The effect of vertical relationships on investment timing

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Abstract

Billette de Villemeur, Ruble and Versaevel (2014. International Journal of Industrial Organization, 33, pp. 110-123) discuss the case of a firm undertaking a project in order to serve an uncertain demand. They show that when the investor requires an outside supplier with market power to provide it with a discrete input the investment occurs too late from an industry stand point. In this paper we extend their work assuming that the input production cost is also uncertain. We show that upstream market power results in dynamic inefficiency also in our framework. We also demonstrate how this inefficiency is affected by the correlation between the two stochastic terms.

KEYWORDS: Irreversible investment; Real options; Vertical relationships

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1 Introduction

The typical model of investment under uncertainty and irreversibility discusses a potential investor who can choose when to invest in a project with uncertain value. Investment bears a fixed

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irreversible cost and the potential investor delays the investment until the project's value hits a sufficiently high threshold level.[1] Billette de Villemeur, Ruble and Versaevel (2014) have incorporated into this framework an outside supplier with market power who is responsible for the provision of a discrete input, e.g. a key piece of equipment, without which the investment cannot take place. They show that the input supplier's market power causes an increase in the input price which is then reflected in an investment that takes place suboptimally late from an industry standpoint.

In this paper we generalize the analysis in Billette de Villemeur et al. explicitly assuming that the input production cost is stochastic and possibly correlated with the downstream demand. The introduction of this second stochastic parameter in the model is motivated by a well known result in the relevant literature according to which, if the value of the project and the corresponding investment cost are highly correlated, the potential investor holds a less volatile investment option and consequently has a reduced incentive to wait before investing (pp. 207-211 in Dixit and Pindyck, 1994). This means that while the vertical relationship between the input supplier and the potential investor favors the postponement of the investment, the stochasticity of the investment cost induces earlier investment if the degree of correlation between the project's value and cost is high enough.

Our contribution in this work is twofold. On one hand we show how the timing effect of correlation is manifested in the two-echelon supply chain described in Billette de Villemeur et al. and, on the other, we discuss how it compares with the upstream market power effect that they identify. We show that upstream market power results in dynamic inefficiency also when both downstream demand and investment cost are uncertain, that is, the results in Billette de Villemeur et al. are preserved under the additional source of randomness. At the same time, we prove that the magnitude of the timing distortion attributed to the supplier's market power is inversely related to the degree of correlation between the two stochastic terms. The rationale is as follows. The input

[1] Dixit and Pindyck (1994).

supplier uses the market power as a hedging mechanism against volatility. When future demand and input production cost are highly correlated the upstream firm is exposed to less risk and has little need to hedge inflating the input price. When instead the correlation coefficient is low the upstream firm is exposed to more risk and has a larger incentive to exercise its market power. We conclude our work deriving the level of correlation that delimits these two cases and present a numerical example.

There are many examples of two-echelon supply chains where upstream market power affects the timing of investments taking place downstream. For instance, Billette de Villemeur et al. discuss an investment in a plant dedicated to the production of vaccines. The specialized piece of equipment in that case is a customized liquid nitrogen refrigeration unit. Investments in large infrastructure projects where an upstream firm with market power is responsible for the provision of some basic infrastructure or input also fit this description. This is the case in the oil and gas extraction industries where the relevant input market is comprised by companies that are providing specialized infrastructure, equipment and know-how needed to explore for, and extract, crude oil and natural gas (see e.g. Gong, 2018). Similarly Pennings (2017) refers to the case where a specialized construction company is responsible for the provision of an input, e.g. a telecommunications network or a building, tailored to the needs of the future user. Large scale climate-friendly investments also fit in this setting. There is empirical evidence suggesting that, for instance, when it comes to the production of electricity from renewable energy sources, the input market can be highly concentrated (see e.g., Pillai and McLaughlin, 2013; Rothwell, 2009 and references therein). Our analysis applies to supply chains with these characteristics.

The remainder of the paper is organized as follows. Section 2 presents the model and makes the connection with the extant literature. In Section 3 we allow for stochastic input production cost. In Section 4 we discuss the effect of the correlation coefficient. Section 5 discusses the results and concludes.

