermissible to judge any controversial validity statements as true. This is of course highly problematic given that controversial validity claims are exactly those claims involving the various notions of logical consequences that a pluralist theory should regard as equally admissible. In this respect, the impermissibility of judging them as true, as predicted by gappy underspecificationism, seems to imply the rejection of the equal admissibility of these notions. Let's call this the *Permissibility Problem*.

What should we say about the normative principle W1? Let's consider a controversial validity sentence such as MOON. We have seen that according to the gappy underspecificationist understanding of logical pluralism, it is not permissible to judge MOON. Given that a subject is not permitted to judge MOON, what is left of the rational requirement expressed by W1? Such a rational requirement tells us that it ought to be the case that if a subject both judges that 2 is even and that 2 is not even she judges that the moon is made of cheese. Let's consider the relevant instance of W1:

(W1- MOON) If "The moon is made of cheese" follows from "2 is even" and "2 is not even", then S ought to (if S judges that 2 is even and S judges that 2 is not even, then S judges that the moon is made of cheese).

Given that the antecedent of W1- MOON is untrue, its consequent cannot be discharged. Thus, no guidance coming from the normativity of logic is provided. More generally, every controversial validity sentence falls outside the scope of the rational requirement imposed by the normativity of logic. What about the semantic status of W1- MOON? Presumably in those precisifications where ""The moon is made of cheese" follows from "2 is even" and "2 is not even"" is true, the ought-sentence should be true. In the precisifications where the ought-sentence is presumably false, the antecedent of W1 is false, thus making the relevant conditional true in those precisifications. Following this line, there is no precisification where W1- MOON is false, thus making it true simpliciter. We can illustrate the situation by means of the following *informal* reasoning:

(1) If "The moon is made of cheese" follows from "2 is even" and "2 is not even", S ought to (if S judges that 2 is even and S judges that 2 is not even, then S judges that the moon is made of cheese) [W1- MOON]
(2) It is neither true nor false that ""The moon is made of cheese" follows from "2 is even" and "2 is not even"" [Gappy Underspecificationism]
(3) It is not permissible to judge that "The moon is made of cheese" follows from "2 is even" and "2 is not even" [from 2 and TN]
(4) The rational requirement occurring in the consequent of (1) cannot be detached. [1,2]
(5) Suppose S judges that 2 is even and S also judges that 2 is not even.
(6) Can S rationally abstain from judging that the moon is made of cheese?
(7) Nothing from (1) to (3) dictates the contrary. [4]

This reasoning gives rise to a second, potentially troublesome, consequence for indeterminacy pluralism, namely that the subject is left in a situation of complete *normative silence* with respect to any controversial validity sentence.[18] This means that there is no normative constraint operative with respect to controversial validity sentences that could provide some guidance to the subject. Let's call this the *Absence of Guidance Problem.*

6.3. *Classical Underspecificationism and Normativity*

Contrary to gappy underspecificationism, for classical underspecificationist the semantic status of controversial validity sentences is indeterminate since it is indeterminate which proposition they express. What is then the proper attitude to have in such cases? A natural proposal is that it is indeterminate which is the appropriate attitude one ought to have with respect to indeterminate cases (Dorr 2003 and Williams 2017). Classical underspecificationism thus maintains that it is indeterminate whether we ought to/are permitted to judge any of the propositions related to an underspecified sentence. So, classical underspecificationism involves indeterminacy as to what is permissible or impermissible to judge in relation to an indeterminate sentence.

Let's illustrate again the position in relation to MOON. If MOON is indeterminate, then it is indeterminate whether we ought to judge that MOON since there is a multiplicity of propositions to which the act of judging is indeterminately related to (the propositions expressed by the admissible

[18] The notion of normative silence is first introduced and developed by Williams (2012).

precisifications of "follows from") and some of these propositions are true whereas others are false. Following the conventions endorsed in section 5.2, let's call *MOON_x* one of these propositions and *J-MOON* the act of judging that MOON. According to classical-underspecificationism there is no fact of the matter as to whether J-MOON is directed towards MOON_x (Judgement Indeterminacy MOON). But if there is no fact of the matter as to whether J-MOON is directed towards MOON_x, then TN implies that there is no fact of the matter as to whether J-MOON is permissible. This is the Permissibility Problem for Classical Underspecificationism.

