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Analytic model to predict productivity in divisional Seru production environment

Marco Bortolini

Alma Mater Studiorum - University of Bologna, Department of Industrial Engineering, Viale del Risorgimento, 2, 40136 Bologna, Italy

A R T I C L E I N F O

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A B S T R A C T

Advanced production environments emerged as the good solution to address the modern market challenges asking for a wide product mix and low time to market. Within cellular systems, made of independent, modular and flexible working areas, tailored on families of similar products, Serus are of increasing adoption for both manufacturing and assembly tasks. Among them, the so-called divisional Serus are the first step to move from the traditional production lines to a production environment made of a set of identical working areas, parallelising activities and enabling potential productivity increase. Despite their adoption in industry, starting from the electronic sector and moving forward, reference analytic models to predict divisional Seru productivity are rare in the literature, while their formulation and application is a gap to fill. This paper addresses this gap in theory, supporting the transition toward Seru production environment by proposing and proofing the analytic closed-form expressions getting the expected productivity of a divisional Seru made of a generic number of workers and a) one (base case), b) two (extension) and c) a generic number (general case) of product types to produce. Together with the steps to get the productivity expressions for these three cases of immediate practical applicability and not yet proposed by the literature, a case study and sensitivity analysis on the divisional Seru dimension showcase the proposed model industrial use and impact on the expected productivity. Key results highlight a stationary behaviour of the working time for all workers making the Seru productivity dependent on the sum of the workers speed and the product type workloads.

1. Introduction

To be proactive to meet the modern market challenges, asking for highly customised product varieties ready to deliver in very few time after the customers' orders, the modern advanced production environments are evolving from traditional configurations, e.g., flow-shop lines, job-shop systems, etc., toward hybrid, dynamic and reconfigurable smart production systems (Villa and Taurino, 2013; Bortolini et al., 2019), also called Next Generation Manufacturing Systems (Benjaafar et al., 2002; Molina et al., 2005; Lee et al., 2013; Bortolini et al., 2018). Their goal is to work efficiently in presence of dynamic demand, short life cycle, frequent product redesign, high variety, low and variable production volumes (Behzad et al., 2016; Yu and Tang, 2019). Among the existing advanced production environments, the big class of cellular manufacturing (CM) systems deserve attention due to its wide application in industry and its potential already demonstrated in multiple contexts (Singh, 1993; Hosseinabad and Zaman, 2020). The basic idea behind CM is to create autonomous working areas, operating in parallel, and tailored to families of product variants having similar components and process cycles (Bortolini et al., 2011; Singh and Rajamani, 2012; Doroudyan and Khoshghalb 2021). In the early '90s, an evolution of traditional CM systems emerged in the Japanese industrial context, called Seru, *'cell'* in Japanese, or, more extensively, Seru Seisan – *'production cell'* (Yin et al., 2008). Leading companies in the electronic sector as Canon, Sony and Fujitsu are good examples of pioneers in the adoption of the Seru production environment (Yin et al., 2017; Wang et al., 2019). Multiple definitions of Seru are proposed. As example, Isa and Tsuru state that a Seru system is *'a production system in which a single worker or small team of production workers perform multiple production jobs (multitasking) in short segment lines'* (Isa and Tsuru, 2002), while Yamada and Kataoka define Serus as *'manufacturing organisations (in most cases, manufacturing/assembly organizations) which consists of one or several workers and all assembly tasks of a product are completed within them'* (Yamada and Kataoka, 2001). All Authors emphasise three common attributes belonging to all Seru production environments, characterising them:

E-mail address: marco.bortolini3@unibo.it.

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– its independence from other production systems. Called *'kanketsu'* in Japanese, Serus must be able to cover the whole production process, so that all the necessary resources must be present inside each Seru, staring from the equipment and, as a pillar, one or a small group of cross-trained workers;
– its self-managing, continuous improvement and evolution. Called *'jiritsu'* in Japanese, Serus do not require a top-down external control and they are based on workers' self-management, attention to any suitable improvement action and continuous learning-by-doing;
– its compactness, order and efficiency. Called *'majime'* in Japanese, Serus minimise their space occupation, reducing travels and handlings, easing product and tool handover among workers, removing any type of unnecessary resource, facilitating the flow of information to reach high levels of smoothness of the physical and informative flows.

These attributes relate to the well-known lean thinking (Womack and Jones, 2003) that is the background philosophy of this advanced production environment.

The literature outlines three types of Serus: divisional Serus, rotating Serus and Yatais (Liu et al., 2011; Stecke at al., 2012; Yin et al., 2017; Yin et al., 2018; Yu and Tang 2019; Bortolini and Galizia, 2022).

– In divisional Serus, known as the first stage of the transition from traditional production lines to advanced production environments (Yu et al., 2017), the working space is shared among a small group of workers that cooperate to complete each product. Typically, all workers work all products assigned to the Seru. The workload is dynamically divided, according to the workers' skills and speed that can vary, significantly, because of the training role of this type of Seru (Ying and Tsai, 2017);
– The second evolution stage is by rotating Serus. At this stage, workers are trained to perform all the tasks of, at least, a subset of all the product types, so that, typically, different workers work different products completely even if the working space is shared and reciprocal support is frequent;
– The so-called *'perfect'* Seru is called Yatai (Yin et al., 2008), where a fully cross-trained worker is responsible and works alone the whole mix of the assigned products with no bottlenecks and delays caused by congestion inside the Seru or, simply, by slower workers co-hosted in the working space.

Focusing on divisional Serus, because of they are the first step of the introduced evolution and they represent a frequent stage of development of these systems, especially in the western world industry, (Wang at et al., 2019), inspired by previous works (Bartholdi and Eisenstein, 1996; Bortolini and Galizia, 2022), this paper contributes to theory in advanced production systems' engineering proposing and proofing, by the use of algebra, the analytic closed-form expressions getting the expected productivity of a divisional Seru made of a generic number of workers and a) one (base case), b) two (extension) and c) a generic number (general case) of product types to produce. A case study applies the proposed model stressing its industrial usability and relevance. As far as the Author knowledge, lack of easy-use analytic models to predict the dynamic and target, i.e., stationary after a long series of worked products, behaviour of divisional Serus exists in the literature, while this paper aims at contributing to its fill.

According to the introduced context and goal, the reminder of this paper is organised as follows: Section 2 shortly reviews the literature on models and Seru applications, Section 3 states the problem, listing the assumptions and notations, while Sections 4, 5 and 6 describe the introduced base case, extension and general case of the proposed model. The case study description is in Section 7 together with some discussion of the results, while the last Section 8 concludes this paper with final remarks and next steps.

2. Literature review

Extensive descriptions of the fundamentals, origin and Seru production schemes are in the past and recent literature (Liu et al., 2011; Stecke et al., 2012; Villa and Taurino, 2013; Kaku, 2017; Kaku, 2019) together with a comprehensive review on Seru production (Yu and Tang, 2019). Despite a holistic analysis of works on Serus is out of this paper target, to place the proposed model in the scientific literature four streams of research are considered:

– Transition to Seru production;
– Workers and their cross-training;
– Models for Serus' balancing and planning;
– Evidence from the industrial field.

