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PRICING CLIMATE-RELATED RISKS IN THE BOND MARKET

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Abstract

We develop a model for defaultable bonds incorporating both uncertainty about corporate earnings and uncertainty due to climate-related risks, which determine downward jumps in the firm value. In particular, we study how bond pricing is affected by transition risks, such as those coming from an abrupt change of climate policies. We show how the issuer's credit quality changes as a result of its engagement in projects funded by green bonds and study the impact of green bonds on investors' portfolio allocation. The way 'green' bonds may contribute to financial stability is also discussed.

Key words:

Sustainable finance; Defaultable bonds; Climate-related risks; Compound Poisson process.

1. Introduction

Climate change is recognized as a significant and complex challenge currently facing our economies and is expected to have effects on global financial assets (Dietz et al, 2016), financial assets' valuation (Battiston et al, 2017) and the stability of financial systems (Batten et al, 2016, Battiston et al, 2017). Recently, the Governor of the Bank of England, Mr Mark Carney, acknowledged that climate change represents a big 'tragedy of the horizon' (Carney, 2015) that could harm political and financial stability, as well as imposing heavy costs on future generations. The potential exposure of the financial system to climate change risks has been discussed within

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various international financial institutions[2] to develop measures limiting systemic losses across the global market.

Two broad climate-related financial risks have been identified, that is, physical risks and transition risks (Bank of England, Prudential Regulation Authority, 2018). Physical risks can be acute, therefore related to specific weather events, or incremental and chronic, related to longer term and more permanent effects, such as changes in precipitation patterns or oceans level rise. Some physical risks can be insured, in which case they affect insurance firms through higher claims. Alternatively, the burden of non-insured losses will fall on households and companies, impairing asset values and reducing the value of assets held by financial institutions. In particular, the fat-tailed probability distributions of several climate variables are such that the possibility of extreme values cannot be ruled out (Weitzman, 2011). This could impact on the solvency of financial institutions which might lack sufficient capital to absorb climate-related losses.

Transition risks have mainly to do with re-pricing of carbon-intensive, or brown assets, as a result of a sudden transition to low-carbon, or green economies. Transition risks also include risks from policy changes, such as the abrupt introduction of stringent carbon-pricing policies and other mitigation policies that make fossil-fuel intensive technologies unprofitable and may make assets 'stranded' (e.g., van der Ploeg and Rezai, 2020). Eventually, as Carney (2015) states, "too rapid a movement towards a low-carbon economy could materially damage financial stability" and thus destabilize markets, while "financial policy-makers have a big interest in making the financial sector resilient". (See also Monnin, 2018).

Our paper deals with climate-related risks that produce a sudden substantial reduction in asset values and, in particular, we study the valuation adjustment of financial securities as a result of transition risks, such as those coming from an abrupt change of climate policies. We focus on the valuation of bonds, comparing the effects on 'brown' bonds, that is, conventional bonds from issuers that derive a large part of their revenues from carbon intensive and pollutant projects, and on 'green' bonds, that is, bonds whose proceeds are directed to projects with environmental benefits, primarily climate change mitigation and adaptation. While climate-related physical risks have often no differential effect on green projects compared to brown projects, transition risks are likely to determine different impacts. Ambitious climate change policies entail drastic changes in the production structure of the economy, with policy-driven increases in the operational costs of carbon-intensive technologies and relatively lower and rapidly decreasing costs of greener projects. Other categories of transition risk include "technology breakthroughs, shifts in investors or public sentiments and disruptive business model innovations" (NGFS, 2020). These will affect the

[2] See e.g., NGFS (2020).

operations of businesses unevenly and may create additional financial risks for lenders and investors[3].

Recently an increasing attention has been paid by financial regulators on the disclosure of climate-related risks to progress assessment of exposures to climate change. For example, the Network for Greening the Financial System (NGFS) launched by eight Central Banks and supervisors last December 2017 brings together 46 members and 9 observers whose priority is the identification and disclosure of existing exposures in the financial sector (NGFS, 2019). A huge amount of work has already been done by the Task Force on Climate-related Financial Disclosures (TCFD), which has proposed a methodology and recommendations for disclosure of climate-related financial risks in investors' portfolios. The TCFD recommendations urge firms to use scenario analysis to disclose the 'actual and potential impacts' of climate-related risk and opportunities on their business as well as how they identify, assess and manage climate risks. In March 2018 also the European Commission announced a comprehensive Action Plan on sustainable finance with the task of developing metrics for climate-related risk disclosure and in June 2019 it released a proposal for an EU green bond standard. Despite this increasing interest by capital markets regulators and organizations, the academic research on the way climate-related risks are reflected in asset pricing is still in its infancy. It is not fully understood whether risks associated to holding assets that could lose value because of climate change policies are correctly priced in financial markets. In consequence, mispricing could hamper investments to flow to low-carbon projects, while investors could even increase their exposures to carbon-intensive technologies, with negative implications on sustainability and financial stability (Battiston and Monasterolo 2019).

Karydas and Xepapadeas (2019) study how climate-related physical and transition risks are reflected in asset prices. They extend Wachter (2013) and Tsai and Wachter (2015) by adding Poisson shocks due to environmental disasters with stochastically time-varying intensity and compute both the equity premium due to climate change and the change in the participation of carbon intensive assets in market portfolios, as a result of climate change risks.

Dafermos et al (2018) study an ecological macroeconomic model addressing the issue of physical risk of climate change on financial stability. They examine the transmission mechanisms of the increase in temperature and catastrophic effects caused by climate change employing a complex stock-flow setting through the balance sheets of households, firms, banks, governments and central banks. Some papers discuss the potential use of monetary policies to address climate-related issues (Murphey and Hines, 2010; Campiglio et al, 2018, Monasterolo and Raberto, 2018, Barkawi and Monnin, 2015). Dafermos et al (2018) examine the financial stability implications of a specific form

[3] We refer to NGFS (2020), Table 1, for a schematic illustration of possible transmission channels of transition risks.

of a green quantitative easing (QE) programme whereby central banks at the global level commit to hold a certain proportion of green bonds over the next decades.

There are very few theoretical papers studying the effects of climate-related risks on the bond market. And yet a deeper understanding of how the price of bonds are affected by climate change policies is necessary to be able to predict the impact of such measures on financial markets and financial stability and the way they interact. Also the consequences of the suggested green QE programmes cannot be properly predicted without a clear understanding of the effects of climate-related risks on bond pricing and yields. Battiston and Monasterolo (2019) studied climate transition risk in sovereign bonds portfolios by employing the CLIMAFIN framework (Battiston, Mandel and Monasterolo, 2019) that combines forward-looking climate transition scenarios as from climate economic models and climate financial risk metrics as from Battiston et al (2017).

