

Third-Degree Price Discrimination in Two-Sided Markets

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Abstract. We investigate the welfare effects of third-degree price discrimination by a two-sided platform that enables interaction between buyers and sellers. Sellers are heterogeneous with respect to their per-interaction benefit, and, under price discrimination, the platform can condition its fee on sellers' type. In a model with linear demand on each side, we show that price discrimination (i) increases participation on both sides, (ii) enhances total welfare, and (iii) may result in a strict Pareto improvement, with both seller types being better off than under uniform pricing. These results, which are in stark contrast to the traditional analysis of price discrimination, are driven by the existence of cross-group network effects. By improving the ability to monetize seller participation, price discrimination induces the platform to attract more buyers, which then increases seller participation. The Pareto improvement result means that even those sellers who pay a higher price under discrimination can be better off, because of the increased buyer participation.

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1. Introduction

Online marketplaces often resort to third-degree price discrimination when dealing with a heterogeneous population of sellers. For instance, Amazon and eBay charge different commission rates depending on the product category (electronics, clothes, etc.).¹ Payment card systems such as Mastercard and Visa apply different fees based on the sector in which a merchant operates, and/or based on its size.² In their application stores, Apple and Google discriminate between large and small developers by charging a higher commission rate (30% instead of 15%) for developers with more than \$1 million (\$1m) annual revenue.³ Several platforms that use participation fees also engage in price discrimination. For example, French e-commerce platforms such as seloger.fr, paruvendu.fr, and leboncoin.fr charge advertising fees to professional sellers but not to nonprofessional ones, who represent a substantial portion of the population.

What are the distributional and welfare consequences of such practices? Although the effects of third-degree

price discrimination have been widely studied (see our literature review below), an interesting feature of marketplaces is that they are two-sided markets, in which the presence of buyers and sellers generates cross-side (sometimes called indirect) network effects. To what extent do the lessons from the standard analysis of thirddegree price discrimination apply to two-sided markets? How should a platform design its pricing policy in the presence of network effects? What are the managerial and policy lessons that can be learnt?

To answer these questions, we study a simple model of monopoly price discrimination by a two-sided platform. There are two groups of agents, buyers and sellers. All buyers obtain the same per-seller benefit, but sellers are heterogeneous with respect to their revenue: high-type sellers obtain a larger revenue for each buyer present on the platform than do low-type sellers. The platform charges participation fees to buyers and sellers. Agents also differ with respect to their exogenous participation cost (or outside option), which is distributed in such a way as to have linear demand on both sides of the market. We compare the situation where the platform charges the same participation fee to all sellers (uniform pricing) to one in which it can set different fees for high- and low-type sellers (third-degree price discrimination).

Our first result is that seller-side price discrimination leads to an increase in the participation of both buyers and sellers. Intuitively, allowing the platform to charge different seller fees allows it to capture more efficiently seller value from buyer participation, thereby giving it an incentive to attract more buyers. This in turn attracts more sellers, resulting in overall larger participation on both sides. Second, we show that total welfare increases with price discrimination. Third, we show that price discrimination can constitute a *strict* Pareto improvement: because of increased buyer participation, high-type sellers may be better off even if they end up paying a higher fee than under uniform pricing.

These results stand in sharp contrast to the "traditional" analysis of third-degree price discrimination. Indeed, with linear demands and no network effects, total output remains constant and welfare goes down, unless the weak market is not served under uniform pricing, in which case discrimination leads to a *weak* Pareto improvement.

Our analysis also delivers insights related to the platform's optimal pricing strategy. We identify several regimes, depending on parameter values. In the first, "typical" case, price discrimination leads to an increase in the fee paid by high-type sellers and to a decrease in the fee paid by low-type ones. The buyers' fee diminishes compared with uniform pricing when buyers' network benefits are relatively small, as the platform needs to increase their participation. When buyers obtain large benefits from sellers' participation, the platform can raise their fee without inducing a drop in participation. Other, more surprising patterns may also emerge in equilibrium. In the second regime, buyers are subsidized under uniform pricing, and price discrimination leads to an increase in the amount of subsidies. Surprisingly, price discrimination leads to an increase in fees for both groups of sellers. There is a third regime, in which both groups of sellers are subsidized. In that case price discrimination leads to an increase in subsidies for both groups, compensated by a fee increase for buyers. Note that whatever regime we are in, each side benefits (on aggregate for sellers) from discrimination.

For analytical tractability, the baseline model relies on some simplifying assumptions, in particular that buyer benefits are independent of sellers' types, that the platform charges participation fees, and that price discrimination occurs only on one side of the platform market. As we show in Section 5, our main insights do not hinge on these assumptions. First, we consider the case in which buyer surplus depends on the seller type. There again, price discrimination increases participation on both sides and may constitute a Pareto improvement. Even though welfare no longer always increases, numerical results indicate that, when it decreases, the loss is very small, whereas welfare gains can be more substantial. Second, we further investigate the role of network effects in order to compare our results to the extant literature. We show that, though price discrimination still enhances participation on both sides, a sufficiently high degree of network effects is necessary for our main welfare results to hold. In the third extension, the platform sets ad valorem instead of participation fees. Analytical results are more difficult to obtain, but our main findings concerning the welfare-enhancing effect as well as the possibility of strict Pareto improvement under price discrimination continue to hold. In the fourth extension, we investigate the situation in which the platform cannot charge buyers. Welfare is no longer always higher under price discrimination because the platform has fewer instruments to attract buyers and sellers. We confirm, however, the existence of a parameter region in which price discrimination leads to a strict Pareto improvement. Finally, our results hold when the platform can also price-discriminate between different buyer groups, resulting in price discrimination on both sides. As before, Pareto improvement occurs when the value of network benefits is high.

2. Relevant Literature

The analysis of third-degree price discrimination by a monopolist has a long tradition in economics (Pigou 1920, Robinson 1933, Schmalensee 1981, Varian 1985, Aguirre et al. 2010, Bergemann et al. 2015). Because it tends to lead to higher prices in some markets and to lower prices in others, its welfare effects are a priori ambiguous. As shown by Schmalensee (1981) and Varian (1985), a necessary condition for welfare to increase is that total output increases.⁴ Failing this, having different consumers face different prices leads to an inefficient "maldistribution of resources" (Robinson 1933). A case of particular interest for its tractability is that of linear demands. There, Pigou (1920) shows that, provided the firm made positive sales to each market under uniform pricing, output would remain the same under price discrimination, and welfare would decrease.

In traditional markets (i.e., without network effects), price discrimination may result in a Pareto improvement for several reasons: when it allows serving a new market,⁵ when profit functions are not single peaked (Nahata et al. 1990), or in the presence of economies of scale (Hausman and MacKie-Mason 1988). Given the connection between network effects and economies of scale, the latter paper is particularly relevant, but the logic is different: in Hausman and MacKie-Mason (1988), price discrimination makes it "cheaper" to attract consumers in the weak market, which reduces the cost to serve the strong market and can induce the firm to lower its price. In our paper, the ability to pricediscriminate sellers increases the firm's ability to monetize buyer participation, which in turn leads to higher seller participation. A notable difference is that Pareto improvement may happen even when one of the prices increases.

A few recent papers study price discrimination in two-sided markets, though of either the first- or seconddegree kind. Liu and Serfes (2013) show that firstdegree price discrimination can soften competition in a setup where the opposite would happen absent crossgroup network effects. In the context of second-degree price discrimination, Böhme (2016) shows that some properties of the optimal contract in traditional markets (e.g., no distortion at the top) no longer hold in twosided markets. Jeon et al. (2022) provide conditions for pooling to be optimal, and for second-degree price discrimination to increase or decrease welfare. In a related setup, Lin (2020) shows that price discrimination is complementary across sides. In a context where sellers use second-degree price discrimination, D'Annunzio and Russo (2024) study fee discrimination by a platform (or government), based on the quantity purchased by consumers, and show that it can alleviate the distortion induced by sellers' market power. Gomes and Pavan (2016) study price discrimination in matching markets and characterize the optimal many-to-many matching mechanism in the presence of two-sided asymmetric information. Chang et al. (2022) empirically find that price discrimination by Uber increases welfare.^o

Weyl (2006) and Rysman (2009) informally discuss third-degree price discrimination on one side of a twosided platform, and touch upon some of the themes presented here. In particular they conjecture that price discrimination on one side leads to a lower price on the other side, which we show is not true in general, even though participation on each side must increase.

Motivated by the app store controversies, Bhargava et al. (2022) study differential revenue sharing schemes, which bear some resemblance but are not equivalent to price discrimination. Indeed, they consider a platform returning to sellers a higher share for revenue contributions up to a predetermined threshold, and a smaller share above that. They find that the platform offering better terms to small developers may benefit large developers (a Pareto improvement), but do not consider the possibility of the platform raising its commission for one group of developers. Also because of this constraint, the platform does not always gain from adopting a differential sharing scheme, and this represents another difference in comparison with our analysis.

Tremblay (2021) also considers a model of price discrimination by a monopolistic platform (in the absence of network externalities) that charges unit fees to merchants, and finds that perfect fee discrimination is likely to reduce welfare. This result, opposite from what we obtain, stems from a different set of modelling assumptions: we consider a model featuring network externalities, elastic participation on all sides, and a platform that is allowed to charge (or subsidize) buyers, whereas Tremblay (2021) views the platform as an upstream supplier that only charges merchants, and emphasizes the double marginalization problem.