2.1. Transition to Seru production

Because of Serus rose up to enhance traditional production systems, the literature pays deep attention in analysing the transition toward this advanced production environment. A first reference contribution is by Kaku et al. (2009) defining and modelling a line-cell conversion problem, sizing the number of cells and workers. Three years later, Liu et al. (2012) proposed and applied an improved model on the same problem benchmarking it to the existing literature. Yu et al. (2012) discussed on how to carry out assembly line-cell conversion by proposing a bi-objective model to minimise the throughput time and labour hours. Yu et al. (2013) and Yu et al. (2014), using a multi-objective line-cell NP-hard conversion model, addressed the goal of jointly reduce the number of workers and increasing the production system productivity. Validation of their model was by several numerical simulation experiments and a critic discussion of the obtained Pareto-fronts. In 2016, Sun et al. (2016) deepened the line-pure Seru conversion proposing different algorithms and comparing their computational performances, while Shao et al. (2017) focused on the time, cost and efficiency improvements coming from a Seru conversion. An organic synthesis and reference framework on this topic is by Yu et al. (2017) clearly stating, on one side, the complexity of the line-cell transition, on the other, the benefits of this enhancement in the production environment structure. In their work, the Authors outlined models, complexities, properties, solutions and insights to support industry in this disruptive conversion.

2.2. Workers and their cross-training

The literature agrees on the cornerstone role of human workers in Seru production environment to get flexibility, dynamism and high performances (Stecke et al., 2012; Yu and Tang, 2019; Ayough et al., 2020; Bortolini and Galizia, 2022). Serus are worker-based production systems, workers are responsible of operative and managerial issues and their constant cross-training and valorisation is among the crucial goals set by Seru production and, in general, by the lean production philosophy (Womack and Jones, 2003; Lolli et al., 2016; Afshar-Nadjafi, 2021). Ying and Tsai (2017) investigated the multi-skilled worker training and assignment in Seru production systems to minimise the total training and balancing cost, while Wu at al. (2018) tackled the cross-trained worker assignment problem in divisional and rotating Serus using OR models and some numerical experiments to conclude about the conditions making a Seru configuration most suitable respect to the other ones. Additionally, Liu et al. (2021a) stressed the cross-trained worker assignment as a vital problem in Seru production addressing it when divisional and rotating Serus coexist, i.e., the so-called hybrid Seru production systems. A model formulation and NSGA-II-based memetic algorithm are proposed and applied. Finally, Torkul et al. (2022) proposed a conceptual model based on deep learning for the real-time monitoring and controlling of the production process factors, as the workers, the tools and the production environment. In their models, all Authors converged in classifying the workers' skills choosing among one or more of these metrics: types of tasks they can do, types of products they can work and working speed to perform their activities (Luo et al., 2016; Yu et al., 2018; Yu and Tang, 2019; Bortolini and Galizia, 2022).

2.3. Models for Serus' balancing and planning

Focusing on the last few years, a major set of models contributed to support the balancing and planning of Serus. In 2016, Luo et al. (2016) focused on the Seru loading problem and worker-operation assignment proposing a single period model and heuristic stepwise solving algorithm. Luo et al. (2017) modelled the Seru loading problem under uncertainty, while Yu at al. (2018) formulated and solved a Seru system balancing problem providing useful relations for the workload balancing. Han et al. (2018) presented a model to assess the reliability of divisional Serus under stochastic capacity, while Yilmaz (2020) explored the problem of the workforce scheduling in Seru production through a comprehensive optimisation model based on flexible workforce, i.e., the so-called *'shojinka'* in Japanese. Production time balancing and energy consumption minimisation were the two goals addressed by the production planning multi-objective model proposed by Escobar Forero and Amaia Guio (2020), while the challenging issue of responding to uncertain demand is addressed by Ye and Tang (2020) and Fujita et al. (2022) formulating robust Seru production system optimisation models. Jiang et al. (2021) developed an exact method for the scheduling problem including setup time and learning effect, while Bortolini and Galizia (2022) focused on divisional Seru presenting a recursive model and simulation to balance the workers' effort. Li et al. (2022) proposed a multi-objective cooperative coevolution algorithm for Seru formation and scheduling, while Zhang et al. (2022) faced the same issue using a logic-based Benders decomposition method. Fu et al. (2022) focused on dynamic Seru production studying its total tardiness when product information changes. Finally, despite not directly related to the Seru production environment, the pioneering study on *'bucket brigades'* by Bartholdi and Eisenstein (1996) and recent studies on *'walking workers'* (Şahin and Kellegöz, 2019; Bortolini et al., 2021; Calzavara et al., 2021) deserve attention due to similarities of these production systems' behaviour and modelling approaches.

2.4. Evidence from the industrial field

The industrial field perspective is a further stream of interest to both drive Seru production implementation and to collect experience and feedbacks. Liu et al. (2014) provided a general framework and some basic principles to follow when implementing Serus in practice, while Yin et al. (2017) reported lessons learnt from the Seru implementation by two electronic leading companies, i.e., Sony and Canon. More recently, Wang et al. (2019) collected and compared 81 cases of applications of CM and Seru systems drawing conclusions and suggestions for practitioners. Liu et al. (2021b) provided a contingency-based framework and explored 357 cases in the Chinese context, while Sari and Erdem (2021) gave a focus on Seru production systems in the clothing industry. Authors stress the potential of Seru production environment to increase productivity in manufacturing and assembly and, at the same time, the key role and need of reference models and easy-use quantitative approaches to predict the Seru performances, as powerful tools feeding feasibility studies on their effective implementation in industry.

Following the literature trend, given the current need of easy-use analytic models to predict the dynamic and target, i.e., stationary after a long series of worked products, behaviour of divisional Serus, and contributing to theory in Seru production systems' engineering, Section 3 states the problem and modelling assumptions.

3. Problem statement and assumptions

The reference industrial scenario focused by the proposed model is a divisional Seru environment made of a given number of partially cross-trained workers, equipped with tools and all the auxiliaries needed to produce a given number of product types.

Fig. 1 shows the divisional Seru, further introducing its key entities and some notations.

Products to work enter from an edge (*In* in Fig. 1) as soon as the worker closest to *In* is idle, they flow the one-way Seru up to the exit position (*Out* in Fig. 1). Workers cooperate inside the Seru and they handover products to the forward worker as soon as available, taking the next product from the backward worker. In this way, out of an initial transitory, all workers work each product.

The following notations are introduced.

Indices and counters

c – Index of couples/ordered sequences of product types (numbered sequentially in order of entrance)

i – Index of workers (numbered sequentially starting from the end of the Seru)

j – Index of products (numbered sequentially in order of entrance)

u – Index of product types

Input parameters

m – Number of product types

($m = 1$ for the base case, $m = 2$ for the model extension to two products)

n – Number of workers inside the Seru, with $n > 1$

$\widetilde{Q}^u$ – Workload of product type u, supposed continuous [tasks]

v_i – Average speed of worker i [tasks/minute]

Output variables

Q_i^j – Processed workload of worker i on product j [tasks]

t_i^j – Working time of worker i on product j [minutes]

t_i^∞ – Target (stationary) working time of worker i on the generic product/couple/ordered sequence of product types [minutes]

η – Target (stationary) productivity of the Seru [products/hour]

The model is based on the following assumptions:

- Tasks are supposed to be continuous and their stop to handover product between workers can be at any time;
- All workers are partially cross-trained. They can work all tasks of all product types but at different speeds (Luo et al., 2016; Yu et al., 2018; Yu and Tang, 2019; Bortolini and Galizia, 2022);

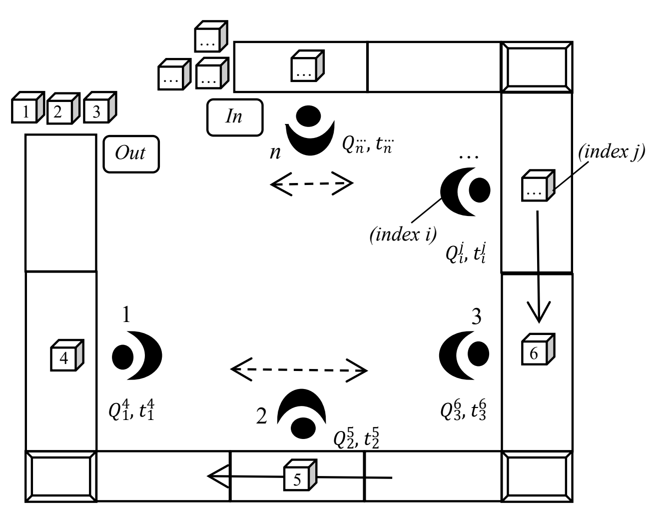

Fig 1. Reference divisional Seru production environment, with notations.

