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# Chapter 41
# A Graph Signal Processing Technique for Vibration Analysis with Clustered Sensor Networks

**Federica Zonzini, Alberto Girolami, Davide Brunelli, Nicola Testoni, Alessandro Marzani and Luca De Marchi**

**Abstract** The modal analysis of large structures, because of spatial and electrical constraints, generally requires cluster-based networks of sensors. In such solutions, dedicated procedures are required to reconstruct the global mode shapes of vibration starting from the local mode shapes computed on individual groups of sensors. Commonly adopted strategies are based on overlapped schemes, in which at least one sensing position is shared among neighbour clusters. In this paper, a non-overlapping monitoring approach is proposed. It relies on the intrinsic capability of graph signal processing to encode structural connectivity on edge weights and exploits the maximization of the global graph signal smoothness to define the best set of scaling factors between adjacent networks. Experiments on a pinned-pinned steel beam in condition of free vibrations proved that the proposed method is consistent with respect to numerical predictions, showing great potential for distributed monitoring of complex structures.

**Keywords** Graph signal processing · Cluster-based modal analysis · Mode shape assembly

F. Zonzini (✉) · N. Testoni · A. Marzani · L. De Marchi
ARCES—Advanced Research Center of Electronic Systems, 40136 Bologna, Italy
e-mail: federica.zonzini@unibo.it

N. Testoni
e-mail: nicola.testoni@unibo.it

A. Marzani
e-mail: alessandro.marzani@unibo.it

L. De Marchi
e-mail: l.demarchi@unibo.it

A. Girolami
DEI, University of Bologna, 40136 Bologna, Italy
e-mail: alberto.girolami2@unibo.it

D. Brunelli
DII, University of Trento, 38123 Trento, Italy
e-mail: davide.brunelli@unitn.it



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## 41.1 Introduction

Operational Modal Analysis (OMA) is commonly applied to inspect the dynamic behaviour of structures, spanning from civil engineering to industrial applications [1]. The extraction of modal parameters, such as natural frequencies and mode shapes, is complicated in large scale monitoring scenarios, where the huge amount of data combined with the intrinsic structural complexity requires advanced and versatile solutions.

In such a context, clustered sensor networks, thanks to their capability to easy adapt to the geometric characteristics of the inspected structure, have been gradually considered as viable solutions to reduce the computational and energy budget associated to the gathering of sensor data and their transmission to a central processing unit. Nevertheless, this network architectural approach implies the development of dedicated post-processing methods to assemble the locally extracted modal information.

With reference to mode shape reconstruction, after modal coordinates have been obtained for each group of sensors, an optimal set of scaling factors between adjacent clusters must be computed. State-of-the-art solutions are based on overlapped sensor configurations, therefore at least one sampling location is shared among neighboring clusters. In [2], three covariance-driven methods were compared for modal shapes merging, showing similar satisfying performances in reconstructing vertical and lateral bending modes of bridges. Similarly, a least-squares minimization algorithm was implemented in [3] to assemble the modal coordinates of a bi-dimensional fan-shaped slab. Alternatively, a joint state space model was proposed in [4] to combine modal information from overlapping network configurations. All the above mentioned approaches suffer from some drawbacks, the most important of them concerning the increase in power consumption and computational efforts inherently related to the presence of superimposed sampling locations.

In this paper, a novel strategy based on non-overlapping clusters of sensors is proposed. Taking advantage of the Graph Signal Processing (GSP) techniques, the connections between the modal parameters extracted by different clusters are dealt with by purposely defining edge weights between adjacent sensors and then by maximizing the global graph signal smoothness. Beyond the obvious reduction in the number of sensors to be employed and the consequent energy saving, such a technique clearly encompasses some other electrical advantages. In detail, while considering large or even harsh environments, sometimes it might be difficult to install overlapped clusters due to physical or communication limitations (i.e. maximum distance between the closest devices, admitted connectivity ranges, geometrical obstacles). In addition, there are also some computational benefits associated to the minimization of data dimension while preserving the accuracy of the measurements. The implemented mode shape assembly algorithm was experimentally tested on a steel beam instrumented by means of clustered and irregularly spaced accelerometers. The results show satisfactory accuracy performances and perfect coherence with respect to the numerical predictions.

## 41.2 Graph-Defined Mode Shape Assembling

The analysis of signals defined on graphs has been gaining increasing attention due to its capability of modeling inherent patterns coded in the acquired data as similarities between adjacent vertices on a graph [5, 6]. Several application fields have recently benefited from this emerging signal representation, including smart cities, traffic networks and environmental processes [7]. Furthermore, a number of mathematical techniques have been developed, including the Graph Fourier Transform (GFT) and the Graph Laplacian (GL) operators, which can be used to transpose classical spectral characterization methods in equivalent tools for the vertex-frequency domain [8].