2 The model

2.1 The basic set-up

The potential investor D holds the option to enter a final market. In order to do so, she needs to invest in an indispensable input that is produced by an upstream supplier U.[2] The production cost of the input at time $t > 0$ is I_t and its price, P_t, is chosen by U as a markup on top of the production cost: $P_t = mI_t$. The production cost of the input fluctuates over time according to the following geometric Brownian motion:

$$\frac{dI_t}{I_t} = \alpha_I dt + \sigma_I dz_I, I_0 = I \tag{1}$$

where α_I is the drift, σ_I is the instantaneous volatility and dz_I is the standard increment of a Brownian motion where $E(dz_I) = dt$. The structural parameters and the realization of I_t over time are assumed to be common knowledge.

As soon as D pays the input price P_t to U, she enters the final market instantaneously. From that point on, D gains access to a project with operating value X_t:

$$\frac{dX_t}{X_t} = \alpha_X dt + \sigma_X dz_X, X_0 = X \tag{2}$$

where α_X and σ_X are respectively the drift and the instantaneous volatility and dz_X is the standard increment of a Brownian motion where $E(dz_X) = dt$. We also assume that $E(dz_I dz_X) = \rho dt$ where $\rho \in [-1, 1]$ denotes the coefficient of contemporaneous correlation between dz_I and dz_X.

[2] For the rest of the paper we use female pronouns for D and male pronouns for U.

We assume that U cannot observe the magnitude of X_t at any time point but can observe the structural parameters of Eq. (2).[3] For simplicity, D and U are assumed to be risk neutral with the risk-free interest rate denoted by r. For convergence we assume $r > \alpha_i$, $i \in \{I, X\}$.[4]

2.2 In-house production of the input under a fixed I

Suppose that D can produce the needed input in-house or, equivalently, can buy it from a competitive input market. Suppose also that the input has a fixed production cost, i.e., $\alpha_I = \sigma_I = 0$. In this case, the investment cost is just I. Given this, D needs to decide when it is optimal to pay this investment cost in order to gain access to the cash flow generated by the project.

According to the real options approach, D should delay the investment until the project's expected return is higher than the cost of the investment by a margin equal to the option value of further postponing the investment decision (Dixit and Pindyck, 1994). Additionally, since in our set-up all the information about the future evolution of process (2) is embodied in X_t, there exists an optimal investment rule of the form: "invest immediately if X_t is at, or above, the critical threshold X^* and wait otherwise".[5]

D's value of the option to invest is:[6]

$$F(X_t) = \max \left\{ X_t - I, \frac{1}{1 + rdt} E_t \left[F(X_t + dX_t) \right] \right\} \tag{3}$$

The difference $X_t - I$ corresponds to the termination value of the investment, i.e., the net present value of the project if the investment takes place at time t. On the other hand, the term $\frac{1}{1+rdt} E_t \left[F(X_t + dX_t) \right]$ is the value associated with the postponement of the investment decision

[3] If we relax this assumption allowing for a U who can continuously and verifiably observe the state of X_t, U can then dictate a pair of X_t and I_t appropriating all the benefits generated by the project. See Section 3 in Billette de Villemeur et al. for more details.

[4] This is a standard assumption. See for instance, Dixit and Pindyck (1994, p. 138).

[5] See Dixit, Pindyck and Sødal, (1999) for more details.

[6] See Dixit and Pindyck (1994), pp. 140-142.

after t.

When the starting point of the cash flow is sufficiently low so that future, rather than immediate, investment is preferred, Eq. (3) reduces to $F(X_t) = \frac{1}{1+rdt} E_t [F(X_t + dX_t)]$ which can be written as:[7]

$$\frac{1}{2}\sigma_X^2 X_t^2 F_{XX} + \alpha_X F_X - rF = 0 \tag{4}$$

This differential equation needs to be solved subject to the following conditions:

$$F(0) = 0 \tag{5}$$

$$F(X^*) = X^* - I \tag{6}$$

$$F_X(X^*) = 1 \tag{7}$$

Condition (5) arises from the observation that if X_t goes to zero, then the net present value of the project will become negative and consequently the value of the option to invest in it should be equal to zero. Conditions (6) and (7) come from the consideration of the optimal investment threshold X^*. Eq. (6) is the value matching condition and suggests that as soon as D decides to exercise the investment option, he will receive exactly $X^* - I$. Eq. (7) is a standard smooth pasting condition. Unless Eq. (7) holds at X^*, D would do better by exercising the investment option at a different time point.