A theoretical explanation of the relationship between climate-related risks and bond pricing is still an open question. An exception is Agliardi and Agliardi (2019), where a structural model is developed to explain the formation of green bond prices. They adopt a traditional one-factor model and assume a specific penalty tailored to the firm income in case of 'brown' assets[4], while our paper applies to very general situations. Their model does not specify climate-related risks as abrupt jumps that have an impact on corporate earnings and may determine sudden reassessment on financial assets and severe reduction in the firm market value, which instead is examined here. While the main focus of Agliardi and Agliardi (2019) is the "greenium", as markets expanded and more data and applied research on the green bond market became available (see, among others, Alessi et al, 2019, Baker et al, 2018, Karpf and Mandel, 2018), further issues and additional sources of risk came to the fore, thus demanding a more comprehensive model, which can capture multiple risks, including jumps related to transition risks. This is the objective of our paper, which also addresses green rating and allows us to derive some implications on the stabilization effects of green bonds. We also discuss the impact of some policy measures, such as taxation aimed at expanding the green bond market.

Our model, albeit stylized, is the first introducing both uncertainty about corporate earnings and uncertainty due to climate policy risks and their impact on bond pricing. We specify climate policy risks through a compound Poisson process, where changes in the policy materialize in the form of downward jumps in the firm value[5]. Section 2 provides an overview of the economic literature branches on which this paper is built on and highlights the main differences. Section 3 lays down

[4] An example is provided by the car industry, where the EU regulation sets fines to automakers that do not conform to a scheduled standard aimed at reducing vehicle CO_2 emissions.

[5] Although climate transition risk is also subject to ambiguity, our model does not handle this form of "deep uncertainty".

the main setting and derives an explicit expression for bond and equity values (Proposition 1). As bond prices and yields crucially depend on the jump size parameter of the compound Poisson process - and this can be related to the effect of mitigation policies, because a strong and immediate action to mitigate climate change would increase transition risks - our model provides a simple tool to assess bonds' prices in terms of the relative resilience to transition risks. Furthermore, our framework allows us to classify bonds according to their 'shade' of greenness. Section 4 discusses the impact on portfolio allocation and shows how investors' decisions influence the green bond market. Finally, Section 5 suggests some policy measures to expand the green bond market, in order to facilitate the transition to a low-carbon economy and, at the same time, promote financial tools that carry some ancillary benefits to the issuers and contribute to financial stability.

2. Related literature

Our paper is related to three strands of literature. First, our methodology is related to bond pricing models with jump risks. In contrast to the Merton-Black-Cox-Longstaff-Schwartz approach, where the evolution of firm value follows a simple diffusion process, Zhou (2001) develops a simple yet flexible structural approach to valuing risky debt by modeling the evolution of firm value as a jump-diffusion process. Jump processes in pricing risky bonds have been proposed as a possible solution to credit default risk puzzles. In particular, the existence of a jump-premium over the premium associated with diffusion risk helps explaining why classical structural models generate lower bond yields than the observed ones, as suggested by Driessen (2005) where jumps are empirically confirmed to be a necessary addition to diffusion-based term structure models. Under a jump-diffusion process, a default can happen expectedly because of slow but steady declines in firm value, but it can also occur unexpectedly because of a sudden drop in firm value.

Wong and Hodges (2002) extend Zhou's model and show several implications. Further developments include jump-diffusion pricing models for structured bonds, for example, convertible bonds (Gapeev and Kühn, 2005). A unique dimension of our model is that we incorporate also jump risks due to climate policy changes: our jump-diffusion model extends the previous framework for green bonds by adding Poisson shocks due to changes in climate policies and combine some features of existing models to the issue of climate-related financial risks. Moreover, we come up with an explicit solution for bond prices.

A second strand of literature we are referring to is portfolio choice. We apply a dynamic model, as pioneered by Merton (1971), but extend the standard analyses by adding jumps and adopting a Stochastic Differential Utility framework à la Duffie and Epstein (1992). The Stochastic Differential Utility framework is generally considered to be the appropriate tool to address portfolio consumption problems in models with environmental issues and overcomes various shortcomings of simpler utility functions (see Epstein and Zin, 1989, Duffie and Epstein, 1992). In contrast to the power-utility formulation that is often used in the integrated assessment models of climate change, recursive preferences of Kreps-Porteus or Epstein-Zin type and their continuous-time analogue (Epstein and Zin, 1989) allow for a separation between willingness to substitute consumption over time and across different states of nature, thus disentangling risk aversion and elasticity of intertemporal substitution, and provide a framework to resolve asset pricing puzzles, e.g., the equity premium puzzle, the risk-free rate puzzle, the excess volatility puzzle, and the credit spread puzzle (Bansal and Yaron, 2004). Therefore, they have become a workhorse specification in the recent literature both in macro-finance and in climate change, where risks persist in the long-run (Bansal, Kiku and Ochoa, 2019). Our paper is the first employing these tools to study the problem of optimal portfolio allocation between green and conventional bonds.

Finally, we relate to research on the impact of abrupt macroeconomic events on financial markets. We elaborate on this literature to study the impact of stringent climate policies. Earlier work studying rare disasters/catastrophic jumps include Barro (2009), Gourio (2012) and Bansal, Kiku and Yaron (2010) in the context of climate change. Climate change induced disasters are described in terms of large and abrupt destructive changes in the environment, whose occurrence has a small probability but with possibly large negative effects on the economy. Extensions to the early disaster/jump risk models are the introduction of time-varying disaster probabilities and multi-period (i.e. persistent) disasters (Wachter, 2013; Tsai and Wachter, 2015). Different from this literature, focusing on the effects on growth and consumption and where temperature is a source of economic risk, we focus on the effects of drastic climate policy changes on the financing decisions of firms. The above-mentioned contributions do not study how climate-related risks are reflected in asset prices and, specifically, they do not deal with bond pricing which is the focus of our work.

There is no unanimous measure for climate policy stringency, especially when different jurisdictions and enforcement regimes are considered, as is the case with climate change mitigation policy. It is acknowledged that more stringent climate policies increase compliance costs and may slow down productivity growth. For example, Greenstone (2002) studies the economic costs of the Clean Air Act Amendments and find that total factor productivity declined by 4.8% for polluting plants in strictly regulated countries, although the impact is short-termed. Rubashkina et al. (2015) use a different proxy for carbon emission regulation and show that it negatively affects the productivity of European manufacturers, even though the effect dissipates in two years. Fofrich et al (2020) state that as a consequence of ambitious climate mitigation policies "the premature retirement of power generating infrastructure could result in the loss of trillions of dollars of capital investment and future returns" and, in particular, show that coal-fired power plants in scenarios consistent with international climate targets will retire one to three decades earlier than historically has been the case. In contrast, various studies confirm that strict climate policies trigger the development of new technologies and encourage green investments (see, for example, Ambec et al., 2013).

In what follows, we represent abrupt stringent climate policies through a doubly stochastic Poisson process. As we explain in the next section, the arrival rate of the Poisson shock reflects the intensity of the transition risk due to climate change policy and the impact of such policy on firm value is modelled through a stochastic process measuring the amplitude of shocks to the firm earnings.

