An Efficient Ray-Based Modeling Approach for Scattering from Reconfigurable Intelligent Surfaces

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Abstract—Reconfigurable Intelligent Surfaces (RISs), which can be implemented using metasurface technology or reflect/transmit antenna array technology, have garnered significant attention in research studies focused on both their technological aspects and potential applications. While various modeling approaches have been proposed - ranging from electromagnetic simulations and analytical integral formulations to simplified approaches based on scattering matrix theory - there remains a great need for efficient and electromagnetically-consistent macroscopic models that can accurately simulate scattering from RISs, particularly for realistic simulations of RIS-based wireless networks. Building on previous work based on the characterization of the RIS through a surface impedance (or "spatial modulation") function and a few parameters, in the present paper we propose a fully ray-based approach for the computation of the re-radiated field that can be easily embedded in efficient, forward ray tracing (also known as "ray launching") models. We validate the proposed model by comparison to well established methods available in the literature. Results show that, although the considered method is based on a completely different formulation and is much more efficient than integral formulation methods, results are almost indistinguishable in some benchmark cases.

Index Terms—macroscopic modeling, ray tracing, ray launching, reconfigurable intelligent surface (RIS), metasurface

I. INTRODUCTION

Until recently, the design of wireless systems has been based on a probabilistic approach where the propagation channel was considered a largely unknown, random process that engineers had to cope with during the design of transmitter and receiver chains or network architecture. In the last years, Reconfigurable Intelligent Surface (RIS) technology has been proposed as an opportunity to broaden the design approach, allowing for the first time to engineer the wireless propagation channel. Interesting applications for 6G networks are envisioned to ease coverage limitations at mm-wave and THz frequencies and to perform basic operations on the signal "at the speed of

light", limiting therefore the use of active repeaters and digital signal processing, with a reduction in latency and energy consumption [1].

A RIS is an electrically thin slab that can be realized either as a metasurface using electrically small, printed scattering elements, or as a reflect-array or a transmit-array with half-wavelength spaced printed antenna elements. Using control networks employing PIN diodes, varactors or other methods, a RIS can dynamically tailor its local reflection or transmission properties and therefore can manipulate the reradiated field characteristics and wavefront shape [1], [2].

The scattering behaviour of a RIS can be accurately simulated using *microscopic modeling* approaches (e.g. using Electromagnetic simulation or microwave network theory), that are based on a detailed description of the RIS microstructure. Unfortunately, microscopic models are complicated to use and require considerable computational resources. Therefore, they cannot be used for efficient, large-scale simulation of wireless links or systems employing RIS technology [3], [4].

Thus, path-loss models or channel models for RIS-assisted links have been developed and used for performance evaluations in recent years, see for example [5]–[7]. Such models however, being based on a discrete periodic approach that assumes independent scattering elements (unit cells) characterized by a given scattering coefficient and pattern, either overlook or only approximately take into account coupling, parasitic modes and other non-idealities [3].

Several other approaches have been proposed in the literature to try to solve the above-mentioned limitations and to achieve a good trade-off between good electromagnetic consistency and low computational complexity. Some Authors propose hybrid approaches where electromagnetic simulation is used to derive a far-field radar cross section of the RIS to be inserted in ray tracing simulation [8]. Such approaches, although efficient, cannot be used to model near-field effects such as focusing, which represents one of the most important RIS applications. Very promising are *macroscopic modeling* approaches that overlook the microscopic structure of the RIS in order to directly address the specific wave transformation it realizes [9]–[13]. These approaches assume that the metasurface can be homogenized and described in terms of an effective surface function - e.g. a surface impedance or a surface (or spatial) modulation function - that determines such a wave transformation based on Maxwell's equations. The function can be derived from theory, i.e. from the wave transformation the RIS is intended to realize, or from experiment, i.e. from measurements on the wave transformation that an existing RIS

actually realizes.

In particular, in [13], a realistic macroscopic model for evaluating multi-mode reradiation from generic, finite-size RIS is introduced. The model is based on a hybrid approach combining a Huygens-based method to model anomalously reradiated modes with well-established ray-based methods to model specular reflection and diffuse scattering that are inevitably present in real-world non-ideal metasurfaces. In particular, diffuse scattering can model the noise-like reradiation effect of mechanical and electrical non-idealities such as deviation of the RIS from a flat surface, phase-tuning errors or even phase-discretization effects due to the use of a limited number of bits in the control circuit. Specifically, the Huygens-based and the ray-based methods are combined through a parametric power-balance constraint that ensures energy conservation between the incident field, scattered field and dissipation inside the slab.

In the present work we build on the foregoing macroscopic and parametric approach to develop a ray-based, efficient approach also for anomalous reradiation, therefore achieving a fully-ray based macroscopic RIS model that can be easily integrated into efficient Ray-Launching (RL) algorithms for large-scale simulation such as the one proposed in [14]. In particular, we suitably extend Geometrical Optics (GO) theory [15] to the case of a reflective RIS illuminated with an astigmatic wavefront. Diffraction is modeled through the Uniform Geometrical Theory of Diffraction (UTD) [16], [17], and a new formulation of the UTD diffraction coefficients is proposed that can be applied to any reradiation mode of a RIS and easily implemented in RL tools, following the same approach of [17].

Differently from the study in [18], where a ray-based description of reradiation from locally-periodic, finite metasurfaces was first proposed, we assume to model reradiation with a forward ray tracing approach, therefore avoiding the complex and time-consuming *critical point* search step.

In the next section of this paper we describe our approach more in detail, briefly addressing the model's parametric foundation, which is shared with the model described in [13], some basic concepts of Geometrical Optics that we have used, and then describing how the ray reradiated from a generic surface location (*anomalously reflected ray*) or from the surface edge (*anomalously diffracted ray*) are computed in terms of direction and field. The model is then validated in section III by comparison with some reference models available in the literature.

II. THE PROPOSED APPROACH

We propose a macroscopic, ray-based approach that uses the *spatial modulation function* introduced in [3], [13], to model RIS reradiation. In the following, we make use of Geometrical Optics (GO) concepts such as ray, locally plane wave, local wavefront curvature, spreading factor etc. [15]. In [12], [18], an image-ray tracing approach for locally periodic metasurfaces is proposed, where "critical points" are identified using an iterative procedure, in order to trace reflected or transmitted rays for specific transmitter and receiver positions. In the present work, we propose a discrete RL approach where

the RIS is discretized into surface elements, rays are launched toward each one of them, reflected/diffracted according to the spatial modulation function at the considered position, and re-launched in space without any need for a critical-point search phase. Therefore, the method can be inserted into a discrete, parallelized RL algorithm as the one presented in [14] for efficient field-prediction over an area or volume: in this case the sub-set of RIS-reradiated rays hitting the desired target area or volume will have to be determined and their field can be mapped onto the target domain using some efficient computer graphics method. Surface discretization resolution, i.e. the size of each surface element (or "tile"), determines the spacing of the rays and therefore the resolution of the computed field, similarly to what shown in [19], Fig. 4, for the traditional discrete ray launching model therein described, and should be therefore chosen according to a trade-off between accuracy and computation speed. However, discrete RL computation time can be drastically reduced using parallelization techniques [14], [19]. In this work, we leverage the macroscopic approach presented in [13], section II, but we propose a more efficient, fully ray-based reradiated-field computation method in place of the Huygens-based methods there described. The basic assumptions, as in [13], are the following:

1) the homogenized surface properties vary slowly at the wavelength scale (slowly modulated RIS)
2) because of 1), we use the concept of *Spatial Modulation Coefficient* often called also "reflection coefficient"
3) the reradiated field can be described as a discrete set of reradiation modes (e.g. Floquet's modes of a locally periodic metasurface)
4) we address the computation of radiative near field and far field, but we neglect for the time being the effect of evanescent modes (i.e. surface waves) and vertex diffraction.

On the base of the foregoing assumptions, we describe each reradiating mode field as a set of rays reflected or diffracted at each surface element. In the rest of this section, after recalling basic GO concepts, the computation of:

 (i) Reradiation angle
 (ii) Field
(iii) Spreading factor

is described for a reflected or edge-diffracted ray of a single re-radiation mode. For a complete field computation, the procedure will have to be iterated for all the propagating modes. For the sake of brevity, only reflecting RISs are considered: although extension to *transmissive* RISs, that reradiate mainly in the forward half-space, is quite straightforward, it will be addressed in future work.

A. Relevant Geometrical Optics background

According to GO theory, a propagating wave in free space can be described in terms of rays, i.e. lines that are everywhere orthogonal to the wavefront and therefore represent wave *paths*. In the high-frequency regime, the Electromagnetic field of a propagating wave can be approximated as [15]:

$$\mathbf{E}\left(\mathbf{r}\right) \simeq \mathbf{E}_0\left(\mathbf{r}\right) e^{-jk_0\,\psi(\mathbf{r})}$$
$$\mathbf{H}\left(\mathbf{r}\right) \simeq \mathbf{H}_0\left(\mathbf{r}\right) e^{-jk_0\,\psi(\mathbf{r})} \tag{1}$$

where $\mathbf{r}$ is the position vector of the generic observation point P, $k_0 = 2\pi c_0 / f$ is the free space wavenumber, $\mathbf{E}_0(\mathbf{r})$, $\mathbf{H}_0(\mathbf{r})$ are slowly varying complex vectors, representing the local amplitude and polarization of the wave, and $\psi(\mathbf{r})$, also called eikonal function, is an optical-length function that depends on the actual shape of the wavefront. In particular, the gradient of the eikonal function $\nabla\psi$, is normal to the wavefront and then defines the local ray direction, while the Hessian matrix of ψ, indicated as $\nabla\nabla\psi$, takes into account the local curvature of the wavefront. The Hessian matrix of the eikonal function is often indicated with the symbol $\underline{\mathbf{Q}}$ and called the *curvature matrix* of the local wavefront [20], [21]. By substituting (1) in Maxwell's equations, the following relations are obtained (*locally plane* TEM wave) [15]:

$$\nabla\psi \cdot \mathbf{E} = 0 \qquad \nabla\psi \cdot \mathbf{H} = 0 \qquad \eta\mathbf{H} = \nabla\psi \times \mathbf{E} \qquad (2)$$