Ding and Wright (2017) study price discrimination by a payment card issuer, and find ambiguous welfare effects. The key driver of inefficiency in that model is the possibility of excessive intermediation (too many transactions being carried through payment cards), which is absent from ours.

A few papers study third-degree price discrimination in one-sided platforms: Adachi (2005) considers a model where agents from each group enjoy the presence of agents from the same group, and shows that welfare can increase with price discrimination even though total output remains the same. Belleflamme and Peitz (2020) analyze the monopoly provision of a network good where users care about the overall level of participation; they show that, under particular circumstances, third-degree price discrimination is equivalent to versioning (second-degree price discrimination). Peitz and Reisinger (2022) demonstrate that operating multiple platforms allows distinction between single-homing and multihoming sellers, which enables the platform owner to price-discriminate between high-valuation and low-valuation sellers. Closer to us, Hashizume et al. (2021) consider third-degree price discrimination in a one-sided market in which the platform sells a network good in two separate markets. They provide conditions for price discrimination to constitute a Pareto improvement, but do not fully characterize its total welfare effects. Our model allows investigation of interaction between sellers and buyers who connect via the platform, and consideration of the effect of increased participation on both sides.

Finally, moved by recent regulatory interventions and proposals, a stream of research theoretically investigates how to regulate platform fees, broadly suggesting the imposition of fee caps (Wang and Wright 2022, Bisceglia and Tirole 2023, Gomes and Mantovani 2024). The results of our paper suggest that fee regulation, especially if too rigid, may accidentally reduce the benefits brought by price discrimination in the presence of network effects.

3. Model

Consider a monopolist two-sided platform that orchestrates interactions between two groups, which we call buyers and sellers. The structure of the model is similar to Armstrong (2006): all pairs of agents interact, and the platform charges participation fees. Whereas most of the examples of transaction platforms involve ad valorem fees, we focus on participation fees for pedagogical and tractability reasons.⁷ We allow the platform to subsidize participation by offering negative fees, which can be interpreted as nonmonetary perks.⁸ Alternatively, in a model with positive marginal costs, the same results could be obtained with positive fees below the marginal cost.

3.1. Sellers

There are two categories of products, denoted *L* and *H*. Each category has a mass one of independent products, and each product is offered by a single seller.⁹ We do not explicitly model sellers' pricing decisions. Instead, we assume that the seller of a product in category $j \in \{L, H\}$ achieves a variable profit of θ_j for each buyer with whom the seller interacts. We assume that $\theta_H > \theta_L > 0$, and refer to θ_j as the type of sellers in category *j*.

If we denote the number of buyers on the platform by N_B , the profit of a seller of type θ_j is $\theta_j N_B - f_j$, where f_j is the participation fee paid to the platform. We assume that sellers have an outside option whose value is uniformly distributed over [0, 1], independently of their type. Assuming that all demands are interior (we provide conditions later on), the demand of the type *j* seller is

$$D_j(N_B, f_j) = \theta_j N_B - f_j.$$

Total seller participation is

$$D_S(N_B, f_L, f_H) = (\theta_L + \theta_H)N_B - f_L - f_H$$

We will compare two regimes: under uniform pricing, the fees must adhere to the constraint $f_L = f_H$, whereas no such restriction applies under price discrimination.

3.2. Buyers

There is a mass one of buyers. Each buyer obtains a stand-alone value v from using the platform. Furthermore, they receive an additional benefit b for each seller present on the platform.¹⁰ For tractability reasons, in the baseline model we assume that the benefit b is independent of the type of seller the buyers interact with.¹¹ In Section 5 we allow for buyers to care about sellers' types (Section 5.1) and for benefit heterogeneity among buyers with two-sided price discrimination (Section 5.5).

If N_S sellers join the platform, a buyer obtains a utility $v + bN_S - p$ from joining the platform, where p is the participation fee set by the platform. Assuming that buyers also have an outside option whose value is uniformly distributed over [0,1], the participation level of buyers is

$$D_B(N_S, p) = v + bN_S - p. \tag{1}$$

3.3. Equilibrium Demands and Platform's Profit

In an equilibrium with rational expectations, participation levels must satisfy

$$N_B = D_B(N_S, p)$$
 and $N_S = D_S(N_B, f_L, f_H)$. (2)

Instead of solving the above system to obtain participation levels as a function of fees, we use inverse demands and assume that the platform chooses participation levels and that fees adjust accordingly. In a monopoly setup the quantity approach amounts to assuming away coordination problems and it allows us to convey the logic of our arguments more clearly.¹²

Under uniform pricing, inverting the system (2) with the additional constraint that $f_L = f_H$ leads to the inverse demand system

$$P^{U}(N_{B}, N_{S}) = v + bN_{S} - N_{B} \quad \text{and}$$

$$F^{U}(N_{B}, N_{S}) = \frac{(\theta_{L} + \theta_{H})N_{B} - N_{S}}{2}.$$
(3)

The platform then chooses N_B and N_S to maximize

$$\Pi^{U}(N_{B}, N_{S}) = N_{B}P^{U}(N_{B}, N_{S}) + N_{S}F^{U}(N_{B}, N_{S}).$$
(4)

Under price discrimination, on the other hand, inverting the system (2) leads to the following inverse demands, where N_L and N_H denote participation by sellers of type θ_L and θ_H , respectively:

$$P^{D}(N_{B}, N_{L}, N_{H}) = v + b(N_{L} + N_{H}) - N_{B},$$

$$F^{D}_{H}(N_{B}, N_{H}) = \theta_{H}N_{B} - N_{H},$$

$$F^{D}_{L}(N_{B}, N_{L}) = \theta_{L}N_{B} - N_{L}.$$
(5)

Price discrimination enables the platform to choose the participation level of each seller type independently, whereas under uniform pricing the platform can only choose the overall level of seller participation, without being able to change the composition of the set of sellers. The platform then chooses N_B , N_L , and N_H to maximize

$$\Pi^{D}(N_{B}, N_{L}, N_{H}) = N_{B}P^{D}(N_{B}, N_{L}, N_{H}) + N_{L}F_{L}^{D}(N_{B}, N_{L}) + N_{H}F_{H}^{D}(N_{B}, N_{H}).$$
(6)

3.4. Interior Solutions

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In the first part of the paper we focus on equilibria where all participation levels are strictly between zero and one. To ensure this, we impose the following parameter restrictions:

Assumption 1. (i) $0 < v < 4 - (b + \theta_L)^2 - (b + \theta_H)^2$; (ii) $\frac{\theta_H - 3\theta_L}{2} < b$.

Under Assumption 1, parameters v, b, θ_L , and θ_H are in an intermediate range: with large values of the network effects the platform would choose full participation for at least some of the groups (buyers, high-type sellers, or low-type sellers). For future use, we define $\overline{b}(\theta_H, \theta_L, v) \equiv \frac{1}{2} \left(\sqrt{8 - 4v - (\theta_H - \theta_L)^2} - \theta_H - \theta_L \right) \text{ as the largest value of } b \text{ compatible with condition (i) above.}$ On the other hand, positive low-type seller participation under uniform pricing requires θ_L or b to be large enough relative to θ_H (condition (ii)).

In Subsection 4.4 we show that condition (i) is necessary for our results to hold, whereas failure of condition (ii) would reinforce them.

4. Analysis

4.1. Participation

4.1.1. Uniform Pricing. Under uniform pricing, the platform chooses N_B and N_S in order to maximize $\Pi^U(N_B, N_S) = N_B P^U(N_B, N_S) + N_S F^U(N_B, N_S)$. Dropping the arguments to lighten notations, the first-order conditions are

$$\frac{\partial \Pi^{U}}{\partial N_{B}} = 0 \Longleftrightarrow P^{U} + N_{B} \frac{\partial P^{U}}{\partial N_{B}} + N_{S} \frac{\partial F^{U}}{\partial N_{B}} = 0, \tag{7}$$

$$\frac{\partial \Pi^{U}}{\partial N_{S}} = 0 \Longleftrightarrow F^{U} + N_{S} \frac{\partial F^{U}}{\partial N_{S}} + N_{B} \frac{\partial P^{U}}{\partial N_{S}} = 0.$$
(8)

Beyond the standard marginal revenues, captured by the first two terms on the left-hand side of (7) and (8), the third terms in each equation capture the idea that attracting an extra agent on one side allows the platform to increase its revenue on the other side.

4.1.2. Price Discrimination. Under price discrimination, the platform chooses N_B , N_L , and N_H to maximize $\Pi^D(N_B, N_L, N_H) = N_B P^D(N_B, N_L, N_H) + N_L F_L^D(N_B, N_L) + N_H F_H^D(N_B, N_H)$. The first-order conditions are

$$\frac{\partial \Pi^{D}}{\partial N_{B}} = 0 \Longleftrightarrow P^{D} + N_{B} \frac{\partial P^{D}}{\partial N_{B}} + N_{L} \frac{\partial F_{L}^{D}}{\partial N_{B}} + N_{H} \frac{\partial F_{H}^{D}}{\partial N_{B}} = 0, \quad (9)$$

$$\frac{\partial \Pi^D}{\partial N_L} = 0 \Longleftrightarrow F_L^D + N_L \frac{\partial F_L^D}{\partial N_L} + N_B \frac{\partial P^D}{\partial N_L} = 0, \tag{10}$$

Figure 1. Equilibrium Participation

and

$$\frac{\partial \Pi^D}{\partial N_H} = 0 \Longleftrightarrow F_H^D + N_H \frac{\partial F_H^D}{\partial N_H} + N_B \frac{\partial P^D}{\partial N_H} = 0.$$
(11)

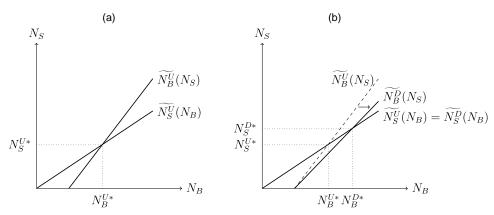
4.1.3. A First Result. We are now ready to state our first main result:

Proposition 1. Under price discrimination, the equilibrium number of both buyers and sellers increases compared with uniform pricing.

