A Trigonometric Solution to the Problem of Overmodulation in Five-Phase Inverters

Luca Vancini
Dept. of Electrical, Electronic and Information Engineering "G. Marconi"
University of Bologna
Bologna, Italy
https://orcid.org/0000-0003-1897-8755

Michele Mengoni
Dept. of Electrical, Electronic and Information Engineering "G. Marconi"
University of Bologna
Bologna, Italy
https://orcid.org/0000-0002-6639-3017

Giacomo Sala
Dept. of Electrical, Electronic and Information Engineering "G. Marconi"
University of Bologna
Bologna, Italy
https://orcid.org/0000-0002-6374-7713

Gabriele Rizzoli
Dept. of Electrical, Electronic and Information Engineering "G. Marconi"
University of Bologna
Bologna, Italy
https://orcid.org/0000-0003-0119-3679

Luca Zarri
Dept. of Electrical, Electronic and Information Engineering "G. Marconi"
University of Bologna
Bologna, Italy
https://orcid.org/0000-0002-5630-2126

Angelo Tani
Dept. of Electrical, Electronic and Information Engineering "G. Marconi"
University of Bologna
Bologna, Italy
https://orcid.org/0000-0003-2347-8480

Abstract— **The exploitation of the dc-link in multiphase inverters raises the problem of overmodulation, which has some peculiar traits compared to three-phase inverters. Typically, overmodulation allows exploiting the dc-bus voltage but suffers from the harmonic distortion of the output voltage. In multiphase inverters, overmodulation causes the pollution of the harmonic subspaces, which are usually independent of each other in the linear modulation range. The solution to the problem of the overmodulation is not unique and affects the performance of the drive. Several overmodulation techniques, which differ in the optimized quality index, have already been proposed for five-phase inverters in the literature. The contribution of this paper is to show with a rigorous analysis that the overmodulation problem can be transformed into a set of trigonometric inequalities, which can be easily solved. Experimental tests on a five-phase induction motor drive are carried out to verify the feasibility of the developed techniques and the quality of the current waveforms.**

Keywords—Overmodulation, Multiphase induction Motor Drive, Pulse-Width Modulation, Five-Phase Induction Motor, Multiphase Machines.

I. Introduction

Multiphase drives are generally used in high-reliability and high-power applications, where a higher number of phases (higher than three) brings benefits such as better fault tolerance and lower phase current. Furthermore, multiphase systems have features that make them worthy of consideration in many industrial areas, such as the multi-motor applications. A multiphase motor drive is usually fed by a multiphase inverter, controlled with a carrier-based Pulse Width Modulation (PWM). Whereas the analysis of a three-phase system usually involves only two degrees of freedom, the representation of a multiphase system can be much more complicated. This complexity is usually overcome through the Vector Space Decomposition (VSD), a mathematical tool that extends three-phase Clarke's transformation to a higher number of phases. The main advantage of the VSD is that each spatial harmonic of the magnetic field of the electric machine can be mapped into a specific subspace [1]. The machine equations can be split into separate groups, which describe the magnetic interactions between stator and rotor through each spatial component of the field.

The VSD approach allows calculating the voltage limits of the converter. The linear modulation region for multiphase inverters operating with rotating voltage vectors in all subspaces can be analytically calculated [2]. However, to fully exploit the dc-link voltage, an inverter must operate in the so-called overmodulation region. In this operating zone, the mean value of the output voltage over a switching period does not match the reference voltage. Thus, different methods can be developed to approximate the reference voltage space vector or improve the performance of the motor drive [3]-[5].

An overmodulation technique for the PWM control of multiphase inverters with an odd number of phases, based on James A. Cadzow's iterative technique to find the minimum infinity-norm solution of an under-determined linear system, is already available in the literature [6]. This method has already been applied to five-phase inverters, illustrated in Fig. 1 [7]. Alternative solutions based on Space Vector Modulation, have also been developed for five-phase drives [8]–[10] and inverters with for a generic number of phases [11].