where $\eta = \sqrt{\mu_0/\epsilon_0}$ is the free-space impedance, and the symbols $"\cdot"$ and $"\times"$ stand for the dot scalar product and the cross vector product, respectively. Moreover, it can be proved that in a homogeneous medium, the ray trajectories are rectilinear. In particular, in free space the generic ray has constant direction $\hat{\mathbf{s}} = \nabla\psi$, whereas the wavefront has an astigmatic shape so that the E-field propagating along a single ray can be written as [20]:

$$\begin{aligned} \mathbf{E}(s) &= \mathbf{E}(0) A(s) e^{-jk_0 s} = \\ &= \mathbf{E}(0) \sqrt{\frac{\mathrm{pdet}\{\underline{\mathbf{Q}}(s)\}}{\mathrm{pdet}\{\underline{\mathbf{Q}}(0)\}}} e^{-jk_0 s} \end{aligned} \qquad (3)$$

where s is the local coordinate along the ray, i.e. the distance between the current point and the reference point $s = 0$, $\mathbf{E}(0)$ is the field at the reference point, and $A(s) = \sqrt{\mathrm{pdet}\{\underline{\mathbf{Q}}(s)\}/\mathrm{pdet}\{\underline{\mathbf{Q}}(0)\}}$ is the so-called *spreading (or divergence) factor*, that derives from power conservation on a ray tube and depends on the actual wavefront's shape. In (3), the notation $\mathrm{pdet}\{-\}$ stands for the *pseudo-determinant* of the square matrix, i.e. the product of its non-zero eigenvalues.

The curvature matrix of an astigmatic wave can be expressed in the following way [17], [20]:

$$\underline{\mathbf{Q}}(s) = \frac{1}{\rho_1 + s}\hat{\mathbf{X}}_1\hat{\mathbf{X}}_1 + \frac{1}{\rho_2 + s}\hat{\mathbf{X}}_2\hat{\mathbf{X}}_2 \qquad (4)$$

with ρ_1 and ρ_2 being the principal curvature radii at the reference point, corresponding to the two principal curvature directions $\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2$. In (4) and in the following, the dyadic product is used for ease of notation, which is equivalent in linear algebra to the multiplication of a column vector by a row vector, i.e.

$$\mathbf{ab} \equiv \mathbf{ab}^T$$

where the superscript $()^T$ stands for the transpose operator.

According to (4), in free space the wavefront diverges as it propagates without changing shape, i.e., the two principal directions remain the same while the curvature radii linearly increase with s as $\rho_1 + s$ and $\rho_2 + s$.

By definition (4), $\underline{\mathbf{Q}}(s)$ is a rank-2 symmetric matrix, and the wave principal curvatures $\kappa_1 = (\rho_1 + s)^{-1}$, $\kappa_2 = (\rho_2 + s)^{-2}$ are its non-zero eigenvalues, while the principal

directions $\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2$ are the corresponding eigenvectors. This means that, by adopting the local ray-fixed reference system $(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2, \hat{\mathbf{s}})$, $\underline{\mathbf{Q}}(s)$ is diagonalized in the form:

$$\underline{\mathbf{Q}}(s) = \begin{bmatrix} \dfrac{1}{\rho_1 + s} & 0 & 0 \\ 0 & \dfrac{1}{\rho_2 + s} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (5)$$

As a consequence of (4), (5) the following property holds, as the principal directions $\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2$ lay on the transverse plane with respect to the ray direction $\hat{\mathbf{s}}$:

$$\underline{\mathbf{Q}}\,\hat{\mathbf{s}} = \hat{\mathbf{s}}^T \underline{\mathbf{Q}} = \mathbf{0} \qquad (6)$$

Finally, according to (3) and (5), the spreading factor for an astigmatic wave can be expressed as a function of the principal curvature radii in the form:

$$A(s) = \sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}} \qquad (7)$$

As it can be seen from (7), the GO field has singularities on the wave *caustics*, i.e. for those points along rays so that $s = -\rho_1$ or $s = -\rho_2$. GO theory cannot be applied to compute the field in the vicinity of a caustic: in such a case, different methods based on asymptotic evaluation need to be applied [22].

In the following, according to the GO approach, we leverage the locally plane wave assumption to model reflection and diffraction at each surface element of a RIS, while we account for the wavefront's actual shape through the spreading factor, that gives the actual attenuation-trend of field's intensity with distance. In practice, we linearize both the incident wavefront - with a local plane - and the effect of the RIS on it – with the local phase gradient – in order to simplify computation steps (i) and (ii) above, whereas the actual curvatures of the wavefront are considered for step (iii).

B. Anomalous ray reflection

Anomalous ray reflection is modeled according to a 3D version of the *generalized law of reflection* [23], which takes into account that, in general, incidence plane and reflection plane can be different. For the sake of simplicity, we limit the analysis to flat surfaces, but the extension to the case of a curved RIS is possible.

Let us then consider a flat RIS of normal $\hat{\mathbf{n}}$, and let be $\mathbf{r}'$ the position of the generic surface element. The position vector can be expressed as a function of 2 local coordinates on the RIS plane (see Figure 1), i.e. $\mathbf{r}' = \mathbf{r}'(u, v)$. When a ray impinges on the surface with propagation direction $\hat{\mathbf{s}}^i$ so that $-\hat{\mathbf{s}}^i \cdot \hat{\mathbf{n}} = \cos\theta_i$, where θ_i is the incidence angle, the field acquires an *incidence phase gradient* on the surface due to the inclination of the locally-plane wavefront of the ray with respect to the RIS. This phase gradient is:

$$\nabla\chi^i = -k_0 \sin\theta_i \, \hat{\mathbf{s}}_\tau^i \qquad (8)$$

where the unit vector $\hat{\mathbf{s}}_\tau^i$ defines the orientation of the incidence plane with respect to the RIS surface (see Fig. 1).
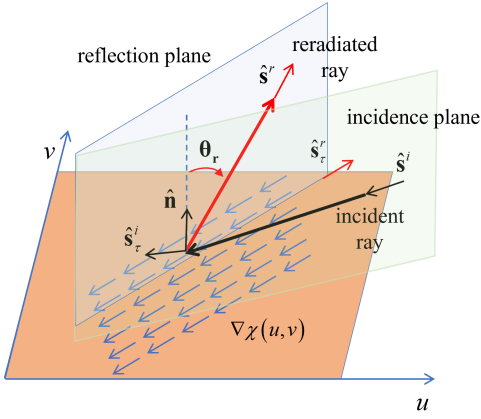
Fig. 1. Total phase gradient and anomalous ray reflection at the generic surface element

Eq. (8) can be rewritten in the equivalent form:

$$\underline{\mathbf{P}}_\tau \, \hat{\mathbf{s}}^i = \hat{\mathbf{s}}^i - \hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \hat{\mathbf{s}}^i) = \sin\theta_i \, \hat{\mathbf{s}}^i_\tau = -\frac{\nabla\chi^i}{k_0} \qquad (9)$$

where $\underline{\mathbf{P}}_\tau$ is the tangent projection operator, defined as:

$$\underline{\mathbf{P}}_\tau = \underline{\mathbf{1}} - \hat{\mathbf{n}}\hat{\mathbf{n}} \qquad (10)$$

and $\underline{\mathbf{1}}$ is the identity matrix.

Then, according to a macroscopic approach, the RIS applies the additional phase gradient $\nabla\chi^m$ of the considered reradiation mode so that the *total phase gradient* at the considered surface location becomes:

$$\nabla\chi = \nabla\chi^i + \nabla\chi^m \qquad (11)$$

Anomalous reflection direction takes place according to that total phase gradient. The reflection plane is parallel to the phase gradient direction (see Fig. 1); however, as surface points with a greater phase will reradiate before those with a phase lag, the resulting locally-plane wavefront will have opposite orientation with respect to the total phase gradient $\nabla\chi$. It can be easily shown (see Appendix A) that the projection of the reflected ray direction on RIS plane is given by:

$$\underline{\mathbf{P}}_\tau \, \hat{\mathbf{s}}^r = \sin\theta_r \, \hat{\mathbf{s}}^r_\tau = \underline{\mathbf{P}}_\tau \, \hat{\mathbf{s}}^i - \frac{\nabla\chi^m}{k_0} = -\frac{\nabla\chi}{k_0} \qquad (12)$$

where θ_r is the reradiation angle, and $\hat{\mathbf{s}}^r_\tau$ defines the reflection (or reradiation) plane, that generally for a RIS is different from the incidence plane, as shown in Fig. 1.

Observing that $|\hat{\mathbf{s}}^r| = 1$, the reflection unit vector can be written using a single compact equation:

$$
\begin{aligned}
\hat{\mathbf{s}}^r =& \underline{\mathbf{P}}_\tau \, \hat{\mathbf{s}}^i - \frac{\nabla\chi^m(\mathbf{r}')}{k_0} + \sqrt{1 - \left|\underline{\mathbf{P}}_\tau \, \hat{\mathbf{s}}^i - \frac{\nabla\chi^m(\mathbf{r}')}{k_0}\right|^2} \; \hat{\mathbf{n}} = \\
=& -\frac{\nabla\chi(\mathbf{r}')}{k_0} + \sqrt{1 - \left|\frac{\nabla\chi(\mathbf{r}')}{k_0}\right|^2} \; \hat{\mathbf{n}}
\end{aligned}
$$
$$(13)$$

which expresses the *generalized law of reflection*.