- New products to work and the required tools are always supposed immediately available;
- The speed of each worker does not depend on the product type (Yu and Tang, 2019; Bortolini and Galizia, 2022);
- Workers cannot overtake others and they are positioned in order of speed, the slowest is close to *In*, the fastest is close to *Out* (see Fig. 1). Follows that, $v_i > v_{i+1}$;
- Handover of products is supposed to be no time consuming;
- Workers' moving speed is higher than their working speed so that workers' moving is always fully in parallel to working, taking no more additional time;
- Product types are scheduled following a round robin manner, e.g., for $m = 3$, the entering sequence will be according to Table 1, further introducing the concept of ordered sequence of product types as the ordered set of products, one per type, composing each round;
- At the beginning, i.e., if the Seru is empty, an initial transitory takes place. n products are loaded together, one per worker. The first product ($j = 1$) is assigned the fastest worker ($i = 1$), the second product ($j = 2$) to the second fastest worker ($i = 2$) up to product $j = n$ that is assigned to the slowest worker ($i = n$). Then, the divisional Seru working behaviour starts.

Starting from the introduced notations and assumptions the model studies the Seru behaviour predicting the worker loads and the target, i.e., stationary, after working a high number of products, productivity. The base case, with one product type, is in the next Section 4, while extension to two products and generalisation to m products follow.

4. Single product type model for divisional Seru (base case)

The base case considers a divisional Seru made of n workers and one product type, i.e., $m = 1$. For simplicity, the index u is omitted in this case.

According to the assumptions, during the initial transitory the first product ($j = 1$) enters the Seru and it is worked by the fastest worker ($i = 1$), only, i.e., $Q_1^1 = \widetilde{Q}$. The working time is $t_1^1 = \widetilde{Q}/v_1$. The second product ($j = 2$) is started by the second fastest worker ($i = 2$) and completed by the fastest worker ($i = 1$) after the finishing of the first product, i.e., handover time. Because of the first and second product entry time is the same, the second fastest worker works the second product for the same time the fastest worker works the first product. Analytically: $t_2^2 = t_1^1 = \widetilde{Q}/v_1$, while $Q_2^2 = v_2 \times t_2^2 = v_2 \times \widetilde{Q}/v_1$. Then, the first product leaves the Seru and the handover of the second product takes place. Because of $v_1 > v_2$, the second product is not yet finished by the second fastest worker. The remaining tasks to complete, by the fastest worker, are $Q_1^2 = \widetilde{Q} - Q_2^2 = \widetilde{Q} - v_2 \times \widetilde{Q}/v_1 = \widetilde{Q} \times (1 - v_2/v_1)$. The fastest worker working time follows, $t_1^2 = Q_1^2/v_1 = (\widetilde{Q}/v_1) \times (1 - v_2/v_1)$.

Similarly, even if formulas become longer, the third product ($j = 3$) is started by the worker $i = 3$, if present, continued by the worker $i = 2$ and finished by the fastest worker ($i = 1$). Its first handover takes place when the worker $i = 2$ becomes idle, due to the handover of the second product to the fastest worker, while the second handover takes place when the second product is finished and the fastest worker becomes idle. Consequently, $t_3^3 = t_2^2 = t_1^1 = \widetilde{Q}/v_1$ and $Q_3^3 = v_3 \times t_3^3 = v_3 \times \widetilde{Q}/v_1$. Concerning the second fastest worker and the third product, follows that $t_2^3 = t_1^2 = (\widetilde{Q}/v_1) \times (1 - v_2/v_1)$ and $Q_2^3 = v_2 \times t_2^3 = v_2 \times (\widetilde{Q}/v_1) \times (1 - v_2/v_1)$. Finally, the fastest worker must finish the remaining tasks:

$$\begin{aligned} Q_1^3 &= \widetilde{Q} - Q_2^3 - Q_3^3 \\ &= \widetilde{Q} - v_2 \times \left(\widetilde{Q}\Big/v_1\right) \times (1 - v_2/v_1) - v_3 \times \widetilde{Q}/v_1 \\ &= \widetilde{Q} \times \left(1 - v_2\Big/v_1 + (v_2/v_1)^2 - v_3\Big/v_1\right) \end{aligned}$$

$$t_1^3 = Q_1^3/v_1 = (\widetilde{Q}/v_1) \times \left(1 - v_2/v_1 + (v_2/v_1)^2 - v_3/v_1\right).$$

This sequential pattern is the same for all products and can be repeated, iteratively.

Two general properties follow.

Property 1. *Within divisional Serus, each worker i works tasks of all products $j \geq i$;*

Property 2. *Within divisional Serus, handover of products between all couples of consecutive workers is synchronous and driven by the working time of the fastest worker.*

Proof of Property 1 follows, immediately, from the last adopted assumption, dealing with the initial transitory, coupled with the one-way direction of the product flow inside the Seru.

Proof of Property 2 comes from the divisional Seru behaviour and the handover of products. As discussed above for the first three products, when the fastest worker completes the current product all products inside the Seru make a step forward passing from their current worker to the next one, while the slowest worker starts working a new product. Because of the handover of products is synchronous, the working time on product j by worker i must equal the working time on product $j-1$ by worker $i-1$, i.e., the product and worker that are a step forward. The recursive Equation (1) follows (Bortolini and Galizia, 2022):

$$t_i^j = t_{i-1}^{j-1} = t_{i-2}^{j-2} = \cdots = t_1^{j-i+1} \tag{1}$$

allowing to refer the working time of any worker to the working time of the fastest one.

Additionally, t_1^j depends on the remaining tasks to do to complete the product workload. Analytically,

$$\begin{aligned} t_1^j &= \frac{\widetilde{Q} - \sum_{i=2}^{\min\{j,n\}} Q_i^j}{v_1} = \frac{\widetilde{Q} - \sum_{i=2}^{\min\{j,n\}} v_i \times t_i^j}{v_1} = \frac{\widetilde{Q} - \sum_{i=2}^{\min\{j,n\}} v_i \times t_1^{j-i+1}}{v_1} \\ &= \frac{\widetilde{Q}}{v_1} - \sum_{i=2}^{\min\{j,n\}} \frac{v_i}{v_1} \times t_1^{j-i+1} \end{aligned} \tag{2}$$

where the sum is extended to all workers that worked the product (see Property 1). After the initial transitory, i.e., for all products $j \geq n$, the previous Equation (2) becomes the following:

$$t_1^j = \frac{\widetilde{Q}}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times t_1^{j-i+1} \tag{3}$$

4.1. Target (stationary) working time of each worker on each product

Equation (3) links the working time of the fastest worker on product j to the working time on the previous products. Because of this time directly affects the Seru throughput rate and productivity, much interest rises on its value after a major set of products are processed, i.e., when $j \to \infty$, looking for a tentative stationary behaviour and easy to use analytic closed-form expression to predict t_1^∞ and, generally, t_i^∞. To this purpose, let us consider Equations (4) and (5), that show Equation (3)

Table 1
Product scheduling rule, example for three product types.