However, all these solutions are not equivalent to each other but differ in terms of distortion of the output voltage, operating range, efficiency, complexity, and calculation burden. Apparently, no solution has prevailed over the others and has
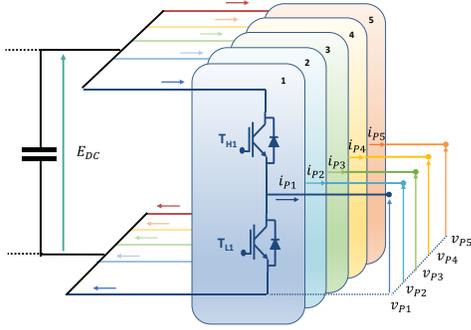
Fig. 1 – Schematic of a five-phase inverter.

been recognized as preferable. Therefore, the present paper aims to contribute to this research topic and illustrate a new formulation of the overmodulation problem for five-phase inverters, which is converted into a set of trigonometric equations. These equations not only reveal the nonlinear nature of the problem but offer a coherent solution. Compared to the previous solutions, the developed algorithm does not require iterations and generates the lowest voltage distortion.

II.   MODULATION PROBLEM IN FIVE-PHASE INVERTERS

The analysis of five-phase systems is usually based on the definition of multiple space vectors and zero-sequence component. For a given set of real voltages $v_1, ..., v_5$, a new set of variables $v_0, \bar{v}_1$ and $\bar{v}_3$ can be defined through the following symmetrical linear transformations:

$$v_0 = \frac{1}{5}\sum_{k=1}^{5} v_k \tag{1}$$

$$\bar{v}_\rho = \frac{2}{5}\sum_{k=1}^{5} v_k \bar{\alpha}_k^\rho \tag{2}$$

where $\bar{\alpha}_k = e^{j\frac{2}{5}\pi(k-1)}$.

The real quantity v_0 defined in (1) is the zero-sequence component, whereas the so-called voltage space vectors $\bar{v}_\rho$ ($\rho = 1, 3$) are complex quantities. The inverse transformation turns out to be as follows:

$$v_k = v_0 + \bar{v}_1 \cdot \bar{\alpha}_k + \bar{v}_3 \cdot \bar{\alpha}_k^3 \tag{3}$$

where the dot operator "$\cdot$" is defined as the real part of the product of the first operand and the complex conjugate of the second operand.

A. Modulation Constraints

The voltage domain of five-phase inverters includes two overmodulation regions, depicted in Fig 2. In the first one the converter can generate the reference voltage vector $\bar{v}_{1,ref}$ in the fundamental subspace α₁-β₁ if the voltage vector $\bar{v}_{3,ref}$ in subspace α₃-β₃ is different from zero and conveniently chosen. In the second one, the match between the reference voltage and the actual voltage can be only approximative, even in the fundamental subspace.

In general, if the electric machine fed by the inverter has a sinusoidal back-emf and an adequate value of the stator inductance, the voltage distortion in the overmodulation operation does not significantly affect the motor operation. Under these assumptions, it is possible to consider the voltage $\bar{v}_3$ as a degree of freedom for the modulation strategy because it is not necessary to compensate for the circulation of a significant current in subspace α₃-β₃.

The link between the modulating signals and the phase voltages is through the following equations:

$$m_0 = \frac{v_0}{E_{DC}} \tag{4}$$

$$\bar{m}_\rho = \frac{\bar{v}_\rho}{E_{DC}} \qquad \rho = 1, 3 \tag{5}$$

where E_{DC} is the dc-link voltage.