The proof of Proposition 1 is detailed in Appendix A.1. Here we provide the intuition for it, illustrated in Figure 1. In the figure, $N_B^U(N_S)$ is the profit-maximizing participation level for buyers under uniform pricing when N_S sellers participate (i.e., the solution to max_{N_B} $\Pi^U(N_B, N_S)$). The other curves are defined similarly. The proof proceeds in three steps.

First, the participation levels of buyers and sellers are strategic complements from the platform's point of view: increasing the participation level of sellers makes it more profitable for the platform to attract new buyers, and reciprocally. Indeed, as the number of sellers increases, not only can each buyer be charged a higher price (e.g., term $N_B(\partial P^U/\partial N_S)$ in (8)), but attracting a new buyer also allows the platform to increase its price to a larger base of sellers (e.g., term $N_S(\partial F^U/\partial N_B)$ in (7)).¹³ In Figure 1(a) the equilibrium under uniform pricing is given by the intersection between the two increasing functions $N_B^U(N_S)$ and $N_S^U(N_B)$.

Second, for a given level of buyer participation, the profit-maximizing total number of sellers is the same under uniform pricing and discrimination. This result aligns with the traditional analysis of price discrimination without network effects and with linear demands: as long as both markets (here, both groups of sellers) are served under uniform pricing, discrimination does not affect total output (Pigou 1920). Formally, this follows from the fact that adding (10) and (11) gives (8). In



Note. (a) Uniform pricing; (b) price discrimination.

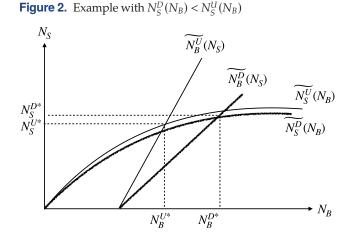
Figure 1(b), this observation means that $N_S^U(N_B) = \widetilde{N_S^D}(N_B)$.

Third, for a given level of seller participation $N_S = N_H + N_L$, switching to the discrimination regime induces the platform to attract more buyers. Intuitively, being able to discriminate among sellers allows the firm to fully extract the value generated by each additional buyer on the seller side, which makes it more profitable to attract new buyers. In Figure 1(b), this corresponds to the shift from $\widetilde{N}_B^U(N_S)$ to $\widetilde{N}_B^D(N_S)$.

Put together, these observations imply that equilibrium participation of both sides is higher under price discrimination, driven by the extra incentive to attract buyers.

4.1.4. Discussion and Generalization. Linear demands offer an ideal benchmark to compare our model to one without cross-side network effects. In a one-sided market (for instance, if the number of buyers was fixed and the fee they pay was exogenous), output (i.e., seller participation) would remain the same. In this case, welfare would go down with price discrimination provided both markets were served under uniform pricing (i.e., positive participation from both types of sellers). Proposition 1 already shows that the output result is no longer true in two-sided markets.

But actually the proposition holds under weaker assumptions. Indeed, its main result about increased participation on both sides continues to be valid if (i) N_B and N_S are strategic complements, (ii) $N_B^D(N_S) >$ $\widetilde{N^U_B}(N_S)$, and (iii) $\widetilde{N^D_S}(N_B)$ is not that much smaller than $N_{S}^{U}(N_{B})$. Conditions (i) and (ii) are fairly natural: having more buyers tends to make attracting an extra seller more profitable (and reciprocally), and being able to extract more profit from sellers through price discrimination makes attracting extra buyers more profitable. Condition (iii) relates to a standard concern in the traditional analysis of third-degree price discrimination, namely, the effect of discrimination on total output: for a given number of buyers, would discrimination increase or decrease output (i.e., seller participation)? Aguirre et al. (2010) provide conditions for welfare and output to increase or decrease under discrimination when the demand function is not necessarily linear. In particular, they show (proposition 4) that output increases with discrimination if demand in the weak market (here, the low-type sellers) is "more convex" than in the strong market. In our model, if that is the case, then Proposition 1 continues to hold. Note that an output increase $(N_S^D(N_B) > N_S^U)$ (N_B)) is a sufficient but not necessary condition for total participation to increase. If the inequality is reversed, but the difference is small enough, then the



increase in N_B may dominate and lead to an overall increase in participation. This is illustrated in Figure 2. However, Proposition 1 does not hold if $N_S^D(N_B)$ is *sufficiently* smaller than $N_S^U(N_B)$, which is the case when demand by low-type sellers is sufficiently less convex than that by high-type ones.

4.2. Equilibrium

In order to provide welfare results, we need to explicitly compute the equilibria under uniform pricing and price discrimination.

4.2.1. Uniform Pricing. Solving the system of first-order conditions (7) and (8), we obtain¹⁴

$$N_{S}^{U} = \frac{2v(2b + \theta_{H} + \theta_{L})}{8 - (2b + \theta_{H} + \theta_{L})^{2}}, \quad N_{B}^{U} = \frac{4v}{8 - (2b + \theta_{H} + \theta_{L})^{2}},$$

which corresponds to equilibrium prices

$$p^{U} = \frac{v(4 - (\theta_{H} + \theta_{L})(2b + \theta_{H} + \theta_{L}))}{8 - (2b + \theta_{H} + \theta_{L})^{2}}$$
$$f^{U} = \frac{v(\theta_{H} + \theta_{L} - 2b)}{8 - (2b + \theta_{H} + \theta_{L})^{2}},$$

and a profit for the platform equal to

$$\Pi^{U} = \frac{2v^{2}}{8 - (2b + \theta_{H} + \theta_{L})^{2}}.$$
 (12)

4.2.2. Price Discrimination. Solving the system of first-order conditions (9), (10), and (11), we obtain¹⁵

$$N_{S}^{D} = \frac{v(2b + \theta_{L} + \theta_{H})}{4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L})},$$
$$N_{B}^{D} = \frac{2v}{4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L})},$$
(13)

which implies prices

$$p^{D} = \frac{v(2 - \theta_{H}^{2} - \theta_{L}^{2} - b(\theta_{H} + \theta_{L}))}{4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L})},$$

$$f_{j}^{D} = \frac{v(\theta_{j} - b)}{4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L})} \text{ for } j \in \{H, L\}.$$
(14)

The platform's profit is then

$$\Pi^{D} = \frac{v^{2}}{4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L})}.$$
 (15)

4.3. Comparison

4.3.1. Welfare Analysis. Our main results concern the welfare effects of price discrimination. They are summarized in the proposition below, whose proof is in Appendix A.2. Considering interior solutions (see Assumption 1), the following results hold:

Proposition 2.

(i) The platform, buyers, and low-type sellers are better off under price discrimination.

(ii) Total welfare is higher under price discrimination.

(iii) High-type sellers are better off under price discrimination if and only if $b > \hat{b}(\theta_H, \theta_L) \equiv \frac{\sqrt{32-7(\theta_H - \theta_L)^2} - 3\theta_H - \theta_L}{4}$. In this case, price discrimination constitutes a strict Pareto improvement over uniform pricing.

Part (i) of Proposition 2 follows naturally from Proposition 1. That the platform is better off follows from a revealed preference argument. Buyers are better off, as revealed by their increased participation. Interestingly, this may happen even if they pay more (see Proposition 3 for more details), as the augmented seller participation compensates for possible fee increases. The result that low-type sellers are better off follows from inspection of their surplus, as there are instances in which they may end up paying a higher fee (see again Proposition 3).

Part (ii) stands in stark contrast with the traditional analysis of price discrimination. Recall that, when demands are linear and both markets are served under uniform pricing, third-degree price discrimination always lowers total welfare. The result is overturned in a two-sided context, thanks to the platform's incentive to increase participation on both sides, as we already explained. As we show in the proof, the increase in participation trumps the misallocation of resources due to price discrimination, so that the welfare effects are positive.

Part (iii) goes even further: when network effects (measured by *b* and θ_H) are large enough, even high-type sellers benefit from price discrimination. Note that high-type sellers can benefit even though the price they pay increases ($f_H^D > f^U$). This is because the increased participation of buyers more than offsets the price increase.

Figure 3 plots the region where price discrimination leads to a Pareto improvement (dotted area), as indicated in part (iii) of Proposition 2.¹⁶ Following Assumption 1, we focus on the region where $\frac{\theta_H - 3\theta_L}{2} < b < \overline{b}(\theta_H,$ $\theta_L, v)$. We fix v = 0.1 and consider two possible values for θ_H to show that the area with Pareto improvement increases with θ_H (this is formally demonstrated in Appendix A.2).¹⁷

4.3.2. Prices. Having stated our main result, it is instructive to take a closer look at the platform's optimal pricing strategy. In Appendix A.3 we formally prove the following result:

Proposition 3.

(i) There exists $\tilde{b}(\theta_H, \theta_L) > 0$ such that $p^D > p^U$ if and only if $b > \tilde{b}(\theta_H, \theta_L)$.