Entry position	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	…
Index j	1	2	3	4	5	6	7	8	9	…
Index u	1	2	3	1	2	3	1	2	3	…
Index c		1			2			3		…

for two consecutive products $j-1$ and j, not belonging to the initial transitory ($j \geq n+1$):

$$t_1^{j-1} = \frac{\widetilde{Q}}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times t_1^{j-i} \tag{4}$$

$$t_1^{j} = \frac{\widetilde{Q}}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times t_1^{j-i+1} \tag{5}$$

Subtracting the correspondent terms of the two equations follows that:

$$t_1^{j} - t_1^{j-1} = \sum_{i=2}^{n} \frac{v_i}{v_1} \times \left(t_1^{j-i} - t_1^{j-i+1}\right) \tag{6}$$

that can be reshaped as in Equation (7).

$$\sum_{i=1}^{n} \frac{v_i}{v_1} \times \left(t_1^{j-i+1} - t_1^{j-i}\right) = 0 \tag{7}$$

where the sum includes, also, the fastest worker. This equation links the working times of the first worker on a set of $n+1$ consecutive products and it is the basis to predict t_1^{∞}.

4.1.1. Proof of the stationary behaviour of the working time

Starting from Equation (7), the t_1^j addenda are isolated and ordered.

$$t_1^{j} + \sum_{i=1}^{n-1} \left(\frac{v_{i+1}}{v_1} - \frac{v_i}{v_1}\right) \times t_1^{j-i} - \frac{v_n}{v_1} \times t_1^{j-n} = 0 \tag{8}$$

In Equation (8) the sequence of $t_1^j, t_1^{j-1}, t_1^{j-2}, \cdots, t_1^{j-n}$ consecutive working times appears making it a linear recurrence relation with constant coefficients, i.e., an equation that expresses the j^{th} term of a sequence as a function of the preceding terms. To solve it, the so-called characteristics polynomial is considered in Equation (9):

$$r_1^{j} + \sum_{i=1}^{n-1} \left(\frac{v_{i+1}}{v_1} - \frac{v_i}{v_1}\right) \times r_1^{j-i} - \frac{v_n}{v_1} \times r_1^{j-n} = 0 \tag{9}$$

where r_1 is the variable and the superscripts are exponents of powers. Dividing all terms by r_1^{j-n} the final characteristics polynomial of grade n follows in Equation (10):

$$r_1^{n} + \sum_{i=1}^{n-1} \left(\frac{v_{i+1}}{v_1} - \frac{v_i}{v_1}\right) \times r_1^{n-i} - \frac{v_n}{v_1} = 0 \tag{10}$$

Equation (10) has the following characteristics:

- $r_1 = 1$ is a root;
- Except on the coefficient of the first term, r_1^n, equal to 1, all the other coefficients are negative (in the round bracket, $v_i > v_{i+1}$ according to the assumptions).

According to the Routh-Hurwitz criterion, follows that, among the n roots of Equation (10), one of them is positive, i.e., $r_1 = 1$, while all the others are negative or, if complex numbers, their real parts are negative. Additionally, because of the sequence of positive and negative coefficients in Equation (10), such negative values are in the range $(-1,0)$.

Let us call $\rho_{1,a}, \rho_{1,b}, \cdots, \rho_{1,n}$ the n solutions of Equation (10), with $\rho_{1,a} = 1$. The linear recurrence relation returns:

$$t_1^{j} = A \times \rho_{1,a}^{j} + B \times \rho_{1,b}^{j} + \cdots + N \times \rho_{1,n}^{j} = A + B \times \rho_{1,b}^{j} + \cdots + N \times \rho_{1,n}^{j} \tag{11}$$

with $A, B, \cdots, N$ real coefficients, unknown at this stage.

Given Equation (11), Property 3 follows.

Property 3. *Within divisional Serus, after a major set of products are processed, i.e., when $j \to \infty$, each worker i works all products for the same amount of time.*

Proof of Property 3 is by calculating the following limit:

$$t_1^{\infty} = \lim_{j\to\infty} t_1^{j} = \lim_{j\to\infty}\left(A + B \times \rho_{1,b}^{j} + \cdots + N \times \rho_{1,n}^{j}\right) = A \tag{12}$$

because of $\rho_{1,b}^j, \cdots, \rho_{1,n}^j$ are powers with bases in the range $(-1,0)$, so that their limit is null.

Finally, according to Equation (1), Property 3 can be extended to all workers.

4.1.2. Proof of the analytic expression for the target (stationary) productivity

The availability of the analytic closed-form expression for t_i^{∞} is a milestone to predict the Seru productivity. The starting point is previous Equation (11) extended to the generic worker i:

$$t_i^{j} = A + B \times \rho_{i,b}^{j} + \cdots + N \times \rho_{i,n}^{j} \tag{13}$$

and the goal is to calculate the value of A, according to the result in Equation (12).

Following the linear recurrence relation theory, $A, B, \cdots, N$ real coefficients are to compute given a same number of initial conditions. To this purpose, without loss of generality, the worker $i = n$ and the first n products are considered. In this case, $t_n^1 = t_n^2 = \cdots = t_n^{n-1} = 0$ and $t_n^n = \widetilde{Q}/v_1$. The $n \times n$ system of linear equations in Equation (14) follows (written with matrices):

$$\begin{bmatrix} 1 & \rho_{n,b} & \cdots & \rho_{n,n} \\ 1 & \rho_{n,b}^2 & \cdots & \rho_{n,n}^2 \\ \cdots & \cdots & \cdots & \cdots \\ 1 & \rho_{n,b}^n & \cdots & \rho_{n,n}^n \end{bmatrix} \times \begin{bmatrix} A \\ B \\ \cdots \\ N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ \widetilde{Q}/v_1 \end{bmatrix} \tag{14}$$

and must be solved to get the unknown A, only, according to Equation (12). By solving it, the following Property 4 is proofed.

Property 4. *Within divisional Serus, after the initial transitory, each worker i works all products for $t_i^{\infty} = \widetilde{Q}/\sum_{i=1}^{n} v_i$.*

Algebra to proof Property 4 is in Appendix A.

Finally, given t_i^{∞}, the target (stationary) productivity of the Seru, in products per hour, can be calculated as follows:

$$\eta = 60/(n \times t_i^{\infty}) = \frac{60}{n \times \widetilde{Q}} \times \sum_{i=1}^{n} v_i \tag{15}$$

5. Two product type model for divisional Seru (extension)

The model extension described in this Section considers a divisional Seru made of n workers and two product types, i.e., $m = 2$. Their workloads are $\widetilde{Q}^1$ and $\widetilde{Q}^2$, with $\widetilde{Q}^1 \neq \widetilde{Q}^2$, while their scheduling follows the round robin manner, starting from a product of type 1. This means that odd values of j are for products of type 1 and even values of j are for products of type 2.

Similarly to the base case, the divisional Seru behaviour is the following: the first product ($j = 1$ and $u = 1$) enters the Seru and it is worked by the fastest worker ($i = 1$), only, i.e., $Q_1^1 = \widetilde{Q}^1$. The working time is $t_1^1 = \widetilde{Q}^1/v_1$. The second product ($j = 2$ and $u = 2$) is started by the second fastest worker ($i = 2$) until the fastest worker ($i = 1$) has finished the first product. Because of the first and second product entry time is the same, the second fastest worker works the second product for the same time the fastest worker works the first product. Analytically: $t_2^2 = t_1^1 = \widetilde{Q}^1/v_1$, while $Q_2^2 = v_2 \times t_2^2 = v_2 \times \widetilde{Q}^1/v_1$. At this stage, two possibilities occur:

- if $\widetilde{Q}^2 \leq Q_2^2$ the second fastest worker already finished the second product, so that $Q_1^2 = t_1^2 = 0$. In this case, the second product (and the second worker) wait for the fastest worker that, simply, takes the product out of the Seru;
- if $\widetilde{Q}^2 > Q_2^2$ the second fastest worker has not enough time to finish the second product that will be finished by the fastest worker after the handover. $Q_1^2 = \widetilde{Q}^2 - Q_2^2 = \widetilde{Q}^2 - v_2 \times \widetilde{Q}^1/v_1$ and $t_1^2 = Q_1^2/v_1 = (\widetilde{Q}^2 - v_2 \times \widetilde{Q}^1/v_1)/v_1 = (v_1 \times \widetilde{Q}^2 - v_2 \times \widetilde{Q}^1)/(v_1)^2$

In practice, the second possibility in the most frequent, the Reader is suggested to focus on it and it will be considered in the following. Nevertheless, by setting $Q_2^2 = \min\left\{\widetilde{Q}^2, v_2 \times \widetilde{Q}^1/v_1\right\}$ the first possibility falls into the second one.