As in [3], [13], the reradiated field can be computed using the Spatial Modulation Coefficient (SMC), that takes into account the overall reradiation properties of the RIS. According to this macroscopic approach, we assume that the following boundary condition holds for every point of the RIS surface:

$$\mathbf{E}^r(\mathbf{r}') = \underline{\mathbf{\Gamma}}(\mathbf{r}') \, \mathbf{E}^i(\mathbf{r}') \qquad (14)$$

In (14), instead of a scalar coefficient we make use of the *Spatial Modulation Dyadic* (SMD) coefficient $\underline{\mathbf{\Gamma}}$, in order to take into account the polarimetric effect of the RIS. Such coefficient is defined as:

$$
\begin{aligned}
\underline{\mathbf{\Gamma}}(\mathbf{r}') &= \underline{\mathbf{\Gamma}}_0(\mathbf{r}')\, e^{j\chi^m(\mathbf{r}')} = A^m(\mathbf{r}')\, e^{j\chi^m(\mathbf{r}')} \cdot \underline{\mathbf{R}}^m = \\
&= A^m(\mathbf{r}')\, e^{j\chi^m(\mathbf{r}')} \cdot \left(R^m_\parallel \, \hat{\mathbf{e}}^i_\parallel \hat{\mathbf{e}}^r_\parallel + R^m_\perp \, \hat{\mathbf{e}}^i_\perp \hat{\mathbf{e}}^r_\perp\right)
\end{aligned}
$$
$$(15)$$

where A^m and χ^m are the amplitude and phase modulation of the considered reradiation mode, while the matrix $\underline{\mathbf{R}}^m = R^m_\parallel \, \hat{\mathbf{e}}^i_\parallel \hat{\mathbf{e}}^r_\parallel + R^m_\perp \, \hat{\mathbf{e}}^i_\perp \hat{\mathbf{e}}^r_\perp$ is used to account for the polarization transformation realized by the RIS [17], [24]. The unit vectors $\hat{\mathbf{e}}^{i,r}_\perp$ and $\hat{\mathbf{e}}^{i,r}_\parallel$ are used to decompose the incident/reflected field into perpendicular (TE) and parallel (TM) components with respect to the incidence/reflection plane on a ray-fixed reference system (see Fig. 1), and are easily computed as:

$$\hat{\mathbf{e}}^{i,r}_\perp = \hat{\mathbf{s}}^{i,r}_\tau \times \hat{\mathbf{n}} \qquad \hat{\mathbf{e}}^{i,r}_\parallel = \hat{\mathbf{e}}^{i,r}_\perp \times \hat{\mathbf{s}}^{i,r} \qquad (16)$$

Usually, the phase modulation coefficient χ^m varies on the wavelength scale, while the amplitude modulation coefficient A^m varies on a larger scale and may take into account non-local effects along the RIS surface. In general, A^m and χ^m cannot be arbitrarily chosen, but must satisfy proper constraints in order to be representative of a realistic RIS design, as discussed in detail in [3]. A RIS able to control the polarization, here modeled through the dyadic $\underline{\mathbf{R}}^m$, can be alternatively represented as a tensor impedance sheet (see for example [25, pp. 57-59]). Additional terms may be introduced in (15) to take into account additional losses caused by parasitic effects and diffuse scattering, as discussed in [13]. In the following, we assume that the SMD coefficient $\underline{\mathbf{\Gamma}}$ is known, either from the design stage, or estimated through measurements.

After defining the local reflection direction $\hat{\mathbf{s}}^r$ and applying the SMD coefficient to the incident field, the last step consists in the computation of the field along the reflected ray, including the spreading factor. This can be derived through the curvature matrix by applying a local phase matching procedure on the RIS surface, following a method similar to the one presented in [20], and also used in [17] to derive the spreading factor of the reflected wave from a curved PEC surface. Differently from [20], here we express wave curvatures using 3x3 non-diagonal curvature matrices, avoiding the use of a ray-fixed local reference system on the incident and reflected wave to diagonalize them, that would need multiple matrix transformations. As a starting point, by substituting (1) and (15) in (14), we get:

$$
\begin{aligned}
\mathbf{E}^r_0(\mathbf{r}') e^{-jk_0\psi^r(\mathbf{r}')} &= \\
= \underline{\mathbf{\Gamma}}_0(\mathbf{r}') \, \mathbf{E}^i_0(\mathbf{r}') & \, e^{j[\chi^m(\mathbf{r}') - k_0\psi^i(\mathbf{r}')]}
\end{aligned}
$$
$$(17)$$

and then:

$$\mathbf{E}_0^r\left(\mathbf{r}'\right) = \underline{\boldsymbol{\Gamma}}_0\left(\mathbf{r}'\right)\mathbf{E}_0^i\left(\mathbf{r}'\right) \tag{18a}$$

$$k_0\psi^r\left(\mathbf{r}'\right) = k_0\psi^i\left(\mathbf{r}'\right) - \chi^m\left(\mathbf{r}'\right) \tag{18b}$$

Eq. (18b) is a phase-matching relation that involves the phase of incident and reflected fields, and the phase χ^m imposed by the RIS. By expressing each phase term through its Taylor series expansion about a reference point $\mathbf{r}_0'$ on the RIS, a simple relation between the local curvature matrices of the incident and reflected fields can be derived, as shown in Appendix A:

$$\mathbf{t}\cdot\underline{\mathbf{Q}}^r\mathbf{t} = \mathbf{t}\cdot\left\{\left[\underline{\mathbf{Q}}^i - \frac{1}{k_0}\nabla\nabla\chi^m\left(\mathbf{r}_0'\right)\right]\mathbf{t}\right\} \tag{19}$$

where $\mathbf{t} = \mathbf{r}' - \mathbf{r}_0'$ is any vector tangent to the RIS surface at $\mathbf{r}_0'$, and $\nabla\nabla\chi^m$ is the Hessian matrix of the phase profile χ^m imposed by the RIS, computed in $\mathbf{r}_0'$.

In order for (19) to be satisfied, the tangent projection of the incident curvature matrix, plus the curvature imposed by the RIS, must equate the tangent projection of the reflected curvature matrix, i.e.:

$$\underline{\mathbf{P}}_\tau\underline{\mathbf{Q}}^r\underline{\mathbf{P}}_\tau = \underline{\mathbf{P}}_\tau\left[\underline{\mathbf{Q}}^i - \frac{1}{k_0}\nabla\nabla\chi^m\right]\underline{\mathbf{P}}_\tau \tag{20}$$

Eq. (20) only provides the tangential component of $\underline{\mathbf{Q}}^r$. The normal component is determined by imposing (6) for the reflected ray, i.e.

$$\underline{\mathbf{Q}}^r\,\hat{\mathbf{s}}^r = \mathbf{0}$$

In particular, by combining (6) and (20), through simple algebraic manipulations the final expression of $\underline{\mathbf{Q}}^r$ can be found:

$$\underline{\mathbf{Q}}^r = \underline{\mathbf{L}}^T\left[\underline{\mathbf{Q}}^i - \frac{1}{k_0}\nabla\nabla\chi^m\right]\underline{\mathbf{L}} \tag{21}$$

where $\underline{\mathbf{L}}$ is a linear transformation operator, having the following form:

$$\underline{\mathbf{L}} = \left(\underline{\mathbf{1}} - \frac{\hat{\mathbf{s}}^r\,\hat{\mathbf{n}}}{\hat{\mathbf{s}}^r\cdot\hat{\mathbf{n}}}\right) \tag{22}$$

According to GO rules, after reflecting on the RIS the wave continues to propagate along a rectilinear trajectory and the curvature radii increase proportionally to the path length, as the medium above the RIS surface is homogeneous. If the incident ray hits the RIS in the point $\mathbf{r}'$, the GO field on a observation point $\mathbf{r} = \mathbf{r}' + s\,\hat{\mathbf{s}}^r$ along the reflected ray is then expressed by:

$$\mathbf{E}^r\left(\mathbf{r}\right) = \underline{\boldsymbol{\Gamma}}\left(\mathbf{r}'\right)\mathbf{E}^i\left(\mathbf{r}'\right)A^r(s)\,e^{-jk_0|\mathbf{r}-\mathbf{r}'|} =$$
$$= \underline{\boldsymbol{\Gamma}}\left(\mathbf{r}'\right)\mathbf{E}^i\left(\mathbf{r}'\right)\sqrt{\frac{\rho_1^r\rho_2^r}{(\rho_1^r+s)(\rho_2^r+s)}}\,e^{-jk_0s} \tag{23}$$

with ρ_1^r, ρ_2^r being the principal curvature radii of the reflected wave at $\mathbf{r}'$, i.e. the reciprocals of the non-zero eigenvalues of the reflection curvature matrix $\underline{\mathbf{Q}}^r$, computed through (21), and s the local coordinate along the reflected ray.

It is worth noting that generally, according to (21), RIS reflection changes the wavefront shape into an astigmatic wave even in simple cases, like with a spherical wave incident on a constant phase gradient RIS, i.e. with $\nabla\nabla\chi^m = 0$.

For instance, let us consider the case of an anomalous reflector configured with constant anomalous angle θ_r, and reradiation plane coincident with the incidence plane (i.e. $\Delta\phi = 0$ in Fig. 1). If such a RIS is illuminated by a spherical wave, i.e. with $\rho_1^i = \rho_2^i = s'$, using (21) it can be easily shown that the reflected wave has curvatures $\rho_1^r = s'$ and $\rho_2^r = s'\cos^2\theta_r/\cos^2\theta_i$: the curvature perpendicular to the incidence plane is unchanged, while the one laying in the incidence plane is modified by anomalous reflection. Therefore, the spherical wavefront shape is unchanged only in the case of specular reflection.

C. Anomalous ray diffraction

Beside the GO contributions for the RIS scattered field, edge diffracted ray-fields are also included in the model. This type of contribution is important to smooth out the abrupt field discontinuity predicted by GO when crossing the shadow boundaries, and to predict a nonzero field in the GO shadow region. Since an exact solution for the truncated RIS canonical problem is not available, the edge diffracted field has to be evaluated by resorting to an approximate solution. Similarly to the approach adopted for the diffraction from arbitrary impedance wedges, two methods are possible; one can either resort to a PO approximate formulation and derive ray contributions from its asymptotic evaluation [26], or develop heuristic solutions [27], [28] by modifying the UTD coefficient [17]. While in [18] the former methodology was pursued, here we follow the latter which is more popular and effective for the application to ray-tracing and propagation prediction because of its simplicity. Since the total phase progression along the RIS edges results from the combination of both the incident wave illumination and the surface impedance modulation, edge diffracted rays are launched toward anomalous directions, similarly to what happens for GO reflected rays.