(ii) $f_L^D < f_H^D$ always holds, yet depending on the parameter values we can have: $f^U \le f_L^D$, $f^U \in (f_L^D, f_H^D)$, or $f^U \ge f_H^D$.

 $\overline{\theta}_I$

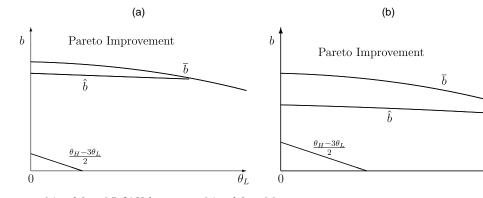


Figure 3. Regions with Pareto Improvement

Notes. (a) Values are v = 0.1 and $\theta_H = 0.5$. (b) Values are v = 0.1 and $\theta_H = 0.8$.

When $f^{U} \leq f_{L}^{D}$, we have $p^{D} < p^{U} < 0$; when $f^{U} \geq f_{H}^{D}$ we have $f^{U} < 0$.

Part (i) of Proposition 3 reveals that, if buyers place a high value on seller participation ($b > \tilde{b}(\theta_H, \theta_L)$), the platform raises their price under price discrimination. In spite of this, buyers are still better off because of the increased number of sellers under price discrimination. Such a strategy may require subsidizing seller participation, especially if their value for buyer participation is relatively low. Conversely, if *b* is smaller ($b < \tilde{b}(\theta_H, \theta_L)$), the platform needs to lower its price to buyers in order to trigger the positive feedback loop leading to more participation on each side.

Part (ii) considers the price paid by sellers. Even though the typical case is such that $f_L^D < f^U < f_H^D$, there are regions in the parameter space such that both fees increase or decrease under price discrimination. On the one hand, when sellers scarcely value buyer interaction, the platform may decide to subsidize them more (i.e., lowering their negative fees) under price discrimination in order to attract them, thus explaining the region where $f_L^D < f_H^D < f^U < 0$.

On the other hand, when sellers highly value buyer interaction, the platform may increase the fees for both of them when it can price-discriminate, leading to a situation in which $f^{U} < f_{L}^{D} < f_{H}^{D}$. We provide more precise conditions in Appendix A.3, together with a figure illustrating the different cases.

4.3.3. Negative Fees. So far, we have allowed the platform to charge negative fees, and indeed it is sometimes optimal for the platform to do so. With positive (and high-enough) marginal costs, the fees would be positive but the platform's mark-up over some group(s) would be negative. However, in environments with low marginal costs for the platform, it may not be possible for it to charge negative fees, and one may wonder if our results would hold true. Indeed, we can demonstrate that this is the case, provided that the "unconstrained" fees do not reach excessively negative levels. Specifically, if the nonnegative price constraint (NNPC) binds for the relevant group, welfare effects (including the possibility of Pareto improvement) would still hold. Only when the unconstrained fees reach highly negative levels does the possibility arise for welfare to decrease under price discrimination with an NNPC. We provide more details in Online Appendix Section A.

4.4. Noninterior Solutions

Assumption 1 guarantees that, for each group of agents, some but not all individuals participate under both uniform and discriminatory pricing. Here we briefly discuss cases where this does not hold.

4.4.1. Exclusion of the Low-Type Sellers. Consider first the scenario in which low-type sellers would be excluded under uniform pricing. In a traditional market without network effects, if the weak market is excluded, price discrimination leads to a *weak* Pareto improvement by enabling service of the weak market without affecting the strong one. Instead, in the presence of network effects, when low-type sellers are excluded under uniform pricing, that is, when $b < \frac{\theta_H - 3\theta_L}{2}$, we obtain the following result, which is formally proven in Appendix A.4:

Proposition 4. *Suppose that only part* (i) *of Assumption* 1 *holds. Then price discrimination leads to a strict Pareto improvement over uniform pricing.*

Strict Pareto improvement implies that participation by all groups as well as total welfare increases, just as in the analysis above. Compared with the standard analysis without network effects, the novelty here is the strictness of the Pareto improvement. High-type sellers benefit from the platform's ability to price-discriminate, because attracting a new type of seller leads to an increase in the participation of buyers.

4.4.2. Full Buyer Participation. Suppose now that part (i) of Assumption 1 does not hold. Then the platform serves all buyers under uniform pricing. Then we must also have $N_B^{D*} = 1$, so that price discrimination does not increase buyer participation. Because of fixed participation on the buyer side, the analysis mirrors the traditional one: price discrimination leaves total participation on the seller side unchanged (by the linearity of demand), but welfare goes down because of the misallocation due to sellers facing different prices.

Proposition 5. If Assumption 1 (i) does not hold, then price discrimination lowers welfare.

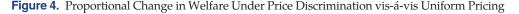
A formal proof can be found in Appendix A.5.

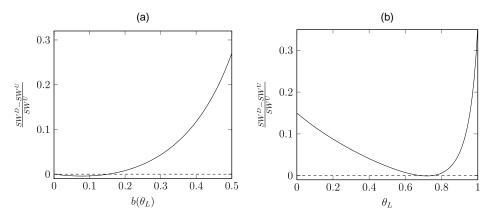
5. Extensions

5.1. Seller-Specific Buyers' Benefits

In order to obtain analytical results, we have assumed that buyers are indifferent with respect to the type of sellers they interact with. Although this assumption can be microfounded, a more plausible assumption is that buyer surplus depends on the type of the seller, $b(\theta)$, and that buyers prefer to interact with high-type sellers: $b(\theta_H) > b(\theta_L)$. Although in the baseline model high-type sellers always pay a higher fee than their low-type peers, this is no longer necessarily the case when $b(\theta_H) > b(\theta_L)$, as the following result shows (formal proof in Online Appendix Section B):

Lemma 1. Under price discrimination, when $\theta_L - b(\theta_L) > \theta_H - b(\theta_H)$, the platform charges a higher participation fee to the low-type sellers.





Notes. (a) Change in welfare as a function of $b(\theta_L)$. Parameter values: v = 0.1, $\theta_L = 0.5$, $\theta_H = 1$, and $b(\theta_H) = 0.5$. (b) Change in welfare as a function of θ_L . Parameter values: v = 0.1, $b(\theta_L) = 0.25$, $\theta_H = 1$, and $b(\theta_H) = 0.5$.

When the above condition holds, even though hightype sellers are willing to pay more, they also generate more benefits to buyers, so that the platform seeks to attract them by charging a relatively lower price.¹⁸ This is consistent with some strategies used in practice. For example, Steam's commission rate is 30% for games earning less than \$10m, then 25% for earnings between \$10m and \$50m, and 20% for earnings above \$50m.

We then have the following result, which is formally demonstrated in Online Appendix Section B:

Proposition 6. Suppose that parameters are such that the equilibrium is interior. When $b(\theta_H) > b(\theta_L)$, participation on both sides increases under price discrimination.

Proposition 6 is a generalization of Proposition 1. Recall that, in Proposition 1, part of the reasoning relied on seller participation being constant across pricing regimes (for a given N_B). When buyers care about seller type, we need to take into account that, even though N_S is the same for a given N_B , the composition of the set of sellers is different, so that buyers may be worse off, everything else being equal. The crux of the proof consists in showing that this composition effect is not enough to offset the platform's incentive to attract more buyers following the improvement of its ability to extract surplus from sellers.

Obtaining clean analytical results in this more general setup is difficult, but numerical simulations indicate that our main insights continue to hold. Even though total welfare may go down with price discrimination, we find that the magnitude of welfare losses is generally small (see Figure 4). There are also parameter regions such that price discrimination leads to a Pareto improvement.

5.2. Importance of Network Effects

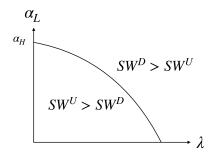
Our baseline model highlights how the existence of network effects can overturn some standard results on the effects of price discrimination. A natural question is whether there is a discontinuity, in that arbitrarily small network effects would be enough to make price discrimination socially desirable. One issue with our baseline model is that taking θ_H and θ_L to zero eliminates any heterogeneity across sellers, thus rendering the analysis of price discrimination meaningless.

In this subsection we look at a generalization of the baseline model, where the demand by sellers of type $j \in \{L, H\}$ is $D_j(N_B, f_j) = \alpha_j + \theta_j N_B - f_j$, with $\alpha_H \ge \alpha_L$. This would be the case if sellers' participation costs followed a type-specific distribution.¹⁹ With this specification, it is possible to take θ_H and θ_L to zero while maintaining some heterogeneity across the two groups.

Proposition 1 still applies in this model, as the result that total seller participation $N_H + N_L$ is the same under uniform pricing and discrimination for a given N_B continues to hold.

To study the role of network effects on the welfare consequences of price discrimination, we use a scaling parameter λ so that network effects are λb , $\lambda \theta_L$, and $\lambda \theta_H$. Figure 5 illustrates how price discrimination affects welfare depending on the strength of network effects (λ) and a measure of homogeneity across groups (α_L/α_H).

Figure 5. Effects of Discrimination on Welfare



Note. Parameter values: $\theta_L = 0.7$, $\theta_H = 1$, b = 0.2, v = 0.1, $\alpha_H = 0.3$.

When $\alpha_L = \alpha_H$, welfare always increases with price discrimination, as in our baseline model. When $\alpha_L < \alpha_H$, there is a tension between the standard distortion due to price discrimination and the effects identified in this paper. Discrimination increases welfare when network effects (measured by λ) are sufficiently large, and decreases it otherwise.