3. The model

Let us consider a firm with assets-in-place that generate uncertain earnings before interests and taxes (EBIT) described by a stochastic process of the form:

$$dV_t = \mu V_t dt + \sigma V_t dW_t \tag{1}$$

where W_t is a standard Wiener process with respect to an assigned filtration $\{I_t\}_{t\geq 0}$. Let r be the market risk-free interest rate and let the asset growth rate, μ, satisfy $\mu < r$, as usually in the financial literature on corporate defaultable debt, where this assumption is adopted for convergence reasons. The production process generates environmental damage, because the employed technology is 'brown'. The firm is supposed to issue conventional (or 'brown') bonds to finance its activity. Alternatively, the firm may decide to go 'green' and to finance this low-carbon policy the firm may decide to issue a 'green' bond.

There is no generally accepted 'green' bond standard and the existing schemes build on different taxonomies classifying environmentally sustainable economic activities. In the absence of uniform criteria, for the purpose of this paper, we stick to ICMA's definition, where green bonds are defined as any type of bonds where 'the proceeds will be exclusively used to finance or re-finance, in part or in full, new and /or existing eligible green projects' (ICMA , 2017, p 2f), that is, environmentally or climate-friendly projects, such as renewable energy, green buildings, clean transportation, sustainable waste management, sustainable land use, biodiversity and clean water. In a way, green bonds are the same as any conventional bond, but they are labelled 'green' because the issuer pledges to use their proceeds for environmental-friendly or climate-focused projects in accordance with sustainability standards (CBI, 2018). With green bonds the issuer gets the capital to finance green projects, while the investors receive fixed income in the form of interest.

In what follows, bonds are supposed to be perpetual bonds, as is becoming more frequent in climate bond markets[6] (see CBI, 2018). Moreover, a significant portion of the green bond universe consists of long-dated bonds with tenor bracket over 20 years, mainly in water, transport and energy sectors, and, therefore, perpetual bonds provide a pretty good approximation for these types of bonds. Bonds pay a continuous coupon, unless the issuer goes bankrupt.

The adopted mitigation policy will result in a smaller environmental damage, even though the firm will incur additional expenditures for reporting, monitoring and accounting which are needed to guarantee accomplishment of a Green Standard[7]. Default on debt obligation is possible and in case V falls below a level, V*, then the firm goes bankrupt and the original debt holders take over and obtain the firm's unlevered assets net of proportional bankruptcy costs α ($0 \leq \alpha \leq 1$). Let τ denote the corporate tax rate that the firm faces on its income after servicing interest payments on its debt.

The main assumption of our model is the introduction of climate-related risks. They are modelled as a result of mitigation policies, which reduce corporate earnings for carbon-intensive projects because of downward jumps, which occur according to a compound Poisson jump process. We may expect that such earnings reduction will hit the business lines according to the different degree of climate alignment of issuers and bonds. As green bond issuers are often issuers of conventional bonds as well, it may be difficult to disentangle the effects of the green policy on different business lines. However, there are cases where green bonds are structured as project bonds, the originated proceeds are deposited in segregated accounts and are allocated only to the eligible projects, for example, to specific car models in the automotive industry, as is the case with Toyota's green bonds[8]. The Green Bond Principles recommend that the net proceeds of green bonds are

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[6] The first green perpetual was issued in 2016 by Xinjiang Goldwind, a Chinese wind turbine manufacturer. In 2018 six Chinese corporations have issued perpetual bonds for investments in water, wastewater management and renewably energy. Other energy companies include Iberdrola (Spain), Engie (France) and Tenne T Holdings (The Netherland) – CBI (2018).

[7] Some voluntary initiatives, such as the Green Bond Principles and the Climate Bond Standards, have been made up to underpin the definitions of which projects and assets are consistent with an environmentally friendly economy and are therefore eligible for inclusion in a Certification Scheme providing a framework for assurance of conformance and a screening tool for labeling that avoids subjective or expensive judgments on the green attributes of investments. The development of process guidelines (e.g. Green Bond Principles) promoted transparency and integrity in the green bond market. An EU green bond label was planned to be developed by Q3 2019 (CBI, 2018).

[8] TMCC green bonds are designed to support the sales of low-carbon vehicles to contribute to Toyota's carbon strategy and to mitigate the environmental risks stemming from disposal of batteries of electric or hybrid vehicles.

credited to a sub-account, moved to a sub-portfolio or otherwise tracked by the issuer. In China, the PBOC rule requires that the issuer shall open a special account or establish a special ledger to manage the transfer and payback of green bond proceeds.

As from the recent CBI Report on climate-aligned issuers (CBI, 2018), bonds from issuers that derive more than 95% of revenues from 'green' business lines are defined fully-aligned bonds; bonds from issuers that derive 75-95% of revenues from 'green' business lines are defined strongly-aligned bonds, and so on. Therefore, one can differentiate among various degrees of 'greenness' in the green bond universe.

In our model the different shades of 'greenness' are obtained as a result of the issuer's commitment to the mitigation policy. By explicitly modelling the climate policy shock as we do here, we can study the effects of climate-related shocks on bond pricing, credit spreads, default, equity and firm values, and relate them to the 'greenness' level of the financial security.

The climate policy affects the value of the firm by reducing it because of downward jumps. The dynamics of the value of the firm is described by a jump-diffusion process as follows:

$$\frac{dV_t}{Vt^-} = \mu dt + \sigma dW_t + d\sum_{i=1}^{N_t} (S_i - 1) \qquad (2)$$

where N_t is a Poisson process with intensity λ and S_i denotes a sequence of iid non negative random variables. It implies that between two jumps the process is just a geometric Brownian motion, while at the jump occurring at τ_N the firm value becomes $S_{\tau_N} V_{\tau_{N-}}$ that is:

$$V_t = Vo \exp[(\mu - \frac{\sigma^2}{2})t + \sigma W_t + \sum_{\tau_i \le t} Y_i], \text{ where } Y_i = \ln(S_i).$$

Note that the Poisson process allows us to model unexpected losses which come as a surprise, while losses related to business operations (modelled through the Wiener process) are predictable to the extent that they can be announced by observing the realizations of the firm value.

More precisely, if τ denotes a default time for the reference process, then τ is predictable whenever there exists an increasing sequence of stopping times, τ_j, with respect to the given filtration, $\{I_t\}_{t\ge 0}$, such that $\tau_j < \tau$ and τ_j converges to τ a.s. On the contrary, τ is totally inaccessible if for every predictable stopping time T the probability that $\tau = T < \infty$ is null, that is, τ occurs as a surprise for the observer which is endowed with market information. Totally inaccessible stopping times are suitable to be used when one is interested in modelling situations characterized by unexpected losses in the value of defaultable assets as those triggered by exogenous risk factors. We refer to

Jeanblanc and Rutkowski (2000) to shed light on the relationship between these mathematical concepts and the behaviour of assets at the ‘jump to default’.