Namely, according to a *generalized law of diffraction*, the diffracted ray direction $\hat{\mathbf{s}}^d$ must obey to (see Appendix B for the proof):

$$\cos\beta = \hat{\mathbf{s}}^d\cdot\hat{\mathbf{e}} = \hat{\mathbf{s}}^r\cdot\hat{\mathbf{e}} =$$
$$= \left(\hat{\mathbf{s}}^i - \frac{\nabla\chi^m}{k_0}\right)\cdot\hat{\mathbf{e}} = \cos\beta' - \frac{1}{k_0}\frac{\partial\chi^m}{\partial e} \tag{24}$$

where β is the aperture angle of the Keller-Rubinowicz diffraction cone, β' is the incidence angle with respect to the edge and $\hat{\mathbf{e}}$ is the unit vector along the edge, as shown in Fig. 2.

Looking at (24), it is evident that the additional term corresponding to the spatial modulation modifies the cone aperture with respect to the standard case, thus leading to an *anomalous diffraction*. Moreover, the transverse component of the reflection direction $\hat{\mathbf{s}}^r$ with respect to the edge direction $\hat{\mathbf{e}}$ gives rise to an Anomalous Reflection Shadow Boundary (ARSB). This applies of course to any reradiation mode of the RIS.

Therefore, one can proceed similarly to the standard UTD case, by recalling that the diffracted wave is astigmatic with one caustic on the edge, and that the diffracted field is computed as [17]:
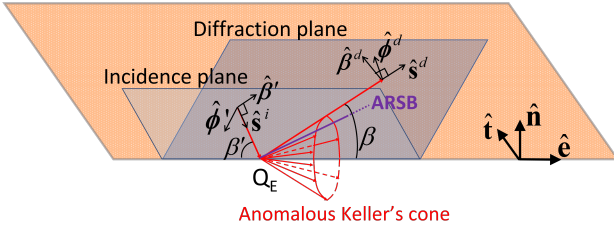
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Fig. 2. Anomalous Keller's cone and edge-fixed reference system for incident and diffracted ray.

$$\mathbf{E}^d\left(s\right) = \underline{\mathbf{D}} \cdot \mathbf{E}^i\left(Q_E\right) \sqrt{\frac{\rho^d}{s\left(\rho^d + s\right)}}\, e^{-jk_0 s} \tag{25}$$

In (25), $\underline{\mathbf{D}}$ is the dyadic diffraction coefficient, and ρ^d is the edge-caustic distance, i.e. the distance between the caustic at the edge and the second caustic of the diffracted ray.

For a straight edge, ρ^d is related to the incident wave curvature radius on the edge-fixed incidence plane, i.e. ρ_e^i, through the following equation (see Appendix B for the proof):

$$\frac{1}{\rho^d} = \frac{1}{\rho_e^i}\frac{\sin^2\beta'}{\sin^2\beta} - \frac{1}{k_0\sin^2\beta}\frac{\partial^2\chi^m}{\partial e^2} \tag{26}$$

Looking at (26), it is evident that the curvature of the incident wave on the edge-fixed diffraction plane is modified by anomalous diffraction, similarly to what happens for anomalous reflection. If the RIS has a constant phase gradient along the edge, such curvature ρ^d is also constant along the edge.
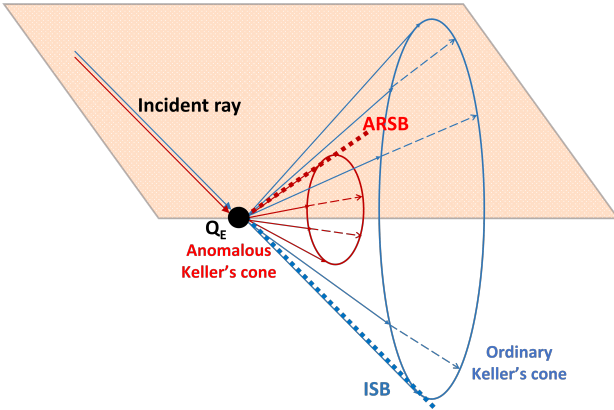

Fig. 3. Ordinary and anomalous Keller's cones for a given point along the edge.

Diffraction must also compensate for the incident field that vanishes at the Incidence Shadow Boundary (ISB). However, the incident ray boundary is not modified by the surface impedance modulation of the RIS across the edge. This means that, in addition to the anomalous Keller's cone, also an *ordinary* Keller's cone of diffracted rays originates at the diffraction point Q_E (see figure 3), i.e.

$$\cos\beta' = \hat{\mathbf{s}}^i \cdot \hat{\mathbf{e}} = \hat{\mathbf{s}}^{d_O} \cdot \hat{\mathbf{e}} \tag{27}$$

with $\hat{\mathbf{s}}^{d_O}$ direction of the diffracted ray laying on the ordinary Keller's cone. For those diffracted rays, the incident field

curvature is not modified by diffraction on a straight edge, i.e. $\rho^d = \rho_e^i$ [17].

Regarding the dyadic diffraction coefficient $\underline{\mathbf{D}}$ in (25), it is expressed as a combination of unit vectors parallel and perpendicular to the incidence and diffraction edge-fixed planes, as in [17]. However, differently from the standard UTD, two separate diffraction coefficients are defined for the anomalous and ordinary diffracted rays. Therefore, we extend the formulation of the UTD diffraction coefficient to the case of a RIS in the following way:

$$\underline{\mathbf{D}}^i = D^i\left(-\hat{\boldsymbol{\beta}}\hat{\boldsymbol{\beta}}' - \hat{\boldsymbol{\phi}}\hat{\boldsymbol{\phi}}'\right) \tag{28}$$

$$\underline{\mathbf{D}}^r = \left(-D_s^r\,\hat{\boldsymbol{\beta}}^d\hat{\boldsymbol{\beta}}' - D_h^r\,\hat{\boldsymbol{\phi}}^d\hat{\boldsymbol{\phi}}'\right)\underline{\boldsymbol{\Gamma}} \tag{29}$$

where D^i is the scalar diffraction coefficient that applies to the diffracted rays on the ordinary Keller's cone, D_s^r and D_h^r are the "soft" and "hard" scalar diffraction coefficients [17] for the anomalous diffraction. In (29), D_s^r and D_h^r are also multiplied by the spatial modulation coefficient $\underline{\boldsymbol{\Gamma}}$ to properly compensate for anomalous reflection on the ARSB, following the heuristic approach adopted in [27], [28] for a non-perfectly conducting wedge.

The unit vectors $(\hat{\boldsymbol{\phi}}', \hat{\boldsymbol{\beta}}')$ form a right-handed triplet with the incidence direction $\hat{\mathbf{s}}^i$ (see Fig. 2) and similarly, $(\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\beta}})$ and $(\hat{\boldsymbol{\phi}}^d, \hat{\boldsymbol{\beta}}^d)$ form a right-handed triplet with the ordinary and anomalous diffraction directions $\hat{\mathbf{s}}^{d_O}$ and $\hat{\mathbf{s}}^d$, respectively. Therefore, they are easily computed as:

$$\hat{\boldsymbol{\phi}}' = -\frac{\hat{\mathbf{e}} \times \hat{\mathbf{s}}^i}{|\hat{\mathbf{e}} \times \hat{\mathbf{s}}^i|} \quad \hat{\boldsymbol{\phi}} = \frac{\hat{\mathbf{e}} \times \hat{\mathbf{s}}^{d_O}}{|\hat{\mathbf{e}} \times \hat{\mathbf{s}}^{d_O}|} \quad \hat{\boldsymbol{\phi}}^d = \frac{\hat{\mathbf{e}} \times \hat{\mathbf{s}}^d}{|\hat{\mathbf{e}} \times \hat{\mathbf{s}}^d|}$$
$$\hat{\boldsymbol{\beta}}' = \hat{\boldsymbol{\phi}}' \times \hat{\mathbf{s}}^i \quad \hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\phi}} \times \hat{\mathbf{s}}^{d_O} \quad \hat{\boldsymbol{\beta}}^d = \hat{\boldsymbol{\phi}}^d \times \hat{\mathbf{s}}^d \tag{30}$$

In order to express the scalar diffraction coefficients D^i, D_s^r, D_h^r in (28),(29) in a similar form to the one introduced in [17] for standard UTD, we need to define the ray angular coordinates with respect to the edge (see Fig.s 2-4). They can be computed with the following equations [29]:

$$\beta = \arccos\left(\hat{\mathbf{s}}^r \cdot \hat{\mathbf{e}}\right) \tag{31}$$

$$\beta' = \arccos\left(\hat{\mathbf{s}}^i \cdot \hat{\mathbf{e}}\right) \tag{32}$$

$$\phi^d = \pi - \left[\pi - \arccos\left(\frac{\hat{\mathbf{s}}^d \cdot \hat{\mathbf{t}}}{\sin\beta}\right)\right]\mathrm{sgn}\left(\frac{\hat{\mathbf{s}}^d \cdot \hat{\mathbf{n}}}{\sin\beta}\right) \tag{33}$$

$$\phi'^r = \pi - \left[\pi - \arccos\left(\frac{-\hat{\mathbf{s}}^r \cdot \hat{\mathbf{t}}}{\sin\beta}\right)\right]\mathrm{sgn}\left(\frac{\hat{\mathbf{s}}^r \cdot \hat{\mathbf{n}}}{\sin\beta}\right) \tag{34}$$

$$\phi = \pi - \left[\pi - \arccos\left(\frac{\hat{\mathbf{s}}^{d_O} \cdot \hat{\mathbf{t}}}{\sin\beta'}\right)\right]\mathrm{sgn}\left(\frac{\hat{\mathbf{s}}^{d_O} \cdot \hat{\mathbf{n}}}{\sin\beta'}\right) \tag{35}$$

$$\phi' = \pi - \left[\pi - \arccos\left(\frac{-\hat{\mathbf{s}}^i \cdot \hat{\mathbf{t}}}{\sin\beta'}\right)\right]\mathrm{sgn}\left(\frac{-\hat{\mathbf{s}}^i \cdot \hat{\mathbf{n}}}{\sin\beta'}\right) \tag{36}$$

with $\hat{\mathbf{n}}$ denoting the unit vector normal to the RIS and $\hat{\mathbf{t}} = \hat{\mathbf{n}} \times \hat{\mathbf{e}}$ the unit vector tangent to the RIS and orthogonal to the edge.

