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Designing and implementing materials on quantum computing for secondary school students: The case of teleportation

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The article sets out to contribute to the educational challenge launched by the second quantum revolution. An approach to quantum computing has been outlined for secondary school students. The approach is shaped as a set of four principles that have been pointed out to design instructional materials to enhance the educational and cultural potential of quantum computing, beyond the technical aspects. In the paper, following a presentation of the design principles, we focus on an activity on quantum teleportation protocol. The activity is described in detail to show how the principles have been concretely implemented. A pilot study with a small sample of secondary school students has been carried out to evaluate the potential of the approach. The results, although preliminary, show that the approach appears to be promising in creating an inclusive and productive learning environment, in which students feel encouraged to search for personal ways to combine different levels of discourse (narrative, logical, mathematical, technical-experimental) and generate meaningful descriptions of the phenomenon.

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I. INTRODUCTION

"We are currently experiencing a 'second quantum revolution.'" This is the opening sentence of the European quantum technologies flagship. The Quantum flagship is a document aimied principally at promoting the knowledge and awareness needed for making Europe a dynamic and attractive region for stimulating research, business, and investments in quantum technologies [1].

The word "revolution" is used to stress the quality and entity of the change that quantum technologies are driving.

In this paper, we present a research-based educational approach that we designed in order to produce materials on quantum computing for secondary school students. In Sec. II, we frame the study within the current debate on the relevance of quantum technologies in our society and the need to develop educational approaches to reach a K–12 audience. After the presentation of the context of the study (Sec. III), the research background and fundamental ideas of our educational approach are presented (Sec. IV). Sections V and VI represent the core of the paper, since they describe the procedure we followed to implement the approach. The procedure is exemplified through the specific use of a teleportation case study. By referring to this case, we show how advanced and hyper-specialized research papers have been transformed into secondary school teaching materials aligned with the four design principles. Section VII reports the results of a pilot study carried out to collect and analyze preliminary reactions of students to the materials.

II. TEACHING QUANTUM TECHNOLOGIES: CONTEXTS AND OPEN QUESTIONS

The second quantum revolution is already taking place from the point of view of research, and it is expanding into many other dimensions including education. Until recently, quantum technologies were mainly an academic issue. In Europe, at least, they were mainly addressed in advanced master's degree physics courses or, sometimes, master's degree programs in computer science. The request from the quantum flagship to expand the workforce (QF agenda) has moved attention to the need to definitively shift the topic of quantum technologies (from an educational point of view) out of the physics departments and rethink it to involve and prepare the next generations of quantum technology experts [2]. One of the biggest challenges, as the strategic agenda for the quantum flagship asserts, "consists in developing and evaluating effective training and educational modules for a variety of learners in the areas that traditionally do not get in touch with quantum physics (e.g., engineering, computer science, mathematics)." [3].

This is a pivotal point not only in Europe. For example, the National Quantum Initiative (NQI) Act states its primary purpose as "to expand the number of researchers,

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educators, and students with training in quantum information science and technology to develop a workforce pipeline." [4].

In the following we present an overview of the books, materials, and tools on quantum technologies that have been produced for higher education and for secondary school.

In higher education, there are an increasing number of books dealing with quantum computing and quantum information that are suitable for different categories of university students. Among the most popular are *Quantum Computation and Quantum Information* by Nielsen and Chuang [5], *An Introduction to Quantum Computing* by Kaye, Laflamme and Mosca, *The Physics of Quantum Information* by Bouwmeester and Zeilinger, and *Quantum Computer Science* by Mermin. There are also different online resources designed for physicists, mathematicians, computer scientists, and engineers. One of the most comprehensive is the online course by Preskill [6].

In the last few years, the strong and intrinsic interdisciplinary character of these new technologies has been increasingly discussed and many books have been published on the subject (e.g., quantum science and technology book series by Djordjevic and Lele).

Regarding secondary school level, despite the challenges and political requirements encountered to "run educational programs for a new generation of technicians, engineers, scientists, and application developers in quantum technologies" [2], there are still few materials and books. The books mainly focus on developing the basic mathematical and physical knowledge to prepare students who have not previously studied quantum physics to approach quantum technologies. These books usually start by introducing basic mathematical tools, such as complex numbers, vectors, and matrices. Then, the books focus on the transition from binary to quantum logic, by stressing the difference between bit and qubit. The difference is illustrated by introducing the basic concepts of quantum physics (superposition principle, state, evolution of the state, measurement, and entanglement), and by describing the postulates of the theory [7,8].

Research centers, like Qutech (an advanced research center for Quantum Computing and Quantum Internet), and universities, like Aarhus University and the University of St Andrews, are developing precious materials to promote comprehension [9–12] by leveraging students' needs of visualization [13–17]. The materials include quantum simulations, visualization tools, and videos on concepts and advanced quantum processes, like cryptography and quantum distillation. These interactive tools on quantum physics are suitable both for physics and nonphysics students, and for secondary school students.

From the point of view of research on teaching quantum technologies at the high school level, there are still few papers. Some papers concern teaching quantum physics and consider quantum technologies as a context in which to apply the concepts of quantum physics and/or to reflect about technological perspectives [18,19]. The most recent papers published on the teaching of quantum technologies focus, mainly, on quantum computation, and the qubit and new logic (also in terms of circuits and logic gates) are the pillars on which modules are developed for curricular and extracurricular courses [20,21].

Our study intends to contribute to the goal of producing research-based materials while also addressing these broader research questions:

a. Which teaching approach can be developed to enhance the educational potential of technologies even beyond technical training?
b. Which educational potential and educational principles can be highlighted? For what purposes?

The questions are challenging since they do not only require the design of approachable resources for teaching effectively knowledge and skills on quantum technologies. They also require us to find ways to make students and citizens aware of the quantum revolution. This deep change induced by quantum technologies touches both the relationship between science and society [22–24] and the foundations of our thinking. Indeed, quantum physics challenges the classical Aristotelian logic that reached its peak with Boolean algebra [25–29]. In this sense, quantum computing conveys a profound conceptual change since it challenges the binary logic on which classical computers are based.

As we will explain, our approach centralizes the logical layer of reasoning and the educational potential of quantum technologies to move beyond the idea that all information can be codified in a bit and processed in terms of classical logic gates. After the presentation of the approach and the educational principles that characterize it, we will show their implementation in the design of a specific activity on teleportation that has been tested with secondary school students.

III. THE EDUCATIONAL CONTEXT OF THE STUDY

The study reported in this paper is part of a broader research task aimed at developing a 20-hour module on quantum computing.

The module has been developed within a European Erasmus + Project titled "Inclusive STEM Education to Enhance the capacity to aspire and imagine future careers" (I SEE). The project finished in 2019 and involved seven partners, with the University of Bologna acting as coordinator. The module was first designed and implemented by the I SEE Finnish partner and then revised by the physics education team in Bologna, Italy. In this paper, we discuss the approach developed by the Italian group [30,31].

The module consists in 6 extracurricular meetings of about three hours each. The whole module was implemented twice (February–March 2019 and January–February 2020), each time with approximately 20 secondary school students (17–19 years old; grade 12 or 13) who were presumed never to have studied quantum physics before. In Appendix A, the timetable of the activities schedule is reported. Two kinds of activities were carried out each day: concept-oriented activities future-oriented activities designed to show the impact of the topic on different dimensions (such as research, politics, and the economy) and to develop *future-scaffolding* skills such as scenario thinking, foresight, back-casting, and action competence. The course teachers included experts in quantum physics, researchers in physics education and mathematics education, and a high school teacher.

The activity on teleportation represents the core part of the conceptual path and the paper. As shown in Table III in Appendix A, it was scheduled for the third meeting in the first implementation, and for the fourth in the second round.

In the next sections, we first describe the overall approach we designed for the module before focusing on the activity of teleportation to show the process of educational reconstruction [32,33] that led us to the design of this activity, according to methodological criteria deriving from research in science (physics) education.

IV. THE APPROACH: RESEARCH BACKGROUND AND DESIGN PRINCIPLES

The approach is comprised of a set of design principles. Some principles are specific to the module on quantum computers developed in the I SEE project. Others are broader and refer to the comprehensive approach of I SEE, which included the development of modules on other advanced STEM topics such as climate change and artificial intelligence.

In designing the approach, we started from an "asymmetry" that we perceived in attending popular seminars and conferences about classical and quantum computers. In fact, we guess that rarely, if ever, a popular conference on computing would start by explaining the physical laws according to which hardware and logical gates are realized and operate. On the contrary, most public seminars and conferences include usually an introduction to the new unit base and its features (superposition principle and entanglement) and a discussion about, for example, the better techniques to create qubits and manipulate them.

The design started from this asymmetry and from the decision to change the way of talking about classical computers, by taking a step back to their hardware and emphasizing the role and features of the "deterministic and linear logics of classical physics" they follow. In such a way, students could be guided to compare (more closely) the binary and nonbinary logic that remains at the core of classical and quantum computers and, through this comparison, to develop (without becoming bogged down in too many technical details) an idea of what we mean today by quantum simulators, quantum computers and how quantum logic gates and quantum circuits work.

This idea represents our design principle #1: to foster a close comparison between classical and quantum computers through an analysis of *the different logic* underlined in the basic mechanisms on which the hardware is built.

This principle leads directly to the second principle and the issue represented by the relation between hardware and physical experiments. In quantum computing, a Mach-Zehnder or a Stern-Gerlach apparatus are conceptualized as quantum simulators [34]. What does this mean? How can an experiment be considered a quantum simulator? What about the corresponding case in classical computers?

Design principle #2 regards the reconceptualization of the foundational experiments in terms of computation, so as to discuss why experiments can be considered as "simulators" or devices to process information (circuits). This means, operationally, re-reading the three main phases of an experiment—*state preparation, state evolution, measurement*—in terms of *input–processing–output information* (see Fig. 1).

Operationally, the principles were implemented from the design of the first lesson of the module concerning a brief history of computer evolution onward (see Figs. 25 and 26 in Appendix A). From the beginning, the comparison between "experiment and computation" (Fig. 1) was used to provide the following take-home message: behind classical computers there are logical gates that follow a Boolean logic and are *materially* built on devices that follow physical laws. The current opacity of the physical behaviors of these devices is the result of a technological development that led to the software becoming in some sense "THE computer."

Concretely, teaching started by acquainting the students with the definition of computer science, given in the 1980s, as the science that concerns the development and use of *structures and procedures for processing information* [35]. The information is put into the computer in a certain form and is then returned in a different form (which is generally easier to use), following the sequence "*input, processing and output information*" (right part of Fig. 1). The input information can be defined as the raw material of the process, the output information as the finished product [35].

Students attending grades 12 or 13 have usually already encountered the concept of bit, binary system, truth tables, and logic gates. Thus, after a brief recap of these concepts, examples of *materials* devices were shown in order to move the discussion on to the concrete realization of the logic gates through the hardware. In order to stress the logic and the working principles without entering into the functioning of complicated devices, we showed the students very special logic gates created through systems of rope and pulleys, as described in the fantasy tale reported in a 1988 paper by Dewdney (see Fig. 2) [36].

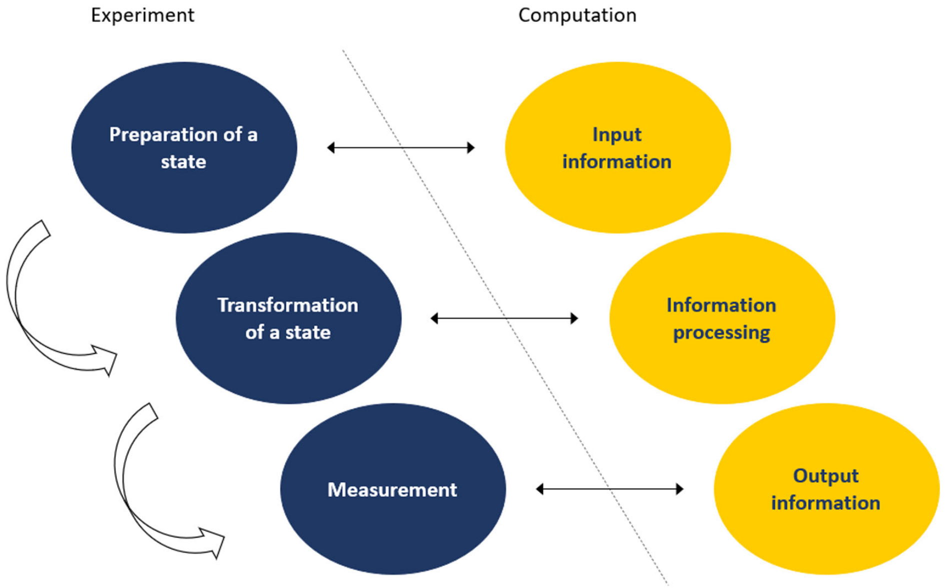

FIG. 1. The essence of the approach: the “experiment-computation” comparison.

This story aimed also at highlighting the deterministic and linear paradigm that is implemented in the logic of the algorithm.

After this demonstration, the historical evolution of computers was briefly described. The narrative started by mentioning the hardware made of vacuum tubes (first generation), which then moved to transistors (second generation), to integrated circuits (third generation), and, finally, to microchips (fourth generation). The discourse was built to emphasize that the changes led to an impressive increase of the calculus power and a progressive miniaturization without, however, changing the Von Neumann architecture of the computers and the basic logic they follow. Furthermore, the last passage was stressed as the step that led to the explosion of the software era, whereby, thanks to the software sophistication, it became possible to *force* the computers to follow probabilistic and nondeterministic algorithms even though their hardware still followed classical logic.

The “experiment-computation comparison” approach was then applied in the quantum case, in the opposite direction: from left to right of Fig. 1, i.e., from experiment to computation.

We started from one of the most famous fundamental experiments, the Stern-Gerlach experiment, and, using the *spin-first* approach, we introduced the basics concepts of quantum physics. The Stern-Gerlach experiment was used as a context to re-conceptualize the simplest superposition state, $a|\uparrow\rangle + b|\downarrow\rangle$, in terms of qubit, $a|0\rangle + b|1\rangle$, the new basic unit of quantum computers. Then, as with the classical case, we conceptualized the experimental operations of switching on and off the magnetic field on a particular axis (state evolution) in terms of information processing through quantum logic gates (information processing) and we read the experimental process of particle detection (measurement) in terms of reading the output information.