5.3. Ad Valorem Pricing

Even though many platforms use ad valorem fees, in our baseline model we focus on participation fees. The main reason is that the use of ad valorem fees in itself constitutes a form of price discrimination, because different sellers end up paying different amounts per transaction. In fact, Wang and Wright (2017) show that ad valorem fees achieve efficient price discrimination when demand is proportional to marginal costs. In addition, in our model with independent firms and no asymmetric information, ad valorem fees would induce a distortion (when marginal costs are positive) and would not be optimal.

These arguments notwithstanding, in this subsection we show that our main welfare results continue to hold when the platform charges ad valorem fees.

Suppose that the platform charges a fee proportional to sellers' revenue, $r_i\theta_i$, for $i \in \{L, H\}$, and that sellers face zero marginal costs, so that their optimal price (and thus gross revenue θ) is not affected by the fee. We compare the uniform pricing regime where the platform charges the same ad valorem fee to all sellers ($r_H = r_L = r$) to the one where it sets discriminatory prices $r_H \neq r_L$.

Platform profits when employing uniform pricing and discriminatory pricing regimes, respectively, are

$$\max_{r,p} \Pi^{U} = (p + r(\theta_{H}N_{H} + \theta_{L}N_{L}))N_{B},$$
$$\max_{r_{H}r_{L}r_{L},p} \Pi^{D} = (p + r_{H}\theta_{H}N_{H} + r_{L}\theta_{L}N_{L})N_{B}.$$

A detailed analysis is presented in Online Appendix Section C, where we also provide the conditions for obtaining an interior solution and the relevant threshold value of b that appears below. In the following proposition, we present the ad valorem fee counterpart to the results in Proposition 2.

Proposition 7. *In comparison with uniform pricing, under price discrimination we obtain that*

(i) Total buyer and seller participation increases.

(ii) *The platform, buyers, and low-type sellers are better off.*

(iii) Total welfare is higher.

(iv) High-type sellers are better off if and only if $b > \hat{b}^{ad}$ (θ_H, θ_L). In this case, price discrimination constitutes a Pareto improvement over uniform pricing. This confirms that our welfare results hold when the platform employs an alternative pricing structure for sellers. The intuitions for these results are similar to those after Proposition 2.

5.4. One-Sided Pricing

We now consider the case in which the platform does not charge buyers, whereas it still charges the participation fee f to sellers. Most digital platforms, such as the major app stores, search engines, and social networks, usually grant free access to users. This is also common in the lodging sector, in which online travel agencies (OTAs) such as Booking.com and Expedia only charge hotels and lodging establishments. We are therefore interested in the case in which the platform faces an additional constraint in terms of possible cross-subsidization between buyers and sellers.

We compare uniform pricing with price discrimination. The formal analysis can be found in Online Appendix Section D, where we also provide the relevant threshold values of b and the conditions ensuring we obtain interior solutions. We present the main results in Proposition 8.

Proposition 8. *In comparison with uniform pricing, under price discrimination:*

(i) Total buyer and seller participation increases.

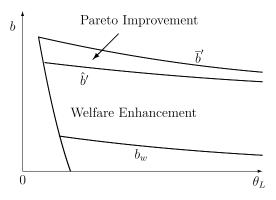
(ii) *The platform, buyers, and low-type sellers are better off.*

(iii) Total welfare is higher if and only if $b > b_w(\theta_H, \theta_L)$.

(iv) High-type sellers are better off if and only $b > \hat{b}'(\theta_H, \theta_L)$, with $\hat{b}'(\theta_H, \theta_L) > b_w(\theta_H, \theta_L)$. In this case, price discrimination constitutes a Pareto improvement over uniform pricing.

Figure 6 provides the interval regions of interest characterized by $\max\{0, \frac{2\theta_H}{\theta_L(\theta_H - \theta_L)}\} < b < \overline{b}'(\theta_H, \theta_L, v)$. It is plotted for v = 0.1 and $\theta_H = 0.5$. Most of the results of the benchmark case continue to hold, the most relevant exception being that price discrimination does not always enhance welfare. In fact, when buyer valuation

Figure 6. Pareto Improvement and Welfare Enhancement



for sellers' participation is not strong enough ($b < b_w$ (θ_H , θ_L)), total welfare is lower under price discrimination. Intuitively, when *b* is small, buyers do not react much to seller participation. Because the platform cannot charge buyers either, their participation is almost fixed, so that the setup is close to the standard model of price discrimination without network effects, where we know that welfare goes down.

5.5. Price Discrimination on Both Sides

In the baseline model we examined the case in which the platform was able to price-discriminate across sellers, but not across buyers. We now introduce heterogeneity on the buyer side as well, by assuming the existence of two types of buyers, denoted by l and h, each with a population of mass one. Each buyer of type $i \in \{l, h\}$ obtains the same stand-alone value v from using the platform, and a type-specific benefit $b_h > b_l >$ 0 for each seller present on the platform. A buyer of type i obtains therefore a utility $v + b_i N_S - p$ from joining the platform, and assuming an outside option which is again uniformly distributed over [0,1], the participation level of type i is $D_i(N_S, p_i) = v + b_i N_S - p$.

We compare uniform pricing on both sides with price discrimination on both sides. A detailed analysis is presented in Online Appendix Section E, where we demonstrate the following:

Proposition 9. *In comparison with uniform pricing, under price discrimination:*

(i) Total buyer and seller participation increases.

(ii) *The platform, low-type buyers, and low-type sellers are better off.*

(iii) Total welfare is higher.

(iv) High-type sellers and high-type buyers are both better off for sufficiently high values of both θ_H and b_H . In this case, price discrimination constitutes a Pareto improvement over uniform pricing.

This confirms that our results hold when the platform can adopt price discrimination on both sides. The intuitions for these results are similar to those after Proposition 2, the only difference being the fact that now we need sufficiently high valuations for both high types (buyers and sellers) in order for price discrimination to give rise to a Pareto improvement.

6. Managerial and Policy Implications

In this section, we summarize our main results and discuss their managerial and policy implications.

6.1. Managerial Insights

1. Price discrimination among sellers should be accompanied by a pricing policy geared toward increasing buyer participation. 2. Price discrimination need not alienate any group of users, even those discriminated against. Indeed, network effects may generate a positive feedback loop such that all groups are better off under price discrimination.

The first insight captures the idea that the platform has an incentive to attract more buyers when it can extract more of sellers' surplus through price discrimination, because each extra buyer generates more revenue for the platform on the seller side. Attracting more buyers often entails offering them a lower price, but that is not always the case, in particular when network effects *b* are particularly large. In that case, buyers are attracted by the larger participation of sellers.

The second insight is a consequence of the Pareto improvement result. A platform may worry that implementing price discrimination may lead to an exodus of high-type sellers, potentially jeopardizing its attractiveness. On the contrary, we showed that, when network effects are large, a carefully designed price discrimination scheme may benefit all participants, thereby increasing participation of all groups.

6.2. Policy Insights

1. Price discrimination among sellers results in an increase of buyer surplus.

2. The existence of network effects makes it more likely that price discrimination increases total welfare.

The first policy insight is the counterpart of the first managerial insight: because the platform seeks to attract more buyers, their surplus has to increase.

Regarding the second policy insight: It is well established in the literature that third-degree price discrimination has ambiguous welfare effects. A main insight of this paper is to show that two-sided network effects make it more likely that price discrimination increases total welfare. We showed that this is always the case when demands are linear, but the forces at play are more general and would apply to nonlinear demands, though in that case welfare is not guaranteed to increase.

7. Conclusion

In this paper, we argue that third-degree price discrimination in markets featuring network effects is not only welfare enhancing but can also be Pareto improving. This result arises because of the presence of network externalities, as in their absence our analysis would reproduce the well-known findings from traditional markets. In particular, cross-sided network externalities render the multiple sides of a platform interdependent and changes in welfare on one side can have relevant repercussions on the other side.

In the presence of two types of sellers, high type and low type, price discrimination enables a platform to profitably and more efficiently extract higher surplus from sellers. This is achieved by enhancing their value on the platform by boosting buyer participation. Because demands on the two sides are elastic, the platform only extracts a portion of this increased seller value which results in increased total participation of sellers. Ultimately, we find that price discrimination enhances platform profit, increases buyer surplus and the surplus of low-type sellers, and can even generate a higher surplus for high-type sellers, thus resulting in a Pareto improvement.

Our analysis is carried out in a simplified setting in which players on both sides pay participation fees to join, and buyers equally value the presence of sellers on the platform. However, we proved that our main results hold when more complex settings are taken into account, such as heterogeneity in buyer valuation of sellers, ad valorem fees on the seller side, and buyers freely joining the platform. Finally, we used linear demands for tractability, but the mechanism underpinning our results, namely, the increases in the participation of both buyers and sellers generated by price discrimination, holds more generally, as we explained at the end of Subsection 4.1.

Notwithstanding the limitations, the results that we obtain bear important managerial implications for executives of large platforms catering to a wide variety of demand segments. They also offer policy makers precious indications about the possible advantages that platforms can create for society at large when cross-sided network externalities are present. Neglecting these forces may have unintended effects for platform managers contemplating price discrimination strategies as well as for policy makers when designing platform regulation.