Compared to the standard UTD, in (31)-(36) 3 additional angles are introduced, namely β, ϕ^d and ϕ'^r: β is the angle formed by the anomalous Keller's cone with the edge
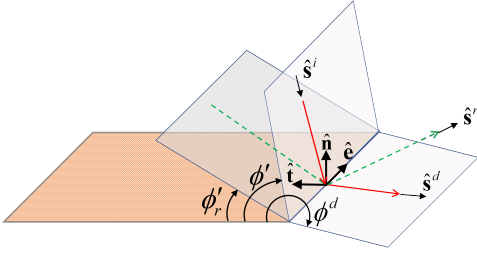
Fig. 4. Diffraction angles for anomalous diffraction. Red: incident ray and one anomalous diffracted ray with corresponding incidence and diffraction planes; Green: anomalous reflected ray and its opposite (back-specular) direction.

direction $\hat{\mathbf{e}}$, and is different from the incidence angle β', in accordance with (24); ϕ^d defines the observation angle on the anomalous Keller's cone, projected on the transverse plane to the edge, while $\phi'^{,r}$ is the transverse angle defining the specular direction of $\hat{\mathbf{s}}_r$ (see fig. 4) so that the diffraction coefficient exhibits its transition at the ARSB, i.e. when $\phi^d + \phi'^r = \pi$ or $\phi^d + \phi'^r = 3\pi$.

The Incidence Shadow Boundary (ISB) condition is $|\phi - \phi'| = \pi$ and is unchanged w.r.t. the standard UTD.

Of course, if the RIS has a specular radiation mode, this gives rise to a standard UTD diffraction, where all the diffracted rays lay on the ordinary Keller's cone, i.e. $\beta = \beta'$, and the ARSB becomes the ordinary Reflection Shadow Boundary (RSB), i.e. $\phi^d = \phi$ and $\phi'^r = \phi'$.

In the standard UTD from a wedge, the diffraction coefficient is formed by two couples of cotangent terms (one for each face of the wedge), that maximize the coefficient on the ISB and on the RSB, respectively. However, since a RIS is a diffracting half-plane, each couple of cotangents degenerates into a single secant term [17]. Moreover, in the case of a RIS these secant terms are not summed together to form a single coefficient as in standard UTD, as they are applied separately to diffracted rays that belong to different Keller's cones.

Therefore, for a RIS properly designed in order to have a single significant (anomalous) reradiation mode, while the other propagating modes including the specular one are negligible, the scalar UTD coefficients to be used in (28),(29) are expressed by:

$$D^i = \frac{-e^{-j\pi/4}}{2\sqrt{2\pi k_0}\sin\beta'} \frac{F\left[k_0 L^i a\left(\phi - \phi'\right)\right]}{\cos\left[\left(\phi - \phi'\right)/2\right]} \qquad (37)$$

$$D^r_{s,h} = \mp \frac{-e^{-j\pi/4}}{2\sqrt{2\pi k_0}\sin\beta} \frac{F\left[k_0 L^r a\left(\phi^d + \phi'^r\right)\right]}{\cos\left[\left(\phi^d + \phi'^r\right)/2\right]} \qquad (38)$$

where

$$F(X) = 2j\sqrt{X}\, e^{jX} \int_{\sqrt{X}}^{\infty} e^{-ju^2}\, du \qquad (39)$$

is the UTD Fresnel Transition function, with arguments

$$a\left(\phi \pm \phi'\right) = 2\cos^2\left(\frac{\phi \pm \phi'}{2}\right) \qquad (40)$$

and distance parameters

$$L^i = \frac{s\left(\rho^i_e + s\right)\rho^i_1 \rho^i_2}{\rho^i_e\left(\rho^i_1 + s\right)\left(\rho^i_2 + s\right)} \sin^2\beta', \qquad (41)$$

$$L^r = \frac{s\left(\rho^d + s\right)\rho^r_1 \rho^r_2}{\rho^d\left(\rho^r_1 + s\right)\left(\rho^r_2 + s\right)} \sin^2\beta. \qquad (42)$$

In (41), ρ^i_1, ρ^i_2 are the principal curvature radii of the incident wave, while ρ^i_e is the curvature radius of the incident wave on the edge-fixed incidence plane (see Fig. 2).

Instead, in (42) ρ^r_1, ρ^r_2 are the principal curvature radii of the reflected wave, computed through (21), while ρ^d is the edge-caustic distance, computed through (26). In the small argument limit $F(X \to 0) \simeq \sqrt{j\pi X}$, it is easy to verify that the factor $\sqrt{L^{i,r}}$ transforms the diffracted field spreading factor into the GO one. As a consequence, the distance parameters in (41), (42) ensure that, at the relevant SB where the arguments (40) vanish, the edge diffracted field exhibits a jump discontinuity compensating the GO abrupt disappearance, thus providing a continuous total field across the SB. This property of the standard UTD, is here suitably extended to the ARSB.

As a last remark, it must be noted that the diffraction coefficients (37),(38) apply to different diffracted rays (ordinary and anomalous) that originate from the same point Q_E on the edge, and the corresponding diffracted fields also have different spreading factors and propagate in different directions, which corresponds to a forward ray-tracing perspective. Therefore, the diffraction coefficients cannot be summed, unlike in standard UTD. Conversely, by assuming a backward ray-tracing perspective, a fixed observation point P in the space might be hit by diffracted rays that originate at two different diffraction

---

Algorithm 1 Computation of the RIS reradiated field

1: Read TX information
2: Define RX grid
3: **for** $k \leftarrow 1$ to N_{tiles} **do**
4:     Calculate incident field $\mathbf{E}^i$ on tile k
5:     **for** $n \leftarrow 1$ to N_{modes} **do**
6:         Use (13) to find the reradiation direction
7:         Intersect the reradiated beam with the RX grid
8:         Compute the SMD $\underline{\mathbf{\Gamma}}$ on tile k using (15)
9:         Calculate curvature matrix using (21)
10:         Calculate reflected field $\mathbf{E}^r$ for mode n using (23)
11:         Add ray contribution to total field at RX
12:     **end for**
13:     **if** *tile k is a "border tile"* **then**
14:         Calculate incident field on the tile edge
15:         **for** $n \leftarrow 1$ to N_{modes} **do**
16:             Find the anomalous Keller's cone with (24)
17:             Intersect Keller's cone with the RX grid
18:             Calculate UTD coefficient using (29)
19:             Calculate ρ^d using (26)
20:             Calculate diffr. field $\mathbf{E}^d$ for mode n using (25)
21:             Add ray contribution to total field at RX
22:         **end for**
23:         Find the ordinary Keller's cone with (27)
24:         Intersect Keller's cone with the RX grid
25:         Calculate UTD coefficient using (28)
26:         Calculate diffracted field $\mathbf{E}^d$ using (25)
27:         Add ray contribution to total field at RX
28:     **end if**
29: **end for**

points (or "critical points") on the edge [30], i.e. those points which satisfy (24) and (27). In such a case, the total diffracted field in P is expressed by

$$\mathbf{E}_{TOT}^d (P) = \underline{\mathbf{D}}^i \cdot \mathbf{E}^i (Q_{E1}) \sqrt{\frac{\rho_e^i}{s_1 (\rho_e^i + s_1)}} e^{-jk_0 s_1}$$
$$+ \underline{\mathbf{D}}^r \cdot \mathbf{E}^i (Q_{E2}) \sqrt{\frac{\rho^d}{s_2 (\rho^d + s_2)}} e^{-jk_0 s_2} \quad (43)$$

where s_1, s_2 are the distances between the critical points Q_{E1}, Q_{E2} and the observation point P, respectively, and ρ^d, $\underline{\mathbf{D}}^i$, $\underline{\mathbf{D}}^r$ are computed using (26), (28) and (29). If the RIS has multiple reradiation modes, additional critical points arise, and additional terms of anomalous diffraction are added to (43).

D. Computation of the overall reradiated field

The procedure for the computation of the reradiated field from a finite-size RIS using the proposed ray approach is summarized in Algorithm 1. The RIS is first subdivided into tiles, and the procedure is iterated over the different tiles and over the different RIS reradiation modes: finally, the reflected and diffracted fields are coherently summed to get the overall reradiated field.

III. APPLICATION EXAMPLES

As a first simple benchmark case, we consider a "perfect" anomalous reflector [23], [31], illuminated with a plane wave at normal incidence. The RIS has size $7 \times 7 \, m^2$, is centered in the origin of an orthogonal reference system $Oxyz$, and lays on the xy plane. Furthermore, the RIS is designed for an anomalous reflection angle $\theta_r = 60°$, and a normal incident wave with perpendicular (TE) polarization with respect to the xz plane, at the frequency of $3.5 \, GHz$. This can be accomplished by setting the following expressions in the SMD coefficient (15):

$$\chi^m = k_0(\sin \theta_i - \sin \theta_r)x$$
$$A^m = \sqrt{\cos \theta_i / \cos \theta_r}$$
$$\underline{\mathbf{R}}^m = \hat{\mathbf{y}}\hat{\mathbf{y}}$$

This means that the RIS imposes a constant phase gradient $\nabla \chi^m = k_0(\sin \theta_i - \sin \theta_r)\hat{\mathbf{x}}$ along the x axis, the wave polarization is perpendicular to the reradiation plane and is not altered by the RIS, while the term $A^m = \sqrt{\cos \theta_i / \cos \theta_r}$ accounts for global power conservation [31]. Such kind of "perfect" anomalous reflector with a single reradiation mode and global power conservation requires a non-local design of the surface impedance through excitation of additional auxiliary evanescent fields or by carefully engineering the surface reactance profile, and can be achieved for example with a nonuniform array of metal patches separated by a dielectric layer from a ground plane, as described in [23].