The design approach not only allows us to introduce the core ideas at the basis of quantum technologies, but also to show that the information can be manipulated from experimental (tools), mathematical (rotation of a vector in the Bloch sphere), and circuital (logic gates) points of view. This plurality of perspectives helps students to understand the analogy between the experimental and algorithmic representations and start to grasp how the

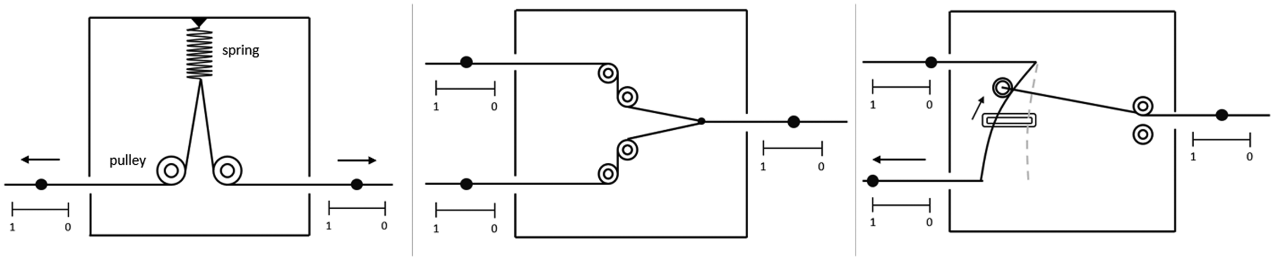

FIG. 2. Mechanical realization of, from left to right, NOT, OR, and AND logic gates [36].

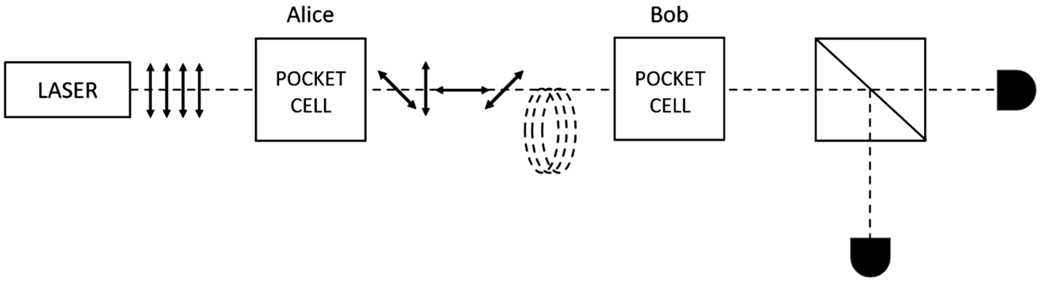

FIG. 3. Schematic experimental representation of quantum cryptography protocol BB84 using PC.

new technologies work, while taking a step toward the concept of simulation.

After discussion of the Stern-Gerlach experiment, we introduced the concept of entanglement, which is fundamental for teleportation protocol. In order to communicate to students its importance and its relevance also from a technological point of view, the concept was applied to the cryptography protocol. We use the simulation of quantum cryptography from the quantum mechanics visualization (QUVIS) project by the University of St Andrews. A schematic representation is reported in Fig. 3. We showed students how the protocol BB84 works from an experimental point of view (using Pockels cells that randomly vary the polarization of the photon) and, from a logical point of view. A circuit made by an opportune combination of logic gates X; Y; Z was presented and discussed to show how it makes the polarization random.

Before describing in detail how the approach was applied to the case of teleportation protocol (Sec. VI B), we briefly describe the further design principles that concern specific aspects of the approach. The two broader principles, that characterize all the I SEE modules, refer to goals of making the materials inclusive and relevant from a societal point of view.

Design principle #3: to keep the quantum technicalities as simple and clear as possible and foster deep understanding of the *essential physical concepts that are needed*.

Modern quantum experiments (such as quantum teleportation or boson sampling) are really complicated both for the experimental setup and for the physical and mathematical formalism. While it was not our aim to acquaint the students with these aspects, we wanted them to be able to recognize the inner phenomenon they refer to and the core concepts of quantum mechanics needed to grasp the inner logic and potentialities of the new technologies.

In our approach, the core concepts necessary and sufficient to grasp the essence of these new technologies include the only ones we introduced in the first lessons, which proved to be within reach of secondary school students and were efficiently built through a simplified *spin-first* approach [37–41]: the concepts of state, superposition principle, manipulation of a state, measurement, and entanglement. Also, the reconstructions of the contemporary experiments were based solely on these concepts. Furthermore, when the experimental apparatus is described, the level of the discourse is kept “light” enough to make the students aware that, behind logical protocols and mathematical expressions, there are concrete physical devices.

Design principle #4: to make the modules as *inclusive* as possible.

This principle is the core of the general educational approach that the research group in physics education of Bologna has been developing to foster students’ understanding of concepts and to make disciplinary learning a locus for identity development [28,42–46]. Broadly speaking, our materials are designed to be multidimensional and multiperspective so as to engage as many students as possible by nurturing their idiosyncratic interests, intellectual and aesthetic tastes. In the case of quantum technologies, it was not difficult to imagine that they could be of interest for many different reasons. They are based in quantum physics that is *per se* an intriguing theory, or even represent current technological frontiers that are opening up new future scenarios. Also, they can be fascinating for their social or political implications, or for the intellectual challenges they pose. In Italy, these advanced modules are set within orientation courses. For many students, quantum technologies become a benchmark test regarding their abilities, interest, and even talent, in order to successfully complete the physics degree course.

For the design of the activity of teleportation, inclusiveness has been fostered by choosing to articulate the discourse along different levels: narrative, logical, mathematical, and technical-experimental. The articulation of the discourse was also needed to avoid students considering quantum logic and mathematics as just a “mere mechanism” to play with [47]. The levels articulation, indeed, has to maintain a strong link with reality and support the comparison between classical computer and quantum protocol.

More specifically, as for the narrative level, the contents have been represented through the story of two characters, Alice, and Bob, who are charged with solving the problem of teleporting the state of a photon from one position to another in the world. The narrative is supposed to activate the students' imagination and help them to build a comprehensive view—a storyboard—to effectively situate the various steps of reasoning.

The logical level is the backbone of the argumentation on which the comparison between the classical and quantum computers is built. This level refers to the logic that lies at the basis of classical and quantum computing and sets the "rules" and truth tables on which the logic gates are built and combined in circuits to solve a problem (algorithm).

The mathematical level refers to the two-state Dirac formalism for quantum physics and is used to formalize the superposition states and their manipulation throughout the logic gates of the circuits.

The technical–experimental level refers to the implementation, to the experimental devices used to realize the logic gates. From an educational point of view, this level was supposed to bestow concreteness to the logical level and provide the students with an idea of what it means, today, to create a quantum computer.

The different levels were supposed to play different roles and to allow students with different interests and tastes to find the level that resonated most with them. The levels were explicitly introduced to the students in two senses: the teachers made the students aware of their existence at the beginning of the key lessons and informed them when the various levels were switched on. The specific roles that the levels were supposed to play was instead a metalevel that was kept implicit.

In the following Secs. V and VI, we will describe how all these principles were implemented in the case of the teleportation activity. As a brief anticipation, we will show how design principle #1 has been implemented to emphasize the logic behind the teleportation protocol and its differences from the binary Boolean logic; design principle #2 is reflected in the comparison, which structures the activity, between the experiment and the circuit; design principle #3 has been implemented in the careful choice of conceptual details added by taking into account the research into students' difficulties; design principle #4 has been implemented in articulation of the discourse along the four levels mentioned above (narrative, logical, mathematical, and technical-experimental).

V. THE ACTIVITY ON QUANTUM TELEPORTATION: METHODOLOGY OF THE STUDY

The design of the activity on teleportation was carried out as a case of "educational reconstruction" [32,33]. We referred to this model, elaborated by German researchers, to methodologically frame the whole process of design.

The model of education reconstruction (MER) is based on epistemological assumptions that we shared and that oriented the instruction design: physics is a discipline that, like every human construction, is rich and complex enough to allow its content to be analyzed, elaborated, and re-structured in many different ways according to many different educational goals. Thus, when the contents are analyzed for teaching, the analysis is explicitly assumed to be "never solely influenced by the referring science content and by issues that stem from philosophy and the history of science but as well by educational concerns, such as the aims of teaching the detected elementary features, that is, the basic ideas of a science theory in question." [33].

The model identifies three components of the process of content reconstruction: analysis and clarifications of science content, analysis or research on teaching and learning with emphasis on students' perspectives, and instruction design.

The first component includes "hermeneutic-analytical research on subject matter clarification and analysis of the educational significance of a particular science content. The interconnected set of core ideas of a particular content domain is detected (in the aforementioned sense) from the perspective of key aims of science instruction." [33]. The second component comprises "studies on students' conceptions, i.e., investigations into students' preinstructional conceptions and their development towards the intended science view." [33]. The third component refers to the need to strictly link academic research with school practices and, indeed, "comprises development and evaluation of pilot instructional modules rather early in the process of educational reconstruction." [33].

In our case study, the first phase (analysis and clarifications of science content) involved an extensive and deep analysis of the literature on the teleportation experiment to find the most suitable version of the experiment for the comparison. By "most suitable version" we mean a version of the experiment that could be explained without getting caught up in too many technicalities.

The second phase of educational reconstruction (analysis or research on teaching and learning with emphasis on students' perspectives) was carried out by taking into account the research on students' conceptions in learning in quantum physics and the transformation of the contents to bring them within the reach of secondary students. For this phase we could count on a quite long experience of teaching and learning quantum physics. Throughout multiple iterative phases of design-implementation-revision, we progressively developed teaching materials that were revealed to be effective in developing the basic concepts of quantum physics on which we built the activity (state, superposition principle, manipulation of a state, measurement, and entanglement) [37,48,49].

As for the third phase (instruction design and pilot investigation), the activities and materials we developed were implemented twice (February–March 2019, January–February 2020), with more than 20 students aged 16–17 years old (first cycle of 15 males and 10 females, second cycle of 16 males and 8 females). Both implementations were carried out in the context of Piano Nazionale Lauree

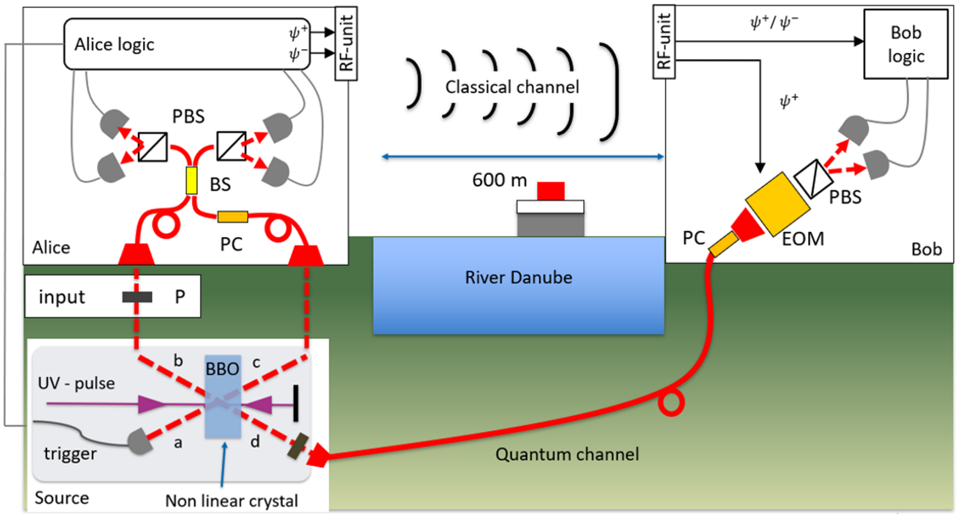

FIG. 4. Set up of teleportation experiment (redrawn by Ref. [50]).

Scientifiche—Italian Scientific Degree Project (PLS) laboratories on a voluntary basis. These laboratories are part of a formal collaboration with schools with the aim of orienting students and assisting them in making an informed university choice.

In both implementations, the group was heterogeneous and comprised of students from different schools with no background in quantum physics. In both cases, the group consisted of more males than females.

In the following section, we illustrate the main results of the work phases. In particular, in Sec. VI A, we describe the experiment we chose and the comparison we made with the circuit. In this section we use an advanced language targeted at physicists. In Sec. VI B we illustrate how we re-elaborated the content so as to make it approachable for secondary students, and the teaching activities we designed.

In the final section of the paper, we report the results of a pilot study aimed to analyze students' reactions to the teleportation activity.

VI. THE RESULTS OF THE STUDY

A. The content analysis

The phase of content analysis led to a conceptual clarification of the teleportation protocol that, according to our design principles, has been analyzed from both experimental and logical or circuital perspectives.

This content analysis is rather technical and can appear unnecessary to grasp the sense of our education approach and the final design of the teaching activities.[1] However, it represents a fundamental step within the model of educational reconstruction: its outcome was crucial not only to evaluate the feasibility of the approach, but also to enable us to recognize the fundamental and structural elements of reasoning, from needlessly complicated details.

As a starting point of the process, we selected one of the first experiments on teleportation, developed by the group of Zeilinger in 2004 [50]. In this experiment, the state of a photon (in term of its polarization) was teleported from one shore to the other of the Danube.

We then considered the physical experiment, whose representation, shown in Fig. 4, is borrowed from Ursin and colleagues [50]. The following description is the result of the analysis: the experiment is clarified according to our design principle #2; i.e., it is described in a form that allows its schematization and its comparison with the protocol.