Similarly, even if formulas become longer, the third product ($j = 3$ and $u = 1$) is started by the worker $i = 3$, if present, continued by the worker $i = 2$ and finished by the fastest worker ($i = 1$). Its first handover takes place when the worker $i = 2$ becomes idle, due to the handover of the second product to the fastest worker, while the second handover takes place when the second product is finished and the fastest worker becomes idle. Consequently:

$$t_3^3 = t_2^2 = t_1^1 = \frac{\widetilde{Q}^1}{v_1}, Q_3^3 = v_3 \times t_3^3 = v_3 \times \widetilde{Q}^1/v_1 \tag{16}$$

$$t_2^3 = t_1^2 = \frac{v_1 \times \widetilde{Q}^2 - v_2 \times \widetilde{Q}^1}{(v_1)^2}, Q_2^3 = v_2 \times t_2^3 = v_2 \times \frac{v_1 \times \widetilde{Q}^2 - v_2 \times \widetilde{Q}^1}{(v_1)^2} \tag{17}$$

Finally, for the fastest worker:

$$Q_1^3 = \widetilde{Q}^1 - Q_2^3 - Q_3^3 = \frac{\widetilde{Q}^1 \times \left((v_1)^2 + (v_2)^2 - v_1 \times v_3\right) - v_1 \times v_3 \times \widetilde{Q}^2}{(v_1)^2} \tag{18}$$

$$t_1^3 = Q_1^3/v_1 = \frac{\widetilde{Q}^1 \times \left((v_1)^2 + (v_2)^2 - v_1 \times v_3\right) - v_1 \times v_3 \times \widetilde{Q}^2}{(v_1)^3} \tag{19}$$

This sequential pattern is the same for all products and can be repeated, iteratively.

Concerning the four properties discussed in the base case, Property 1 and Property 2 remain the same, with the same proofs, while Property 3 and Property 4 require an extension, as discussed in the following.

Previous Equations (2) and (3) need to be split according to the product type. They become Equations (20) and (21), respectively.

$$t_1^j = \begin{cases} \frac{\widetilde{Q}^1}{v_1} - \sum_{i=2}^{\min\{j,n\}} \frac{v_i}{v_1} \times t_1^{j-i+1} \textit{if } j \textit{ is odd} \\ \frac{\widetilde{Q}^2}{v_1} - \sum_{i=2}^{\min\{j,n\}} \frac{v_i}{v_1} \times t_1^{j-i+1} \textit{if } j \textit{ is even} \end{cases} \tag{20}$$

$$t_1^j = \begin{cases} \frac{\widetilde{Q}^1}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times t_1^{j-i+1} \textit{if } j \textit{ is odd} \\ \frac{\widetilde{Q}^2}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times t_1^{j-i+1} \textit{if } j \textit{ is even} \end{cases} \tag{21}$$

Let us consider two couples of consecutive products $j-1$ ($u = 1$), j ($u = 2$) and $j-3$ ($u = 1$), $j-2$ ($u = 2$), with $j \geq n + 3$. Follows that:

$$t_1^{j-1} = \frac{\widetilde{Q}^1}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times t_1^{j-i}, t_1^j = \frac{\widetilde{Q}^2}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times t_1^{j-i+1} \tag{22}$$

$$t_1^{j-3} = \frac{\widetilde{Q}^1}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times t_1^{j-i-2}, t_1^{j-2} = \frac{\widetilde{Q}^2}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times t_1^{j-i-1} \tag{23}$$

Adding the correspondent terms in Equations (22) and (23), Equations (24) and (25) follow.

$$t_1^{j-1} + t_1^j = \frac{\widetilde{Q}^1 + \widetilde{Q}^2}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times \left(t_1^{j-i} + t_1^{j-i+1}\right) \tag{24}$$

$$t_1^{j-3} + t_1^{j-2} = \frac{\widetilde{Q}^1 + \widetilde{Q}^2}{v_1} - \sum_{i=2}^{n} \frac{v_i}{v_1} \times \left(t_1^{j-i-2} + t_1^{j-i-1}\right) \tag{25}$$

Now, subtracting the correspondent terms of these last two equations follows that:

$$\left(t_1^{j-1} + t_1^j\right) - \left(t_1^{j-3} + t_1^{j-2}\right) = \sum_{i=2}^{n} \frac{v_i}{v_1} \times \left(\left(t_1^{j-i-2} + t_1^{j-i-1}\right) - \left(t_1^{j-i} + t_1^{j-i+1}\right)\right) \tag{26}$$

that can be reshaped as in Equation (27).

$$\sum_{i=1}^{n} \frac{v_i}{v_1} \times \left(\left(t_1^{j-i} + t_1^{j-i+1}\right) - \left(t_1^{j-i-2} + t_1^{j-i-1}\right)\right) = 0 \tag{27}$$

In Equation (27), terms $t_1^{j-i} + t_1^{j-i+1}$ and $t_1^{j-i-2} + t_1^{j-i-1}$ in the brackets are the sum of the working times of the fastest worker on the two consecutive couples of consecutive products $j-i$, $j-i+1$ and $j-i-2$, $j-i-1$, namely τ_1^{c-i+1} and τ_1^{c-i} according to the introduced notations. Replacing them, Equation (28) follows:

$$\sum_{i=1}^{n} \frac{v_i}{v_1} \times \left(\tau_1^{c-i+1} - \tau_1^{c-i}\right) = 0 \tag{28}$$

Equation (28) is logically identical to previous Equation (7) replacing the working time of each product to the working time on each *couple of consecutive* products as generalised in the following property.

Property 5. *Within divisional Seru working two types of products, the unitary, i.e., atomic, entity to focus on is each couple of consecutive products. Alternatively, the behaviour of a divisional Seru working two types of products is equivalent to the behaviour of a single product type divisional Seru working a dummy product, with* $\widetilde{Q} = \frac{\widetilde{Q}^1 + \widetilde{Q}^2}{2}$.

According to Property 5 and following what discussed and proofed in previous Sections 4.1.1 and 4.1.2, the extension of Property 3 and Property 4 is in the following:

Property 3. *(extended). Within divisional Serus, after a major set of products are processed, i.e., when $j \to \infty$, each worker i works all couples of consecutive products for the same amount of time.*

Property 4. *(extended). Within divisional Serus, after the initial transitory, each worker i works all couples of consecutive products for* $\tau_i^\infty = \left(\widetilde{Q}^1 + \widetilde{Q}^2\right) / \sum_{i=1}^{n} v_i$.

Finally, given τ_i^∞, the target (stationary) productivity of the Seru, in products per hour, can be calculated as follows.

$$\eta = 2 \times 60/(n \times \tau_i^\infty) = 2 \times \frac{60}{n \times \left(\widetilde{Q}^1 + \widetilde{Q}^2\right)} \times \sum_{i=1}^{n} v_i \tag{29}$$

where the factor 2 states that every τ_i^∞ minutes two products, one per each product type, are finished.