The modulating signals must satisfy the following constraints:

$$m_k \in [0,1] \quad k=1, ..., 5. \tag{6}$$

The constraints in (6) imply the following equivalent set of inequalities:

$$m_h - m_k \leq 1 \quad k = 1, ..., 5; \, h = 1, ..., 5. \tag{7}$$

Inequalities (7) combined with (3) can be rewritten in terms of complex vectors as follows:

$$\bar{v}_{1,ref} \cdot \bar{A}_{1,h,k} + \bar{v}_{3,ref} \cdot \bar{A}_{3,h,k} \leq E_{DC} \qquad \begin{matrix} h = 1, ..., 5; \\ k = 1, ..., 5. \end{matrix} \tag{8}$$

where

$$\bar{A}_{1,h,k} = \bar{\alpha}_h - \bar{\alpha}_k \tag{9}$$

$$\bar{A}_{3,h,k} = \bar{\alpha}_h^3 - \bar{\alpha}_k^3. \tag{10}$$

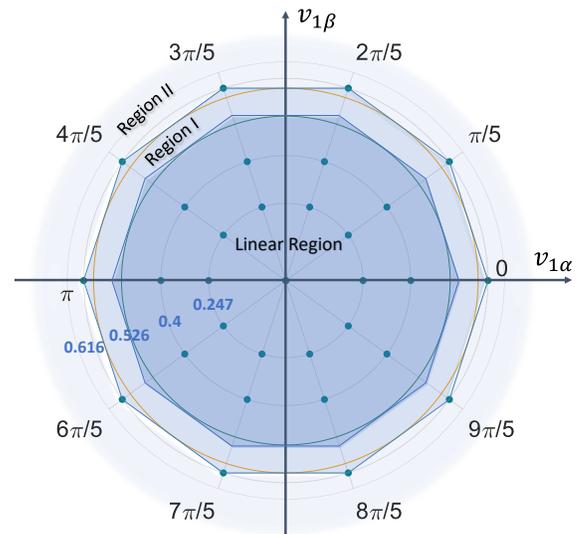Generally, the reference value $\bar{v}_{1,ref}$ is generated by the

Fig. 2 – Modulation regions in a five-phase inverter.

motor control system, whereas v_0 in (4) is a degree of freedom that can be used to maximize the voltage range [12], to optimize some characteristics of the converter, such as the switching losses [13], the spectrum of the output current [14], and the sizing of the dc-link capacitors [15]. Conversely, $\bar{v}_{3,ref}$ in linear modulation region can be chosen in order to reduce the harmonic distortion of the stator current [16], ensure fault tolerance [17] and estimate the motor parameters [18].

B. Range of Linear Modulation

In linear modulation region $\bar{v}_{3,ref}$ can be set to zero if it is not used for other purposes. In this condition, (8) can be easily solved to obtain the maximum amplitude of $\bar{v}_{1,ref}$ which delimits the linear modulation region. In fact, (8) can be equivalently rewritten as follows:

$$cos\left(\theta_1 - arg\bar{A}_{1,h,k}\right) \leq \frac{E_{DC}}{\left\|\bar{A}_{1,h,k}\right\|V_1} \quad \begin{matrix} h = 1, ..., 5; \\ k = 1, ..., 5, \\ k \neq h. \end{matrix} \quad (11)$$

where

$$\bar{v}_{1,ref} = V_1 e^{j\theta_1}. \quad (12)$$

Equation (11) must be verified for all possible combinations of h and k (h=1, …,5, k=1, …, 5) and for any position of the angle θ_1. The worst-case scenario occurs when $\theta_1 = arg\bar{A}_{1,h,k}$ and the amplitude of $\bar{A}_{1,h,k}$ is maximum. In this condition inequality (11) can be written as follows:

$$V_1 \leq \frac{E_{DC}}{2\sin\left(\frac{2\pi}{5}\right)} \approx 0.526. \quad (13)$$

Equation (13) describes the well know voltage constrain for a five-phase inverter in linear modulation region when $\bar{v}_{3,ref}$ is set to zero.

III. Overmodulation in Five-Phase Inverters

In this paper, $\bar{v}_{3,ref}$ is used to extend the linear modulation region of $\bar{v}_{1,ref}$ and go beyond the threshold value given by (13).