A pulsed laser (wavelength 394 nm; rate 76 MHz) is used to pump a β-barium borate (BBO) nonlinear crystal and, hence, to generate the first entangled photon pair c and d by parametric conversion. C is the photon that goes to "Alice's station" and d the photon that goes to "Bob's station." For reflection of the pulsed light on a mirror, another pair of entangled photons, a and b, are produced: a serves as a trigger and b, passing through a polarizer, comes to be in the superposition state $|\psi\rangle_b = (\alpha|0\rangle + \beta|1\rangle)_b$ that Alice wants to teleport to Bob. Therefore, the initial state of the system is

$$\begin{aligned}|\psi\rangle &= |\psi\rangle_b|\beta_{11}\rangle_{cd}\\ &= (\alpha|0\rangle+\beta|1\rangle)_b\left(\frac{|01\rangle-|10\rangle}{\sqrt{2}}\right)_{cd}\\ &= \alpha|0\rangle_b\frac{|0\rangle_c|1\rangle_d-|1\rangle_c|0\rangle_d}{\sqrt{2}}+\beta|0\rangle_b\frac{|0\rangle_c|1\rangle_d-|1\rangle_c|0\rangle_d}{\sqrt{2}}.\end{aligned}$$

After the preparation of the photon b in the form of state to be teleported, b and c are guided into a polarizer

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[1] This section can be skipped by a reader who is not specifically interested in the methodological research aspects or the disciplinary details.

controller and into a single-mode optical-fiber beam splitter (BS). These experimental tools and their connection with polarizing beam splitters (PBS) allow a Bell-state measurement to be realized. Mathematically, in order to better follow the manipulation of the system's state, b and c photons are coupled in the same ket, obtaining

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\alpha|00\rangle_{bc}|1\rangle_d - \alpha|01\rangle_{bc}|0\rangle_d + \beta|10\rangle_{bc}|1\rangle_d - \beta|11\rangle_{bc}|0\rangle_d). \qquad (1)$$

The four Bell states are

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$
$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$
$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}.$$

With simple calculations aimed to make the computational basis explicit, we obtain

$$|00\rangle = \frac{|\Phi^+\rangle + |\Phi^-\rangle}{\sqrt{2}}$$
$$|11\rangle = \frac{|\Phi^+\rangle - |\Phi^1\rangle}{\sqrt{2}}$$
$$|01\rangle = \frac{|\Psi^+\rangle + |\Psi^-\rangle}{\sqrt{2}}$$
$$|10\rangle = \frac{|\Psi^+\rangle - |\Psi^-\rangle}{\sqrt{2}}.$$

Replacing these states in (1), we obtain

$$\begin{aligned}|\psi\rangle =& \frac{1}{\sqrt{2}}\left(\alpha\left(\frac{|\Phi^+\rangle+|\Phi^-\rangle}{\sqrt{2}}\right)_{bc}|1\rangle_d - \alpha\left(\frac{|\Psi^+\rangle+|\Psi^-\rangle}{\sqrt{2}}\right)_{bc}|0\rangle_d \right.\\ &\left. + \beta\left(\frac{|\Psi^+\rangle-|\Psi^-\rangle}{\sqrt{2}}\right)_{bc}|1\rangle_d - \beta\left(\frac{|\Phi^+\rangle-|\Phi^-\rangle}{\sqrt{2}}\right)_{bc}|0\rangle_d\right)\\ =& \frac{1}{2}[|\Phi^+\rangle_{bc}(\alpha|1\rangle_d - \beta|0\rangle_d) + |\Phi^-\rangle_{bc}(\alpha|1\rangle_d + \beta|0\rangle_d)\\ & - |\Psi^+\rangle_{bc}(\alpha|0\rangle_d - \beta|1\rangle_d) - |\Psi^-\rangle_{bc}(\alpha|0\rangle_d + \beta|1\rangle_d)]. \qquad (2)\end{aligned}$$

This replacement and the algebraic passages highlight what it means to mathematically prepare the state of the system in order to perform a Bell-state measurement. The teleportation can occur if, and only if, it is possible to make this Bell-state measurement and if a temporal coincidence is measured, through the detectors, in Alice's station.

Making a Bell measurement on two states means projecting them onto one of the Bell states. Theoretically, the probability of finding each state is

$$P(|\Phi^+\rangle_{bc}) = P(|\Phi^-\rangle_{bc}) = P(|\Psi^+\rangle_{bc}) = P(|\Psi^-\rangle_{bc}) = 25\%.$$

TABLE I. Alice's state and Bob's corresponding state.

Cases	Alice	Bob
1	$|\Psi^-\rangle_{bc}$	$(\alpha|0\rangle_d + \beta|1\rangle_d)$
2	$|\Psi^+\rangle_{bc}$	$(\alpha|0\rangle_d - \beta|1\rangle_d)$

Nevertheless, by construction, for this specific experimental setup, the only two possible Bell states are either $|\Psi^-\rangle_{bc}$ or $|\Psi^+\rangle_{bc}$, which can be distinguished from each other by Alice's logical electronics (Bell state measurement) [50]. Alice's result is then transmitted through a classical microwave channel (rf unit). Table I shows the two possible results of Bell measurement that Alice, with the same probability, can obtain and the corresponding state of Bob's photon.

Knowing the state of Bob's photon, a transformation can be operated with the electro-optic modulator (EOM) to transform the state of photon d into the desired s input state of photon b by Alice, so that the teleportation is complete. The latter are unitary transformations that, in the case of photons, correspond to rotation of polarization or phase displacements, obtained by applying a voltage pulse to the EOM.

As Bennett and colleagues stated in their 1993 paper, "the spin-exchange method of sending full information to Bob still lumps classical and nonclassical information together in a single transmission" [51], as Fig. 4 shows. Indeed, as they demonstrated, the full information of Alice encoded in her state is composed of two parts, "one purely classical and the other purely nonclassical," and is sent to Bob through two different channels. This observation, combined with the fact that the state of Alice is destroyed during the process, ensures that information does not travel at higher speeds than speed of light. Thus, the second principle of relativity is not violated, and it ensures that the state is not cloned, as the no-cloning theorem requires.

After this presentation of the experiment, we can now move on the circuit and read it not only as an abstract representation of the experiment, but also as a special way of conceptualizing the experiment in terms of logic gates. In Fig. 5, the circuit of quantum teleportation is reported.

In this representation it is possible to identify five different moments given by the states $|\psi_0\rangle$, $|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3\rangle$, and $|\psi_4\rangle$.

The three qubits represented by the three registers (3 lines of the circuit) regard, respectively, the state of the photon b ("1"), the state of the photon c ("2"), the state of the photon d ("3").

The state $|\psi_0\rangle$ describes the initial state of the system and it is the product of $|\psi\rangle$ and $|\beta_{11}\rangle$, where the former is the state that has to be teleported ($|\psi\rangle_1 = (\alpha|0\rangle_1 + \beta|1\rangle_1)$ and the latter is one of the four Bell states:

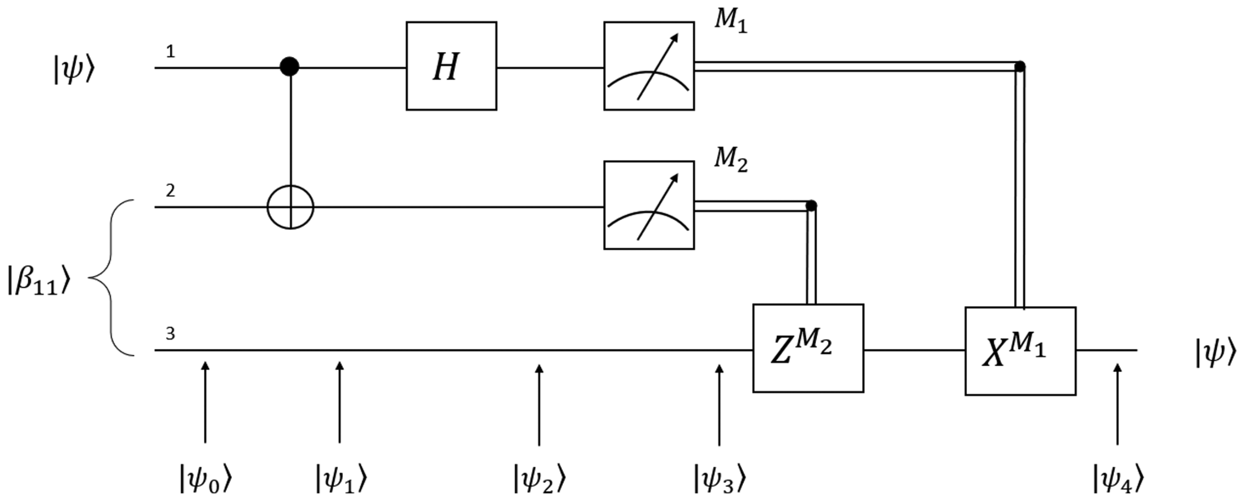


FIG.5. Teleportation circuit (redrawn from Ref. [5]).

$$\begin{aligned}|\psi_0\rangle &= |\psi\rangle_1|\beta_{11}\rangle_{23}\\ &= (\alpha|0\rangle_1+\beta|1\rangle_1)\left(\frac{|01\rangle-|10\rangle}{\sqrt{2}}\right)_{23}\\ &= \frac{1}{\sqrt{2}}[\alpha|0\rangle_1(|01\rangle-|10\rangle)_{23}+\beta|1\rangle_1(|01\rangle-|10\rangle)_{23}].\end{aligned} \tag{3}$$

As well as in the experiment, where it is necessary to make a Bell measurement on the photons *b* and *c* in order to have teleportation, also in the algorithm it is necessary to project photon 1 and 2 in a Bell state. This is possible by means of two logic gates in sequence, a CNOT, having as input photons 1 and 2, and a Hadamard gate on photon 1.

The CNOT gate has two input qubits, known as the *control* qubit and the *target* qubit, respectively. The circuit representation for the CNOT is shown in Fig. 6; the top line represents the *control* qubit, while the bottom line represents the *target*.

The action performed by the logical gate is the following: if the control qubit is set on 0, then the *target* qubit is left as it is; if the *control* qubit is set on 1, then the *target* qubit is flipped. Formally, this means

$$\begin{aligned}|00\rangle &\to |00\rangle, \quad |01\rangle \to |01\rangle,\\ |10\rangle &\to |11\rangle, \quad |11\rangle \to |10\rangle.\end{aligned}$$

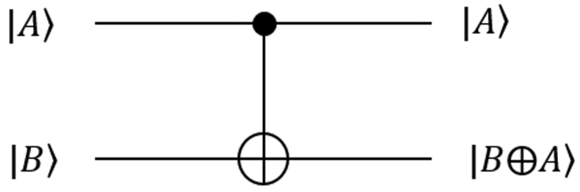


FIG. 6. CNOT gate.

Therefore, if CNOT gate is applied on photons 1 and 2, the Eq. (3) becomes

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle_1(|01\rangle-|10\rangle)_{23}+\beta|1\rangle_1(|11\rangle-|00\rangle)_{23}]. \tag{4}$$

In order to complete the projection on a Bell state, a Hadamard gate is applied to photon 1. This gate is about a single qubit gate and transforms the state as shown in Fig. 7.

Therefore, (4) becomes

$$\begin{aligned}|\psi_2\rangle = \frac{1}{2}[&\alpha(|0\rangle_1+|1\rangle_1)(|01\rangle-|10\rangle)_{23}\\ &+\beta(|0\rangle_1-|1\rangle_1)(|11\rangle-|00\rangle)_{23}].\end{aligned} \tag{5}$$

Reorganizing the terms of (5), we obtain

$$\begin{aligned}|\psi_2\rangle = \frac{1}{2}[&|00\rangle_{12}(\alpha|1\rangle_3-\beta|0\rangle_3)-|01\rangle_{12}(\alpha|0\rangle_3-\beta|1\rangle_3)\\ &+|10\rangle_{12}(\alpha|1\rangle_3+\beta|0\rangle_3)-|11\rangle_{12}(\alpha|0\rangle_3+\beta|1\rangle_3).\end{aligned} \tag{6}$$

In (6) the first term represents Alice's qubit ($|00\rangle_{12},\ldots,|11\rangle_{12}$) and the second Bob's qubit.

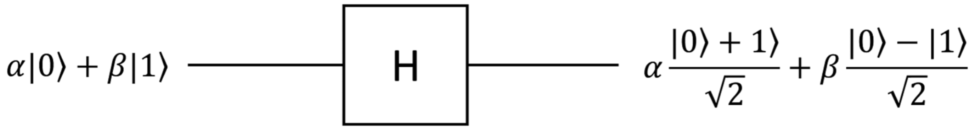


FIG. 7. Hadamard gate.

FIG. 8. Circuital representation of the measurement.

After the Hadamard logic gate, a measurement on the qubit b and c is performed. The circuital representation of the measurement is in Fig. 8.

Depending on Alice's measurement, Bob's qubit will be in one of four possible states:

$$\begin{aligned}
|00\rangle_{12} &\to |\psi_3(00)\rangle \equiv [\alpha|1\rangle_3 - \beta|0\rangle_3] \\
|01\rangle_{12} &\to |\psi_3(01)\rangle \equiv [\alpha|0\rangle_3 - \beta|1\rangle_3] \\
|10\rangle_{12} &\to |\psi_3(10)\rangle \equiv [\alpha|1\rangle_3 + \beta|0\rangle_3] \\
|11\rangle_{12} &\to |\psi_3(11)\rangle \equiv [\alpha|0\rangle_3 + \beta|1\rangle_3].
\end{aligned}$$

As in the physics experiment, also here Bob needs to know the result of Alice's measurement to complete teleportation.

If Alice makes the measurement and obtains $|11\rangle$, Bob will not have to do anything, because his qubit is already in the right state. If, on the other hand, Alice obtains $|10\rangle$, Bob will have to apply the X gate. If Alice obtains $|01\rangle$, Bob will apply the Z gate. Finally, if Alice's result is $|00\rangle$, Bob will apply both X and Z. X and Z are two single-qubit gates that work, respectively, as depicted in Figs. 9 and 10.

In other words, in order to recover the state $|\psi\rangle_4 = \alpha|0\rangle + \beta|1\rangle$ successfully, Bob will have to apply the unitary transformation $Z^{M_2}X^{M_1}$ to his qubit.

To sum up, this phase of content analysis consisted in conceptualizing a teleportation experiment as a computational device. This process allowed the logical structure of the experiment to emerge. Such an overall picture provides criteria to schematize the complex phenomena and organize its main elements within a comprehensive whole. This outcome was the basis for the following phases of the process of educational reconstruction, i.e., for designing and testing teaching activities for upper secondary school students.

B. Making teleportation approachable for secondary school students: Design and implementation of teaching activities

In this section, we report the results of the second phase of educational reconstruction: the transformation of the physical contents, described in the previous section, into knowledge that is culturally and socially relevant, approachable, and inclusive for secondary school students.

In order to reach this goal, the contents have been reconstructed by implementing our four design principles (Sec. IV). As orientation for reading, we anticipate that design principle #1 (to foster a close comparison between classical and quantum computers through an analysis of *the different logic* underlined in the physics of their hardware) has been applied by emphasizing the logic that stays behind the teleportation protocol and those features, e.g., its foundation on entanglement, which does not have a classical analogue and cannot be reconceptualized through the classical Boolean logic. Design principle #2, which regards the reconceptualization of the foundational experiments in terms of computation, is the overarching principle in the design of the activity, built on the comparison between the experiment and the circuit. The Design principle #3 (making the activity approachable and then keeping the quantum technicalities as simple and clear as possible) has been implemented through the careful choice of conceptual details that were needed to structure the discourse and by taking into account physics education research about students' difficulties. Design principle #4 concerning the inclusiveness has been implemented in articulation of the discourse along the four levels mentioned above (narrative, logical, mathematical, and technical-experimental).