Solving Eq. (4) in view of Eq. (5) we obtain $F(X_t) = \Omega X_t^{\beta}$ where $\beta = \frac{1}{2} - \frac{\alpha_X}{\sigma_X^2} + \sqrt{\left(\frac{\alpha_X}{\sigma_X^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_X^2}} > 1$ is the positive root of the characteristic equation $\frac{1}{2}\sigma_X^2 \zeta (\zeta - 1) + \alpha_X \zeta - r = 0$ and Ω is a constant to be determined.

[7] Subscripts denote partial derivatives.

From Eq. (6) and Eq. (7) we obtain $\Omega = (X^* - I)\frac{1}{X^{*\beta}} > 0$ and:

$$X^* = \frac{\beta}{\beta - 1} I \tag{8}$$

Last, the value of the option to invest at $t = 0$ is equal to:

$$F(X) = (X^* - I)\left(\frac{X}{X^*}\right)^{\beta} \tag{9}$$

The terms X^* and $F(X)$ are standard in the real options literature and will serve as our standards of comparison.

2.3 Production of the input by U under a fixed I

Billette de Villemeur et al. reapproach the problem presented above assuming that D purchases the input from an upstream firm U with market power. The only difference with the case presented above is that the price of the input is chosen by U. The price taker D will have to recalibrate the optimal investment threshold accounting for this change and the input manufacturer U will have to choose the optimal price P so that the expected value of $P - I$ reaches its maximum.

We begin with the problem of the downstream firm D. The problem is identical to the one presented above. The only difference is that the price of the input is P instead of I. As a result, the new optimal investment threshold is equal to:

$$X^U(P) = \frac{\beta}{\beta - 1} P \tag{10}$$

Given that D will exercise the investment option as soon as X^U is reached, U needs to choose the

P that will maximize his expected net present value. The problem of U can be written as:

$$\max_P E_0 \left[(P - I)\, e^{-r\tau^U} \right] \tag{11}$$

where $P = mI$ and $\tau^U := \inf \left\{ t > 0 | \, X_t = X^U(P) \right\}$. U maximizes his expected net present value accounting for the fact that the investment will take place at τ^U, i.e., as soon as X_t reaches the investment threshold $X^U(P)$. Expression (11) can alternatively be written as:[8]

$$\max_P (P - I) \left(\frac{X}{X^U(P)} \right)^{\beta} \tag{12}$$

From this we find that the optimal input price is equal to $P_m = \frac{\beta}{\beta-1} I$ which in turn implies $X^U = \frac{\beta}{\beta-1} X^* = \left(\frac{\beta}{\beta-1} \right)^2 I$.

As Billette de Villemeur et al. point out there are two important points to be made here. First, when the input market is non-competitive the input is, as expected, more expensive $(P_m > I)$. Second, a more expensive input results in an investment that takes place inefficiently late in industry terms $\left(X^U > X^* \right)$.

3 The effect of the vertical relationship under a stochastic I

Let us now approach the same problem anew explicitly assuming that the input production cost is not fixed, but is instead fluctuating over time according to (1). As we have seen in subsection 2.1, this means that the price of the input at time t is also a random variable: $P_t = mI_t$.

From Itô's lemma we have:

$$dP_t = \alpha_I P_t dt + \sigma_I P_t dz_I \tag{13}$$

[8] See Dixit and Pindyck (1994), pp. 315-316.

In this case, D's value of the option to invest is:

$$F\left(X_t, P_t\right) = \max\left\{X_t - P_t, \frac{1}{1+rdt} E_t\left[F\left(X_t + dX_t, P_t + dP_t\right)\right]\right\} \tag{14}$$

Focusing on the continuation region we have $F\left(X_t, P_t\right) r = \frac{E_t[dF]}{dt}$. From Itô's lemma we get:

$$\frac{E_t\left[dF\right]}{dt} = \alpha_X F_X X_t + \alpha_I F_P P_t + \frac{1}{2} F_{XX} \sigma_X^2 X_t^2 + \frac{1}{2} F_{PP} \sigma_I^2 P_t^2 + F_{XP} \sigma_X \sigma_I X_t P_t \rho \tag{15}$$

From Eqs. (14) and (15) we obtain:

$$Fr = \alpha_X F_X X_t + \alpha_I F_P P_t + \frac{1}{2} F_{XX} \sigma_X^2 X_t^2 + \frac{1}{2} F_{PP} \sigma_I^2 P_t^2 + F_{XP} \sigma_X \sigma_I X_t P_t \rho \tag{16}$$