Notice that previous literature within the structural approach (Agliardi and Agliardi, 2019) employs a standard continuous stochastic process with a unique source of risk. Our model allows for more flexibility and includes various sources of risk. In particular, we employ a doubly-stochastic jump process to capture the drastic policy-driven changes hitting the business lines of the firm. This will have implications on the bond valuation expression, which completely differs from Agliardi and Agliardi (2019). In the sequel, we use the term “sources of risk” to refer to the three stochastic processes described above.

The arrival rate of the Poisson shock λ reflects the intensity of the transition risk due to climate change policy, which is ultimately determined by cumulative carbon emissions and thus the global temperature anomaly, although the translation of cumulative emissions into transition risks may be less direct because it also depends on complex policy issues. In our model we adopt a reduced form for λ, which however can be modelled with a simplified continuous time representation of the global temperature anomaly dynamics like in Karydas and Xepapadeas (2019). In their model a precise calibration method is provided to obtain real world values for this parameter. We refer to their work for model details and numerical values which can be used in simulations.

In what follows, let us make the following:

Assumption A.1. The log jump sizes Y_i follow an exponential distribution with density: $f(y) = \eta e^{\eta y}$ with $\eta > 0$ (η is the decay parameter of the jump distribution) and y < 0 (i.e., jumps are downwards and reduce the value of the firm).

It is reasonable to think that firms investing in green projects are less exposed to transition risks. We suppose that the firm may decide to finance green projects with the issuance of green bonds. As green bonds are bound to finance green projects, the issuance of green bonds will have an impact on business operations and thus on the carbon intensity of production.

As a first step, we suppose that on average the jump sizes for a brown bond are larger than for a green bond, meaning that the reduction in firm value is bigger for assets that are not financing projects complying with the policy. Therefore, η(brown) < η(green), because the mean of the downward jumps is $1/\eta$. In the sequel we will show that η can be calibrated to market data and thus this parameter can be endogenized.

Under A1, we can calculate the cumulants for the value of the firm:

C1=E(V) = $\mu - \frac{\lambda}{\eta}$

C2=Var(V) = $\frac{2\lambda}{\eta^2} + \sigma^2$

C3= $-6\lambda \frac{1}{\eta^3}$

C4 = $24\lambda \frac{1}{\eta^4}$

Since $\frac{C4}{C2^2} > 0$, it follows that the value of the firm has a leptokurtic feature. Since the feature of having fat tails becomes more pronounced if the jump size expectation $1/\eta$ or the jump rate λ are larger, then it follows that the value of brown bonds is more leptokurtic than the value of comparable green bonds. Since fat tails are often associated to an increased market instability, it follows that green bonds may have a beneficial stabilizing effect in financial markets.

It is worth mentioning that the jump-diffusion process above admits the simplified Lévy-Kintchine (unique) representation (see, for example, Agliardi and Pascucci, 2011) that is, the characteristic exponent is of the form:

G($\xi) = i\psi\xi - \frac{\sigma^2\xi^2}{2} + \int (e^{i\xi y} - 1)\lambda\eta e^{\mu y} 1_{y<0}\, dy = i\psi\xi - \frac{\sigma^2\xi}{2} + i\lambda\xi(\frac{-1}{\eta + i\xi})$.

where ψ is the modified drift of the value of the firm consistent with a risk neutral probability measure.

Let us start computing the value of a bond for given η. Denote by *b* the continuous coupon paid by the bond. We can show that the current value of the defaultable bond and of equity are as follows:

Proposition 1.

The value of a defaultable bond B(V) has the following expression:

B(V) = (1-α)V*, *if V<V**

$\frac{b}{r}$ + A$V^{-\beta}$ +B$V^{-\gamma}$, if V$\geq V^*$

where A = $(V^*(1-\alpha)$- $\frac{b}{r})\frac{V^{*\beta}\,\gamma(\beta-\eta)}{\eta(\beta-\gamma)}$ and B = $(V^*(1-\alpha)$- $\frac{b}{r})\frac{V^{*\gamma}\,\beta(\eta-\gamma)}{\eta(\beta-\gamma)}$, V* is obtained by equity value maximization, that is, $V^* = \frac{\gamma\beta \frac{b}{r}}{(1+\beta)(1+\gamma)}(r+\frac{\lambda}{\eta+1}-\frac{\sigma^2}{2}-\psi)$, and β and γ are the positive roots of $G(x) = \frac{1}{2}\sigma^2 x^2 - \psi x + \lambda(\frac{\eta}{\eta - x} - 1)$.

The value of equity E(V) is:

$E(V) = sV - (1-\tau)\frac{b}{r} + \Omega V^{-\beta} + \Lambda V^{-\gamma}$, where $s = \frac{(1-\tau)}{r+\frac{\lambda}{\eta+1}-\frac{\sigma^2}{2}-\psi}$,

$\Omega = V^{*\beta}\frac{((1-\tau)\frac{b}{r}\gamma - sV^*(\gamma+1))}{\gamma-\beta}$ and $\Lambda = V^{*\gamma}\frac{(-(1-\tau)\frac{b}{r}\beta + sV^*(\beta+1))}{\gamma-\beta}$

Proof:

Let us employ the following notation: x=ln(V). The infinitesimal generator of the jump diffusion process described above is given by:

$$Lu(x) = \frac{1}{2}\sigma^2 u''(x) + \psi u'(x) + \lambda\int_{-\infty}^{0}[u(x+y)-u(x)]f(y)dy \quad (3)$$

where *u(x)* is a twice continuously differential function and ψ is the modified drift of the value of the firm consistent with a risk neutral probability measure. The bond value satisfies the following PDE:

$$Lu(x) - ru(x) + b = 0 \quad (4)$$

We can make the following guess and write the bond value B(V) as a function of x:

$$u(x) = \begin{cases} (1-\alpha)e^{x^*}, & \text{if } x < x^* \\ \frac{b}{r} + Ae^{-\beta x} + Be^{-\gamma x}, & \text{if } x \geq x^* \end{cases} \quad (5)$$

To show that (5) solves (4) we first need to compute $\int_{-\infty}^{0}[u(x+y)-u(x)]f(y)dy$.