To be clear: we are not denying that we can permissibly judge MOON_x type propositions. Once we classically precisify the concept of "follows from" as expressing the classical proposition MOON_c, we can correctly judge that MOON_c is true (i.e. that explosion is valid classically). Rather, we are denying that when we make judgements analogous to what we assert when we assertively utter the English sentence MOON, our mental act J-MOON is determinately permissible. J-MOON is indeterminately permissible because it is indeterminately directed towards a cluster of propositions. This indeterminacy claim is intended to be perfectly analogous to the indeterminacy that we have with the linguistic act of asserting MOON: our assertive utterance of the English sentence MOON is indeterminately permissible since MOON indeterminately expresses a proposition among a cluster of propositions. If we take the language of thought hypothesis on board, we could say that J-MOON is a mental act of accepting as true a sentence of *Mentalese* (the mental analogous of MOON) indeterminately expressing one proposition among a cluster of propositions.

What should we say about W1 in the case of Classical Underspecificationism? Even with this approach it seems that we get an unwelcome consequence for indeterminacy pluralism. Again, the normative principle for logic do not offer any guidance to a subject who believes the premises of a controversially valid argument. In other words, another version of the *Absence of Guidance Problem* can be given:

(8) If "The moon is made of cheese" follows from "2 is even" and "2 is not even", S ought to (if S judges that 2 is even and S judges that 2 is not even, then S judges that the moon is made of cheese) [W1-MOON]

(9) It is indeterminate whether ""The moon is made of cheese" follows from "2 is even" and "2 is not even"" is true [Classical Underspecificationism]

(10) The rational requirement occurring in the consequent of (8) cannot be detached. [8,9][19]

(11) Suppose S judges that 2 is even and S believes that 2 is not even.

(12) Can S rationally abstain from judging that the moon is made of cheese?

(13) Nothing from (8) to (9) dictates the contrary. [10]

Before moving to Glutty Overspecificationism we would like to consider an objection[20] to the normative setting we have chosen for classical underspecificationism. The classical underspecificationist might protest that the proper truth norm for her setting has to be relativized to precisifications:

(LIBERAL) it is permissible to judge that S if, in *some* of the admissible precisifications of "S", "S" expresses a true proposition.[21]

LIBERAL clearly avoids the Permissibility Problem since it allows us to permissibly judge that MOON. However, we think that LIBERAL applied to the classical underspecificationist interpretation of logical pluralism falls prey of a normative interpretation of what is known as the *Collapse Problem*.[22] To appreciate the point, first note that the consequent of (8) expresses the normative requirement for judging the conclusion of a classically valid argument, once the premises are judged true. Since MOON expresses a true proposition in the classical precisification of "follows from", it is always permissible to reason according to MOON. More generally, for every controversial validity sentence there is always a normative preference for reasoning in accordance to classical logic since it is the strongest logic—

[19] In non-standard supervaluationism the inference from "if P, then Q" and "Indeterminate whether P" to "Q" is plausibly invalid. The reason for this relies on the following considerations. First, the proper notion of validity for non-standard supervaluationism is related to truth-preservation, where truth is not equivalent to determinate truth. Second, given the first point, one way to express validity for non-standard supervaluationism is the so-called local notion of validity: an inference A is valid if and only if for any precisifications X, if the premises of A are true in X then the conclusion of A is true in X (see Varzi 2007). Given this notion of validity, the inference from "if P, then Q" and "Indeterminate whether P" to "Q", counts as invalid since there is a precifisification where its premises are true but its conclusion false. In fact, consider a precisification where "Indeterminate P" is true and "if P then Q" is true, and where "P" is false and "Q" is false. In such a precisification "Indeterminate P" can be true because there is another precisification where "P" is true.

[20] Many thanks to Matti Eklund for pressing us on this point.

[21] See Eklund (2010) for proposing such norm in relation to vagueness.