This phase led to the design of teaching activities for pilot implementations (third phase) [33].

The literature on quantum physics teaching and learning shows that a simplified *spin-first* approach appears

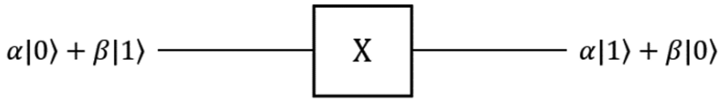


FIG. 9. X gate.

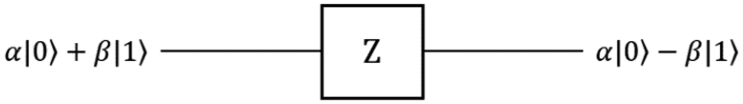


FIG. 10. Z gate.

effective in guiding secondary school students into quantum physics [11,37,38]. More specifically, there is a set of basic concepts that are perfectly comprehensible: the concepts of state, superposition principle, manipulation of a state, measurement, and entanglement [37–41]. Our reconstruction of the teleportation phenomenon is entirely built on these concepts.

The resulted activity is articulated in three parts: (i) presentation of the teleportation experiment created by Ursin and colleagues in 2004 [50]; (ii) presentation of the circuit that carries out the teleportation protocol and analysis of its correspondence with the experiment; (iii) discussion of teleportation applications and their impact on the future.

The lesson started by switching on the narrative level with the story of Alice and Bob: “Alice and Bob, before leaving, exchange a pair of entangled photons; after a few years, Alice (who has obtained a second photon) decides to send Bob the status of her new photon—how can she do so?” The students were fostered to reason why a classical channel cannot be used by Alice to send her status and the discussion revolved around the fact that a qubit contains an infinite number of classical data (its state varies in a continuous space); thus, she would have needed infinite time to communicate such information to Bob: Alice needs quantum teleportation to solve this task. The students were then guided through the physical apparatus (Fig. 4) to see how Alice’s task could be solved by teleportation, i.e., by means of experimental tools that manipulate the state of a photon and teleport it from Alice to Bob.

Since the original experimental setup was too complicated for high school students, we presented them with a simplified version, which resulted by schematizing the apparatus in “five blocks” concerning key moments:

i. the production of two pairs of entangled photons;
ii. the entanglement of two photons that were initially nonentangled;
iii. measurement of Alice’s state;
iv. Alice’s communication to Bob, via classical channel;
v. Bob’s operation to recover the initial status of Alice, after knowing her.

In order to guide the students through the key moments of the apparatus, we described the experiment by referring to Fig. 4, which represents (as mentioned in the previous section) the experiment performed by Zeilinger and colleagues in 2004 [50].

The students were explicitly told that the two goals of the experiment description were (a) to show where and how the narrative of Alice and Bob was attached to reality; (b) to pave the way for the experiment analysis in terms of its logical structure (bullet list above) and, hence, start to build links with the circuital representation.

Both the goals involved guiding the students to focus their attention on “what they had to see” in the experiment. One challenge was to help them to consider only the photons *b*, *c*, and *d*, (where *c* and *d* are the pair of entangled photons that Alice and Bob exchanged previously, and *b* is the photon whose state is going to be teleported), since the photon *a* acts as a trigger, communicating to Alice that the two pair of entangled photons are correctly produced.

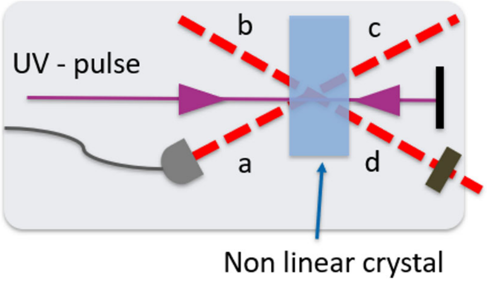


FIG. 11. Schematization of parametric down-conversion—interaction between a UV pulse beam with a nonlinear crystal (BBO).

In the following section, we briefly describe how the “technical-experimental level” was presented to the students. The aim of this description is to show what we mean in our design principle #3, when we say that we do not want the students to understand the details of the actual discourse, but simply become aware that there are physical devices behind logical protocols.

The description refers to Figs. 11, 12, and 13, representing the key moments from Alice’s production of the two pairs of entangled photons to Bob’s recovery of the initial state.

Technically, Fig. 11 schematizes a parametric down-conversion.

Since the parametric down-conversion is based on very complicated phenomena, we highlighted only that the pairs of entangled photons are produced through a double

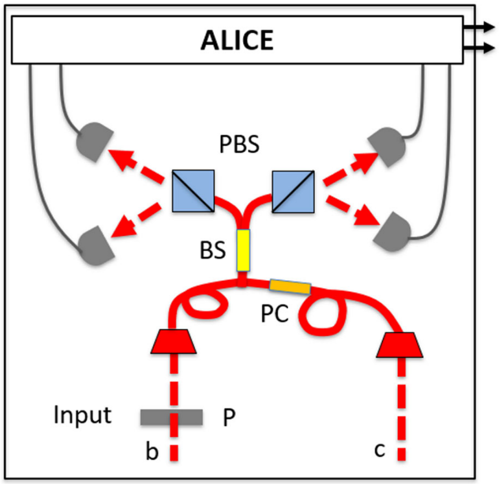


FIG. 12. Schematic representation of the apparatus to project two photons into a Bell state.

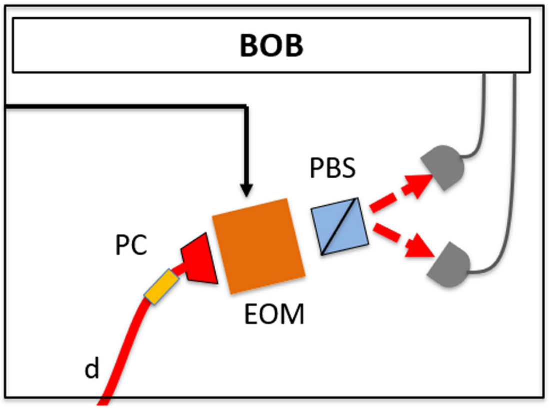


FIG. 13. Schematic representation of Bob's station and tools to recover the input state.

interaction of a pulsed light beam with a nonlinear crystal (first c and d, then a and b). This information was enough, in our opinion, to move to the logical level of the experiment and invite students to focus their attention on the photons b and c which were transported to Alice's station through optical fibers. Here, in order for teleportation to occur, they had to become entangled, i.e., projected into a Bell state (Fig. 12).

In Fig. 12, we explained to the students that photon initially passes through a polarizer, which prepares it in the state to be teleported. A series of tools (including a polarization controller and a beam splitter) manipulate the states so that photons b and c become entangled. The students were then made aware of the role played by the two polarized beam splitters (PBS) and the four detectors represented in the figure: these are needed to know if the two photons were really entangled, indeed, it is possible to know that the two photons have become entangled if, and only if, the detectors reveal them simultaneously. Thus, Alice, through PBS and detectors, measures the state of her two photons and is in the condition to communicate her result to Bob.

This is the most delicate point of reasoning, where narrative, experimental, and logical levels need to be carefully aligned. Following the narrative, the students were told that Alice makes a phone call in order to communicate the results to Bob and, on the basis of the information provided by Alice, Bob can finally recover the initial state and accomplish the process of teleportation. At the technical-experimental level, this means that Alice uses a classical channel, represented by microwave and that, to recover the initial state, Bob has to apply a voltage to the EOM (Fig. 13). Because of the reduced speed of light in the optical fiber channel (two-thirds of the speed of light in the air and through the air), the classic signal reaches the other laboratory 1.5 μs before the arrival of the photon d.

At this state of the discourse, the students did still confuse two key elements of the reasoning: the information that Alice provides to Bob by phone and the teleported quantum state: "if Alice has to make a call, what are the advantages and the sense of teleportation?" was the question that several students posed. In order to distinguish between the two types of information, the logical level represented by the circuit (Fig. 5) and mathematical level have been introduced. Their discussion was the core of the second part of the activity.

In this second part, which was highly engaging, we worked together with the students to reconstruct step by step how teleportation takes place mathematically. The students participated actively and became involved by trying themselves to contribute to the reconstruction of the mathematical passages.

In this dialogue, the narrative level was still present, but the backbone of the discourse was the circuit, which was stressed as representing a way to "transform the experiment into a quantum simulator." The circuit was indeed the way to flesh out the logical structure behind the experiment. The circuit, hence, became the playground where the students became acquainted with the new logic by tackling the concepts seen in the first lectures. For these purposes, the representation of the circuit was shown and step by step, together with the students, the mathematical passages were reconstructed, demonstrating that Alice's status had actually been teleported to Bob. The formalism was simpler than the one shown in the previous section. The entangled photons were chosen in the Bell state $\beta_{00} = (|00 + |11)/\sqrt{2}$, and not in β_{11}, in order to find, by developing the calculation, the initial state $\alpha|0\rangle + \beta|1\rangle$ corresponding to Alice's first measurement ($|00\rangle$). Finally, we decided to present the mathematical steps both to demonstrate formally that the teleportation takes place, and to show that manipulating information formally corresponds to manipulating the states in an equation.

Regarding details of the teaching activity, we needed to connect explicitly the formal representation of the qubit with the photons a, b, c, d, mentioned in the experiment description. In real terms, this involves explaining that: $|\psi\rangle = \alpha|0\rangle_b + \beta|1\rangle_b$ represents the state to be teleported corresponding to the photon b; $|\beta_{00}\rangle$ represents the Bell state that describes the relationship of entanglement between photons c and d, which, in the experiment, was produced through the parametric down-conversion.

We began by explaining that the initial state of the total system, $|\psi_0\rangle$, is the product between $|\psi\rangle$ and $|\beta_{00}\rangle$. As Fig. 14 shows, we immediately reconnected this state to the experiment: the first thing that happened is the "creation of an entangled relationship" between the photons b and c and this, from a circuital point of view, corresponds to the sequence of a CNOT gate and a Hadamard gate.

Step by step and in a dialogic way, the whole class was involved in the calculus of the evolution of the overall state, passing through a CNOT and then to H gates and obtaining

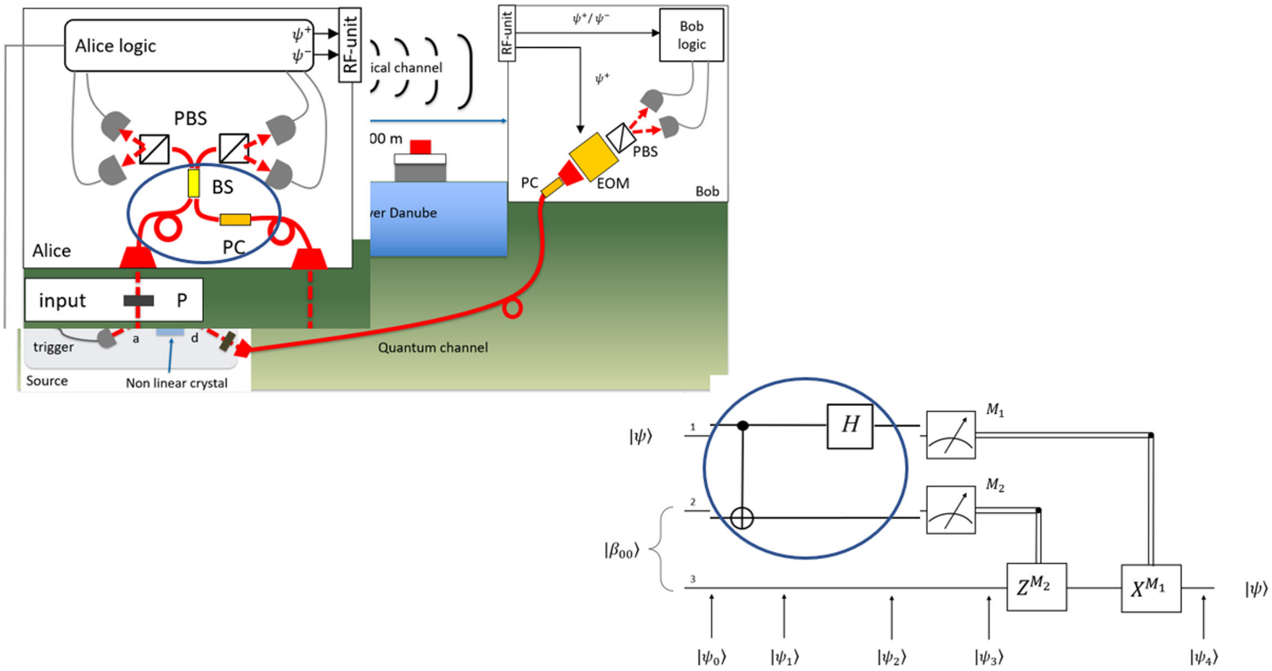


FIG. 14. Comparison between the “experimental and computational” projection of two photons in a Bell state.

$$|\psi_2\rangle = \frac{1}{2}[|00\rangle_{bc}(\alpha|0\rangle_d + \beta|1\rangle_d) + |01\rangle_{bc}(\alpha|1\rangle_d + \beta|0\rangle_d) + |10\rangle_{bc}(\alpha|0\rangle_d - \beta|1\rangle_d) + |11\rangle_{bc}(\alpha|1\rangle_d - \beta|0\rangle_d)].$$

This was the most engaging, stimulating, and easy part for the students, who realized they were able to manipulate the states mathematically. We then returned to the parallelism and showed the students what (in the experiment) corresponds to the symbol of a quantum logical gate for measurement (Fig. 15).

Still following the logic of the experiment, we focused students’ attention on the fact that, once the measurement is complete, Alice has to communicate her outcome to Bob.