We have $\int_{-\infty}^{0}[u(x+y)]f(y)dy = \int_{-\infty}^{x^*-x}(1-\alpha)e^{x^*}\eta e^{\eta y}dy + \int_{x^*-x}^{0}(\frac{b}{r} + Ae^{-(x+y)\beta} + Be^{-(x+y)\gamma})\eta e^{\eta y}dy =$

$$\frac{b}{r} + (e^{x^*}(1-\alpha) - \frac{b}{r})e^{\eta(x^*-x)} + \frac{\eta}{\eta-\beta}A(e^{-\beta x} - e^{-\beta x^*}e^{\eta(x^*-x)}) + \frac{\eta}{\eta-\gamma}B(e^{-\gamma x} - e^{-x^*\gamma}e^{\eta(x^*-x)})$$

Thus, we need to compute :

$$Lu(x) - ru(x) + b = \frac{1}{2}\sigma^2 u''(x) + \psi u'(x) + \lambda \int_{-\infty}^{0} [u(x+y) - u(x)] f(y) dy - ru(x) + b \quad = \qquad (6)$$

$$\frac{1}{2}\sigma^2 (\beta^2 A e^{-\beta x} + \gamma^2 B e^{-\gamma x}) - \psi (\beta A e^{-\beta x} + \gamma B e^{-\gamma x}) - (r+\lambda)(A e^{-\beta x} + B e^{-\gamma x}) +$$

$$+\lambda[\frac{b}{r} + e^{x^*}(1-\alpha) e^{\eta(x^*-x)} + \frac{\eta}{\eta-\beta} A(e^{-\beta x} - e^{-\beta x^*} e^{\eta(x^*-x)}) + \frac{\eta}{\eta-\gamma} B(e^{-\gamma x} - e^{-x^*\gamma} e^{\eta(x^*-x)} - \frac{b}{r} e^{\eta(x^*-x)})) = 0$$

Define G(x) = $\frac{1}{2}\sigma^2 x^2 - \psi x + \lambda(\frac{\eta}{\eta - x} - 1)$. Let β and γ denote the positive roots of G(x) = r . There are 3 solutions, 2 positive and 1 negative, but we discard the negative solution because it is not compatible with the limit condition: B(∞) = b/r (no-bubble condition), that is, the value of a risk-free bond for $V \to \infty$. So expression (6) becomes:

$$\frac{\eta}{\eta-\beta} A(-e^{\beta x^*}) + \frac{\eta}{\eta-\gamma} B(-e^{x^*\gamma}) + e^{x^*}(1-\alpha) - \frac{b}{r} = 0 \qquad (7)$$

Therefore, we can find A and B from equation (7) and the following value-matching condition (8) :

$$(1-\alpha) e^{x^*} = \frac{b}{r} + A e^{-\beta x^*} + B e^{-\gamma x^*} \qquad (8)$$

that is, A= $(e^{x^*}(1-\alpha) - \frac{b}{r}) \frac{e^{\beta x^*}\gamma(\beta-\eta)}{\eta(\beta-\gamma)}$ and B= $(e^{x^*}(1-\alpha) - \frac{b}{r}) \frac{e^{\gamma x^*}\beta(\eta-\gamma)}{\eta(\beta-\gamma)}$.

Thus, if $x \geq x^*$, then

$$u(x) = \frac{b}{r} + (e^{x^*}(1-\alpha) - \frac{b}{r})(\frac{\gamma(\beta-\eta)}{\eta(\beta-\gamma)} e^{\beta(x^*-x)} + \frac{\beta(\eta-\gamma)}{\eta(\beta-\gamma)} e^{\gamma(x^*-x)}) \qquad (9)$$

Similarly, if *v(x)* denotes the equity value at *x=ln V*, then *v* satisfies the following PDE:

$Lv(x) + (1-\tau)(e^x - b) - rv(x) = 0$, where τ is the corporate tax rate.

Thus, v(x) = $se^x - (1-\tau)\frac{b}{r} + \Omega\, e^{-\beta x} + \Lambda\, e^{-\gamma x}$, where s = $\frac{(1-\tau)}{r + \frac{\lambda}{\eta+1} - \frac{\sigma^2}{2} - \psi}$,

$$\Omega = e^{\beta x^*} \frac{((1-\tau)\frac{b}{r}\gamma - se^{x^*}(\gamma+1))}{\gamma-\beta} \quad \text{and} \quad \Lambda = e^{\gamma x^*} \frac{(-(1-\tau)\frac{b}{r}\beta + se^{x^*}(\beta+1))}{\gamma-\beta}$$

, which are obtained by the following boundary conditions: E(e^{x^*}) = 0, E'(e^{x^*}) = 0, ensuring that the equity value equals zero at bankruptcy, due to limited liability, and that equity holders choose the bankruptcy threshold optimally.

Proposition 1 provides an explicit formula for bond prices. It also offers an explicit expression for the default threshold V*.

Remark 1. Since V* is decreasing in η, by a comparison between the threshold values V* for a brown bond and a green bond we get that a brown bond defaults earlier than a green bond with the same characteristics.

As green bonds contribute to fund projects which are less hit by climate change policies, the potential default threshold is lower, which contributes to a better creditworthiness of the issuer.

Figure 1 shows the bond value as a function of V for three different values of the parameter η, associated with different degrees of 'greenness': a lower value of the average jump amplitude $1/\eta$ yields a higher bond value. The lowest value corresponds to the solid line and represents the case we dubbed 'green' bond. As expected, all bond values increase with the level of the underlying asset, V, as default becomes a remote occurrence when V is large.

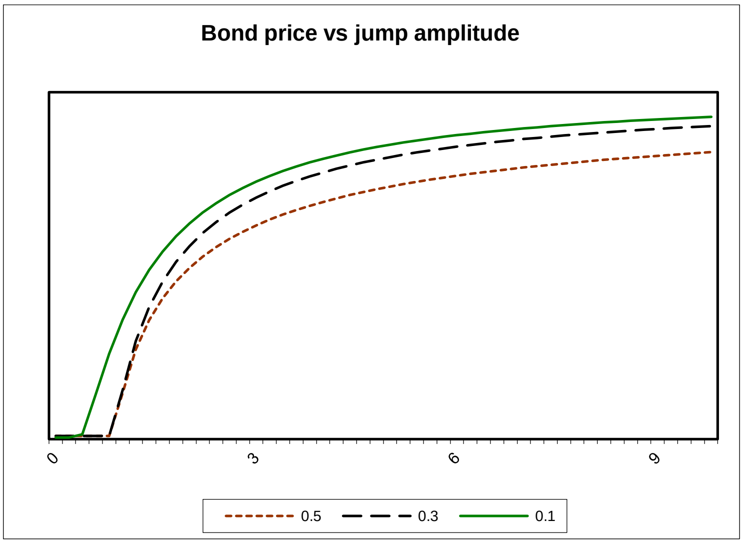

Figure 1. B(V) as a function of V for three different values of 1/η shown in the legend

We point out that the degree of green commitment can be extracted from the periodic reporting published by the issuers of green bonds, as this form of disclosure is part of the green labelling process. Our valuation formula in Proposition 1 shows how the green commitment is reflected in the bond price, and hence, in the yield. Our model can be used as a back-of-envelope to extract the implied 'shade' of greenness from bond market data. A Mathematica script for calibrating η to market data is available in the Appendix. In other words, while the computation in Proposition 1 is performed treating η as an exogenously given parameter, one might infer its implied value from the observation of market data for green and brown bonds by relating them to the relative size of revenues which are derived from the funded ‘green’ business projects. This applies also to the case where different types of bonds (green, brown or an intermediate shade) are issued by the same issuer, if the relative notional value of each bond is taken into account. Then a mapping between the levels of η and bonds’ greenness can be created, which in turn can be used to assess the ‘greenness’ of other issued bonds. Thus, our method provides a parsimonious and effective way to calibrate green bonds and create a ‘green’ rating system based on a continuous scale. This procedure can be performed under the implicit assumptions that firms honor their green commitments and that market price in such information perfectly. Note that, as Proposition 1 shows, the relation between the bond yield and the parameter η measuring greenness is non-linear and requires the knowledge of several other parameters, which makes it hard to draw the information about bond greenness from a mere observation of market yields. Notice that our model does not consider the issue of ‘greenwashing’, where the market valuation does not fully reflect the ‘true’ greenness of a bond[9], for example, in the case of fraudulent reporting by the bond issuer. Of course, our model is not applicable under such pathological circumstances.