[22] The collapse problem can be formulated as follows: if all controversial validity sentences are true, then classical logic dominates over the other logics admissible within B&R's framework since it is stronger than relevant logic and intuitionistic logic. If that's the case, why shouldn't a subject inferring classically from known true premises if classical logic correctly guarantees that a true conclusion follows? Pluralism risks to collapse into monism. For a discussion of this issue see, e.g., Priest (2001), Read (2006), Keefe (2014), Stei (2017). Caret (2017) argues that the collapse problem arises also with a normative framework using wide scope principles for the normativity of logic. Thanks to Nikolaj Pedersen for making us aware of the relevance of the collapse problem in this context.

i.e. the logic that respects every controversial validity sentence and that validates all the validity sentences that are validated by intuitionistic and relevant precisifications. Hence, there is a normative preference for reasoning classically than relevantly or intuitionistically since by reasoning classically LIBERAL says that it is never impermissible to reason in accordance to controversial validity sentences, and it is never forbidden to reason in accordance to principles that are valid in the other two non-classical logics. This situation generalises whenever we have that among the precisifications "follows from" there is a logic that is stronger than the others. Assuming classical underspecificationism in conjunction with LIBERAL, we thus incur in a normative version of the collapse problem for logical pluralism.

6.4. *Glutty Overspecificationism and Normativity*

Last, let's consider glutty overspecificationism. By adopting TN, it follows that for gappy underspecificationism it is permissible to judge a proposition expressed by an overspecified sentence. We should note, however, that an overspecified sentence can only be *merely permissible* to judge.[23] By *mere permissibility* we mean the existence of a kind of permissibility which is incompatible with the existence of a corresponding obligation. Take an overspecified sentence expressing the proposition that p. Suppose that there is an obligation to judge that p. Since not-p is true, by TN, we are permitted to judge that not-p. But a permission to judge that not-p and an obligation to judge that p are clearly inconsistent normative requirements. So, there cannot be an obligation to judge that p. By the same reasoning, there cannot be an obligation to judge that not-p either. Let us call such propositions *merely permissible*. Glutty overspecificationism involves that indeterminate sentences can be merely permissibly judged. In this respect, the situation, though not ideal, looks more promising than underspecificationist models since there is no analogous of the *Permissibility Problem* here. Moreover, we also think that there is no analogous of the *Absence of Guidance Problem* for glutty overspecificationism. However, we think that glutty overspecificationism faces two potentials worries: the first is what we call the *Normative Conflict Problem*, while the second is the normative version of the collapse problem we have mentioned at the end of the last section.

In order to illustrate these two problems, let us consider MOON again. According to glutty overspecificationism MOON is indeterminate because both true and false. It is thus *merely* permissible

[23] Hansson (2013: 199) calls this type of permission "bilateral permission".

to judge MOON and, thus, it is not obligatory to judge MOON. So, what ought a subject to infer given the normativity of logic in such a situation?

(14) If "The moon is made of cheese" follows from "2 is even" and "2 is not even", S ought to (if S judges that 2 is even and S judges that 2 is not even, then S judges that the moon is made of cheese) [W1- MOON]

(15) It is both true and false that "The moon is made of cheese" follows from "2 is even" and "2 is not even" [Glutty Overspecificationism]

(16) S ought to (if S judges that 2 is even and S judges that 2 is not even, then S judges that the moon is made of cheese) [14, 15]

(17) It is merely permissible to judge that "The moon is made of cheese" follows from "2 is even" and "2 is not even" [from 15 and TN]

(18) It is not the case that S ought to judge that "The moon is made of cheese" follows from "2 is even" and "2 is not even" [17]

(19) Suppose S judges that 2 is even and S judges that 2 is not even.

(20) Can S rationally abstain from judging that the moon is made of cheese?

(21) According to (16) she cannot.

(22) Given her judgement [19], S has a normative requirement to judge that the conclusion of the argument holds *and* at the same time it is not the case that S ought to judge that the conclusion logically follows from the premises. [16, 18]

This reasoning highlights an odd normative consequence of the glutty overspecificationist interpretation of indeterminacy pluralism in relation to controversial validity sentences. In fact, there seems to be a tension between the normative prediction expressed by the first conjunct of (22) and the one expressed by the second conjunct. On the one hand, the subject is under the normative requirement of judging the conclusion of a controversial validity sentence—e.g. to judge that the moon is made of cheese. On the other hand, the subject is only merely permitted to judge that the conclusion that the moon is made of cheese follows from the premises. However, since it is compatible also with the mere permissibility to abstain from judging that the conclusion follows from the premises, there seems to be a potential tension in the normative situation of the subject—this illustrates what we call the *Normative Conflict Problem*.