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Appendix A. Omitted Proofs: Baseline Model A.1. Proof of Proposition 1

In order to provide some intuition along with the proof, it is helpful to study the platform's dual problem of choosing the participation level on each side to maximize profit, while prices adjust accordingly. The idea of the proof is the following: writing $N_i(N_{-i})$ to denote the profit-maximizing participation level of side $j \in \{B, S\}$ as a function of the total participation on the other side, we will first show that N_i is increasing in both pricing regimes. Then, we will verify that, as in standard models of third-degree price discrimination in one-sided markets, $N_S^U(N_B) = N_S^D(N_B)$: taking buyers' participation as given, price discrimination leaves "output" on the seller side unchanged. Finally, we will show that, taking N_S as given, $N_B^U(N_S) < N_B^D(N_S)$: because the platform can extract more value from sellers under price discrimination, it has an incentive to attract more buyers. This in turn leads to more sellers joining the platform, and so on until the end of the feedback loop.

A.1.1. Uniform Pricing. Under uniform pricing, the platform chooses the quantity of buyers N_B and the total quantity of sellers N_S , and the participation fees adjust accordingly. Inverse demands and the platform's profit are given by (3) and (4), respectively. From first-order conditions (7) and (8) we derive the expression for $\widetilde{N}_B^U(N_S)$ and $\widetilde{N}_S^U(N_B)$:

$$\frac{\partial \Pi^{U}}{\partial N_{B}} = 0 \iff P^{U} + N_{B} \frac{\partial P^{U}}{\partial N_{B}} + N_{S} \frac{\partial F^{U}}{\partial N_{B}} = 0$$
$$\iff \widetilde{N_{B}^{U}}(N_{S}) = \frac{v + bN_{S}}{2} + \frac{(\theta_{H} + \theta_{L})N_{S}}{4}, \tag{A.1}$$

$$\frac{\partial \Pi^{U}}{\partial N_{S}} = 0 \iff F^{U} + N_{S} \frac{\partial F^{U}}{\partial N_{S}} + N_{B} \frac{\partial P^{U}}{\partial N_{S}} = 0$$
$$\iff \widetilde{N_{S}^{U}}(N_{B}) = \frac{(\theta_{L} + \theta_{H} + 2b)N_{B}}{2}.$$
(A.2)

A.1.2. Price Discrimination. Under price discrimination, the platform has an extra instrument, and can thus choose N_H and N_L independently. Inverse demands are given by (5), and the platform's profit is given by (6). From the first-order conditions with respect to the number of sellers, namely, (10) and (11), we obtain $\widetilde{N_L^D}(N_B)$ and $\widetilde{N_L^D}(N_B)$:

$$\frac{\partial \Pi^{D}}{\partial N_{L}} = 0 \iff F_{L}^{D} + N_{L} \frac{\partial F_{L}^{D}}{\partial N_{L}} + N_{B} \frac{\partial P^{D}}{\partial N_{L}} = 0$$
$$\iff \widetilde{N_{L}^{D}}(N_{B}) = \frac{(\theta_{L} + b)N_{B}}{2}, \tag{A.3}$$

$$\frac{\partial \Pi^{D}}{\partial N_{H}} = 0 \iff F_{H}^{D} + N_{H} \frac{\partial F_{H}^{D}}{\partial N_{H}} + N_{B} \frac{\partial P^{D}}{\partial N_{H}} = 0$$
$$\iff \widetilde{N_{H}^{D}}(N_{B}) = \frac{(\theta_{H} + b)N_{B}}{2}.$$
(A.4)

Note that adding (A.4) and (A.3) gives (A.2), so that $N_S^D(N_B) = \widetilde{N_S^U}(N_B)$.

Solving the first-order condition with respect to the number of buyers (9) yields

$$\frac{\partial \Pi^{D}}{\partial N_{B}} = 0 \iff P^{D} + N_{B} \frac{\partial P^{D}}{\partial N_{B}} + N_{L} \frac{\partial F_{L}^{D}}{\partial N_{B}} + N_{H} \frac{\partial F_{H}^{D}}{\partial N_{B}} = 0$$
$$\iff N_{B} = \frac{v + bN_{S}}{2} + \frac{\theta_{H}N_{H} + \theta_{L}N_{L}}{2}.$$
(A.5)

In (A.5), N_B is obtained as a function of N_S , N_H , and N_L . But from (A.4) and (A.3), we know that $\widetilde{N_H^D}(N_B) = \frac{\theta_H + b}{\theta_H + \theta_L + 2b} \widetilde{N_S^D}(N_B)$ and $\widetilde{N_L^D}(N_B) = \frac{\theta_L + b}{\theta_H + \theta_L + 2b} \widetilde{N_S^D}(N_B)$. Therefore, we can rewrite (A.5) as

$$\widetilde{N_B^D}(N_S) = \frac{v + bN_S}{2} + \frac{\theta_H(\theta_H + b) + \theta_L(\theta_L + b)}{2(\theta_H + \theta_L + 2b)} N_S.$$

Because $\theta_H > \theta_L$, simple algebra reveals that $N_B^D(N_S) > \widetilde{N_B^U}(N_S)$: for a given level of seller participation, the platform wants to serve more buyers in the discrimination regime.

Together, these observations imply that discrimination leads first to an increase in N_B , which leads to an increase in N_S , which further increases N_B , etc., until we converge to a point where both buyer and seller participation levels are higher than under uniform pricing.

A.2. Proof of Proposition 2

We first have to compute buyer surplus, sellers' surplus, and total welfare in both scenarios. Platform profits are given by (12) and (15), respectively.

A.2.1. Uniform Pricing. When the platform sets a unique fee, buyer surplus and type $j \in \{L, H\}$ sellers' surplus are respectively given by

$$\begin{split} CS^{U} &= \int_{0}^{N_{B}^{U}(p^{U},f^{U})} (v + b(N_{H}^{U}(p^{U},f^{U}) + N_{L}^{U}(p^{U},f^{U})) - p^{U} - k^{B}) dk^{B} \\ &= \frac{8v^{2}}{(8 - (2b + \theta_{H} + \theta_{L})^{2})^{2}}, \\ DS_{j}^{U} &= \int_{0}^{N_{j}^{U}(p^{U},f^{U})} (\theta_{j}N_{B}^{U}(p^{U},f^{U}) - f^{U} - k^{S}) dk^{S} \\ &= \frac{v^{2}(2b + 3\theta_{j} - \theta_{-j})^{2}}{2(8 - (2b + \theta_{H} + \theta_{L})^{2})^{2}}, \end{split}$$

for a total welfare of

$$SW^{U} = CS^{U} + \Pi^{U} + \sum_{i=1,2} DS_{j}^{U}$$
$$= \frac{v^{2}(24 - (2b + 3\theta_{H} - \theta_{L})(2b - \theta_{H} + 3\theta_{L}))}{(8 - (2b + \theta_{H} + \theta_{L})^{2})^{2}}.$$

A.2.2. Price Discrimination. When the platform charges two different fees, buyer surplus and type $j \in \{L, H\}$ sellers'

surpluses are respectively given by

$$CS^{D} = \int_{0}^{N_{B}^{D}(p^{D}, f_{H}^{D}, f_{L}^{D})} (v + b(N_{H}^{D}(p^{D}, f_{H}^{D}, f_{L}^{D}) + N_{L}^{D}(p^{D}, f_{H}^{D}, f_{L}^{D})) - p^{D} - k^{S})dk^{S}$$

$$= \frac{2v^{2}}{(4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L}))^{2}}.$$

$$DS_{j}^{D} = \int_{0}^{N_{j}^{D}(p^{D}, f_{H}^{D}, f_{L}^{D})} (\theta_{j}N_{B}^{D}(p^{D}, f_{H}^{D}, f_{L}^{D}) - f_{j}^{D} - k^{B})dk^{B}$$

$$= \frac{v^{2}(b + \theta_{j})^{2}}{(b + \theta_{j})^{2}}.$$
(A.6)

$$= \frac{b(b+b_{f})}{2(4-2b^{2}-\theta_{H}^{2}-\theta_{L}^{2}-2b(\theta_{H}+\theta_{L}))^{2}},$$
 (A.7)

for a total welfare of

$$SW^{D} = CS^{D} + \Pi^{D} + \sum_{i=1,2} DS_{j}^{D}$$
$$= \frac{v^{2}(12 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L}))}{2(4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L}))^{2}}.$$
 (A.8)

We will now prove the three points of Proposition 2, taking into account the admissible parametric region defined by Assumption 1,

(i) By a revealed preference argument, the platform is necessarily better off under price discrimination. Formally:

$$\Pi^{D} - \Pi^{U} = \frac{v^{2}(\theta_{H} - \theta_{L})^{2}}{(4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} - \theta_{L}))(8 - (2b + \theta_{H} + \theta_{L})^{2})} > 0$$

That buyers are also better off is a corollary of Proposition 1. Regarding low-type sellers, one can check that, in the admissible parametric region,

$$DS_{L}^{D} - DS_{L}^{U} = \frac{1}{2}v^{2} \left(\frac{(2b + 3\theta_{L} - \theta_{H})^{2}}{(8 - (2b + \theta_{H} + \theta_{L})^{2})^{2}} - \frac{(b + \theta_{L})^{2}}{(4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L}))^{2}} \right) > 0.$$

(ii) Turning to total welfare, we obtain that

$$SW^D - SW^U$$

$$=\frac{b^{2}(\theta_{H}-\theta_{L})\lambda}{2(4-2b^{2}-\theta_{H}^{2}-\theta_{L}^{2}-2b(\theta_{H}+\theta_{L}))^{2}(8-(2b+\theta_{H}+\theta_{l})^{2})^{2}}$$

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 $(2)^{21}$

where $\lambda = 32 - 24b^4 - 7\theta_H^4 + 2\theta_H^3\theta_L + 28\theta_L^2 - 7\theta_L^4 - 48b^3(\theta_H + \theta_L) - 2\theta_L\theta_H(12 - \theta_L^2) + 14\theta_H^2(2 - \theta_L^2) + 2b(\theta_H + \theta_L)(16 - 13\theta_H^2 + 2\theta_H\theta_L - 13\theta_L^2) + 2b^2(16 - 25\theta_H^2 - 22\theta_L\theta_H - 25\theta_L^2).$

Notice that the sign of λ determines the sign of the difference in total welfare. Equating λ to 0 and solving for *b*, we get four solutions, but only one is positive and given by

$$b^{sol}(\theta_{H}, \theta_{L}) = \frac{\sqrt{6}\sqrt{16 - 7(\theta_{H} - \theta_{L})^{2} + \sqrt{1024 + 256(\theta_{H} - \theta_{L})^{2} + (\theta_{H} - \theta_{L})^{4}}}{12}$$

Further, this $b^{sol}(\theta_H, \theta_L)$ is greater than the upper bound of our feasible region $\overline{b}(\theta_H, \theta_L, v)$.