Remark 2. From Proposition 1 we get that green bonds have normally a lower current yield, and thus a tighter credit spread, if compared with brown bonds of the same issuer and seniority.
This outcome is consistent with most empirical analyses (Ehlers and Packer, 2017, Reboredo, 2018, Zerbib, 2019) that have often found evidence of a ‘greenium’, that is, a negative premium that makes green bonds more expensive to investors who receive a slightly lower yield compared to existing similar bonds. A negative premium is regarded favorably by issuers because it can lower their funding costs.

Remark 3. We conclude our discussion on the beneficial effects of green bond issuance by making explicit the channels through which the issuance of green bonds contributes to mitigate a firm’s exposure to transition risks. As explained above, engagement in green projects affects the degree of the firm’s sensitivity to climate policy shocks (represented by the parameter η), which in turn lowers the default threshold V*, as discussed in Remark 1. As a result, potential losses due to negative transition events are reduced, which benefits the total firm value as the expressions for the

[9] We thank an anonymous referee for pointing out that ‘greenwashing’ may weaken this specific conclusion.

equity and bond value in Proposition 1 show. The size of this effect depends on the relative volume of green projects in place[10] and on the firm's financial policy.

Remark 4. The model can be easily extended to accommodate the costs arising from the requirements for attributing the green labelling to the bond (special reporting, monitoring and accounting, external review and third party certification, etc.). As they act in the opposite direction in comparison to the benefits related to the adoption of the green policy, a weaker greenium effect is obtained. More precisely, let *k* denote an additional cost which adds-up to debt service. Following the argument in Proposition 1 one can find a default threshold of the form:

$$V_k^* = \frac{\beta\gamma\zeta \frac{b+k}{r}}{(1+\beta)(1+\gamma)} \quad \text{where } \zeta = r + \frac{\lambda}{\eta+1} - \frac{\sigma^2}{2} - \psi \ .$$

Let us take $\beta > \gamma$ and suppose that α is not too small. Then from the expression of the green bond value we obtain

$$\frac{\partial B(V)}{\partial k} = \frac{\beta\gamma\zeta}{r\eta(\beta-\gamma)}\left[\left(\frac{V_k^*}{V}\right)^{\beta}(\beta-\eta)H_{\gamma} + \left(\frac{V_k^*}{V}\right)^{\gamma}(\eta-\gamma)H_{\beta}\right]$$

with $H_j = (1-\alpha)\frac{j}{j+1} - \frac{1}{\zeta}$, $j \in \{\beta,\gamma\}$.

Under our model assumptions, $H_\gamma < H_\beta < 0$ and $\frac{\partial B(V)}{\partial k}$ <0. As a consequence, in the presence of extra costs, the green bond yield increases and the greenium tightens. In conclusion, a very precise pricing formula should take into account a careful estimate of these costs as well. As practitioners and market participants concur that a main obstacle in the green bond market is currently the limited supply, compared to a sustained demand from green-minded investors, a reduction of the extra expenses for green bond issuance would partially offset disadvantageous funding costs. On the other hand, the tendency towards the development of a standardized certification model for obtaining and monitoring the green credentials will help reducing the impact of costs related to green bond issuance and transaction costs for investors.

4. Portfolio allocation with bonds subject to climate-related risks

In this section we link investors' preferences to the relative degree of greenness of a bond as defined in our model. In what follows we study the problem of optimal portfolio allocation between

[10] These data can be extracted from the Green Bond Annual Assessment Report released by companies which are green bond issuers.

a defaultable green bond and a money-market account, and a similar problem for a conventional bond. We take the perspective of a representative consumer-investor who splits his wealth between consumption and investment according to his utility function. Let *c(t)* denote the consumption rate and let *R(t)* denote the total wealth at time *t*. Let *x* denote the share of residual wealth (net of consumption) that is invested in a defaultable bond, where the current bond value, *B,* has been computed in Section 3, while the other portion of the residual wealth earns at the risk-free rate, *r*. We confine our argument to the case where the asset value is close to default, which simplifies the discussion and allows for a closed-form expression. As noted in the previous section, this is a case where the bond sensitivity to downward jumps makes a difference. The consumer-investor's budget equation is:

$$dR_t = [x\frac{dB}{B} + (1-x)rdt]R_{t^-} - c(t)dt \tag{10}$$

and in the above-mentioned case it can be written as:

$$dR_t = [(x(\varepsilon\mu - r) + r)R_{t^-} - c]dt + x\varepsilon R_{t^-}[\sigma dW_t + d\Psi_t] \tag{11}$$

where W_t is a Wiener process, Ψ_t stands for the jump process of Section 3 and $\varepsilon = \frac{\beta\gamma}{\eta r}[\hat{b} - r]$.
Here $\hat{b}$ is the current yield on the bond, that is, $\hat{b}$ - r is the credit spread on the bond. In contrast to Agliardi and Agliardi (2019), where a standard exponential utility function is adopted, here we use an Epstein-Zin utility, which allows to separate the intertemporal elasticity of substitution from the degree of risk aversion so that one can disentangle risk aversion effects and substitution effects. This kind of utility seems to be the elective choice in environmental economics, as it 'allows for the possibility that the agent has a preference for early resolution of risk, clearly of relevance in a discussion on climate risks' (Olijslagers and van Wijnbergen, 2019).
Then the value function for the consumer-investor's problem takes the form:

$$J_t = E[\int_t^\infty U(c_s, J_s)ds] \tag{12}$$

where the consumer's utility function is of the form:

$$U(c,J)=\frac{\rho}{1-1/\delta}\frac{c^{1-1/\delta}-((1-\chi)J)^{(1-\chi)/(1-1/\delta)}}{((1-\chi)J)^{(1/\delta-\chi)/(1-1/\delta)}}$$

Here χ denotes risk-aversion, δ describes the elasticity of intertemporal substitution and ρ is the rate of time preference parameter. If $\delta\chi$ =1 the utility specification reduces to standard power utility.