For this aim, (8) can be rewritten as a set of trigonometric inequalities:

$$cos\left(\theta_3 - arg\bar{A}_{3,h,k}\right) \leq \frac{\varepsilon_{h,k}}{V_3} \quad \begin{matrix} h = 1, ..., 5; \\ k = 1, ..., 5, k \neq h. \end{matrix} \quad (14)$$

where

$$\bar{v}_{3,ref} = V_3 e^{j\theta_3} \quad (15)$$

$$\varepsilon_{h,k} = \frac{E_{DC} - \bar{v}_{1,ref} \cdot \bar{A}_{1,h,k}}{\left\|\bar{A}_{3,h,k}\right\|}. \quad (16)$$

Equation (14) must be verified for all possible combinations of h and k (h=1, …,5, k=1, …, 5). Let us suppose that H and K are the indexes that identify the smallest coefficient $\varepsilon_{h,k}$. The *cos* function is never lower than -1, then the set of inequalities (14) admits a solution only if $\varepsilon_{H,K}/V_3$ is greater or equal than -1.

This condition can be equivalently written as follows:

$$V_3 \geq -\varepsilon_{H,K}. \quad (17)$$

Since V_3 is defined as a magnitude of a vector, it cannot be negative, the constraint for V_3 becomes

$$V_3 \geq max\{0, -\varepsilon_{H,K}\}. \quad (18)$$

Inequality (18) defines the beginning of the overmodulation region I and shows that V_3 is necessary to modulate $\bar{v}_{1,ref}$ only if $\varepsilon_{H,K}$ is negative. Otherwise, V_3 can be set equal to zero. In fact, if $\varepsilon_{H,K}$ is positive, even infinitely small values of V_3 could make $\varepsilon_{H,K}/V_3$ greater than 1 and satisfy (14) for any value of θ_3.

Conversely, if $\varepsilon_{H,K}$ is negative, the lowest value of V_3 that satisfies (18) is $-\varepsilon_{H,K}$. With this value for V_3, the inequality with indexes H and K in the set (14) can be rewritten as follows:

$$cos\left(\theta_3 - arg\bar{A}_{3,H,K}\right) \leq -1. \quad (19)$$

Inequality (19) admits a solution, $\theta_3 = arg\bar{A}_{3,H,K} + \pi$, which can be used in (15) to find an explicit expression of the vector $\bar{v}_{3,ref}$ with the minimum amplitude satisfying inequalities (8):

$$\bar{v}_{3,ref} = -\frac{\bar{A}_{3,H,K}}{\left\|\bar{A}_{3,H,K}\right\|}\varepsilon_{H,K}. \quad (20)$$

Unfortunately, it can be verified that this solution is valid only if the magnitude of $\bar{v}_{1,ref}$ is not too high. When the magnitude of $\bar{v}_{1,ref}$ increases, two coefficients $\varepsilon_{h,k}$ tend to become negative. In this case, it is still possible to find a solution to (14). If $\varepsilon_{L,M}$ is the second lowest coefficient, then the following set of equations provides a candidate solution for (14):

$$cos\left(\theta_3 - arg\bar{A}_{3,H,K}\right) = \frac{\varepsilon_{H,K}}{V_3} \quad (21)$$

$$cos\left(\theta_3 - arg\bar{A}_{3,L,M}\right) = \frac{\varepsilon_{L,M}}{V_3}. \quad (22)$$

It is straightforward to verify that equations (21)-(22) are equivalent to (23)-(24).

$$\bar{v}_{3,ref} \cdot \bar{A}_{3,H,K} = \varepsilon_{H,K}\left\|\bar{A}_{3,H,K}\right\| \quad (23)$$

$$\bar{v}_{3,ref} \cdot \bar{A}_{3,L,M} = \varepsilon_{L,M}\left\|\bar{A}_{3,L,M}\right\|. \quad (24)$$