Next, differentiating the expression for λ with respect to *b* and computing it at $b = b^{sol}$, we get

$$\left.\frac{\partial\lambda}{\partial b}\right|_{b=b^{\rm sol}} = -\sqrt{\frac{2}{3}}\sqrt{g(16-7(\theta_H-\theta_L)^2+g)} < 0,$$

with $g = \sqrt{1024 + 256(\theta_H - \theta_L)^2 + (\theta_H - \theta_L)^4}$. Regardless of whether λ is convex or concave in b, for $b < \overline{b}(\theta_H, \theta_L, v) < b^{sol}(\theta_H, \theta_L)$, we must have $\lambda > 0$. We then confirm that the social welfare is higher under price discrimination than under uniform pricing.

(iii) This point follows from the comparison of DS_H^U and DS_H^D . We obtain that

$$DS_{H}^{D} - DS_{H}^{U} = \frac{1}{2}v^{2} \left(\frac{(2b + 3\theta_{H} - \theta_{L})^{2}}{(8 - (2b + \theta_{H} + \theta_{L})^{2})^{2}} - \frac{(b + \theta_{H})^{2}}{(4 - 2b^{2} - \theta_{H}^{2} - \theta_{L}^{2} - 2b(\theta_{H} + \theta_{L}))^{2}} \right) > 0$$

if and only if $b > \hat{b}(\theta_H, \theta_L) = \frac{\sqrt{32 - 7(\theta_H - \theta_L)^2 - 3\theta_H - \theta_L}}{4}$. Further, notice that $\frac{\partial \hat{b}(\theta_H, \theta_L)}{\partial \theta_H} < 0$, thus explaining why the parametric region with Pareto improvement enlarges when θ_H increases, as we can see in Figure 3 when comparing panel (a) with panel (b).

A.3. Proof of Proposition 3

Considering the conditions specified on Assumption 1, which define our feasible parametric region, we compare buyers' and sellers' participation fees across the two regimes.

Starting from buyers, we find that

$$p^D > p^U \Longleftrightarrow b > \tilde{b}(\theta_H, \theta_L) = \frac{\sqrt{16 + (\theta_H + \theta_L)^2} - \theta_H - \theta_L}{4},$$

with $\hat{b}(\theta_H, \theta_L)$ admissible in the feasible region when θ_H is not very large. Hence, provided the high-type seller's valuation for the buyer is not excessive, there exists a threshold value of *b* above which buyers pay a higher price under price discrimination. This represents another novel result of our analysis, as we prove that buyers may end up paying more under price discrimination. Remember that, by Proposition 1, participation of both sides increases under price discrimination. When *b* is low, that is, when buyers do not highly value seller participation, attracting more of them requires lowering their price, and this could even be achieved through subsidization. On the contrary, when *b* is high, the increased seller participation is enough to attract more buyers, and the platform can also increase the price buyers have to pay.

Turning to sellers, we first obtain that

$$\begin{split} f_L^D &< f_H^D < f^U \Longleftrightarrow b > b_H(\theta_H, \theta_L) \\ &= \frac{\sqrt{32 + (\theta_H - \theta_L)(9\theta_H + 7\theta_L)} - 3\theta_H - \theta_L}{4} \end{split}$$

with $b_H(\theta_H, \theta_L)$ admissible in the feasible region when both θ_H and θ_L are sufficiently low. Then,

$$\begin{aligned} f^{U} < f_{L}^{D} < f_{H}^{D} \Leftrightarrow b > b_{L}(\theta_{H}, \theta_{L}) \\ = \frac{\sqrt{32 - 7\theta_{H}^{2} - 2\theta_{H}\theta_{L}} - \theta_{H} - 3\theta_{L}}{4}, \end{aligned}$$

with $b_L(\theta_H, \theta_L)$ admissible in the feasible region when both θ_H and θ_L are sufficiently high.

Finally, $f_L^D < f^U < f_H^D$ for all remaining admissible parameter values, which reproduces a well-known result in the traditional one-sided market literature (Robinson 1933): price discrimination raises the price for the high type, whereas it lowers that for the low type. This applies to the platform context that we consider, provided the sellers' values for buyer participation are neither too small nor too big. Conversely, if sellers show more extreme attitudes toward the presence of buyers, the conventional result can be overturned, as we obtained above. On the one hand, there is a region in which both types of sellers pay less under price discrimination. More precisely, when $b_H(\theta_H, \theta_L)$ is admissible, $f_L^D < f_H^D <$ $f^{U} < 0$ if and only if $b > b_{H}(\theta_{H}, \theta_{L})$: both types of sellers are subsidized to join the platform, and such subsidy increases under price discrimination. On the other hand, when $b_L(\theta_H, \theta_L)$ is admissible, then both types of sellers pay a higher price under price discrimination if $b > b_L(\theta_H, \theta_L)$: 0 < $f^U < f^D_L < f^D_H.$

By considering buyers and sellers together, we summarize our main results on comparing prices across the two regimes as follows (see also Figure 4):

(i) When θ_H and θ_L are relatively low and $b > b_H(\theta_H, \theta_L)$: $f_L^D < f_H^D < f^U < 0$ and $p^D > p^U > 0$. Conversely, when $b < b_H(\theta_H, \theta_L), f_L^D < f^U < f_H^D$ (with subsidies for sellers when b is high enough, and $p^D > p^U$ when $b > \tilde{b}(\theta_H, \theta_L)$).

(ii) For intermediate values of θ_H , we always have $f_L^D < f^U < f^D_H$, and $p^D > p^U$ when $b > \tilde{b}(\theta_H, \theta_L)$.

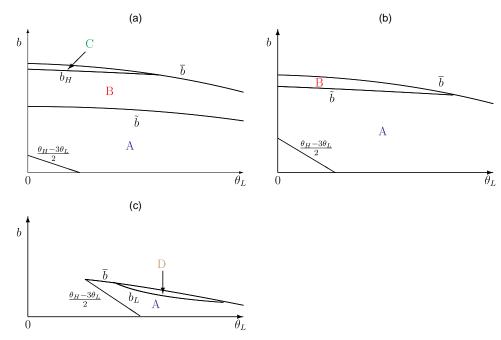
(iii) When θ_H and θ_L are relatively high and $b > b_L(\theta_H, \theta_L)$: $0 < f^U < f^D_L < f^D_H$ and $p^D < p^U < 0$. Conversely, when $b < b_L(\theta_H, \theta_L), f^D_L < f^U < f^D_H$, with $p^D < p^U$ (with subsidies for buyers only when b is high enough).

Starting from point (i), price discrimination enables the platform to charge a high price to buyers, who highly value seller participation, in order to increase the subsidy for both types of sellers. The fact that the fees are negative implies that the platform can subsidize sellers more than under unique pricing in order to attract them, as their value for buyer participation is particularly low. When *b* is lower, we obtain the standard result that $f_L^D < f^U < f_H^D$.

Turning to point (ii), when the high-type sellers' value for buyer participation is intermediate, we obtain the standard result: price discrimination increases the price for high-type sellers, whereas it lowers that for low-type sellers. As per buyers, we still find a region in which they end up paying more with price discrimination (when $b > \tilde{b}(\theta_H, \theta_L)$), but this region shrinks in comparison with point (i).

Finally, in case (iii), when sellers' value for buyer participation is high, we find the interesting case in which price discrimination enables subsidizing buyers more than under uniform pricing, and this is possible as a higher fee is

Figure A.1. (Color online) Pricing Regimes



Notes. (a) Values are v = 0.1 and $\theta_H = 0.5$. (b) Values are v = 0.1 and $\theta_H = 0.8$. (c) Values are v = 0.1 and $\theta_H = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H^U < f_H^U < f_H^U < f_H^U < f_H^U = 1.4$. Uppercase letters: A, $f_L^D < f_H^U < f_H$

imposed on both types of sellers. This occurs when $b > b_L(\theta_H, \theta_L)$. The fact that this scenario requires a sufficiently high value for *b* can be explained by the fact that buyers need to have a sufficiently high value for seller participation in order for the platform to decide to increase their subsidy at the expense of sellers. When $b < b_L(\theta_H, \theta_L)$, we obtain the standard result $f_L^D < f^U < f_H^D$, with $p^D < p^U$; buyers are subsidized only when *b* is high enough. In any case, they pay a lower participation fee (or obtain a higher subsidy) under price discrimination.