The wealth process satisfies the budget equation (11) and the initial wealth endowment, *Ro*, is assigned. The Hamilton-Jacobi-Bellman equation is of the form:

$$\max_{x,c}\left\{U(c,J)+\left[(x(\varepsilon\mu-r)+r)R-c\right]J'(R)+\frac{(x\varepsilon\sigma R)^2}{2}J''(R)+\lambda E[J(\mathrm{Re}^{x\varepsilon Y})-J(R)]\right\}=0$$

Let us search for a solution of the form: $J(R) = HR^{1-\chi}$, where *H* is to be determined. Plugging this function into the HJB equation and optimizing for *c* and *x*, we obtain an explicit expression for *H* and for the optimal consumption and portfolio weights. In particular, the first order conditions yield:

$U'(c^*)=J'(R)$

and $x^*=(z^*-\eta)/(\varepsilon(1-\chi))$ where z* solves the following equation:

$$(z^*)^3-(z^*)^2[\eta+(r/\varepsilon-\mu)(1-1/\chi)/\sigma^2]+\lambda\eta(1-\chi)/(\chi\sigma^2)=0. \quad (13)$$

Equation (13) can be solved numerically and the optimal portfolio weights crucially depend on the jump amplitude size that we associate with the degree of sustainability of the bond investment. As we noted in Section 3, the bond yield is inversely related to η and therefore an investor without any green bias tends to prefer brown bonds. This is confirmed by our portfolio allocation analysis. For example, with the same fundamentals as in the base case of the previous section, we obtain a significant difference between the portfolio weight of the brown and the green bond. Denote by Δ the difference in the portfolio weights between the brown and the green bond. Table 1.a and Table 1.b show how the portfolio weights are modified in the presence of various levels of η for green bonds, and for two different values of the risk aversion parameter, χ. We fixed η = 0.1 for the brown bond. Negative values for Δ imply that the portfolio weight of green bonds exceeds the weight of brown bonds. If η is relatively small (e.g. 0.3), investors prefer brown bonds whose yield is higher. As η increases, and, in particular, when η is very large, the investors prefers the green bond despite its lower yield. Clearly, η plays a significant role in determining the portfolio allocation between the two types of securities. This effect is mitigated in the presence of a higher risk aversion and when credit risk related to the defaultable component of the portfolio is an important concern, i.e. when V is close to the default threshold, as in Table 1.a.

	$\eta = 0.3$	$\eta = 0.5$	$\eta = 1$
$\chi = 2$	3.2%	0.9%	-9.1%
$\chi = 4$	2.1%	1.7%	-1.2%

Table 1.a. Dependence of Δ on η and χ when V is close to the default threshold.

	$\eta = 0.3$	$\eta = 0.5$	$\eta = 1$
$\chi = 2$	32.8%	-3.8%	-57%
$\chi = 4$	3%	2.5%	-9.9%

Table 1.b. Dependence of Δ on η and χ when V is far from the default threshold.

Now consider a consumption-investment problem where the investor chooses between two bonds issued by the same company. We suppose that one bond can be identified as 'green' and the other is a traditional one ('brown') and assume that the issuer maintains separate accounts for each bond, so that the two bonds can be distinguished as in the model presented in the previous section[11]. This form of management of proceeds is adopted by some green bond issuers. For example, Toyota Motor Credit Corporation committed itself to deposit the net proceeds from the 2017 TMCC green bond into segregated accounts and no proceeds can be used to refinance existing loans or leases that were originated prior to the settlement of the green bond. In this problem we study the role of taxation on investors' returns. Therefore, the current yield $\hat{b}$ will be interpreted as the bond yield net of taxation on investors' gain.

Let x_i denote the share of wealth (net of consumption) that is invested in the i^{th} bond, $\hat{b}_i$ the net current yield on the i^{th} bond and $\varepsilon_i = \frac{\beta\gamma}{\eta_i r}[\hat{b}_i - r]$, i=1,2. Assume the residual wealth is invested at the risk-free rate, r. Then the wealth dynamics is as follows:

$$dR_t = [(x_1\varepsilon_1 + x_2\varepsilon_2)\mu - r(x_1 + x_2)) + r)R_{t^-} - c]dt + x_1\varepsilon_1 R_{t^-}[\sigma dW_t + d\Psi_t^{(1)}] + x_2\varepsilon_2 R_{t^-}[\sigma dW_t + d\Psi_t^{(2)}]$$

[11] In general, the separation of accounts is an instrument used to more easily comply with the GBPs and it is an accounting trick that helps to reduce compliance costs. However, separated accounts do not imply that, in the case of default, one income stream (say of the green project) only serves to repay one type of bond (the green bond).

where W_t is a Wiener process, $\Psi_t^{(i)}$ stands for the jump process of Section 3 with parameter η_i. Then the optimal portfolio weights are as follows:

$x_i^* = (z_i^* - \eta_i)/(\varepsilon_i(1-\chi))$ where the z_i^*'s solve the following system of two equations:

$(z_i^*)^3 - (z_i^*)^2[A_i - z_j^*] + \lambda\eta_i(1-\chi)/(\chi\sigma^2) = 0$, i,j=1,2, $j \neq 1$,

where $A_i = \left(\mu - \dfrac{r}{\varepsilon_i}\right)\left(\dfrac{1-\chi}{\chi\sigma^2}\right) + \dfrac{\eta_1 + \eta_2}{\varepsilon_i}$.

We employ this model to show how a taxation policy significantly affects the investors' allocation policy. Adopting the same parameter values as in Table 1.b and taking χ=2 and η=1, we test the effect of a tax rate on the coupons on the portfolio weights. Under a zero tax rate on the coupons the portfolio weight of green bonds is 26.8%, while under a tax regime imposing a tax rate of 10% (or 25%) the weight of green bonds is decreased to 16.8% (or -11.9%, respectively). In other words, investors' preferences are strongly directed by the tax regime on financial returns. We conclude by emphasizing that further specific policies supporting green securities are requested to promote their diffusion. For example, an appropriate taxation policy may substantially modify the investors' allocation strategies and direct the market towards green investments. While most green bonds are taxable, like ordinary bonds, in some countries government incentives such as tax reduction have been experimented with some bond categories, for example, the Qualified Energy Conservation Bonds and the Clean Renewable Energy Bonds in the U.S. The TEG Report on Green Bond Standards issued in March 2019 explicitly recommends 'EU to encourage Member States to assess supporting the green bond market through tax incentives that could either be granted at issuer or investor level'.