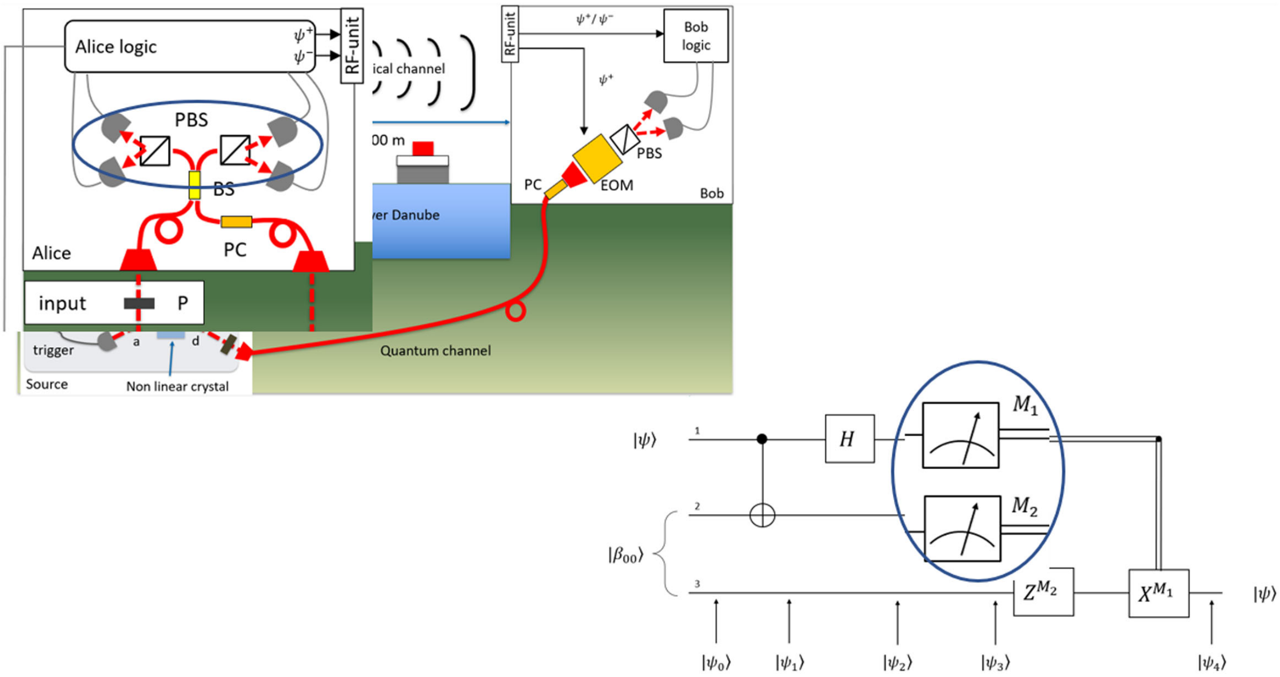


FIG. 15. Comparison between the experimental and computational measurement.

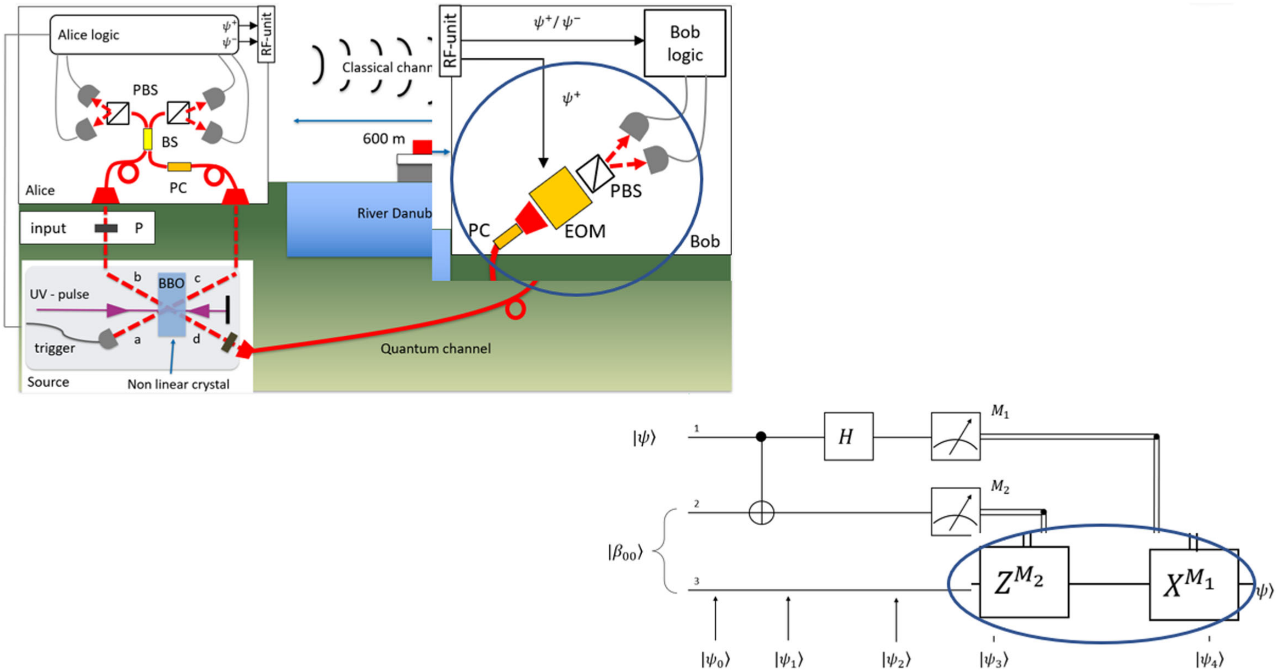


FIG. 16. Comparison between experimental and computational recovery of the input state.

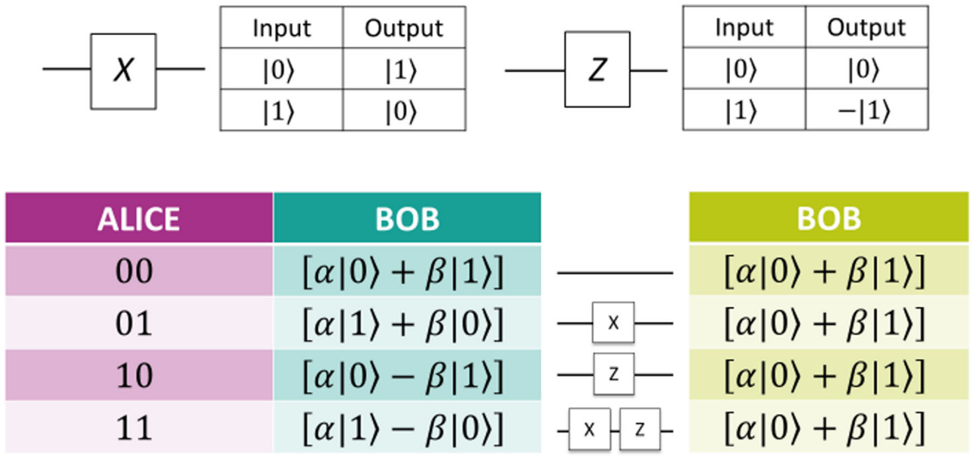
In order to recover the input state in the experimental case, he has to apply a voltage to the EOM, which corresponds to the application of the X and/or Z gates in the circuital case (Fig. 16).

This part of reasoning, as well as proving very engaging, was challenging for the students, since they were asked to apply the learned concept of measurement and state collapse in order to understand what Bob would have obtained if Alice had measured $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. We asked them to recognize which gate had to be applied (X or Z) to complete the teleportation (Fig. 17).

The third and last part of the teaching activity was dedicated to the development of reflections on the implications of teleportation for quantum internet and its potentialities. In order to understand how a quantum network can be created, we introduced the concepts of

(i) maximally entangled states;
(ii) quantum repeater.

We explained to them that the first concept is important because entanglement is fragile, since decoherence caused by the interaction of the quantum system with the environment, quantum noise and absorption, dispersion, and nonlinearity phenomena within the fiber could destroy this quantum bond. Thus, a fairly simple video was presented to the students (produced by QuTech), showing entanglement distillation as a way to make two states optimally entangled and how diamonds, or rather the spins of their carbon atoms, could be used to store information. We then introduced the quantum repeater as something that is able



Input	Output
$\vert 0\rangle$	$\vert 1\rangle$
$\vert 1\rangle$	$\vert 0\rangle$

Input	Output
$\vert 0\rangle$	$\vert 0\rangle$
$\vert 1\rangle$	$-\vert 1\rangle$

ALICE	BOB		BOB
00	$[\alpha\vert 0\rangle+\beta\vert 1\rangle]$		$[\alpha\vert 0\rangle+\beta\vert 1\rangle]$
01	$[\alpha\vert 1\rangle+\beta\vert 0\rangle]$	X	$[\alpha\vert 0\rangle+\beta\vert 1\rangle]$
10	$[\alpha\vert 0\rangle-\beta\vert 1\rangle]$	Z	$[\alpha\vert 0\rangle+\beta\vert 1\rangle]$
11	$[\alpha\vert 1\rangle-\beta\vert 0\rangle]$	X Z	$[\alpha\vert 0\rangle+\beta\vert 1\rangle]$

FIG. 17. Application of logic gates to recover the input state.

to extend the quantum communication interval between sender and receiver. It was then shown that, if you want to transmit information between two network nodes at a distance of 200 km (too far for direct transmission), it is necessary to

- create two entangled qubits between the first end point and the repeater (100 km away) and
- create two further entangled qubits between the repeater and the second end point (100 km away).

By teleportation, the quantum repeater transfers the qubit that is entangled with the first end point to the second end point, forming an entangled link. We showed that the development of a quantum internet is important not only in order to have a secure network, but also because, as quantum computers have such large dimensions and require temperatures close to 0 K, it offers the possibility of remote access to a quantum computer by cloud computing. We concluded the activity by showing students that we are not that far from the realization of quantum internet. Indeed, the research group of Qutech at the University of Delft is expected to produce, by 2020, the first quantum internet that will connect four Dutch cities.

To sum up, the second phase of the educational reconstruction led us to provide a comparison between the teleportation experiment and a circuit with a structure grounded only on those basic concepts that have been proved to be comprehensible to secondary school students. We now move on to the third component of the MER, which concerns the need to carry out and evaluate “pilot instructional modules rather early in the process of educational reconstruction” [33].

VII. PILOT STUDY TO EVALUATE STUDENTS’ REACTIONS

In order to value the effectiveness of the activity on quantum teleportation, we carried out a pilot study that refers to the second implementation of the module. The study refers to a rather small and nonrepresentative sample of secondary school students, since it involves volunteer students, already interested in scientific topics. Therefore, the results are not intended to provide general results and the goal is limited to collecting preliminary signals about the capability of the approach to reach this special target of secondary school students and engage them. Generalizability issues will be addressed in a further step of the research.

To reach our specific goal, we gave the students a questionnaire that included both closed and open-ended questions (see Appendix B). The questionnaire was articulated in three sections, respectively, designed to collect data about (i) engagement and inclusiveness of the students, (ii) students’ reactions to the multilevel structure and the role associated to the four registers of the discourse, (iii) students’ understanding of the main conceptual and epistemological issues represented by the deep changes introduced by quantum computation at the logical level.

The questionnaire was launched after the second implementation (February 2020). Response was purely voluntary. 14 out of 22 students replied (4 female, 10 male students). It was administered through a Google form, and we allowed a week for completion. All the information for management, protection, and data processing was provided, both orally and in paper-based format [52].

For the analysis, a bottom-up approach was adopted, through which we searched for patterns starting from data organized in histograms. The search for patterns was controlled through a triangulation operation, or through a control and discussion process among different researchers. The following analysis has no statistical value, but was conducted to generate an overview of students’ reactions.

The results are presented in a different order from the questionnaire sections, since we think that the quality of students’ understanding (second part of the questionnaire) can be captured more easily if the conceptual responses are framed within a broader picture of their comprehensive attitude toward the class and activities (last part of the questionnaire).

A. Students’ general reaction to the activity

The histogram presented in Fig. 18 refers to students’ answers to these questions: *To what extent [from 1 (not at all) to 5 (very much)] did you find the lesson on teleportation easy, useful to understand quantum technologies, stimulating, fascinating and in accordance with your expectations?*

The graph compares the average ranking of students’ responses. The graph shows that the lesson on teleportation was greatly appreciated and resulted close to their expectations. It was also evaluated useful for understanding quantum technologies.

As we might expect, the lesson was evaluated “not easy”: 1 student out of 14 ranked it as 1, 9 students ranked the level of difficulty as 2, and the other 4 ranked it as 3 (Fig. 19).

The intense and lively discussions that arose during and at the end of the lesson, designed to clarify the most difficult points, confirm the reactions of deep interest and engagement reported in Fig. 18, *in spite of*—or perhaps, *because of*—the innate difficulties. Indeed, it is likely that the perception that they were directly addressing an extremely difficult and advanced topic was itself one reason of interest.

B. Students’ navigation within the multilevel structure and the role associated to the four registers

The third part of the questionnaire set out to check how the students reacted to the multilevel discourse and the role that they associated with the registers. As we have already stated, the teachers told the students explicitly about the

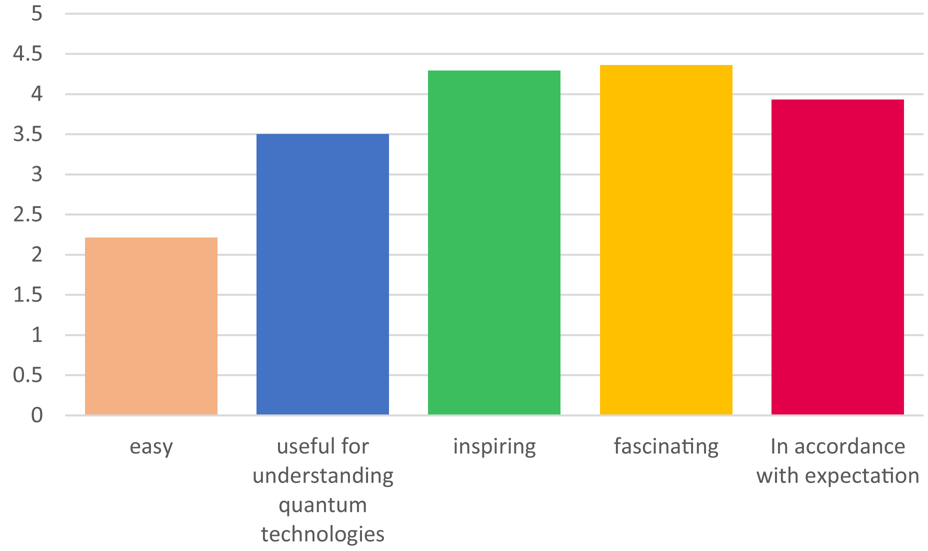


FIG. 18. General evaluation of the activity on teleportation: Average score awarded by students for each aspect.

existence of the levels and informed them when the various levels were switched on. However, the specific roles that we supposed the levels would play were not revealed.