In Figure A.1 we illustrate the different cases.²⁰ The standard results are obtained in region A. In region B, the platform's optimal strategy is to increase the price paid by buyers, while still moving f_L^D and f_H^D in opposite directions. In region C, sellers get a relatively low per-buyer benefit compared with buyers' per-seller benefit, and are subsidized under both regimes. Price discrimination induces the platform to increase the subsidy to both seller types and to charge a higher price to buyers. In region D, θ_H and θ_L are relatively high compared with *b*, and the platform increases fees for both types of sellers, while at the same time increasing the subsidy to buyers.

A.4. Proof of Proposition 4

If Assumption 1 (ii) does not hold, that is, if $b \le \frac{\theta_H - 3\theta_L}{2}$, under uniform pricing we have to impose $N_L = 0$ since the beginning of our analysis and recompute the equilibrium.²¹ Participation fees under uniform pricing are obtained as

$$f^{U} = \frac{v(\theta_{H} - b)}{4 - (b + \theta_{H})^{2}} - 1, p^{U} = \frac{v(2 - \theta_{H}(b + \theta_{H}))}{4 - (b + \theta_{H})^{2}}$$

The profit-maximizing levels of participation for sellers and buyers are

$$N_L^U = 0, N_H^U = \frac{v(b+\theta_H)}{4-(b+\theta_H)^2}, N_B^U = \frac{2v}{4-(b+\theta_H)^2}.$$

Equilibrium profits and relevant welfare measures are as follows:

$$\Pi^{U} = \frac{v^{2}}{4 - (b + \theta_{H})^{2}}, DS_{H}^{U} = \frac{v^{2}(b + \theta_{H})^{2}}{2(4 - (b + \theta_{H})^{2})^{2}}, DS_{L}^{U} = 0;$$
$$CS^{U} = \frac{2v^{2}}{(4 - (b + \theta_{H})^{2})^{2}}, SW^{U} = \frac{v^{2}(12 - (b + \theta_{H})^{2})}{2(4 - (b + \theta_{H})^{2})^{2}}$$

Under price discrimination, as we know from the analysis carried out in Subsection 4.2, equilibrium prices and participation levels are given by (14) and (13), respectively. We recall the expressions for participation levels and specify those for high-type and low-type sellers:

$$\begin{split} N_{L}^{D} &= \frac{v(b+\theta_{L})}{4-2b^{2}-\theta_{H}^{2}-\theta_{L}^{2}-2b(\theta_{H}+\theta_{L})},\\ N_{H}^{D} &= \frac{v(b+\theta_{H})}{4-2b^{2}-\theta_{H}^{2}-\theta_{L}^{2}-2b(\theta_{H}+\theta_{L})},\\ N_{B}^{D} &= \frac{2v}{4-2b^{2}-\theta_{H}^{2}-\theta_{L}^{2}-2b(\theta_{H}+\theta_{L})}. \end{split}$$

The platform profit is given by (15), whereas sellers' surplus and relevant welfare measures are given by (A.7), (A.6), and (A.8), respectively.

First notice that all three participation levels under price discrimination are larger than those under uniform pricing, as can be easily ascertained. This implies that total participation increases. The platform gains under price discrimination, and so do low-type and high-type sellers, even if the latter end up paying a higher price.²² Welfare is therefore higher under price discrimination, which always induces a strict Pareto improvement.

A.5. Proof of Proposition 5

If Assumption 1 (i) does not hold, then we have $N_B^{U*} = N_B^{D*} = 1$. Notice that $N_B^{D*} = 1$ when $b \ge \overline{b}(\theta_H, \theta_L, v)$ and $N_B^{U*} = 1$ when $b \ge \sqrt{2-v} - \frac{(\theta_H + \theta_L)}{2} > \overline{b}(\theta_H, \theta_L, v)$, where \overline{b} is specified below Assumption 1. This is explained by the fact that buyer participation increases at equilibrium under price discrimination, as we know from Proposition 1.

In this case, participation fees under uniform pricing are given by

$$f^{U} = \frac{\theta_{H} + \theta_{L} - 2b}{4}, \ p^{U} = v + b^{2} + \frac{b(\theta_{H} + \theta_{L})}{2} - 1.$$

The associated demands are

$$N_B^U = 1, N_H^U = \frac{2b + 3\theta_H - \theta_L}{4}, N_L^U = \frac{2b + 3\theta_L - \theta_H}{4}.$$

Total welfare is given as

$$SW^{U} = v + \frac{12b^{2} + 7(\theta_{H}^{2} + \theta_{L}^{2}) - 2\theta_{H}\theta_{L} + 12b(\theta_{H} + \theta_{L})}{16} - \frac{1}{2}$$

Under price discrimination, the fees are given as

$$f_L^D = \frac{\theta_L - b}{2}, f_H^D = \frac{\theta_L - b}{2}, p^D = v + b^2 + \frac{b(\theta_H + \theta_L)}{2} - 1$$

The associated demands are

$$N_B^D = 1, N_H^D = \frac{b + \theta_H}{2}, N_L^D = \frac{b + \theta_L}{2}.$$

Total welfare can be easily computed:

$$SW^{D} = v + \frac{3(2b^{2} + \theta_{H}^{2} + \theta_{L}^{2} + 2b(\theta_{H} + \theta_{L}))}{8} - \frac{1}{2}$$

It is straightforward to note that total participation of sellers stays the same in the two pricing regimes:

$$N_H^U + N_L^U = N_H^D + N_L^D = b + \frac{\theta_H + \theta_L}{2}.$$

Subtracting SW^D from SW^U yields

$$SW^{U} - SW^{D} = \frac{(\theta_{H} - \theta_{L})^{2}}{16} > 0$$

Thus, we show that total welfare falls under price discrimination.

Endnotes

¹ See https://sell.amazon.com/pricing and https://www.ebay.co.uk/ help/selling/fees-credits-invoices/fees-business-sellers?id=4809.

² See, for instance, https://www.mastercard.us/content/dam/public/ mastercardcom/na/us/en/documents/merchant-rates-2022-2023apr22-2022.pdf.

³ The Microsoft Store applies different fees to games (30%) and nongame applications (usually 15%). More examples can be found in Borck et al. (2020). ⁴ Cowan (2016) identifies families of demand functions for price discrimination to raise total output and welfare.

⁵ In that case the Pareto improvement is weak, because consumers in the market that is served under uniform pricing are indifferent.

⁶ Bouvard et al. (2022) and Gambacorta et al. (2023) study platform lending as a way to price-discriminate sellers.

⁷ This approach is standard in the literature. See also Jullien et al. (2021), Belleflamme and Peitz (2024), Reisinger (2014), Shekhar (2021), and Carroni et al. (2024), among others. In addition, it is well documented that ad valorem fees represent a form of price discrimination (see Wang and Wright 2017). We elaborate more on this point in Subsection 5.3, where we show that our insights extend to the case of ad valorem fees.

⁸ Amazon subscribers, for instance, enjoy a range of exclusive offers, including free premium delivery, targeted deals and discounts, and access to streaming media content.

⁹ In other words, there is no competition among sellers.

¹⁰ Hence, we assume that the platform only attracts sellers who generate a positive utility for the buyers.

¹¹ A possible microfoundation is as follows: suppose all marginal costs are zero. A buyer's willingness to pay for the product of a seller of type θ is equal to θ with probability x, and to $\theta + b/(1 - x)$ with probability 1 - x, with $x > b/\theta_L$. The optimal price charged by a seller θ is equal to θ . In this case, per-interaction profit is indeed equal to θ whereas buyers' expected per-interaction surplus is b.

¹² Under our assumptions there can be only two equilibria for a given set of fees, one of which has no participation (and is unstable). By focusing on a quantity-setting monopolist, we sidestep this issue, in a way reminiscent of the concept of insulated equilibrium (Weyl 2010).

Formally, we have
$$\frac{\partial \Pi}{\partial N_B \partial N_S} = \frac{\partial P}{\partial N_S} + \frac{\partial F^*}{\partial N_B} > 0$$
.

¹⁴ Participation of sellers of type $j \in \{L, H\}$ under uniform pricing is $N_j^{II} = \frac{v(2b+3\theta_j - \theta_{-j})}{8 - (2b+\theta_H + \theta_L)^2}.$

¹⁵ Participation of sellers of type $j \in \{L, H\}$ under price discrimination is $N_j^D = \frac{v(b+\theta_j)}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}$.

¹⁶ In this figure and in the following ones we omit the arguments of the threshold values of b for simplicity.

¹⁷ Also notice that the feasible region decreases in θ_{H} , as can be derived from Assumption 1.

¹⁸ This principle is familiar (Armstrong 2006, Belleflamme and Peitz 2021). See also Benzell and Collis (2022) for a recent contribution with an application to social media.

¹⁹ For instance, if the participation cost for type *j* sellers is uniformly distributed over $(-\alpha_i, -\alpha_j + 1)$.

²⁰ We omit the arguments of the threshold values of *b* for simplicity.

²¹ It can be easily verified that in this case the platform has no incentives to serve some low-type sellers.

²² More calculations are available upon request, together with the conditions that ensure that all remaining participation levels are between zero and one.

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