Equations (23)-(24) form a set of vector equations that should be solved to find the expression of $\bar{v}_{3,ref}$. This mathematical problem can be simplified by assuming that $\bar{v}_{3,ref}$ can be expressed in the following way:

$$\bar{v}_{3,ref} = C_1 j\bar{A}_{3,H,K} + C_2 j\bar{A}_{3,L,M}. \quad (25)$$

When (25) is substituted in (23)-(24), $\bar{v}_{3,ref}$ can be found solving a linear set of equations where C_1 and C_2 are the unknowns. Therefore, it can be verified that a solution to (23)-(24) is

$$\bar{v}_{3,ref} = 2\frac{\varepsilon_{L,M}\left\|\bar{A}_{3,L,M}\right\|\bar{A}_{3,H,K} - \varepsilon_{H,K}\left\|\bar{A}_{3,H,K}\right\|\bar{A}_{3,L,M}}{\bar{A}_{3,H,K}\bar{A}_{3,L,M}^* - \bar{A}_{3,H,K}^*\bar{A}_{3,L,M}} \quad (26)$$
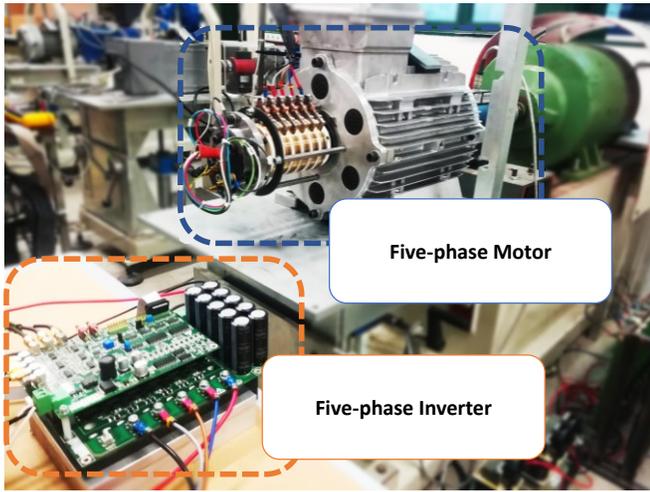
where "*" is the complex conjugate operator.

Fig. 3. Experimental set-up of the electric drive composed by a five-phase wound-rotor induction machine and a five-phase inverter.

Instruments.

Fig. 4 shows the loci of $\bar{v}_{1,ref}$ and $\bar{v}_{3,ref}$ in the overmodulation region $(0.52 < MI < 0.62)$ and the waveforms of the real and imaginary parts of the space vector $\bar{v}_{3,ref}$. As can be seen, $\bar{v}_{1,ref}$ tracks a circular orbit, whereas the path of $\bar{v}_{3,ref}$ is more complicated and strictly dependent on the value of the modulation index MI.

In Figs. 5 and 6, a motor speed transient is presented. If the motor speed is less than about 100 rpm, the inverter operates in a linear region. After this speed, the use of the space vector $\bar{v}_{3,ref}$ allows operating in the first region of overmodulation. As soon as the inverter enters into the overmodulation region, some modulating signals are forced to zero or to one for long periods, which causes a drastic reduction in the number of inverter commutations that is visible in the waveform of the pole voltages of the inverter. Consequently, when the converter operates in the first overmodulation region, there is a significant reduction in the switching losses. The drawback is the presence of a distorted phase current, which get worse as the modulation index rises.

To highlight the significant reduction in the number of switch commutations, Fig. 7 illustrates the average switching frequencies when the motor operates in the rated conditions. The number of commutations begins decreasing shortly before leaving the linear region due to the inverter dead times. In the overmodulation region, the commutations decrease down to 34% of the rated value.

Region II starts when no value of $\bar{v}_{3,ref}$ can simultaneously satisfy all the inequalities (14).

A performance index that can be used to assess the extension of the voltage range is the maximum modulation index (MI), defined as the ratio of the magnitude of $\bar{v}_{1,ref}$ and the dc-link voltage E_{DC}. Under balanced sinusoidal conditions, the modulation index of a five-phase inverter can reach 0.526. Conversely, the developed technique allows increasing the index up to 0.616, which corresponds to a significant improvement of about 17% (Fig. 2) if the resulting current distortion due to the overmodulation can be tolerated.