A graph is a mathematical entity described by a set of vertices connected by edges, whose Algebraic representation is expressed through the Adjacency and Degree matrices [5]. The weighted Adjacency matrix $W$ expresses the vertex connectivity between two generic nodes $n$ and $m$ by means of a correspondent edge weight $w_{nm}$. Conversely, each entry of the Degree matrix $D$ is given by the sum of all the weights incident on a specific vertex. The eigendecomposition of the graph Laplacian operator $L = D - W$ is an extremely useful tool to extract meaningful information from graph signals. In particular, it can be seen as the graph counterpart of the second-order derivative operator. Besides, a Fourier-like transform has been developed for graph spectral characterization, which consists of projecting graph signals on the Laplacian eigenvectors. The eigenvalues of the Laplacian matrix are also inherently related to the global graph signal smoothness of a generic function $f$ sampled on the graph vertices:

$$\lambda = \frac{1}{2} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} w_{nm} (f(n) - f(m))^2 = f^T L f \tag{41.1}$$

which quantifies the cumulative energy of signal changes sensed at different vertices [9].

### *41.2.1 Graph-Based Mode Shape Assembling*

In vibration-based structural monitoring, spatially varying modal coordinates can be mapped as values on the vertices of an undirected arbitrary graph. Once a specific sampling grid has been deployed, edge weights can be defined as the inverse of the sampling points' spatial distance. In this context, no specific requirement about sensor density is additionally required apart from having the minimum cluster-size compliant to the number of modes to be investigated [10]. Given the quasi-sinusoidal dynamic regime typical of civil structures, which corresponds to smooth modal curves independently from the nature of the exciting force, the developed GSP technique iteratively tries to maximize the global graph signal smoothness introduced in Eq. (41.1) by correspondingly adapting a scaling factor $\alpha_c$ for each cluster, where subscript $c = 1, \ldots, N_c$ identifies one of the $N_c$ subsets of sensors. The implemented

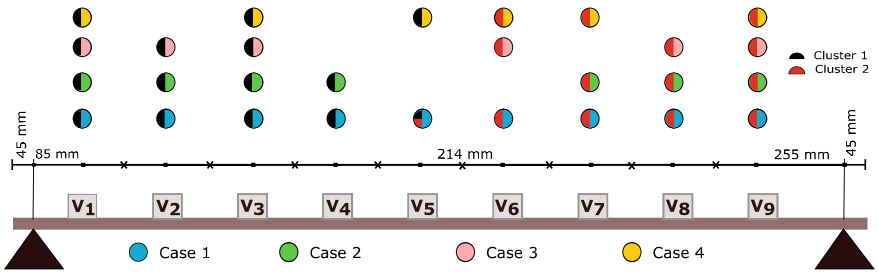


**Fig. 41.1** Experimental setup with pinpointed sampling positions

algorithm comprises the following steps. During the starting phase, (*i*) a vector consisting of unitary scaling values is considered as the initial guess. Then, after the currently assembled mode shapes have been normalized (ii), the fitness function $\lambda$ is computed (iii) according to (41.1). In particular, some of the graph data processing procedures from GSPBOX [11] were exploited. Finally, a prediction phase (iv) updates the scaling coefficients. More specifically, the values $\alpha_{k+1}$ predicted at iteration $k$ are computed as $\alpha_{k+1} = \alpha_k - r_k \nabla f(\alpha_k)$, in which $r_k$ and $\nabla f$ respectively represent the updating ratio and the gradient operator. Steps (ii–iv) are repeated until a convergence criterion is met, which is intended in the current approach as a smoothness variation between subsequent iterations inferior to a predefined threshold $\epsilon$.

## 41.3 Experimental Validation

The effectiveness of the implemented graph-based mode shape assembly algorithm is tested on an instrumented steel beam, which was left to vibrate (free-vibration) after an initial stimulus. An extensive description of the geometric and physical properties of the structure, together with a detailed illustration of the employed electronic equipment, can be found in [12]. In particular, the circuitry consisted of low-cost tri-axial MEMS accelerometers capable of transmitting real-time data in a strictly synchronized manner by means of a CAN bus, each of them embedding an STMicroelectronics STM32L433 microcontroller unit.