6. Multi-product type model for divisional Seru (general case)

In the general case, the divisional Seru is asked to work m types of products, with $\widetilde{Q}^u$ as the required workload of the generic product type u. As for the model extension, the product scheduling follows the round robin manner, starting from a product of type 1 ($j = 1$ and $u = 1$). The m^{th} product to enter will be of type m ($j = m$ and $u = m$), then, the sequence restarts from the product type 1 ($j = m+1$ and $u = 1$). n workers are present and they operate as in the previous cases so that, as for the model extension described in the previous Section 5, Property 1 and Property 2 are still true.

The turning point to study this case is the extension of Property 5, that introduced the concept of *couple of consecutive products* as the atomic entity to study a divisional Seru working two types of products. In the general case, Property 5 becomes as in the following.

Property 5. *(generalised). Within divisional Seru working m types of products, the unitary, i.e., atomic, entity to focus on is each ordered sequence of m consecutive products. Alternatively, the behaviour of a divisional Seru working m types of products is equivalent to the behaviour of a single product type divisional Seru working a dummy product, with* $\widetilde{Q} = \frac{\sum_{u=1}^{m}\widetilde{Q}^u}{m}$.

According to Property 5 and following the same approach and proofs proposed for the base and extended cases, Property 3 and Property 4 are generalised is in the following:

Property 3. *(generalised). Within divisional Serus, after a major set of products are processed, i.e., when* $j\rightarrow\infty$*, each worker i works all the ordered sequence of m consecutive products for the same amount of time.*

Property 4. *(generalised). Within divisional Serus, after the initial transitory, each worker i works all the ordered sequence of m consecutive products for* $\tau_i^{\infty} = \sum_{u=1}^{m}\widetilde{Q}^u / \sum_{i=1}^{n} v_i$.

Finally, given τ_i^{∞}, the target (stationary) productivity of the Seru, in products per hour, can be calculated as follows.

$$\eta = m \times 60/(n \times \tau_i^{\infty}) = m \times \frac{60}{n \times \sum_{u=1}^{m}\widetilde{Q}^u} \times \sum_{i=1}^{n} v_i \tag{30}$$

where the factor m states that every τ_i^{∞} minutes m products, one per each product type, are finished.

The obtained model and closed-form expressions getting the target productivity of a divisional Seru are exemplified through a case study in Section 7.

7. Case study and discussion

A case study is built to exemplify and test the proposed model. $n = 6$ workers of increasing experience and speed are considered and a mounting task is assumed, e.g., equipment production by picking components, assembly tasks and final inspection and packing. The case of toys for kids is taken as reference despite, in this Section, the analysis is left general also due to non-disclosure constraints with the manufacturer. Table 2 presents the field measured average workers' speeds.

Four types of products are analysed as candidates to be produced in the Seru environment. For each of them, the joint analysis of its bill of materials, its technological process and its production cycle allows quantifying the workload, expressed in tasks to complete. The values of $\widetilde{Q}^u$ are in Table 3.

Starting from these input data, prediction of the divisional Seru performances varying the number of products and workers is done, further simulating the trend of the productivity level toward the expected target.

7.1. Single product Seru (base case)

This scenario focuses on a divisional Seru dedicated to the first product type, only, i.e., $m = 1$ and $\widetilde{Q} = \widetilde{Q}^1 = 100$ tasks. Five cases are analysed, varying the number of workers, i.e., $n = 2, \cdots, 6$. For the cases with a number of workers lower than six, the fastest n workers are kept. Table 4 shows the target (stationary) working time of each worker, t_i^{∞}, and the Seru productivity, η.

As expected, a low number of workers asks each of them to work each product for a higher amount of time. Concerning the Seru productivity, having low high-skilled (fast) workers allow increasing the target productivity even if the performance trend is less than linear (+30% passing from 6 to 2 workers). This result could mislead the Reader suggesting cutting the slowest workers. The approach to follow is different, according to the cross-training role of divisional Serus (Ying and Tsai, 2017; Wu et al., 2018; Liu et al., 2021a). The starting point is to have a large divisional Seru with a lot of workers and, dynamically, to remove the fastest one to be assigned to a Yatai (Yin et al., 2008). The remaining workers operating in the divisional Seru, thanks to their learning process, will increase their speed and, all together, they will be able to reach much higher levels of productivity considering the global production system, i.e., divisional Seru + Yatais. An in-depth study of these phenomena, considering the workers' learning curves, is among the expected next steps opened by the present paper.

The graph in Fig. 2 shows the dynamic trend over the finished number of products, of the percentage gap between the current productivity and the target values, varying the number of workers.

The graph highlights wide initial fluctuations due to the initial transitory followed by a smooth trend, up to the stationary target condition proofed before. Fluctuations are wider if the number of workers is higher, despite, in this case, the speed of convergence to target is faster (Bartholdi and Eisenstein, 1996; Bortolini et al., 2021). In all the five cases at the end of product $j = 13$, (~10 min from the work start, i.e., the begin of the work shift with an empty Seru), the percentage gap to the target productivity value is below 5%. Starting from the 14th product, the percentage gap is below 5%, stating a quite fast convergence to a stable divisional Seru behaviour, namely the initial transitory affects the divisional Seru productivity quite low.

7.2. Two product Seru (extension)

To exemplify the model use with two types of products, a divisional Seru made of the previous $n = 6$ workers and product types $u = 1$ and $u = 2$ is considered. Among the possible products to be coupled with the first one, the choice falls on the product type with the largest workload, to stress the production system. The application of the model extension returns the following targets: $\tau_i^{\infty} = \frac{(100+115)}{183.54} = 1.1714$ minutes of work per worker and couple of products, while $\eta = 2 \times \frac{60}{6\times1.1714} = 17.07$ products/hour as the target (stationary) productivity. The target productivity is almost comparable to the single product case but a little lower due to the highest workload of the equivalent *dummy* product (see Property 5), $\widetilde{Q} = \frac{100+115}{2} = 107.5$ tasks.

The graph in Fig. 3 shows the dynamic trend, for the first 30 products, of the working time of each worker, t_i^j. Continuous lines are for odd

Table 2
Case study, average workers' speeds, v_i.

	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$
v_i[tasks/minute]	41.34	38.15	34.49	30.12	24.48	14.94

Table 3
Case study, workload of the product types, $\widetilde{Q}^u$.

	$u = 1$	$u = 2$	$u = 3$	$u = 4$
$\widetilde{Q}^u$[tasks]	100	115	90	95

Table 4
Case study, base case: target (stationary) performances, t_i^{∞} and η.

	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
t_i^{∞}[minutes]	1.2579	0.8773	0.6939	0.5931	0.5448
η[products/hour]	23.85	22.80	21.62	20.23	18.35