IV. EXPERIMENTAL RESULTS

The experimental setup used to test the developed modulation strategy is shown in Fig. 3 and consists of a five-phase wound-rotor induction motor and a five-phase IGBT inverter. The parameters of the machine, experimentally determined, are listed in Table I, whereas those of the inverter are shown in Table II. The control scheme is implemented on a TMS320F28335 development board, produced by Texas

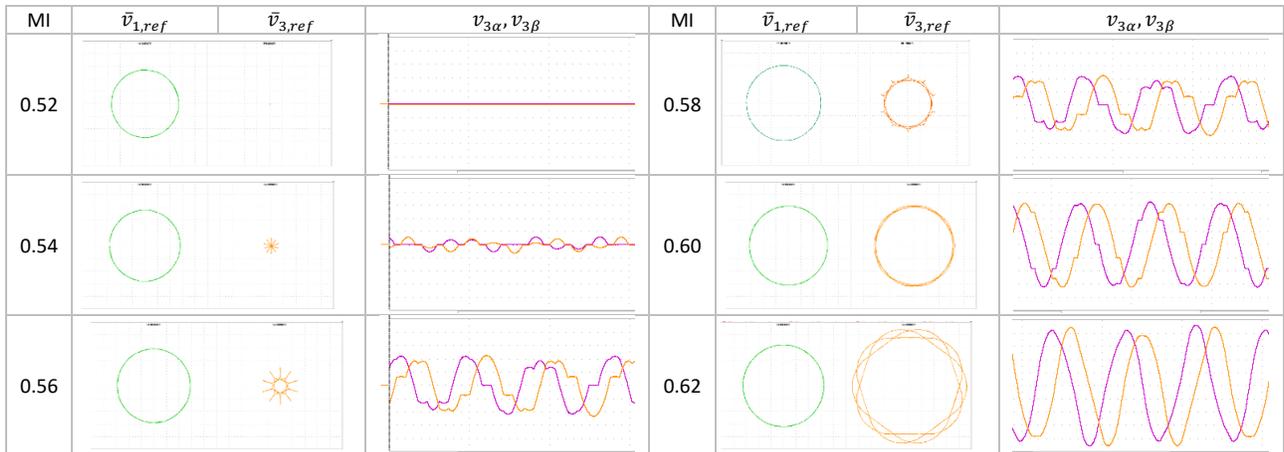| MI	$\bar{v}_{1,ref}$	$\bar{v}_{3,ref}$	$v_{3\alpha}, v_{3\beta}$	MI	$\bar{v}_{1,ref}$	$\bar{v}_{3,ref}$	$v_{3\alpha}, v_{3\beta}$
0.52				0.58			
0.54				0.60			
0.56				0.62			

Fig. 4 – Loci of space vector $\bar{v}_{1,ref}$ (20 V/div) and $\bar{v}_{3,ref}$ (4 V/div) in overmodulation region I for different modulation indices (50 ms/div).
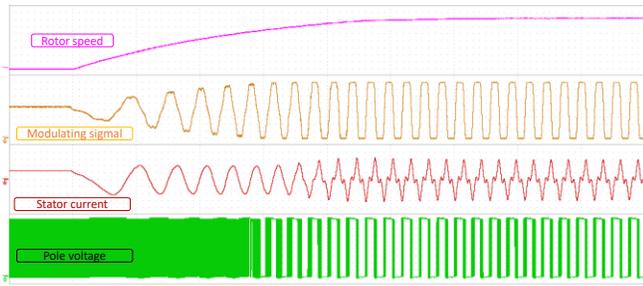
Fig. 5. Speed transient of the motor from 0 to 144 rpm. Mechanical speed ω_m (20 rpm/div), modulating signal of phase 1 (0.125/div), current of phase 1 (2 A/div), and pole voltage v_{p1} of phase 1 (12 V/div).
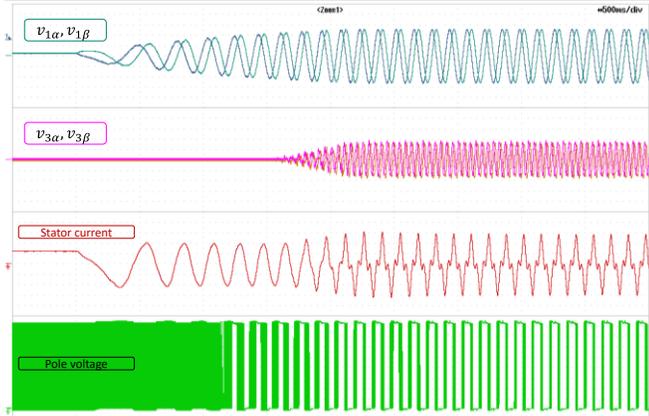