As for the normative version of the Collapse Problem, the reasoning is perfectly analogous to that outlined at the end of the previous section. Note that (16) expresses the normative requirement for endorsing the conclusion of a classically valid argument, once the premises are judged to be true. Hence, there is a normative preference for reasoning classically than relevantly. Assuming glutty overspecificationism, this situation generalises. Since every controversial validity sentence is a glut, there is always a normative preference for reasoning in accordance to the stronger logic—i.e. the logic that validates all controversial validity sentences.

7. *Conclusions*

We have accomplished three things in this paper. First, we have offered a detailed interpretation of B&R's logical pluralism as a thesis of semantic indeterminacy of our concept of logical consequence—i.e. as a form of *indeterminacy logical pluralism*—and we have provided some considerations in favour of this interpretation. Second, we have discussed three models of semantic indeterminacy that we think are fit to capture B&R's indeterminacy logical pluralism. Third, we have raised a series of normative problems for indeterminacy logical pluralism—i.e. the *Permissibility Problem* and the *Absence of Guidance Problem* targeting the gappy and classical underspecificationist interpretations of indeterminacy pluralism and the *Normative Conflict Problem* and (a normative version of) the *Collapse Problem* for the glutty overspecificationist interpretation of indeterminacy pluralism.

The overall conclusion that we have argued for is that B&R's logical pluralism cannot offer an adequate account of the normative guidance that logic is expected to provide us with in ordinary contexts of reasoning. This conclusion depends on one crucial assumption—i.e. that whatever normative constraint logic might exert, it should be understood in terms of some sort of guidance over reasoning. One plausible strategy for indeterminacy pluralist would be to reject this assumption by either denying that logic is normative or by arguing that logic's normative function is of a weaker kind that is not meant to provide any guidance over reasoning. The first option would require us to abandon *normativity* as a core feature of the concept of logical consequence—we take this to be an unhappy choice for indeterminate pluralists.[24] The second option looks more promising. The basic thought would be to distinguish between different dimensions in which logic can be normatively significant for reasoning,[25] and argue that if logic is normative at all, it is normative only in a criterial,

[24] See Russell (2017).
[25] Ferrari (forthcoming) has distinguished between three dimensions of normativity: *criterial, axiological* and *deontic*. See also Steinberger (2015) for some further useful distinctions in normativity.

non-guiding, sense. Of course, much more need to be said about this non-guiding normative way in which logic can be normative—but this is material for another project.

References

Beall, Jc & Restall, G. (2006) *Logical Pluralism*, New York: Oxford University Press.

Caret, C. (2017) "The Collapse of Logical Pluralism has been Greatly Exaggerated", *Erkenntnis*, 82 (4): 739–760.

Carnap, R. (1937) *The Logical Syntax of Language*, London: K. Paul, Trench, Trubner & Co.

Carnap, R. (1950) "Empiricism, semantics, and ontology", *Revue Internationale de Philosophie* 4 (11): 20-40.

Dorr, C. (2003) "Vagueness Without Ignorance", *Philosophical Perspectives* 17 (1):83–113.

Eklund, M. (2010) "Vagueness and Second-Level Indeterminacy", In Dietz, R. & Moruzzi, S. (eds.), *Cuts and Clouds: Vagueness, its Nature and its Logic*, Oxford: Oxford University Press: 63–76.

Eklund, M. (2017) "Making sense of logical pluralism", *Inquiry*, DOI:10.1080/0020174X.2017.1321499.

Ferrari, F. (forthcoming) "Normative (Alethic) Pluralism", in Kellen, N., Pedersen, N.J.L.L., Wyatt, J. (eds), *Pluralisms: Truth and Logic*, Palgrave Macmillan.

Field, H. (2009) "What is the Normative Role of Logic?", *Proceedings of the Aristotelian Society* (SV) 83: 251-68.

Fine, K. (1975) "Vagueness, truth and logic", *Synthese* 30 (3-4): 265–300.

Hansson, S. O. (2013) "The varieties of permission", In Gabbay, D., Horty, J. F., van der Meyden, R. & van der Torre, L. (Eds.), *Handbook of deontic logic and normative systems* (Vol. 1), London: College Publications.