In this part, we asked how much the narrative, logical, technical-experimental, and mathematical level aided them to:

- figure out what quantum teleportation is (general idea about the phenomenon and of the “problem to be solved”);
- understand what the phenomenon of teleportation consists in (understanding of the key moments of the protocol);
- follow the reasoning conducted on teleportation in its entirety (follow the sequence of the key moments to solve the problem);
- grasp the articulation of the reasoning in its different phases (grasp the sense of the sequence);
- understand the details of the reasoning (understand the meaning of the logical and mathematical steps);
- convince themselves of how it happens.

The specific question we posed was

> *What contribution has the register made to your understanding of the phenomenon? Please indicate, from 1 to 5 (1 not at all, 2 a little, 3 quite a lot, 4 very, 5 very much), how much each register has contributed to developing the previous aspects.*

The histogram presented in Fig. 20 shows the mean value of how much the different levels contributed to the six aspects.

As we can see from the graph, the narrative level (in red), together with the mathematical level (in light blue), is the

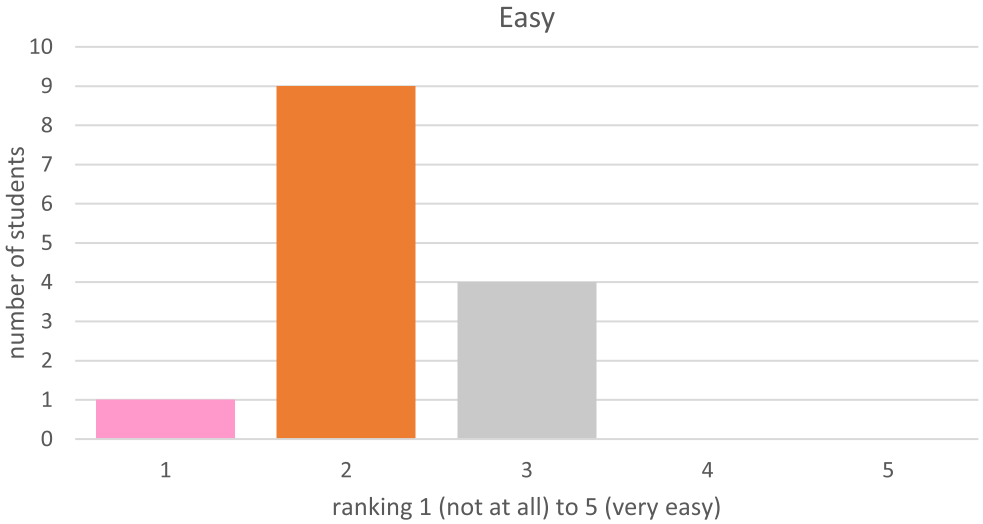


FIG. 19. Distribution of students’ scores (range: 0–5) on the item “easiness”.

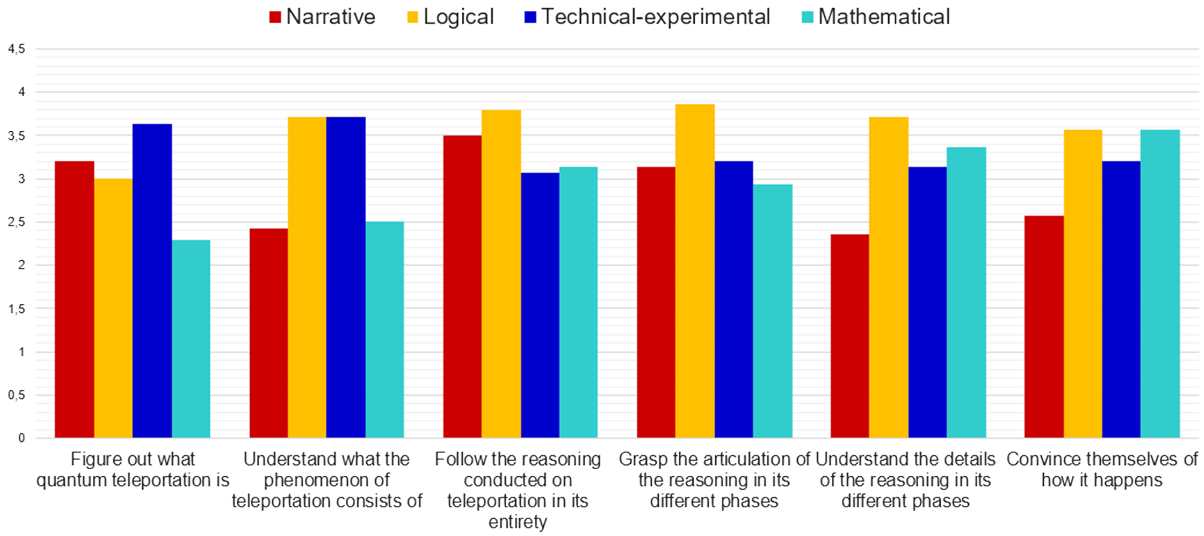


FIG. 20. For the four levels, average of the score attributed to each aspect (range: 0–5).

register that fluctuates most. It proved particularly useful for the aspects that concern the "narration" of the phenomenon: figuring out what quantum teleportation is, following the whole reasoning carried out on teleportation and grasping the articulation of the reasoning in its different phases. All these aspects concur to provide a comprehensive, large-scale picture of the phenomenon.

The logical level (in yellow) appeared to be the most fruitful. This is the register that was more useful for all six aspects and for understanding the phenomenon. In particular, it played an important role in helping students to focus on the details and understand what the phenomenon of teleportation consists of, to follow the whole reasoning carried out on teleportation, to grasp the articulation of the reasoning in its different phases, to understand the details of the reasoning, and to grasp the mechanism that makes teleportation occurs.

The technical-experimental level (in blue) appeared particularly useful for the aspects that concern the physical understanding of the phenomenon. In particular, this register was useful for figuring out what quantum teleportation is and to understand what the phenomenon of teleportation consists of.

Finally, the mathematical level (in light blue) was perceived as useful, in particular, for the aspects concerning "understanding of details": to understand the details of the

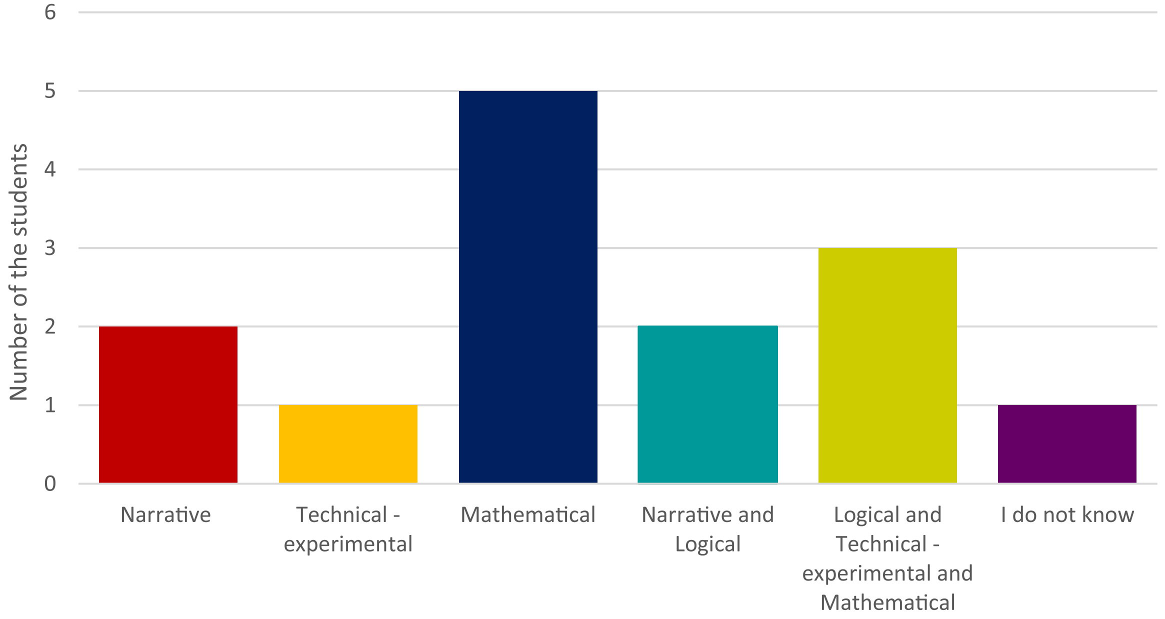


FIG. 21. Distribution of the students' preference of the registers.

reasoning and to clearly grasp the mechanism that makes teleportation occurs.

This result confirms our initial hypotheses. The students have recognized the different roles of the various registers and these roles are consistent to those we attributed to the levels when we designed the activity (Sec. IV). In fact, the narrative level was introduced to promote the formation of a comprehensive view in which the problem could be framed and addressed. The logical level was intended to provide the backbone of the discourse, and indeed the students recognized it as the structure that could reveal and connect the single elements of reasoning and bridge the experiments with the circuital representation. Finally, the technical-experimental level had to give a sense of concreteness and feasibility, while the mathematical level had to foster reasoning, to be developed in its precise and detailed steps.

As well as roles, the various registers received different levels of appreciation. Students' answers to the following question were indeed very varied: "Which register did you prefer?" (see Fig. 21).

It is interesting to note that, although the logical level was retained most useful, it was the favorite one for only three students. The distribution of answers contributes to the supposition that the approach is able to appeal to different tastes and intellectual interests of the students.

However, in the open question about the potential in the intertwining of the four registers, many of them stressed the complementary structural roles played by the different levels: each level had its own role but, by removing even one level, the discourse was no longer complete:

> *"they (the registers) were complementary and filled in the gaps that other reasoning could not; if a register was removed, it was a bit incomplete as reasoning."*
> *"each part, more or less important, takes on its own value, as I don't think a lesson without one of these aspects would be satisfactory"*

Other students highlighted the importance of analyzing a phenomenon from different points of view (multidimensionality) because different points of view provide a more comprehensive or better understanding:

> *"I found that the intertwining of the four registers allowed me to understand the phenomenon in a more complete way, describing the various aspects from different points of view".*
> *"It is always important to compare different aspects of reality to try to understand it as well as possible".*

These reactions to the multilevel structure reveal that the approach is *inclusive*. By inclusiveness we do not refer to the quality of including all the students, thereby ensuring that no one be excluded or marginalized. That is not the case of the students who attended the course, because PLS laboratories are extracurricular activities, and the students are usually very highly motivated and very interested in scientific topics. By inclusiveness we refer to the quality of an approach to resonate with different tastes and interests, to stimulate the students to find "their own way to enter and understand a physics topic." As we stressed in the design principles, this is a crucial aspect for our approach and these results, even though gathered in a pilot study with few students, are very promising. These results are particularly relevant if we consider the last part of our analysis, which allowed us to highlight how the four registers also acted at the level of conceptual understanding. In the following section, we present two cases that show how a special combination of the various registers supported understanding.

C. Students' understanding of the main conceptual and epistemological knots

The questions aimed at investigating students' understanding were formulated as follows:

Images (a) and (b) (Fig. 22) show the experiment and the circuit we have analyzed for teleportation. Try to describe the phenomenon of teleportation, specifying: (i) what is teleported; (ii) what physically corresponds to the moments indicated with M–N–R in figure (a) and (iii) to which parts of the circuit (E–F–G) they correspond.

Most of the students (12 of 14) answered schematically. A typical answer is

1. *The quantum state of photon B is teleported*
2. *R corresponds to the moment in which b and c become entangled, M to the moment of measurement of b and c, N to the moment in which I modifies the photon d knowing the measurements of B and C*
3. *R–E, M–F, N–G*

Although rather synthetic, all the students reached this level and were able to distinguish between the "state of photon" and "photon," by stressing that it is the first that is teleported from one side to the other of the Danube. Moreover, they were able to identify the "logic of the experiment," by recognizing the three main phases, both in the experiment and in the circuit.

Two students gave much more articulated answers, as reported in Table II. When we started to analyze them, we discovered that they differ in three main aspects. The first is the overall structure of the discourse. Indeed, one answer is built on the structure of the circuit representation [Fig. 22(b)] while the other on the experiment [Fig. 22(a)]. The second aspect regards the "main actors" of the discourse: in one case, the discourse follows the narrative of Alice and Bob, while, in the other case, the narrative is focused on description of the concrete experimental steps. The third aspect concerns the language and the linguistic registers and their connection with the different levels along which the discourse is articulated.

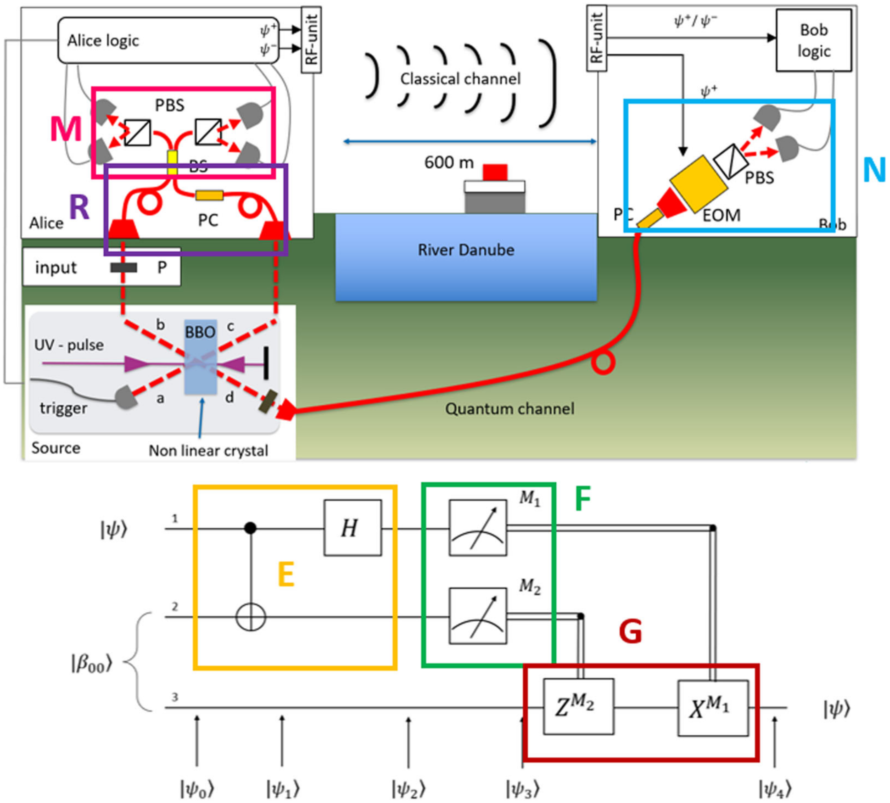


FIG. 22. Experiment and circuit in comparison.