In Fig. 5, the distribution of the reradiated E-field computed with the ray model on the xz plane is shown, assuming a unitary incident field $\mathbf{E}^i = (-1 \, V/m)\hat{\mathbf{y}}$ at the RIS surface. Being an high-resolution image, interference fringes caused by edge diffraction are well visible, both inside and outside

the reflection cone. The result of Fig. 5 is very similar to the one shown in [13, Fig. 4] except for a small scale factor in the values of the reradiated field, as this previous result was obtained using a different model, called "Antenna Array-Like" (AAL) model, and applied to the case of an *ideal* phase-gradient reflector, by using a "locally-specular" reflection assumption, which can cause a small bias error, as mentioned in [13].
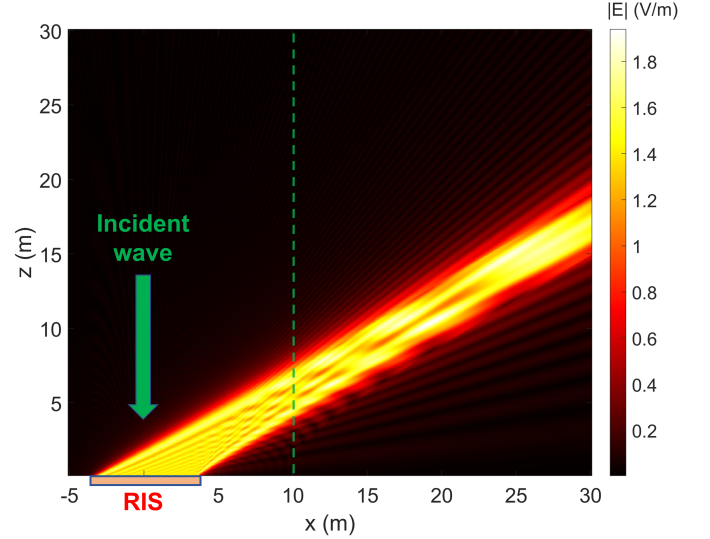

Fig. 5. Field distribution for a perfect anomalous reflector, with $\theta_i = 0°$, $\theta_r = 60°$. Frequency: $f = 3.5 \, GHz$. TE-polarized incident plane wave, with $|\mathbf{E}^i| = 1 \, V/m$ at the RIS surface.

In order to show the effectiveness of the proposed approach, the scattered field computed with the ray model and shown in Fig. 5 is compared with the one computed using the Physical Optics approach, which is well-proven and widely used [10]. The PO field is computed through the following radiation integral:

$$\mathbf{E}_{PO}^s(\mathbf{r}) = -\frac{jk_0}{4\pi} \int_S \frac{e^{-jk_0|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} [\eta \, \hat{\mathbf{r}} \times \mathbf{J}_S(\mathbf{r}') \times \hat{\mathbf{r}} + \mathbf{M}_S(\mathbf{r}') \times \hat{\mathbf{r}}]dS$$
$$(44)$$

where the equivalent surface currents for an impenetrable metasurface are approximated as [10]:

$$\mathbf{J}_S = \hat{\mathbf{n}} \times (\mathbf{H}^i + \mathbf{H}^r)$$
$$\mathbf{M}_S = -\hat{\mathbf{n}} \times (\mathbf{E}^i + \mathbf{E}^r) = -\hat{\mathbf{n}} \times (\mathbf{E}^i + \underline{\mathbf{\Gamma}} \, \mathbf{E}^i) \quad (45)$$

with

$$\mathbf{H}^{i,r} = \frac{1}{\eta}\hat{\mathbf{s}}^{i,r} \times \mathbf{E}^{i,r} \quad (46)$$

By comparing the whole predicted field in Fig. 5 with the one obtained using the PO model on the same Rx grid, one obtains that the Root Mean Square (RMS) distance between the two models is about 2.1% of the unit incident field. The reference PO solution is obtained through numerical computation of the integral (44) with a discretization of the RIS into tiles of length $\lambda/2$, the minimum resolution to have a reliable prediction without grating lobes [13]. It is worth noting that both the ray and the PO solutions are slightly approximate, albeit in different ways, the first being based on

an asymptotic approximation of the field for high frequencies, and the second on the assumption that the total field is zero on the shadow side of the RIS and the radiating currents are not perturbed near the edges. However, the ray-based approach is intrinsically more efficient. Just to have an idea, to produce the high resolution image of Fig. 5 (1.2 Mpixel) with numerical solution of the integral (44), using MATLAB on a workstation with Intel(R) Xeon(R) E5-2620 CPU and parallelization on 8 cores, it takes about 17 hours and 45 minutes. On the contrary, the same result can be obtained with the ray model in about 200 seconds. Results are summarized in Table I in terms of mean error, standard deviation of the error and RMS error with respect to the reference PO model, and computation time. The errors are expressed as a percentage of the unit incident field. Table I also reports the AAL model of [13], which has intermediate performance: the mean error is slightly worse and the error standard deviation is slightly better than the ray model, but its computation time is of about 2 hours and 26 minutes, which is 43 times slower than the ray model.

Fig. 6. Comparison of the ray model with the PO model along the dashed green line in Fig.5

TABLE I
ACCURACY AND COMPUTATION TIME OF RAY MODEL AND AAL MODEL WITH RESPECT TO THE REFERENCE PO MODEL

Model	Mean error	Error std deviation	RMS error	Computation time (s)
PO model	—	—	—	63918
AAL model	0.19%	1.33%	1.35%	8767
Ray model	0.16%	2.07%	2.08%	200

In order to provide a visual comparison between the proposed ray model and the PO reference model, the reradiated field shown in Fig. 5 is sampled along the RX line at $x = 10$, $y = 0$ (green dashed line) and compared with the one obtained using the PO model in the same Rx locations. The comparison is shown in Fig. 6 where the reradiated field obtained through the ray model is represented by the black curve, while red dotted curve corresponds to the PO model. The AAL is not plotted in this case for readability reasons, as the curves are very close each other. It is evident that the 2 curves in Fig. 6 are nearly coincident, thus confirming the validity of the adopted approach. The only small difference that can be appreciated, at $z = 7$, is due to the absence in the model of vertex diffraction, which would allow for a smoother transition when edge ray diffraction ceases to exist.

Anomalous reflectors are usually conceived and designed for the canonical case of an incident plane wave from a given direction, but in a real environment the incident wave is spherical (or astigmatic), unless the illuminating source is very far. This fact causes an impact on RIS performance, as depicted in Fig. 7, where the same RIS of the previous example is considered (perfect anomalous reflector) and the field along the green dashed line in Fig. 5 is computed with the ray model for an illuminating spherical source located along the z-axis, at a distance of $50\ m$ from the RIS center. The incident field is normalized so that its maximum value, at tee center of the RIS, is $1\ V/m$. Fig. 7 shows a significant widening of the reflection cone and a reduction in the amplitude of the reradiated field for a uniform spherical incident wave (red dashed curve) compared to the reference case of plane wave
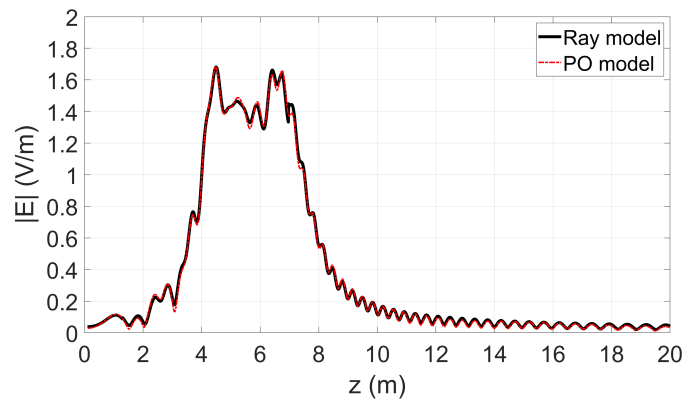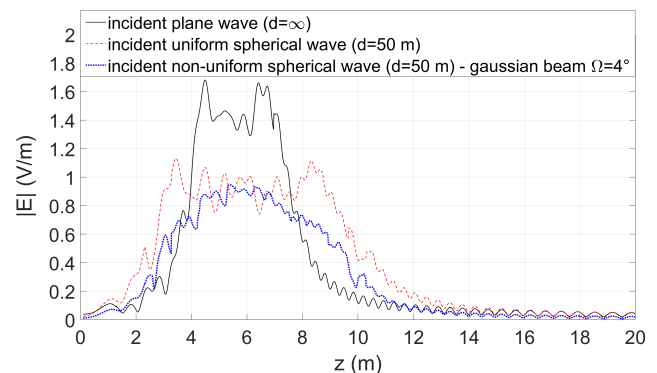

Fig. 7. Comparison of the reradiated field predicted with the ray model along the dashed green line in Fig.5 in 3 different cases: a) incident plane wave (black line), b) incident uniform spherical wave (red dashed line), c) incident non-uniform spherical wave with gaussian profile and divergence $\Omega = 4°$ (blue dotted line)

illumination (black curve). This is mainly due to the fact that the incidence phase gradient is not constant along the RIS surface, and then the phase compensation operated by the RIS is imperfect. Moreover, the reflected wave is astigmatic, as discussed in Section II, and therefore attenuates faster with distance than a spherical wave.

The reradiated field intensity is further reduced if the RIS is illuminated with a directive antenna. As a reference example, Fig. 7 depicts the case of illumination with a circular gaussian beam (blue dotted curve), that can well approximate the main radiation lobe of a pencil-beam directive antenna [32]. The considered gaussian beam has beam waist $w_0 = 0.39\ m$, corresponding to a divergence angle $\Omega = \frac{\lambda}{\pi w_0} \approx 4°$ at $f = 3.5\ GHz$. As the distance from the source ($d = 50\ m$) is far beyond the Rayleigh distance, the incident wave on the RIS surface is a non-uniform spherical wave, and about 86% of its power is contained on a circular spot with radius $R = \Omega d \approx 3.5\ m$ [33]. As expected, in this case the reradiated field intensity further decreases compared to the case of incident uniform spherical wave (red dashed curve), especially in the side regions of the reflection cone, where the reduction is of about 6 dB: in fact, since most of the incident power is concentrated around the RIS center, the contribution of edge diffraction becomes less significant in this case.
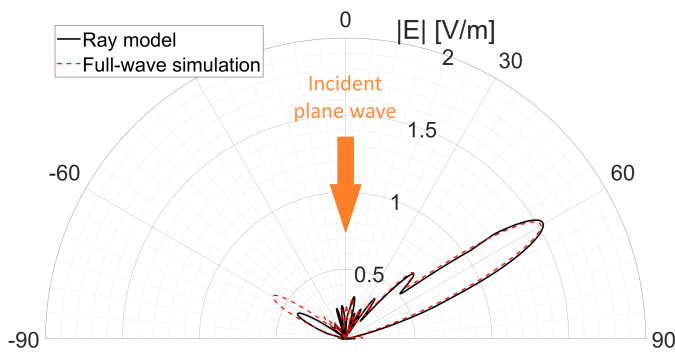
Fig. 8. Comparison between ray model and full-wave simulation along a semicircle on the xz plane centered on the RIS, at a distance $r = 1\,m$.