Fig. 6. Speed transient of the motor from 0 to 144 rpm. Waveforms of real and imaginary part of $\bar{v}_{1,ref}$ (25 V/div), $\bar{v}_{3,ref}$ (10 V/div), current of phase 1 (2 A/div), and pole voltage v_{p1} of phase 1 (12 V/div).

Fig. 8 shows the Total Harmonic Distortion (THD) of the inverter phase current in overmodulation region I. The usage of vector $\bar{v}_3$ to extend the linear region of the vector $\bar{v}_1$ generates the presence of the third and seventh harmonic components in the stator currents deteriorating the THD. The extent of this distortion is strongly dependent on the electrical parameters of the machine, and particularly on the stator inductance L_{S3}.

V. CONCLUSION

The three-phase electric drives that use an inverter operating into the overmodulation region in order to obtain a boost in the dynamic performance of the motor are now widespread.

The main contribution of this paper is to solve the problem of overmodulation for five-phase inverters with an original approach that leads to analytic expressions of the voltage vector $\bar{v}_{3,ref}$ for the generation of the reference voltage vector $\bar{v}_{1,ref}$. The presented solution allows increasing the modulation index by 17%.

At the same time, when the motor operates in the overmodulation region, the average switching frequency decreases down to 34% of the rated switching frequency, which is constant in the linear modulation region.

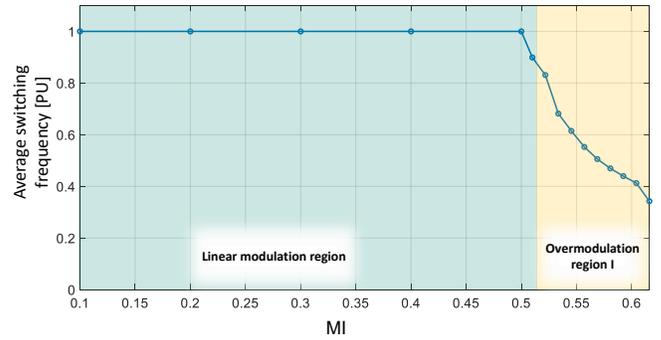The experimental results confirm the feasibility of the proposed method.

Fig. 7. Average switching frequency for different values of the modulation index.
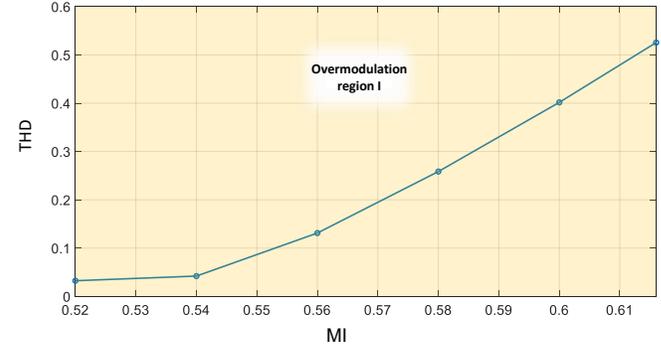

Fig. 8. THD of the stator current in overmodulation region I.

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