Harman, G. (1984) "Logic and Reasoning", *Synthese* 60 (1):107–27.

Harman, G. (1986) *Change in View*, Cambridge(MA): MIT Press.

Hyde, D. (1997) "From heaps and gaps to heaps of gluts", *Mind,* 106 (424): 641–60.

Hyde, D. (2010) "The Prospects of a Paraconsistent Response to Vagueness", In Dietz, R. & Moruzzi, S. (eds.), *Cuts and Clouds: Vagueness, its Nature and its Logic,* Oxford: Oxford University, Press: 385–405.

Iacona, A. (2010) "Saying More (or Less) than One Thing" In Dietz, R. & Moruzzi, S. (eds.), *Cuts and Clouds: Vagueness, its Nature and its Logic,* Oxford: Oxford University, Press: 289–303.

Keefe, R. (2000) *Theories of Vagueness*, Cambridge: Cambridge University Press.

Keefe, R. (2014) "What logical pluralism cannot be", *Synthese,* 191 (7): 1375–90.

Lewis, D. (1970) "General Semantics", *Synthese,* 22: 18–67.

Lewis, D. (1986) *On the Plurality of Worlds*, Oxford: Blackwell.

Lewis, D. (1993) "Many, but almost one", In Cambell, K., Bacon, J. & Reinhardt, L. (eds.), *Ontology, Causality, and Mind: Essays on the Philosophy of D. M. Armstrong*, Cambridge: Cambridge University Press: 23–38.

MacFarlane, J. (2004) "In What Sense (If Any) Is Logic Normative for Thought?", unpublished typescript, (http://johnmacfarlane.net/normativity_of_logic.pdf)

McGee, V. & McLaughlin, B. (1995) "Distinctions Without a Difference", *Southern Journal of Philosophy,* 33 (S1): 203–251.

Mezzadri, D. (2015a) "Frege on the Normativity and Constitutivity of Logic for Thought I", *Philosophy Compass,* 10 (9): 583–91.

Mezzadri, D. (2015b) "Frege on the Normativity and Constitutivity of Logic for Thought II", *Philosophy Compass,* 10 (9): 592–600.

Priest, G. (2001) "Logic: One or many?" In Woods, J. & Brown, B. (eds.), *Logical consequences: Rival approaches*, Oxford: Hermes Scientific Publishers: 23–38.

Read, S. (2006) "Monism: The One True Logic", In de Vidi, D. & Kenyon, T. (eds.), *A Logical Approach to Philosophy: Essays in Memory of Graham Solomon*, Berlin: Springer: 193–209.

Restall, G. (2002) "Carnap's Tolerance, Meaning, and Logical Pluralism", *Journal of Philosophy,* 99 (8): 426–43.

Russell, G. (2017) "Logic isn't Normative", *Inquiry*, DOI: 10.1080/0020174X.2017.1372305.

Shah, N. & Velleman, D. (2005) "Doxastic Deliberation", *Philosophical Review,* 114 (4): 497–534.

Shapiro, S. (2014) *Varieties of Logic*, Oxford: Oxford University Press.

Smith, N. J. J. (2008) *Vagueness and Degrees of Truth*, Oxford: Oxford University Press.

Stei, E. (2017) "Rivalry, normativity, and the collapse of logical pluralism", *Inquiry*, DOI: 10.1080/0020174X.2017.1327370.

Steinberger, F. (2015): "Three Ways Logic May be Normative" unpublished manuscript (http://floriansteinberger.weebly.com/uploads/5/7/9/5/57957573/three_ways_in_which_logic_might_be_normative.pdf)

Steinberger, F. (2016) "Explosion and the Normativity of Logic", *Mind,* 125 (498): 385–419.

Tarski, A. (1956) *Logic, Semantics, Metamathematics*, Oxford: Clarendon Press.

Varzi, A. C. (2007) "Supervaluationism and Its Logics", *Mind,* 116 (463): 633–676.

Williams, J. R. G. (2012) "Indeterminacy and Normative Silence", *Analysis*, 72 (2):217-225.

Williams, J. R. G. (2017) "Indeterminate Oughts", *Ethics,* 127 (3): 645–673.