Fig. 8 shows a more realistic case of a periodic phase-gradient RIS with multiple propagating modes. The ray model is compared with full-wave simulations performed with the frequency-domain solver (FEM) of CST microwave studio. Similarly to the previous cases, we consider the reference case of a normally incident plane wave, with field amplitude $E_0 = 1\,V/m$ on the RIS surface. The RIS is located in the xy plane, centered at the origin, and consists of a reactive impedance sheet $Z_s(x) = j\eta \tan(\frac{\pi x}{D})$ with period D along the x axis, while the incident field is TE-polarized (i.e. along the y axis). The period can be found as $D = \lambda / |\sin\theta_i - \sin\theta_r|$, in accordance with [10], [23]: therefore, it is chosen as $D = 98.91\,mm$ in order to give a reflection angle $\theta_r = 60°$ on the xz plane at $f = 3.5\,GHz$. To limit the computation time, the size of the RIS in the CST simulation was chosen to be $7D \times 7D$, i.e. about $0.7 \times 0.7\,m^2$. The surface impedance profile was sampled at 20 points in each period, so that the RIS model consists of 140 strips of length $7D$ and width $\frac{D}{20}$, each with a constant surface impedance boundary condition. The reradiated field is sampled along a semicircle in the xz plane at a distance $r = 1\,m$ from the center of the RIS.

In order to compare the full-wave simulation with the ray model, the amplitude and initial phase for the propagating modes have been obtained by first simulating in CST a single periodic cell of the RIS with periodic boundary conditions and Floquet port excitation. According to the Floquet theory, in the considered case of a normally incident plane wave, there are three scattered propagating modes $n = -1, 0, 1$ [10]: $n = 1$ corresponds to the desired reradiation mode at $\theta = 60°$, while $n = 0$ and $n = -1$ correspond to the specular mode ($\theta = 0°$) and to the opposite mode at $\theta = -60°$, respectively. The S-parameters calculated by CST directly provide the amplitude A^m and phase χ^m of the scattered modes which are used in (15) to obtain the spatial modulation coefficient $\underline{\Gamma}$ for each mode. Then, the procedure described in Section II is iterated to obtain the total reradiated field for each of the 3 propagating modes, and such fields are coherently summed to obtain the result shown in Fig. 8 (black curve), which is compared with the reference full-wave simulation (red dashed curve). In both curves, the 2 lobes at $\theta = 60°$ and $\theta = -60°$ are clearly visible, whereas the specular mode appears to be almost negligible, except for a few grating lobes. It is apparent that the proposed ray-method can predict the RIS scattering

with good accuracy. Overall, the RMS distance between the 2 curves is equal to 0.019, i.e. 1.9% of the unit incident field. The direction and level of the main lobes are quite well estimated except for the underestimation of the lobe at $\theta = -60°$ corresponding to the mode $n = -1$. Such a difference is due to the fact that the ray-method is based on the PO currents, i.e., on the equivalent currents in the infinite periodic problem, which are only an approximation of the the true currents on the truncated structure calculated by the full-wave method. As the difference between the two currents is mainly concentrated at the plate edges, the edge diffraction as predicted under the PO approximation may differ from exact edge diffraction. However, such a difference generally decreases as the electrical size of the RIS increases. In this moderate-size example it is still noticeable, though not dramatic.
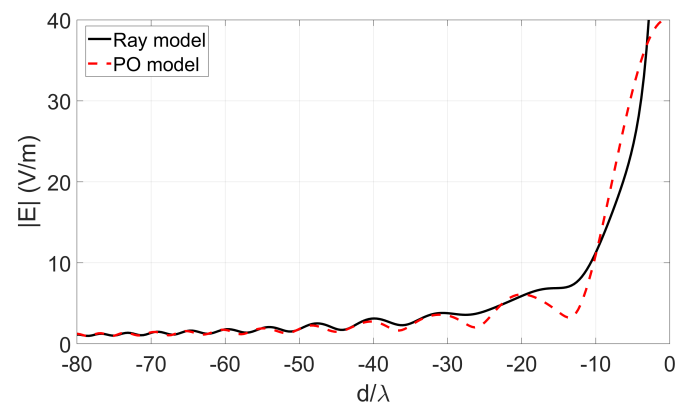

Fig. 9. Comparison between the ray model and the PO model in the case of an ideal focalizing reflector. The distance from the focus is normalized with respect to the wavelength.

As a last example, we consider an *ideal* focalizing reflector, illuminated by a TE-polarized plane wave with $|\mathbf{E}^i| = 1\,V/m$ and incidence angle $\theta_i = \pi/3$ on the xz plane, at the frequency $f = 3.5\,GHz$: this is achieved by setting $A^m = 1$ and $\chi^m = -\chi^i + k_0 |\mathbf{r}_F - \mathbf{r}'|$ in (15), where $\chi^i = -k_0 \sin\theta_i\,x$ is the phase of the incident wave and $\mathbf{r}_F$ is the position vector of the focus point [13]. The RIS has the same size as the one considered in the example of Fig. 5, and it is centered in the point $(0, 0, -10)$, while the focus point is located in the origin of the reference system.

Fig. 9 shows the predicted field along the z-axis, starting from the RIS surface up to the focus point, and compares the proposed ray model with the PO model. In the plot, distance from the focus is normalized to the wavelength, to give a clear idea of the focal-spot size that must be related to the wavelength (radius of about 5λ). Recalling that the GO field has singularities on caustics (or focii), as mentioned in Section II, it can be observed that the ray model provides reliable results and in good agreement with the PO model up to a distance of about 5λ from the focus, then the predicted field value starts diverging. Proper handling of singularities in focal points will have to be addressed in future work, together with the introduction of vertex diffraction and extension of the model to transmissive surfaces.

IV. Conclusions

On the base of the characterization of a finite-size, reflective RIS through a "spatial modulation" dyadic function and a few parameters, in the present work we propose a fully ray-based approach for the computation of the reradiated field that can be easily embedded in efficient, forward ray tracing algorithms. The model is based on the computation of the anomalous direction of the reflected or diffracted ray based on the phase gradient of the spatial modulation function, and on the computation of its spreading factor using the curvature matrix of the local wavefront. We show that a new Keller's cone, the "anomalous Keller cone", has to be taken into account in addition to the ordinary one and a new, original formulation of the UTD diffraction coefficients is proposed inspired by the heuristic approach in [27], [28]. We validate the proposed model by comparison to well established methods available in the literature: results show that the ray model is far more efficient in term of computation time, but corresponding results are very similar in a number of benchmark cases.

Appendix A
Anomalous reflection: computation of the wave curvature matrix

Let's consider a reference point P_0 on the RIS surface, and the corresponding position vector $\mathbf{r}'_0$. The phase of the incident ray in a point P with position vector $\mathbf{r}'$ located in the vicinity of P_0 can be approximated by its Taylor series expansion about P_0, truncated after the 2nd-order term:

$$\psi^i\left(\mathbf{r}'\right) \simeq \psi^i\left(\mathbf{r}'_0\right) + \hat{\mathbf{s}}^i\left(\mathbf{r}'_0\right) \cdot \left[\mathbf{r}' - \mathbf{r}'_0\right] + \frac{1}{2}\left[\mathbf{r}' - \mathbf{r}'_0\right] \cdot \left\{\underline{\mathbf{Q}}^i\left(\mathbf{r}'_0\right)\left[\mathbf{r}' - \mathbf{r}'_0\right]\right\} \tag{47}$$

where the identities $\nabla\psi^i \equiv \hat{\mathbf{s}}^i$ and $\nabla\nabla\psi^i \equiv \underline{\mathbf{Q}}^i$ have been used, as stated in Section IIA.

Similarly, the phase of the reflected field can be locally approximated as:

$$\psi^r\left(\mathbf{r}'\right) \simeq \psi^r\left(\mathbf{r}'_0\right) + \hat{\mathbf{s}}^r\left(\mathbf{r}'_0\right) \cdot \left[\mathbf{r}' - \mathbf{r}'_0\right] + \frac{1}{2}\left[\mathbf{r}' - \mathbf{r}'_0\right] \cdot \left\{\underline{\mathbf{Q}}^r\left(\mathbf{r}'_0\right)\left[\mathbf{r}' - \mathbf{r}'_0\right]\right\} \tag{48}$$

The same principle also applies to the phase χ^m imposed by the RIS:

$$\chi^m\left(\mathbf{r}'\right) \simeq \chi^m\left(\mathbf{r}'_0\right) + \nabla\chi^m\left(\mathbf{r}'_0\right) \cdot \left[\mathbf{r}' - \mathbf{r}'_0\right] + \frac{1}{2}\left[\mathbf{r}' - \mathbf{r}'_0\right] \cdot \left\{\nabla\nabla\chi^m\left(\mathbf{r}'_0\right)\left[\mathbf{r}' - \mathbf{r}'_0\right]\right\} \tag{49}$$

For a generic surface, any point P in the vicinity of P_0 is described by the following relation [20]:

$$\mathbf{r}' = \mathbf{r}'_0 + \mathbf{t} - \frac{1}{2}\left(\mathbf{t} \cdot \underline{\mathbf{C}}\mathbf{t}\right)\hat{\mathbf{n}} \tag{50}$$

where $\mathbf{t} = t_1\hat{\mathbf{u}} + t_2\hat{\mathbf{v}}$ is a vector tangent to the surface in $\mathbf{r}'_0$ and $\mathbf{C} = \kappa_1\hat{\mathbf{u}}\hat{\mathbf{u}} + \kappa_2\hat{\mathbf{v}}\hat{\mathbf{v}}$ is the curvature matrix of the surface. However, in the present work we are considering only flat surfaces ($\mathbf{C} = \mathbf{0}$), so $\mathbf{r}' - \mathbf{r}'_0$ will be a tangent vector to the surface, i.e.