Clusters of sensors were modelled on an undirected path graph of non homogeneous dimensions, the vertices of which holding modal coordinates extracted with conventional mode shape-extraction methods. As already discussed in [13], both classical Time or Frequency Domain Decomposition (TDD/FDD) methods and the unsupervised Second Order Blind Identification (SOBI) approach can be applied for this purpose. Considering that the predicted first three natural frequencies of vibration of the beam were below 50 Hz, a sampling frequency $f_s$ 100 Hz was used; accordingly, clusters comprising at least three sensor nodes were used. Nine sampling positions were uniformly distributed along the beam length at a spatial step of 214 mm.

Four different configurations of two clustered networks were considered with various inter and intra-cluster distances between sensors. The sensor-to-cluster assignment adopted in each considered case is depicted in Fig. 41.1, from which it can be inferred that all the configurations except one (case 1) are non-overlapping. A maximum variation $\epsilon = 10^{-4}$ in successive evaluations of the fitness function was empirically estimated to be sufficient to achieve the best trade-off between the resulting modal accuracy and the convergence velocity.

To numerically quantify the level of superposition between theoretically predicted and graph-assembled modal curves, the Modal Assurance Criterion (MAC) [13] was computed, providing the modal correspondence indexes summarized in Table 41.1. Such quantities may range from 0 to 100, the latter value meaning a perfect recovery. An example of graph-combined mode shapes ($\phi_i$), $i = 1, 2, 3$, is drawn in Fig. 41.2, where raw modal coordinates are extracted through the SOBI technique starting from sensing positions of case 4. Independently from the spatial distribution of the sensors and the adopted clustering scheme, making use of GSP tools a proper graph topology can be derived. As it can be observed, results yield to an almost perfect fitting between graph-assembled curves and numerical expectations, proved by a MAC value always above 95% (see Table 41.1). Additionally, it can be concluded that the sensor distribution and their relative distances seem not to affect the overall quality of the modal shape estimated for each specific mode under investigation. It is also worth noting that the performance of the proposed algorithm attains high scores with supervised (FDD and TDD) and unsupervised (SOBI) modal inspection methods. Furthermore, the number of iterations necessary to meet the convergence condition was always less than 15, thus limiting the required computational effort.

**Table 41.1** MAC percentages between experimental and graph assembled mode shapes from overlapped and disjoint cluster network

| | Case 1 | | | Case 2 | | | Case 3 | | | Case 4 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi_1$ | $\phi_2$ | $\phi_3$ |
| FDD | 95.87 | 99.62 | 99.22 | 99.61 | 99.87 | 99.36 | 97.03 | 99.73 | 99.74 | 99.70 | 99.07 | 98.93 |
| TDD | 99.87 | 99.41 | 99.62 | 99.81 | 99.77 | 99.70 | 96.73 | 99.87 | 99.46 | 99.85 | 98.82 | 99.66 |
| SOBI | 95.29 | 99.77 | 99.34 | 99.79 | 99.94 | 99.43 | 97.47 | 99.86 | 99.55 | 99.83 | 99.24 | 99.09 |

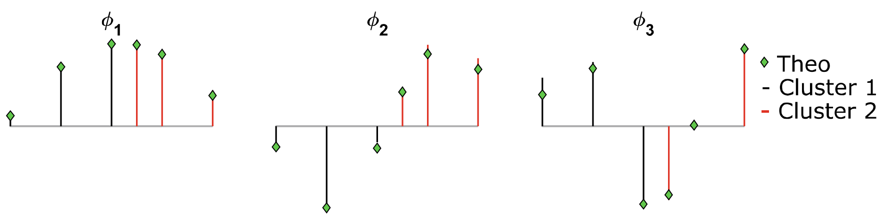


**Fig. 41.2** Graph-assembled mode shapes at sensing locations chosen for case 4 exploiting SOBI modal reconstruction technique

## 41.4 Conclusions

This paper proposes a new approach for mode shape assembly of vibrating structures based on clustered sensor networks. Exploiting the advantages of graph signal domain to account for the underlying connectivity, the described method appears to be a powerful strategy to overcome the current limitation of state-of-the-art overlapped solutions. Different sampling grids were tested on the array of 9 sensors installed on a vibrating steel beam, assessing the robustness of the developed processing scheme in different spatial configurations. The consistency of the obtained results corroborates the possibility to deploy accelerometer sensor networks in large and complex civil structures. Future developments will address the validation of the proposed data fusion method in setups including damaged scenarios, to verify that the proposed approach does not affect the damage detection performance. Concurrently, denser sensor networks will be considered, allowing for a computational evaluation (e.g. convergence time, required processing resources) of the method under more complicated situations.

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