TABLE II. Articulated answers by two students.

Answer #1

“There are three photons b, c and d. Alice has photon c already in entanglement with photon d belonging to Bob, while photon b contains the information that Alice wants to transmit to Bob.

Alice makes photons b and c entangled and then measures. After the measurement, the quantum state collapses and Alice communicates the result of her measurement to Bob through a mechanical channel. Thanks to this measurement, Bob performs transformations on the quantum state of d in order to recover the initial state of b, since, for transitive property, it (photon b) *is entangled with d.*

(I) Only the information contained in the qubit, in the photon b, is teleported, not the matter.

(II) M is the measurement of the states of photons b and c; R is the circuit which creates entanglement between b and c; N is the process to which d is subjected to obtain the initial information of b

(III) M–F; R–E; N–G”

Answer #2

“All this starts with the formation, through a light source that crosses a particular crystal, of two pairs of entangled photons, respectively a-b and c-d. Then the photon “d” will be passed through a channel to an operator that, to simplify, we will call Bob, while, both “b” and “c” remain to Alice (the other operator).

First, Alice makes the two remaining photons entangled and then analyses them (assuming they are therefore entangled).

After the measurement, it is communicated via a classic channel to Bob who, on the basis of what “arrives”, “decides” whether or not to use the two “quantum Boolean operators” X and Z or, based on the response of Alice will modify the state of her photon “d” in four possible ways: using both the “operation” X and Z on the state of the photon, only the X, only the Z or neither. In all cases, a photon with the same superposition characteristics of the “zero or one” state of the “b” photon is obtained, even though it still belongs to Alice.

(i) the state of b is teleported.

(ii) R: an entanglement relationship is created between the photons b and c

M: check that these photons are entangled and analyze them

N: after the communication of the state, the characteristics of the photon “d” are altered.

(iii) $E = R$, $F = M$, $G = N''$

In Fig. 23 we report the analysis of answer #1.

The structure of the discourse is articulated in an introduction (the description of the initial state) and four blocks that are the four phases of the circuit: the creation of a state of entanglement between photons *b* and *cd*; the measurement; the communication through a classical channel, and the transformation to perform in order to recover the initial state.

The answer is provided in a narrative form where Alice and Bob are the main actors and the steps are described in terms of their actions: "*Alice makes photons b and c entangled and then measures," "Alice communicates the result of her measurement to Bob," "Bob performs transformations."* The narrative level, in students' discourse, emerges as playing the role of keeping together what happens and the different actions performed by Alice and Bob.

As for the language, even though the student chose the logic of the circuit to structure her discourse, the words she uses are closer to the language of the experiment. She indeed uses the words *photons*, *transmission*, and *communication through a mechanical channel* to describe what happens in each phase. This leads us to infer that the circuit provided a criterion to select the main pillars of the logical structure, while the experiment and the physical phenomena were used to attach meaning to each step.

The student's answer includes all the registers but the mathematical one, and they mirror a personal articulation of her discourse along the levels. She indeed combines the registers in her idiosyncratic way that is different from the student who provided answer #2 (see Fig. 24), which is described in the following.

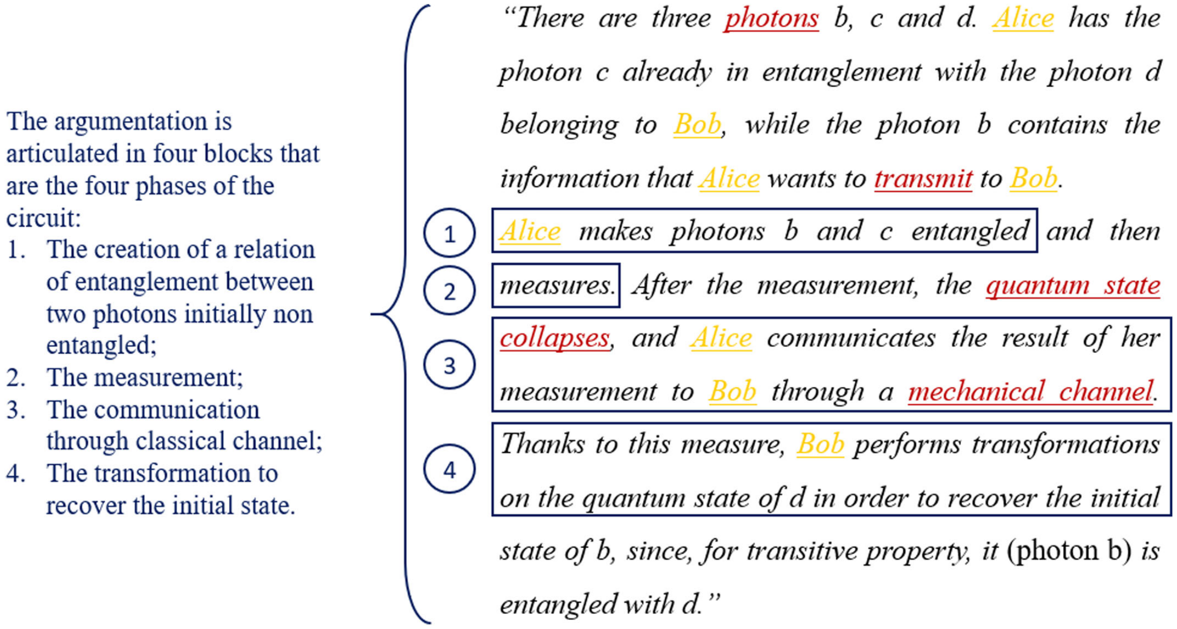
As for the structure of the discourse, this student does not use the separation in blocks of Fig. 22, but instead explicitly starts from the experiment. In fact, the discourse follows the phase of the experiment starting from the production of the two pairs of entangled photons and also describing the process ("*All this starts with the formation, through a light source that crosses a particular crystal, of two pairs of entangled photons, respectively a-b and c-d.*").

The "main actors" of the narrative level are the processes and the physical operations: Alice and Bob are the experimenters.

For the core description of the phenomenon, his language taps mainly into the experimental register, and characteristic expressions are: *the photon "d" will be passed through a channel to an operator, analyses them* (the photons). Instead in the final part, he switches into a highly specialized logical register: "[…] *whether or not to use the two "quantum Boolean operators" X and Z or, based on the response of Alice will modify the state of her photon "d" in four possible ways*: […]".

As with the previous one, this student managed to understand the phenomenon by drawing on the three registers.

While in answer #1 we noticed a logical structure acting as backbone of the discourse, in answer #2 we notice a preference for the technical-experimental level, especially in the description of the processes. Despite initial detachment from the story of Alice and Bob, the narrative register is used to reconstruct the sequence of events, confirming the role that we have attributed to this level: the construction of an overarching picture that allows the building of an overarching idea without becoming lost in details.



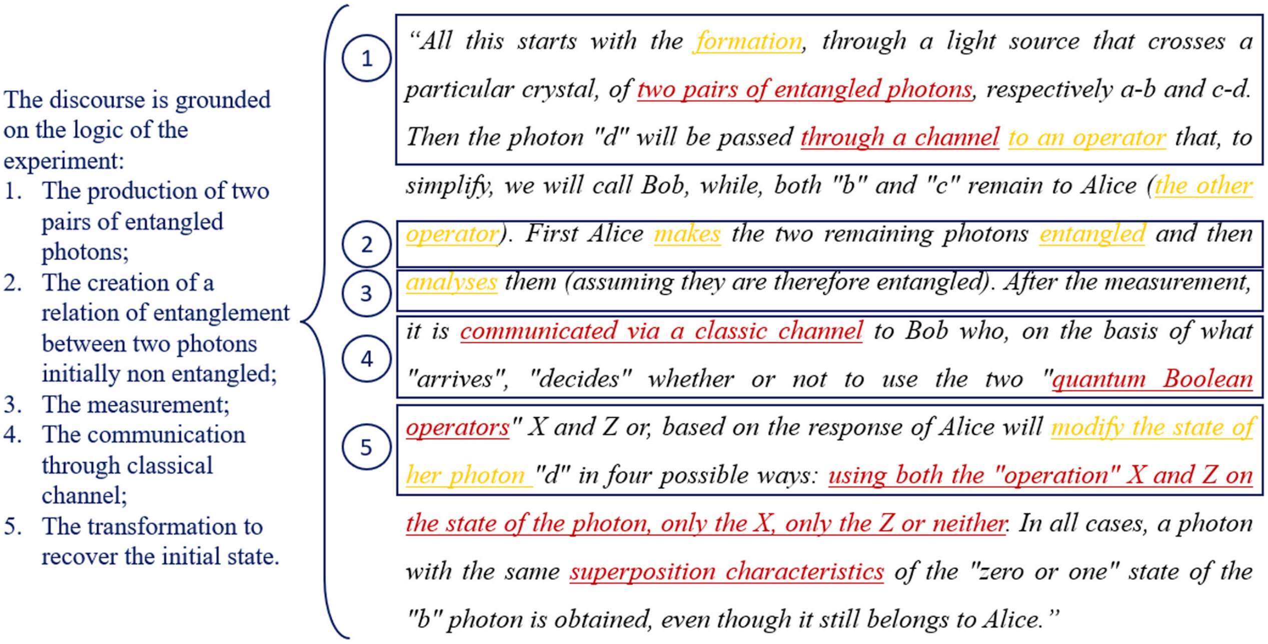
The argumentation is articulated in four blocks that are the four phases of the circuit:

1. The creation of a relation of entanglement between two photons initially non entangled;
2. The measurement;
3. The communication through classical channel;
4. The transformation to recover the initial state.

"There are three photons b, c and d. Alice has the photon c already in entanglement with the photon d belonging to Bob, while the photon b contains the information that Alice wants to transmit to Bob.

① *Alice makes photons b and c entangled and then*
② *measures. After the measurement, the quantum state*
③ *collapses, and Alice communicates the result of her measurement to Bob through a mechanical channel.*
④ *Thanks to this measure, Bob performs transformations on the quantum state of d in order to recover the initial state of b, since, for transitive property, it* (photon b) *is entangled with d."*

FIG. 23. Analysis of answer #1—on the left the structure of the discourse; in yellow the "main actors"; in red the key-words characterizing the registers.



The discourse is grounded on the logic of the experiment:

1. The production of two pairs of entangled photons;
2. The creation of a relation of entanglement between two photons initially non entangled;
3. The measurement;
4. The communication through classical channel;
5. The transformation to recover the initial state.

① *"All this starts with the formation, through a light source that crosses a particular crystal, of two pairs of entangled photons, respectively a-b and c-d. Then the photon "d" will be passed through a channel to an operator that, to simplify, we will call Bob, while, both "b" and "c" remain to Alice (the other* ② *operator). First Alice makes the two remaining photons entangled and then* ③ *analyses them (assuming they are therefore entangled). After the measurement,* ④ *it is communicated via a classic channel to Bob who, on the basis of what "arrives", "decides" whether or not to use the two "quantum Boolean* ⑤ *operators" X and Z or, based on the response of Alice will modify the state of her photon "d" in four possible ways: using both the "operation" X and Z on the state of the photon, only the X, only the Z or neither. In all cases, a photon with the same superposition characteristics of the "zero or one" state of the "b" photon is obtained, even though it still belongs to Alice."*

FIG. 24. Analysis of answer #2—on the left, the structure of the discourse; in yellow, the "main actors"; in red, the key-words characterizing the registers.

In the section of the questionnaire designed to investigate students' understanding, the second question was: *What main differences do you find between the experiment shown in figure (a) and the circuit represented in figure (b)? More generally, what analogies and differences do you see between a description of a phenomenon made in terms of experiment and a description made in terms of circuit?*

In order to investigate the differences and analogies that students find between the circuit and the experiment, we have analyzed the words that the students used to refer to one or to the other. A simple count shows that the words most frequently associated to the circuit are: *trivial*, *schematic*, *sequential* (refers to sequential organization of processes or events), and *simplification*. They are used both in a positive sense—a simplification that shows the essential aspects of the experiment—and in a negative sense—too simple to be significant without a concrete description of it. Regarding the experiment, the most frequent words are *general functioning*, *global vision*, *practicality*, and *utility.*

Some answers that highlight the differences (sequential vs global, schematic or abstract vs practical) are

> *"Both describe the series of changes that occur in the system, but while the description of the phenomenon in terms of the experiment highlights a more global vision, the circuit is more schematic and shows the steps in series"*
>
> *"Both a circuit and an experiment describe the same steps of a phenomenon, but the experiment describes them from a more practical point of view, while the circuit in a more theoretical way".*

A few students focused their answer on the role and usefulness of both representations. For example, two students wrote

> *"The experiment is useful for understanding the general functioning, understanding what is being done […]. The circuit instead simplifies it (the experiment), eliminating the most complex parts and reducing it to the essential form"*
>
> *"The circuit abstracts the concept of the experiment and is important for generalizing the principles of reality also in other conceptual structures. The experiment serves to understand the technologies used and how the measurement is carried out in reality".*

Again, the students stress the potential of the circuit to show the essential structure of the experiment, while the experiment gives concreteness to the circuit, by showing how teleportation takes place "*in reality*," how "*it is contextualized in reality*" (*general functioning*).

The answers reveal the extent to which the experiment and the circuit are different and focus on different aspects of the model of the phenomenon. Particularly importantly, they stimulate the formation of different kinds of imagery and explanations. The experimental approach encourages students to follow the events and photons in a spacetime framework, allowing them to grasp the counterintuitive essence of entanglement as a "spooky action at a distance." The circuital approach, instead, suggests a systemic view of the phenomenon, allowing a student to have a global comprehensive picture of the entire system within which logical steps and details can be coherently placed.

From an educational perspective, this is known to be particularly problematic in quantum physics. As Mannila, Koponen, and Niskanen [9] showed, “students are used to direct their attention to properties of entities (particle, bodies, etc.), create images and draw pictures, where illustrations concentrate on the behavior of entities. A similar approach is very difficult in quantum physics where the properties of basic entities are difficult to approach, and one should really concentrate on properties of phenomena” and foster a proper “‘conceptual shift to form a new ontology’” [9].

VIII. CONCLUSIONS

A module on quantum computation has been developed to respond to the challenges launched by different programs on the research and development of quantum technologies (such as the quantum flagship). In fact, many of them show the dimension of education as a pivotal aspect in order to meet the need of increasing the workforce and reaching quantum awareness and literacy [3].

In this paper, we presented and discussed an educational approach to quantum computing that led us to design a module for secondary school students and, in particular, to design an activity on quantum teleportation.