$$\mathbf{r}' - \mathbf{r}'_0 = \mathbf{t} \tag{51}$$

By imposing the phase matching relation (18b) in the point P, we have then:

$$\psi^r\left(\mathbf{r}'_0\right) + \hat{\mathbf{s}}^r \cdot \mathbf{t} + \frac{1}{2}\mathbf{t} \cdot \underline{\mathbf{Q}}^r\mathbf{t} = \psi^i\left(\mathbf{r}'_0\right) + \hat{\mathbf{s}}^i \cdot \mathbf{t} + \frac{1}{2}\mathbf{t} \cdot \underline{\mathbf{Q}}^i\mathbf{t}$$
$$- \frac{1}{k_0}\left\{\chi^m\left(\mathbf{r}'_0\right) + \nabla\chi^m\left(\mathbf{r}'_0\right) \cdot \mathbf{t} + \frac{1}{2}\mathbf{t} \cdot \left[\nabla\nabla\chi^m\left(\mathbf{r}'_0\right)\mathbf{t}\right]\right\} \tag{52}$$

and then, the following equations must be separately satisfied:

$$\psi^r\left(\mathbf{r}'_0\right) = \psi^i\left(\mathbf{r}'_0\right) - \frac{1}{k_0}\chi^m\left(\mathbf{r}'_0\right) \tag{53a}$$

$$\hat{\mathbf{s}}^r \cdot \mathbf{t} = \left[\hat{\mathbf{s}}^i - \frac{1}{k_0}\nabla\chi^m\left(\mathbf{r}'_0\right)\right] \cdot \mathbf{t} \tag{53b}$$

$$\mathbf{t} \cdot \underline{\mathbf{Q}}^r\mathbf{t} = \mathbf{t} \cdot \left\{\left[\underline{\mathbf{Q}}^i - \frac{1}{k_0}\nabla\nabla\chi^m\left(\mathbf{r}'_0\right)\right]\mathbf{t}\right\} \tag{53c}$$

Eq. (53a) just provides the phase matching on the reference position $\mathbf{r}'_0$. Eq. (53b) means that the tangent components of the 1st-order terms of the Taylor's expansion are equal, as (53b) must be satisfied for any choice of the vector $\mathbf{t}$. So, using the projection operator $\underline{\mathbf{P}}_\tau = \underline{\mathbf{1}} - \hat{\mathbf{n}}\hat{\mathbf{n}}$ we get (12):

$$\underline{\mathbf{P}}_\tau\,\hat{\mathbf{s}}^r = \underline{\mathbf{P}}_\tau\left[\hat{\mathbf{s}}^i - \frac{1}{k_0}\nabla\chi^m\left(\mathbf{r}'_0\right)\right] =$$
$$= -\frac{\nabla\chi^i\left(\mathbf{r}'_0\right) + \nabla\chi^m\left(\mathbf{r}'_0\right)}{k_0} = -\frac{\nabla\chi\left(\mathbf{r}'_0\right)}{k_0}$$

which leads to (13) by imposing $|\hat{\mathbf{s}}^r| = 1$.

Finally, by pre-multiplying and post-multiplying with the projection operator the 2nd-order terms (curvature matrices) in (52), we immediately get (20):

$$\underline{\mathbf{P}}_\tau\underline{\mathbf{Q}}^r\underline{\mathbf{P}}_\tau = \underline{\mathbf{P}}_\tau\left[\underline{\mathbf{Q}}^i - \frac{1}{k_0}\nabla\nabla\chi^m\right]\underline{\mathbf{P}}_\tau$$

which leads to (21) by imposing $\underline{\mathbf{Q}}^r\,\hat{\mathbf{s}}^r = \mathbf{0}$.

Appendix B
Anomalous diffraction: computation of the wave curvature ρ^d

We proceed in a similar way as for reflection, by writing the Taylor series expansion of the phase functions of the incident and diffracted wave, respectively, about a point P_0 on the edge:

$$\psi^i\left(\mathbf{r}'\right) \simeq \psi^i\left(\mathbf{r}'_0\right) + \hat{\mathbf{s}}^i \cdot \left(\mathbf{r}' - \mathbf{r}'_0\right) + \frac{1}{2}\left(\mathbf{r}' - \mathbf{r}'_0\right) \cdot \left\{\underline{\mathbf{Q}}^i\left(\mathbf{r}'_0\right)\left[\mathbf{r}' - \mathbf{r}'_0\right]\right\} \tag{54}$$

$$\psi^d\left(\mathbf{r}'\right) \simeq \psi^d\left(\mathbf{r}'_0\right) + \hat{\mathbf{s}}^d \cdot \left(\mathbf{r}' - \mathbf{r}'_0\right) + \frac{1}{2}\left(\mathbf{r}' - \mathbf{r}'_0\right) \cdot \left\{\underline{\mathbf{Q}}^d\left(\mathbf{r}'_0\right)\left[\mathbf{r}' - \mathbf{r}'_0\right]\right\} \tag{55}$$

and similarly for the phase profile χ^m imposed by the RIS:

$$\chi^m\left(\mathbf{r}'\right) \simeq \chi^m\left(\mathbf{r}'_0\right) + \nabla\chi^m\left(\mathbf{r}'_0\right) \cdot \left(\mathbf{r}' - \mathbf{r}'_0\right) + \frac{1}{2}\left(\mathbf{r}' - \mathbf{r}'_0\right) \cdot \left\{\nabla\nabla\chi^m\left(\mathbf{r}'_0\right)\left[\mathbf{r}' - \mathbf{r}'_0\right]\right\} \tag{56}$$

As we assume that the edge is rectilinear, we have

$$\mathbf{r}' - \mathbf{r}'_0 = ds\,\hat{\mathbf{e}} \tag{57}$$

and then, by imposing the *phase matching* relation

$$\psi^d\left(\mathbf{r}'\right) = \psi^i\left(\mathbf{r}'\right) - \frac{1}{k_0}\chi^m\left(\mathbf{r}'\right) \tag{58}$$

e by substituting (54)-(57) into (58), we get:

$$\psi^i\left(\mathbf{r}'_0\right) + \hat{\mathbf{s}}^i \cdot \hat{\mathbf{e}}\ ds - \frac{\nabla\chi^m\left(\mathbf{r}'_0\right)}{k_0} \cdot \hat{\mathbf{e}}\ ds$$
$$+ \frac{1}{2}\left[\hat{\mathbf{e}} \cdot \underline{\mathbf{Q}}^i\left(\mathbf{r}'_0\right)\hat{\mathbf{e}}\right]ds^2 - \frac{1}{k_0}\left[\hat{\mathbf{e}} \cdot \nabla\nabla\chi^m\left(\mathbf{r}'_0\right)\hat{\mathbf{e}}\right]ds^2 \tag{59}$$
$$= \psi^d\left(\mathbf{r}'_0\right) + \hat{\mathbf{s}}^d \cdot \hat{\mathbf{e}}\ ds + \frac{1}{2}\left[\hat{\mathbf{e}} \cdot \underline{\mathbf{Q}}^d\left(\mathbf{r}'_0\right)\hat{\mathbf{e}}\right]ds^2$$

Finally, by equating separately the 0-order, 1st-order and 2nd-order terms, we obtain:

1) the phase matching in $\mathbf{r}'_0$:

$$\psi^i\left(\mathbf{r}'_0\right) = \psi^d\left(\mathbf{r}'_0\right) \tag{60}$$

2) the *generalized law of diffraction*:

$$\hat{\mathbf{s}}^d \cdot \hat{\mathbf{e}} = \cos\beta = \left(\hat{\mathbf{s}}^i - \frac{\nabla\chi^m}{k_0}\right) \cdot \hat{\mathbf{e}} = \cos\beta' - \frac{\nabla\chi^m}{k_0} \cdot \hat{\mathbf{e}} \tag{61}$$

3) the matching of the wave curvatures:

$$\hat{\mathbf{e}} \cdot \underline{\mathbf{Q}}^d\left(\mathbf{r}'_0\right)\hat{\mathbf{e}} = \hat{\mathbf{e}} \cdot \underline{\mathbf{Q}}^i\left(\mathbf{r}'_0\right)\hat{\mathbf{e}} - \frac{1}{k_0}\hat{\mathbf{e}} \cdot \nabla\nabla\chi^m\left(\mathbf{r}'_0\right)\hat{\mathbf{e}} \tag{62}$$

In (62), we observe that $\hat{\mathbf{e}} \cdot \underline{\mathbf{Q}}^i\left(\mathbf{r}'_0\right)\hat{\mathbf{e}}$ gives the ray curvature of the incident wave on the edge-fixed incidence plane (i.e. $1/\rho_e^i$), projected along the edge, i.e. multiplied by $\sin^2\beta'$, as the component of the edge direction along the ray gives no contribution, in accordance with (6). Therefore:

$$\hat{\mathbf{e}} \cdot \underline{\mathbf{Q}}^i\left(\mathbf{r}'_0\right)\hat{\mathbf{e}} = \frac{1}{\rho_e^i}\sin^2\beta' \tag{63}$$

and a similar relation holds for the diffracted wave:

$$\hat{\mathbf{e}} \cdot \underline{\mathbf{Q}}^d\left(\mathbf{r}'_0\right)\hat{\mathbf{e}} = \frac{1}{\rho^d}\sin^2\beta \tag{64}$$

Finally, recognizing that $\hat{\mathbf{e}} \cdot \nabla\nabla\chi^m\left(\mathbf{r}'_0\right)\hat{\mathbf{e}}$ is the 2nd order derivative of χ^m along the edge direction, i.e.

$$\hat{\mathbf{e}} \cdot \nabla\nabla\chi^m\left(\mathbf{r}'_0\right)\hat{\mathbf{e}} \doteq \frac{\partial^2\chi^m}{\partial e^2} \tag{65}$$

and by substituting (63)-(65) into (62) we immediately get (26):

$$\frac{1}{\rho^d} = \frac{1}{\rho_e^i}\frac{\sin^2\beta'}{\sin^2\beta} - \frac{1}{k_0\sin^2\beta}\frac{\partial^2\chi^m}{\partial e^2}$$

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