The smooth pasting conditions when the investment threshold is reached are:

$$F_X = 1 \tag{17}$$

$$F_P = -1 \tag{18}$$

Homogeneity of degree one allows us to rewrite F in the following way:[9]

$$F\left(X_t, P_t\right) = P_t f\left(\frac{X_t}{P_t}\right) = P_t f\left(x_t\right) \tag{19}$$

From Eq. (16) and (19) we obtain the following differential equation:

$$\frac{1}{2}\left(\sigma_X^2 + \sigma_I^2 - 2\sigma_X \sigma_I \rho\right) f_{xx} x^2 + \left(\alpha_X - \alpha_I\right) x f_x - f\left(x\right)\left(r - \alpha_I\right) = 0 \tag{20}$$

[9] See p. 210 in Dixit and Pindyck (1994).

As for the value matching condition, we have:

$$f\left(x^{**}\right)=x^{**}-1 \tag{21}$$

The smooth pasting condition is:

$$f_x\left(x^{**}\right)=1 \tag{22}$$

Solving Eq. (21), and thanks to $f(0)=0$, we obtain $f(x_t)=\Gamma_1 x_t^{\lambda}$ where $\lambda=\frac{1}{2}+\frac{\alpha_I-\alpha_X}{\sigma_X^2+\sigma_I^2-2\sigma_X\sigma_I\rho}+\sqrt{\left(\frac{\alpha_I-\alpha_X}{\sigma_X^2+\sigma_I^2-2\sigma_X\sigma_I\rho}+\frac{1}{2}\right)^2+\frac{2(r-\alpha_I)}{\sigma_X^2+\sigma_I^2-2\sigma_X\sigma_I\rho}}>1$ is the positive root of the characteristic equation $\frac{1}{2}\left(\sigma_X^2+\sigma_I^2-2\sigma_X\sigma_I\rho\right)\lambda(\lambda-1)+(\alpha_X-\alpha_I)\lambda-(r-\alpha_I)=0$ and Γ_1 a constant that needs to be determined.[10]

Finally, from the smooth pasting and the value matching conditions we have:

$$\Gamma_1=\left(x^{**}-1\right)\frac{1}{x^{**\lambda}} \tag{23}$$

and

$$x^{**}=\frac{\lambda}{\lambda-1} \tag{24}$$

The optimal investment threshold is reached when the value of the project becomes $\frac{\lambda}{\lambda-1}$ times higher than the price of the input.

The problem for the input supplier U is:

$$\max_m E_0\left[(m-1)\,e^{-r\tau^{**}}\right] \tag{25}$$

[10] One can easily check that λ reduces to β when $\alpha_I=\sigma_I=0$.

where $\tau^{**} := \inf\{t > 0 | \, x_t = x^{**}\}$. Expression (25) can alternatively be written as

$$\max_{m} (m-1)\left(\frac{x(m)}{x^{**}}\right)^{\lambda} \tag{26}$$

which is equivalent to:

$$\max_{m} \frac{m-1}{m^{\lambda}} \tag{27}$$

This results in $m^* = \frac{\lambda}{\lambda-1}$.

Summing up, the investment takes place when the project's value becomes $(\lambda/(\lambda-1))^2$ times larger than the cost of manufacturing the input since $x^{**}m^* = (\lambda/(\lambda-1))^2$. The wedge $(\beta/(\beta-1))^2$ presented in Billette de Villemeur et al. is a subcase of this corresponding to a fixed investment cost ($\alpha_I = \sigma_I = 0$).

4 The effect of the correlation coefficient

The finding of Billette de Villemeur et al. regarding an investment time distortion attributed to upstream market power is robust in the presence of a second stochastic term in the model. This follows from $m^* > 1$ and implies that an input supplier exercises his market power distorting the optimal investment threshold also when I is stochastic. This is of course unsurprising since the presence of an additional source of uncertainty leaves unaffected the way the two parties interact.