5. Concluding remarks

This paper proposes a structural model for defaultable bonds incorporating both uncertainty about corporate earnings and uncertainty due to climate-related risks. We derive explicit expressions for bond values affected by sudden climate policy shocks and study the interplay among the various risk drivers. As bond prices and yields crucially depend on the jump size parameter, which can be related to the effect of sudden or disorderly mitigation policies, our model provides a simple tool to assess bonds' prices in terms of the relative sensitivity to transition risks. Furthermore, our model provides a simple tool to classify bonds according to their 'shade' of greenness. Despite the numerous underway initiatives to harmonize green finance definitions and standards, a unified classification framework is far from being achieved. There are some attempts toward a classification of green bonds based on the percentage of funds directed to projects that conform to general green standards, leading to a distinction between fully-aligned bonds, strongly-aligned bonds, and so on (CBI 2018) where information comes from the issuers' annual reporting documents. Unfortunately, the annual assessment reports include a mixture of information on the use of proceeds as they often refer to diverse categories of green projects (fight against climate change, air quality, clean water, sustainable waste management, etc.) and to target figures that are rarely comparable (avoided CO2 emissions, net produced energy from renewable sources, quality of discharged water, ratio of sorted waste to overall managed, etc). Furthermore, the effectiveness of the green projects is difficult to gauge. The quality of the implied 'greenness' depends on the quality of the disclosure and the ability of market players to process the information. Our model suggests a one-factor method to rate green bonds according to their market-implied 'greenness'. Of course, since our model belongs to the structural approach for bond evaluation, it shares all the limitations of this methodology, specifically it requires a perfect knowledge of the relevant firm's parameters along with the parameters needed to model the 'greenness' of securities, in particular of η, whose role and limitations have been discussed in Section 3. While suffering from some limitations, our model allows us for a better understanding of the determinants of bond pricing and the mechanisms behind corporate green policies. The needed inputs are the firm's fundamentals and the intensities of climate-related events, including the adoption of climate policy stringency, which can be extracted from the specialist literature. Although some subtle differences are lost because of a stylized framework, one can avoid many complexities and come up with a synthetic picture of the green bond universe.

Another outcome of our paper concerns the indirect benefit to the issuer's credit quality as a result of its commitment in reducing the damages from climate-related shocks (Remark 1). Although the green label does not imply a credit rating enhancement[12], its indirect beneficial effect may contribute to reduce systemic risk and thus improve financial stability in the worldwide bond market. Our model clarifies the channels through which it contributes to reduce the risk of financial distress and default (Remark 3) generated by shocks in the transition policy. Financial distress at a given firm has negative spillover effects for the remainder of the financial system, resulting in systemic risk, as this involves the collapse of financial markets through multiple defaults or widespread reduction of liquidity. Thus, there is an important role for green bonds in limiting these risks and thus mitigating the dysfunctionality of financial markets. Another beneficial effects of green bonds comes from the reduction of fat tails in the value distribution of financial assets. We showed that the value of brown bonds is more leptokurtic than the value of comparable green

[12] In some practical cases green credit is considered a factor enhancing credit merit and thus lowering capital risk requirements, e.g., the People's Bank of China with its scoring system MPA (Macro Prudential Assessment).

bonds. Since fat tails are often associated to an increased market instability, it follows that green bonds may have a beneficial stabilizing effect in financial markets. Our model, however, does not capture systemic risks since it does not examine interconnections within the non-financial and financial corporate sectors as well as macro-financial feedback loops.

Despite the impressive rise of green bond issuance in the recent years, there is still a limited supply of green debt and the question remains how to stimulate capital reallocation towards climate-friendly investments. As our Section 4 shows, the answer cannot be left to merely financial considerations and policy makers should take measures to scale-up the green bond markets, for example, favorable tax rates to green bondholders. Other suggestions may include central warehouses to facilitate the aggregation of large volumes of green loans and the harmonization of green standards to enhance transparency and investors' appetite for environment-friendly bonds. Besides stimulating private investments, public sector can play a key role in supporting green securities through the implementation of such policies as those proposed by Murphey and Hines (2010), Monasterolo and Raberto (2018) and Dafermos et al, 2018). The final goal is to facilitate the transition to a low-carbon economy and to add credibility to the long-term climate goal and, at the same time, to promote some financial tools that, as our model highlights, carry some ancillary benefits to the issuers and contribute to financial stability.

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APPENDIX. Mathematica Script

Green bond price

```
P[V_,b_,a_,r_,\[Sigma]_,p_,n_,l_]:=
    b/r+A[b,a,r,\[Sigma],p,n,l]/V^bet[\[Sigma],p,n,l]+ B[b,a,r,\[Sigma],p,n,l]/V^gam[\[Sigma],p,n,l];
A[b_,a_,r_,\[Sigma]_,p_,n_, l_]:=(Vstar[b,r,\[Sigma],p,n,l]*(1-a)-b/r)*(Vstar[b,r,\[Sigma],p,n,l])^
        bet[\[Sigma],p,n,l]* gam[\[Sigma],p,n, l]*(bet[\[Sigma],p,n,l]/n-1)/(bet[\[Sigma],p,n,l]-
gam[\[Sigma],p,n,l]);
B[b_,a_,r_,\[Sigma]_,p_,n_, l_]:=(Vstar[b,r,\[Sigma],p,n,l]*(1-a)-b/r)*(Vstar[b,r,\[Sigma],p,n,l])^
        gam[\[Sigma],p,n,l]* bet[\[Sigma],p,n,l]*(1-gam[\[Sigma],p,n,l]/n)/(bet[\[Sigma],p,n,l]-
            gam[\[Sigma],p,n,l]);
G[x_,\[Sigma]_,p_,n_,l_]:=0.5*(\[Sigma]*x)^2-p*x+l*(n+x)/(n-x);
bet[\[Sigma]_,p_,n_,l_]:=  x /.FindRoot [G[x,\[Sigma],p,n,l]\[Equal]0, {x, {0.01,1}}];
gam[\[Sigma]_,p_,n_,l_]:= x /.FindRoot [G[x,\[Sigma],p,n,l]\[Equal]0, {x, {1.01,100}}];
Vstar[b_,r_,\[Sigma]_,p_,n_,l_]:=
    gam[\[Sigma],p,n,l]*bet[\[Sigma],p,n,l]*
b/(r*(1+bet[\[Sigma],p,n,l])*(1+gam[\[Sigma],p,n,l]))*(r+
          l/(1+n)-0.5*\[Sigma]^2-p);
```

Backing-out Eta

Here y0 denotes the current yield of a brown bond.

```
Y[V_,b_,a_,r_,\[Sigma]_,p_,n_,l_,y0_]:=y0-b/P[V,b,a,r,\[Sigma],p,n,l];
Eta[V_,b_,a_,r_,\[Sigma]_,p_,n_,l_,y0_,perc_]:=
    n/.FindRoot [ Y[V_,b_,a_,r_,\[Sigma]_,p_,n_,l_,y0_]-perc\[Equal]0, {n, {0.01,100}}];
```

Replace parameters with numbers

```
Print[perc,Eta[V,b,a,r,\[Sigma],p,n,l,y0,perc]]
Null
```

HIGHLIGHTS

- A new model for climate-related risks and corporate bond pricing is derived
- Climate transition risk is modelled through a doubly-stochastic jump process
- The benefits of green investment on the firm creditworthiness are analysed
- A bond classification in terms of relative ‘greenness’ is proposed