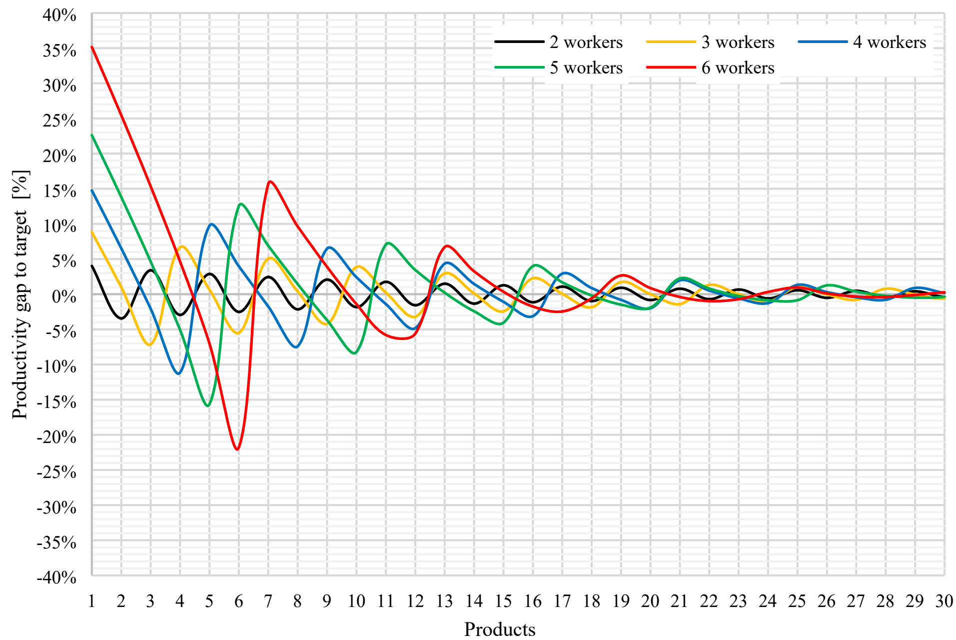

Fig. 2. Case study, dynamic trend of the productivity gap over the finished products.

workers, i.e., $i = 1,3,5$, while dashed lines are for even workers, i.e., $i = 2,4,6$.

The graph highlights for all workers the effect of the initial transitory and the stationary condition reached after the first 25 products, approximately. In addition, once the stationary condition is reached, the working time of the three odd workers (and, separately, of the three even workers) per product converges to a same trend. Products with an odd value of j are worked, in prevalence, by the even workers, while products with an even value of j are worked, in prevalence, by the odd workers. As expected, the total working time, sum of the six contributions of the odd and even workers, becomes constant according to the proofed target (stationary) condition. Nevertheless, the analytic study of the dynamic (harmonic) behaviour of the working time per worker is a further relevant next step enabled by this paper and expected by the literature (Jiang et al., 2021).

7.3. *Four product Seru (general case)*

The last step of the case study includes all the four product types, exemplifying the application of the multi-product type model proposed in Section 6 when $m = 4$, according to the total number of products of the proposed case study. The sensitivity analysis on the number of workers proposed for the base case is repeated following the same approach. Results are in Table 5.

Results follow the same trend discussed before, while, in this case, the workload of the equivalent *dummy* product (see Property 5 *(generalised)*) is $\widetilde{Q} = \frac{100+115+90+95}{4} = 100$ tasks, justifying the equal values of the target productivity in the present and the base cases (see Tables 4

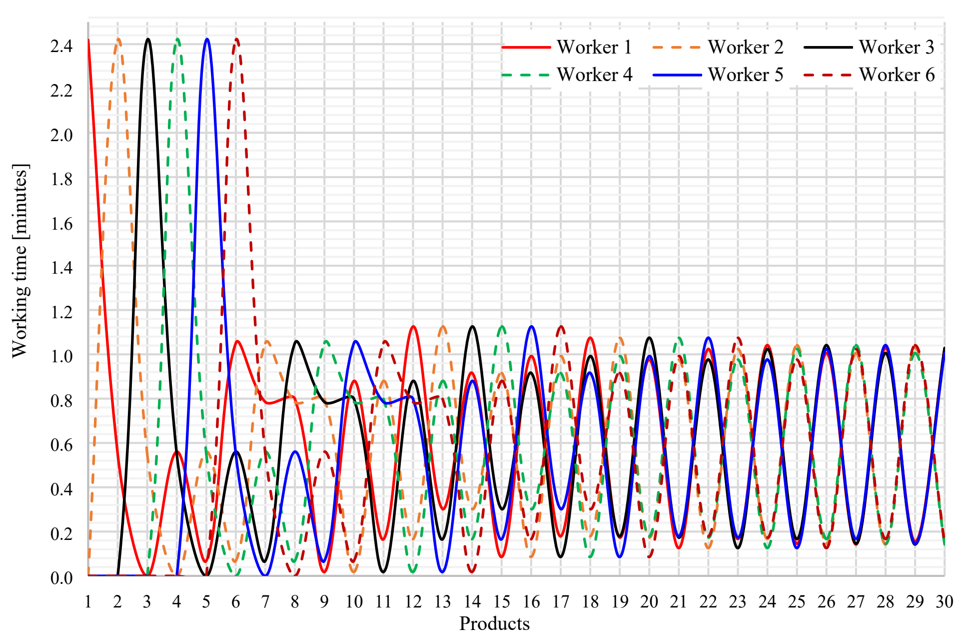

Fig. 3. Case study, dynamic trend of the working time of each worker for the first products.

and 5).

Finally, for a divisional Seru with 6 workers, a sensitivity analysis increasing the workers speed is done to highlight its linear impact on productivity. Starting from the values in Table 2, a 1% step increase of all workers' speed, up to +20%, is simulated calculating the correspondent target productivity level. Results are in Table 6.

As stated before, the direct impact of the workers' performances on divisional Serus encourages, as a next step, to investigate the correlation between the learning curves of different workers, trained using divisional Serus, and the overall production performances, as a key driver to encourage industry in embracing this production environment.

7.4. Managerial implications

The results coming from the application of the model to the proposed case study allow drawing some managerial implications for industrial practitioners, as points of attention when implementing a divisional Seru production environment. In summary:

– Despite the number of products to consider is sometimes out of the direct control because of the market constraints and the demand mix, the dimension of the divisional Seru, in terms of number of workers, is relevant and it should best balance the positive effect on productivity of small Serus to the cross-training role of divisional Serus (Ying and Tsai, 2017; Wu et al., 2018; Liu et al., 2021a);
– The relatively fast convergence to a stable behaviour of the Seru productivity, i.e., the small length of the initial transitory, (see Fig. 2) matches with the need of small production batches and dynamic production set by the modern market trend (Villa and Taurino, 2013; Bortolini et al., 2019). Practitioners can plan frequent product mix changes without a relevant decrease of the productivity levels. This evidence makes Serus an effective solution within Next Generation Manufacturing Systems;
– Fluctuations on the product type workloads and the scheduling of product types inside each production batch are self-balanced by the Seru working behaviour allowing the practitioners to focus their attention to the batch composition and sequencing among different production batches.

8. Conclusions and next steps

The interest in predicting performances of advanced production environments is a pillar of industrial systems' engineering to encourage industry to convert traditional production lines into Next Generation Manufacturing Systems. Within cellular manufacturing (CM) environment, Serus emerged in the Japanese electronic sector as a powerful solution to increase productivity, decrease the labour intensity and enhance the workers' cross-training and learning. Divisional and rotating Serus are the first two steps of the transition toward the *'perfect'* Yatai configuration. Focusing on divisional Serus, this paper contributes to theory finding and proofing the analytic closed-form expressions getting their expected productivity given a generic number of partially cross-trained workers and a) one (base case), b) two (extension) and c) a generic number (general case) of product types. As key finding, the expressions providing the dynamic trend of the working time for each worker on each product are derived and properties stating the target Seru behaviour, after a long series of worked products, are presented and proofed using algebra filling a literature gap and providing to practitioners an easy-use analytic model to develop feasibility studies on the expected productivity coming from the switch to Seru production environment. Results show a stationary behaviour of the working time for all workers, balancing fluctuations of the workers' speed and of the different workloads of the product types. Seru productivity depends on the sum of the workers speed and on the sum of the product type workloads. Additionally, the proposed case study allowed concluding that after few tens of products the target conditions are reached, with a gap lower then 5%. For this case study, taken from the toys for kind industry, in 10 to 30 min the target conditions are obtained, depending on the considered configuration.