The markup m^* is sensitive to the correlation between the two stochastic terms X_t and I_t since it is a function of λ. More precisely, since the expression $(y/(y-1))^2$ is decreasing and convex in y for $y > 1$, we have $(\lambda/(\lambda-1))^2 \lesseqgtr (\beta/(\beta-1))^2$ when $\lambda \gtreqless \beta$ which is the case when $\rho \gtreqless \rho^*$ where

$$\rho^* = \min\left\{\frac{\sigma_X^2+\sigma_I^2}{2\sigma_X\sigma_I} - \frac{\frac{r-\alpha_I}{\beta}+\alpha_I-\alpha_X}{\sigma_X\sigma_I(\beta-1)}, \frac{\sigma_X^2+\sigma_I^2}{2\sigma_X\sigma_I} - \frac{\frac{\alpha_I-\alpha_X}{2\beta-1}}{\sigma_X\sigma_I}\right\}. \tag{28}$$

This means that when the correlation coefficient ρ is high enough, that is, higher than the critical level ρ^*, the wedge $(\beta/(\beta-1))^2$ overestimates the investment trigger which is instead $(\lambda/(\lambda-1))^2 \left(< (\beta/(\beta-1))^2\right)$ times higher than the investment cost at the time of the investment. When instead the correlation coefficient is lower than the critical level ρ^*,[11] the wedge $(\beta/(\beta-1))^2$ underestimates the investment trigger which is $(\lambda/(\lambda-1))^2 \left(> (\beta/(\beta-1))^2\right)$ times higher than the investment cost. The effect of upstream market power is clearly inversely related to the correlation coefficient ρ.

Billette de Villemeur et al. conclude their paper presenting a numerical exercise suggesting that dynamic double marginalization leads to delayed entry in the vaccine industry. Using the same values, namely, $\alpha_X = 0.05$, $\sigma_X = 0.21$, $\alpha_I = 0$ and $r = 0.2$, we obtain $\beta = 2.44$.[12] Assuming $\sigma_I = \sigma_X$, for $\rho > 0.954$ we have $\lambda > \beta \rightarrow (\lambda/(\lambda-1))^2 < (\beta/(\beta-1))^2$ which means that if the production cost and the operating value of the project have the same volatility and are highly correlated, the wedge $(\beta/(\beta-1))^2$ overestimates the optimal investment threshold. For lower or higher values of σ_I, for instance, $\sigma_I = 0.1$ or $\sigma_I = 0.4$, we find that $\lambda < \beta \rightarrow (\lambda/(\lambda-1))^2 > (\beta/(\beta-1))^2$. In this case the wedge $(\beta/(\beta-1))^2$ underestimates the optimal investment threshold.

5 Conclusion

Billette de Villemeur et al. is to the best of our knowledge the first paper to describe a dynamic analog to the well-known static effect of double marginalization. Here we pursue their analysis to its natural end by considering stochastic, rather than constant, input production cost. We present a two echelon supply chain comprised by an input supplier (upstream firm) and a potential investor (downstream firm). The upstream firm has market power and faces a stochastic input production cost. The downstream firm faces a stochastic demand and has the option to delay the investment

[11] Since $\rho \in [-1,1]$, $\rho \gtrless \rho^*$ makes sense as long as $\max\left\{\frac{\frac{r-\alpha_I}{\beta}+\alpha_I-\alpha_X}{\beta-1}, \frac{\alpha_I-\alpha_X}{2\beta-1}\right\} \gtrless \frac{(\sigma_X-\sigma_I)^2}{2}$.

[12] For the calibration the authors use data from the WHO. See p. 119 of their paper for more details.

for some future time point.

In this setting, we show that the upstream firm's pricing decision is driven by two forces. The first is the one analytically discussed in Billette de Villemeur et al. and has to do with the upstream firm's market-power. The second force has to do with the stochasticity of the input production cost. The input manufacturer is interested both in the price that he will receive and in the production cost that he will have to pay in order to manufacture the input.

If the stochastic demand of the downstream firm and the stochastic input production cost are correlated, then a high coefficient of contemporaneous correlation implies a lower level of uncertainty and hence a reduced incentive for the input manufacturer to distort the price of the investment $\left((\lambda/(\lambda-1))^2 < (\beta/(\beta-1))^2\right)$. If instead the correlation coefficient is low, then the project becomes more uncertain and this makes the input manufacturer more willing to increase the input price in an attempt to hedge against volatile future revenues. This is reflected in a more distorted investment threshold $\left((\lambda/(\lambda-1))^2 > (\beta/(\beta-1))^2\right)$.

Our work generalizes the analysis by Billette de Villemeur et al. showing that upstream market power results in dynamic inefficiency, i.e. in delayed investment, not only when downstream demand is uncertain but also when the input production cost is. Furthermore, we prove that this inefficiency depends on the correlation between the two stochastic terms. When the correlation is high, the inefficiency is toned down because the motive of the input supplier to exercise his market power is relaxed. When instead correlation is low, the effect of market power is more pronounced since the input supplier will use this power as a mechanism to hedge against volatility.

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