In our opinion, the second quantum revolution is primarily a cultural revolution. So, even before the more technical aspects, it is important to build and provide lenses to understand and interpret it. The four design principles have been pointed out in order to answer the two research questions and to pursue the goal of stressing that the quantum revolution is not only a matter of calculus power and military strategies, but that it also conveys deep changes at the level of the foundations and at the inner logic that quantum computers follow. Furthermore, the principles aimed to expand physics learning along many dimensions so as to be inclusive and diversity responsive.

The four principles have been implemented not only in the design of the whole module, but also in the design on the specific activity of teleportation. This design was the result of a process of “educational reconstruction” [33] and, consistently, it emerged from three analytical and design phases: analysis and clarifications of science content, content reconstruction in light of the research on teaching and learning quantum physics, with emphasis on students’ perspectives, and pilot study on designed instructional materials.

The results of the pilot study show that the activity was not only within the reach of the specific target of secondary school students we involved but also very engaging. Moreover, two basic choices were revealed to be particularly fruitful for fostering understanding of the phenomenon and the essence of quantum technologies: (a) the choice to base the module on the comparison between experiments and circuits and, hence, to base the activity on the reconceptualization of the teleportation experiment as a computational device (design principle #2); (b) the choice to articulate the discourse on the narrative, logical, technical-experimental, and mathematical levels (design principle #4).

Regarding the comparison, the students found the circuit and experiment to be two different but “*complementary*” perspectives. The two “versions” of the quantum protocol have effectively leveraged on different aspects of the phenomenon and provided two different images, both needed to build a comprehensive view: an image spatially organized in terms of experimental equipment and, hence, in terms of experimental operations; and an image logically organized in terms of logical gates and, hence, in terms of states manipulation.

As for the four-level discourse, the students recognized the structural articulation along different levels, and said recognition was fruitful and effective in allowing them to move back and forth between the global picture and specific details. More specifically, the data analysis confirms that the narrative level served to promote the formation of a comprehensive view in which the problem can be framed and addressed, while the logical level acted as *backbone* of the discourse. The third discourse level, technical experimental, restored a sense of concreteness and feasibility, while the mathematical level guided the students to understand and follow the calculation details that created the logical steps.

The most interesting result concerns students’ acknowledgment both of the role of logic and of the multi-level discourse structure. This means that the logical structure was indeed crucial in order to emphasize the single elements of reasoning, connect them, and bridge the experiment with the circuit. In particular, the logical level showed its potential to exploit the diversity of the students and resonate with different personal ways of understanding. In fact, we were able to recognize the signal, from a couple of students, that they felt encouraged to use and combine various registers to elaborate personal descriptions of the phenomenon. This signal, although emerging from a small sample, is consistent with the research that we have carried out on other topics, i.e., thermodynamics [53], quantum physics [49], and, from this point of view there is good reason to retain that it exerts a real effect on the approach. However, further implementations are going to be carried out to deeply investigate the meaningfulness and robustness of the approach with a larger sample of students.

Indeed, the results achieved in this pilot study are the basis for further studies that we are planning to expand the span of activities on quantum protocols and the target groups. As for the activities, we have already implemented in the second round an activity on the classical and quantum random walk, but further protocols will be investigated to check the educational potential of the approach.

Regarding the target groups, we are testing the teaching module in three further contexts: (a) summer schools for university students who come from STEM backgrounds but do not necessarily have any knowledge of quantum physics; (b) summer or winter school for preservice physics teachers who are attending masters' courses in physics education, mathematics education, or computer science education, (c) in-service courses, and (d) further courses for secondary school students that involve entire classes (and not only volunteer students). The last subgroup will allow us to check the generalizability of the results and the span of the students reachable by the approach, beyond the highly motivated and volunteer students who have been involved in this pilot.

Comparison of the results will allow us to merge student reactions that are intrinsically due to the approach with those which are, instead, context dependent.

ACKNOWLEDGMENTS

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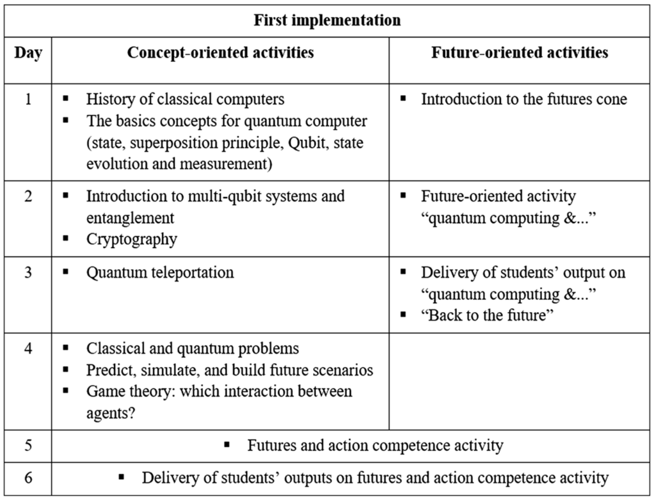
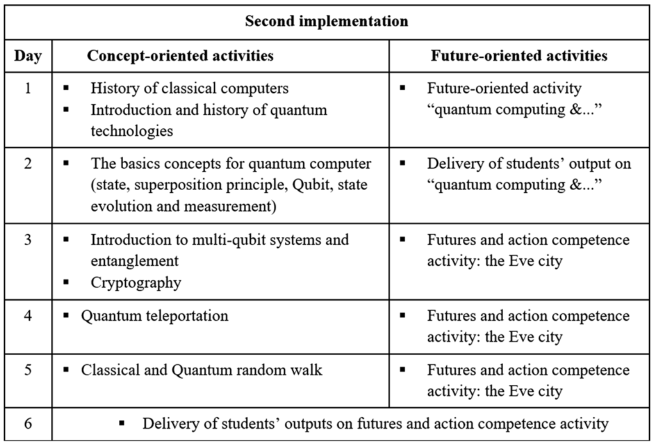
APPENDIX A: THE MODULE'S TIMETABLE

The timetables in Figs. 25 and 26 show the activities for the first two implementation. The section dedicated to the history of classical computing was held by a secondary school teacher with professional expertise in classical computing architectures and algorithms. The introductory lectures on the physics of quantum computers and the lecture on the quantum cryptography were held by E. E. The lesson on teleportation was held by S. S. The description of the future-oriented activities in the first implementation is reported by Spada [54].



First implementation		
Day	**Concept-oriented activities**	**Future-oriented activities**
1	▪ History of classical computers ▪ The basics concepts for quantum computer (state, superposition principle, Qubit, state evolution and measurement)	▪ Introduction to the futures cone
2	▪ Introduction to multi-qubit systems and entanglement ▪ Cryptography	▪ Future-oriented activity "quantum computing &..."
3	▪ Quantum teleportation	▪ Delivery of students' output on "quantum computing &..." ▪ "Back to the future"
4	▪ Classical and quantum problems ▪ Predict, simulate, and build future scenarios ▪ Game theory: which interaction between agents?	
5	▪ Futures and action competence activity	
6	▪ Delivery of students' outputs on futures and action competence activity	

FIG. 25. First implementation's timetable.



Second implementation		
Day	**Concept-oriented activities**	**Future-oriented activities**
1	▪ History of classical computers ▪ Introduction and history of quantum technologies	▪ Future-oriented activity "quantum computing &..."
2	▪ The basics concepts for quantum computer (state, superposition principle, Qubit, state evolution and measurement)	▪ Delivery of students' output on "quantum computing &..."
3	▪ Introduction to multi-qubit systems and entanglement ▪ Cryptography	▪ Futures and action competence activity: the Eve city
4	▪ Quantum teleportation	▪ Futures and action competence activity: the Eve city
5	▪ Classical and Quantum random walk	▪ Futures and action competence activity: the Eve city
6	▪ Delivery of students' outputs on futures and action competence activity	

FIG. 26. Second implementation's timetable.

APPENDIX B: THE QUESTIONNAIRE

In this appendix we attach the complete text of the questionnaire designed for collect data on (i) the engagement and inclusiveness of the students, (ii) the students' reactions to the multilevel structure and the role associated to the four registers of the discourse, (iii) the students' understanding of the main conceptual and epistemological issues represented by the deep changes introduced by quantum computation at the logical level.

1. Part 1: The experiment and the circuit

a. The images (a) and (b) in Fig. 27 show the experiment and circuit that we analyzed. Try to re-describe the teleportation, specifying: (i) what is teleported; (ii) what physically corresponds to "themoments" indicated in figure (a) with M–N–R and (iii) to which parts of the circuit (E–F–G) they correspond.

b. What main differences do you find between the experiment shown in figure (a) and the circuit represented in figure (b)?

c. More generally, what analogies and differences do you see between a description of a phenomenon made in terms of experiment and a description made in terms of circuit?

d. What meaning do you attribute to the phrase repeated throughout the course: "an experiment can be seen as a device that manipulates information and therefore also be represented in circuital terms"?

e. Which of the two representations do you prefer and why?

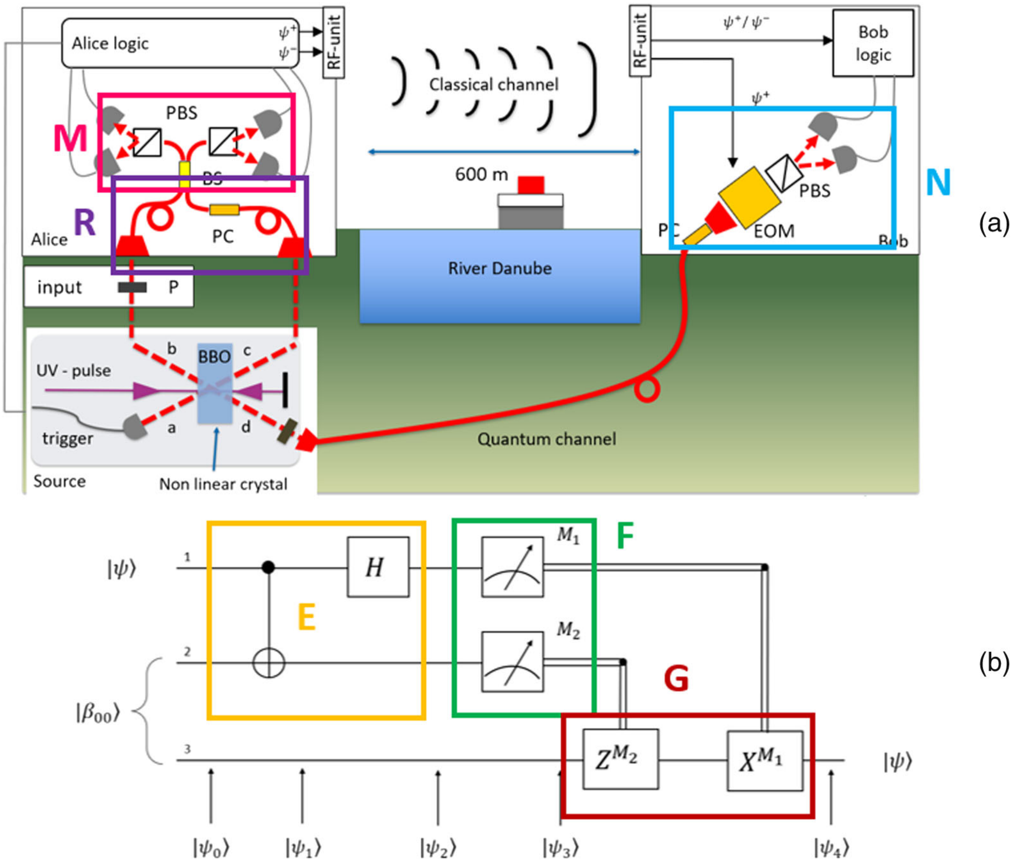


FIG. 27. Experiment and circuit in comparison.

2. Part 2: The four registers

a. As we discussed together, we divided the discourse on teleportation into 4 registers: narrative, logical, technical–experimental (the register referred to the description of the experiment) and mathematical. Each of the registers was chosen to play a different role. What idea did you form about the role attributed to each of them? Complete the following tables about the narrative (Fig. 28), the logical (Fig. 29), the technical-experimental (Fig. 30) and the mathematical level (Fig. 31).

b.

NARRATIVE LEVEL: what contribution has this register made to your understanding of the phenomenon? Specifically, on a scale of 1 to 5, how much has this register contributed to:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Figuring out what quantum teleportation is					
Understanding what the phenomenon of teleportation consists in					
Following the whole reasoning carried out on teleportation					
Grasping the articulation of the reasoning in its different phases					
Understanding the details of the reasoning					
Having a clear understanding of the mechanism that makes teleportation occur					

FIG. 28. About the narrative level.

c.

LOGICAL LEVEL: what contribution has this register made to your understanding of the phenomenon? Specifically, on a scale of 1 to 5, how much has this register contributed to:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Figuring out what quantum teleportation is					
Understanding what the phenomenon of teleportation consists in					
Following the whole reasoning carried out on teleportation					
Grasping the articulation of the reasoning in its different phases					
Understanding the details of the reasoning					
Having a clear understanding of the mechanism that makes teleportation occur					

FIG. 29. About the logical level.

d.

TECHNICAL–EXPERIMENTAL LEVEL: what contribution has this register made to your understanding of the phenomenon? Specifically, from 1 to 5, how much this register has contributed to:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Figuring out what quantum teleportation is					
Understanding what the phenomenon of teleportation consists in					
Following the whole reasoning carried out on teleportation					
Grasping the articulation of the reasoning in its different phases					
Understanding the details of the reasoning					
Having a clear understanding of the mechanism that makes teleportation occur					

FIG. 30. About the technical-experimental level.

e.

MATHEMATICAL LEVEL: what contribution has this register made to your understanding of the phenomenon? Specifically, on a scale of 1 to 5, how much has this register contributed to:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Figuring out what quantum teleportation is					
Understanding what the phenomenon of teleportation consists in					
Following the whole reasoning carried out on teleportation					
Grasping the articulation of the reasoning in its different phases					
Understanding the details of the reasoning					
Having a clear understanding of the mechanism that makes teleportation occur					

FIG. 31. About the narrative level.

f. Have you found any potentialities in the intertwining of the four registers? If so, which?

g. Which register did you prefer?

i. Do you have any suggestions for improving the lesson?

j. Do you want to add any comments?

h.

To what extent did you find the activity about teleportation:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Easy					
Useful for understanding quantum technologies					
Inspiring					
Fascinating					
Within expectations					

FIG. 32. About the teleportation activity answer to the following question (Fig. 32).

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