Next steps enabled by this paper and based on the limitations coming from the adopted assumptions need to further improve the model studying the dynamic (harmonic) behaviour of the working time per worker for multi-product Serus, including the effect of learning curves on the Seru behaviour and productivity and introducing turbulences such as workers' pauses, tool failures and other disruptive events. In this way, together with the performance of the Seru as a whole, the trend of each worker behaviour can be analysed and predicted driving their assignment in the case of multiple Serus working in parallel. Finally, applications and field experience from the model use in industry are expected and of certain interest.

Table 5
Case study, general case example: target (stationary) performances, τ_i^∞ and η.

	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
τ_i^∞ [minutes]	5.0318	3.5092	2.7757	2.3726	2.1794
η[products/hour]	23.85	22.80	21.62	20.23	18.35

Table 6
Case study, general case example: sensitivity analysis on the workers' speed.

Worker speed increase [%]	τ_i^∞ [minutes]	η [products/hour]
–	2.1794	18.35
+1%	2.1578	18.54
+2%	2.1367	18.72
+3%	2.1159	18.90
+4%	2.0956	19.09
+5%	2.0756	19.27
+6%	2.0560	19.46
+7%	2.0368	19.64
+8%	2.0179	19.82
+9%	1.9994	20.01
+10%	1.9813	20.19
+11%	1.9634	20.37
+12%	1.9459	20.56
+13%	1.9287	20.74
+14%	1.9117	20.92
+15%	1.8951	21.11
+16%	1.8788	21.29
+17%	1.8627	21.47
+18%	1.8469	21.66
+19%	1.8314	21.84
+20%	1.8162	22.02

CRediT authorship contribution statement

Marco Bortolini: Conceptualization, Methodology, Investigation, Data curation, Validation, Writing – original draft, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A

In the $n \times n$ system of linear equations in Equation (14), the unknown A comes from the following ratio:

$$A = \frac{\det(\mathscr{A}|\mathscr{B})}{\det(\mathscr{A})} \tag{A1}$$

where

$$\mathscr{A}\,|\mathscr{B} = \begin{bmatrix} 0 & \rho_{n,b} & \cdots & \rho_{n,n} \\ 0 & \rho_{n,b}^2 & \cdots & \rho_{n,n}^2 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{Q}{v_1} & \rho_{n,b}^n & \cdots & \rho_{n,n}^n \end{bmatrix}$$
$$\mathscr{A} \;= \begin{bmatrix} 1 & \rho_{n,b} & \cdots & \rho_{n,n} \\ 1 & \rho_{n,b}^2 & \cdots & \rho_{n,n}^2 \\ \cdots & \cdots & \cdots & \cdots \\ 1 & \rho_{n,b}^n & \cdots & \rho_{n,n}^n \end{bmatrix} \tag{A2}$$

To get $\det(\mathscr{A}|\mathscr{B})$ the Doolittle decomposition of the matrix $\mathscr{A}|\mathscr{B}$ is of help by applying the Gaussian row reduction methodology. The Doolittle decomposition transform a matrix into the product of a lower matrix and upper matrix, $\mathscr{A}|\mathscr{B} = \mathscr{L} \times \mathscr{U}$, where the triangular lower matrix has 1 along the main diagonal and the Gaussian row reduction coefficients below the diagonal, while the triangular upper matrix is the reduced row echelon form of the initial matrix. Additionally, because of the determinant of a triangular matrix is the product of its elements along the main diagonal, it follows that $\det(\mathscr{L}) = 1$ so that $\det(\mathscr{A}|\mathscr{B}) = \det(\mathscr{L}) \times \det(\mathscr{U}) = \det(\mathscr{U})$. The steps of the Gaussian row reduction methodology are omitted for brevity. It follows that:

$$\det(\mathscr{A}|\mathscr{B}) = (\widetilde{Q}/v_1) \times \prod_{h=b}^{n}\left(\rho_{n,h} \times \prod_{k=b}^{h-1}(\rho_{n,k} - \rho_{n,h})\right) \tag{A3}$$

The matrix $\mathscr{A}$ has a structure that matches to the so-called squared Vandermonde matrix (Hoffman and Kunze, 1971; Aldrovandi, 2001), $\mathscr{V}$, whose terms make a geometric progression. Analytically:

$$\mathscr{V} = \begin{bmatrix} 1 & \rho_{n,b} & \rho_{n,b}^2 & \cdots & \rho_{n,b}^{n-1} \\ . & . & \cdots & . & . \\ 1 & \rho_{n,n} & \rho_{n,n}^2 & \cdots & \rho_{n,n}^{n-1} \end{bmatrix} \tag{A4}$$

and

$$\mathscr{A}^{\mathbf{T}} = \mathscr{V} \times \begin{bmatrix} 1 \\ \rho_{n,b} \\ \cdots \\ \rho_{n,n} \end{bmatrix}^{\mathrm{T}} \tag{A5}$$

It is known that $\det(\mathscr{V}) = \prod_{h=a}^{n}\prod_{k=a}^{h-1}(\rho_{n,h} - \rho_{n,k})$, where $\rho_{n,a} = 1$. It follows that:

$$\det(\mathscr{A}) = \prod_{h=a}^{n}\left(\rho_{n,h} \times \prod_{k=a}^{h-1}(\rho_{n,h} - \rho_{n,k})\right) = \prod_{h=a}^{n}\left((-1)^{h-1} \times \rho_{n,h} \times \prod_{k=a}^{h-1}(\rho_{n,k} - \rho_{n,h})\right) = (-1)^{n-1} \times \prod_{h=a}^{n}\left(\rho_{n,h} \times \prod_{k=a}^{h-1}(\rho_{n,k} - \rho_{n,h})\right) \tag{A6}$$

Finally,

$$A = \frac{\det(\mathscr{A}|\mathscr{B})}{\det(\mathscr{A})} = \frac{(\widetilde{Q}/v_1) \times \prod_{h=b}^{n}\left(\rho_{n,h} \times \prod_{k=b}^{h-1}(\rho_{n,k} - \rho_{n,h})\right)}{(-1)^{n-1} \times \prod_{h=a}^{n}\left(\rho_{n,h} \times \prod_{k=a}^{h-1}(\rho_{n,k} - \rho_{n,h})\right)} = (\widetilde{Q}/v_1) \times \frac{(-1)^{n-1}}{\prod_{h=b}^{n}(\rho_{n,h} - 1)} \tag{A7}$$

To relate the $\rho_{n,h}$ values, that are unknown, to the coefficients of the starting Equation (10), the Vieta's formulas are considered setting the following group of equations (Hoffman and Kunze, 1971):

$$\begin{cases} \rho_{n,b}+\rho_{n,c}+\cdots+\rho_{n,n}=-\dfrac{v_2}{v_1} \\ (\rho_{n,b}\rho_{n,c}+\rho_{n,b}\rho_{n,d}+\cdots+\rho_{n,b}\rho_{n,n})+(\rho_{n,c}\rho_{n,d}+\rho_{n,c}\rho_{n,d}+\cdots+\rho_{n,c}\rho_{n,n})+\cdots+\rho_{n,m}\rho_{n,n}=\dfrac{v_3}{v_1} \\ \cdots \\ \rho_{n,b}\rho_{n,c}\cdots\rho_{n,n}=(-1)^{n-1}\dfrac{v_n}{v_1} \end{cases} \tag{A8}$$

By calculating the terms of the ratio $\frac{(-1)^{n-1}}{\prod_{h=b}^{n}(\rho_{n,h}-1)}$ and replacing time by time the correspondent Vieta's formulas, follows that $\frac{1}{\sum_{i=1}^{n}(v_i/v_1)}$, so that:

$$A=(\widetilde{Q}/v_1)\times\frac{1}{\sum_{i=1}^{n}(v_i/v_1)}=\widetilde{Q}/\sum_{i=1}^{n}v_i \tag{A9}$$

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