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The influence of linguistic background on early representations of multiplicative comparison

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Linguistic aspects involved in young children's interpretation of multiplicative comparison situations are investigated by comparing the answers provided by children aged 5-6 years old from three different countries: Australia, Italy, and Israel. Two tasks with different wording are used to analyse how language differences affect children's performance in terms of number of correct answers and most common wrong answers. The study shows that the number of children who can provide correct answers to multiplicative comparison tasks before their formal introduction change significantly according to the linguistic background. Furthermore, the analysis of wrong answers suggests that several possible linguistic factors may intervene including vocabulary knowledge, familiarity with phrases, and differences in the way that languages convey multiplicative meanings.

Keywords: Kindergarten, language, multiplicative comparison, representations.

Introduction

There is a growing (but still scarce) body of literature about how young children represent intuitively multiplicative semantic structures before their formal introduction in school. Through an exploratory study, the first and third authors of this contribution found that by the age of 6, many children can represent correctly different multiplicative situations including equal groups, rectangular arrays, multiplicative comparison, and allocation situations (Downton & Maffia, 2023). In the context of that study, multiplicative comparison (MC) problems were more difficult for the children than the other multiplicative situations. The authors attributed this to children's lack of familiarity with the phrase 'times-as-many', which Nesher (1988) also found. Evidence of this was that the Italian children (for whom this phrase is very unfamiliar, see the Method section) found MC situations more difficult to interpret than Australian children. This observation may lead to the relevant educational implication that familiarity with the words used to describe MC could be a prerequisite for accessing MC problems (Vanluydt et al., 2020), but this remains a conjecture.

Seeking to better understand this phenomenon, this contribution presents a study, which replicated interviews similar to those by Downton and Maffia (2023) but compared the Italian and Australian contexts with another linguistic background: the Hebrew language. Our research question is: To what extent do linguistic aspects influence young children's answers to MC word problems?

Multiplicative comparison and language

Multiplicative comparison, commonly referred to as 'times-as-many' in research undertaken in English, is a different application of multiplication from that of Equal Groups or Rectangular Array, as it refers to a comparison or an enlargement that may apply to either discrete objects or to a

continuous medium, rather than an iteration of equal groups (Anghileri & Johnson, 1992; Greer, 1992). In short, MC involves two different groups being compared to each other. For example, *John has 3 times-as-many apples as Mary. If Mary has 4 apples, how many apples does John have?* In this instance the number of apples Mary has is being compared to how many John has. In this instance the multiplicative factor (3) may be considered as the multiplier. In essence the MC relationship includes three elements that relate to each other: one quantity is a unit of measure (Mary's apples-4), another is the measured quantity (John's apples-12) and the third is the coefficient (the multiplied measure, which in this context is 3) (Polotskaiša & Savard, 2020). The same problem may also be viewed in terms of ratio or a many-to-one correspondence (3 apples of John's for every one apple of Mary's) which makes 4 the multiplier. Greer (1992) purported that *multiplicative comparison* or '*n times-as-many as*' to be a preliminary stage to ratio and has the advantage over the *groups of* aspect as it relates directly to the nature of multiplication.

While some studies found that young children (6 and 7 years old) had difficulty interpreting this semantic structure (Anghileri, 1989; Mulligan & Mitchelmore, 1997), Downton and Sullivan (2017) found that a high proportion (64%) of Grade 3 children (8 and 9 years old) successfully solved comparison problems using multiplicative strategies.

From a linguistic perspective, Nesher (1988) maintained that an understanding of the textural structure of word problems is critical to interpreting the underlying mathematical structure of the problem. From her analysis of the differences in the propositional structure of three types of multiplicative word problems, Nesher noted that for MC (scalar problems), both the order of the text and the language of 'times-as-many' must be considered. Further, the language of this phrase may confuse students (Anghileri, 1989; Nesher, 1988). Others suggested that it is not uncommon for students to add the two given numbers rather than multiply to solve one-step MC word problems (González-Calero et al., 2015; Nunes et al., 1993).

Several studies have shown that language development and numeracy development are related, and that language skills may account for variation when predicting early numeracy skills (Purpura & Ganley, 2014). This makes sense as understanding and "doing" mathematics necessitates the use of language, and specifically the use of mathematical terms such as more, less, and number names. Studies have also shown that different languages may assist or deter children from understanding number concepts. For example, Kung et al. (2019) found that structural language differences between Chinese and English provide different supports for learning mathematics in preschool, especially since numeracy-related words are embedded in Chinese natural language. Indeed, studies have shown that one of the challenges of teaching mathematics at all ages is bridging between natural, sometimes called informal, language, and the more formal, or technical, language of mathematics. For example, in Greek, the mathematical term for set can have three different meanings in their natural language (Antonopoulos, 2023).

Related to the linguistic perspective are the cultural differences, which also impact on children's interpretation of the phrase '*n times as many*' in relation to MC. For example, in Spanish the translation is '*n times more than*' (increasing comparison) and '*n times less*' (decreasing comparison), which is the case for other languages (e.g., Hebrew, Czech, and Arabic) (Nesher et al.,

2003). These subtle differences indicate that MC is expressed by a different construction to that used in English. In fact, these alternative phrases would be considered grammatically incorrect or confusing to English speakers (González-Calero et al., 2015). Another issue related to the current study is that the same concept, in this case multiplication, may be expressed verbally using different terminology, giving rise to different meanings. Language such as 4 times 6 does not activate conceptual meaning, while saying instead, 4 groups of 6, infers the equal groups semantic structure of multiplication (Downton & Maffia, 2023). Hence, the available literature suggests that vocabulary and wording can affect young children’s understanding of comparison tasks. In this paper we investigate to what extent such linguistic aspects influence children’s responses to MC problems. We will ascertain this by comparing answers provided by children from three different countries to two different MC word problems.

Methods

We conducted the same interview in three countries: Australia, Israel, and Italy. The interview consisted of several tasks including different multiplicative semantic structures. In this report we will focus only on the MC tasks presented in Table 1.

Table 1: The two tasks in the three languages involved in the study

	English version	Hebrew version	Italian version
Apples task	Mary has 4 apples and John has double the amount. How many apples does John have?	לגל יש 4 תפוחים ולתומר יש כמות לתומר. כמה תפוחים יש כפולה?	Maria ha 4 mele e Giovanni ne ha il doppio . Quante mele ha Giovanni?
Towers task	Mary made a tower using three blocks. Max made a tower three times as tall . How many blocks did Max use?	גל בנתה מגדל באמצעות שלוש פי קוביות. דן, בנה מגדל בגובה יותר מגל. בכמה קוביות שלושה השתמש דן?	Maria ha fatto una torre usando tre blocchi. Max ha fatto una torre tre volte più alta . Quanti blocchi ha usato Max?

Both tasks present a MC situation but they differ in several aspects. Different numbers are involved and, while both situations involve discrete quantities, the Towers task can also be interpreted in terms of linear comparison (in terms of the height of the towers). In the Apples tasks, the word ‘double’ is used in all three languages; in the Towers tasks we did our best to translate the English ‘times-as-many’ into similar phrases in Hebrew and Italian.

Children in the study were aged between 5 and 6 years old. Convenience sampling was used to select the schools that already had been in contact with the researchers for past projects. Children for whom parent consent was provided were interviewed. The Australian sample consisted of 39 children, the Israeli sample 37 children, and the Italian sample 43 children. Each sample was coded according to the country in which they attended school and where English, Hebrew, and Italian are respectively – the language of teaching and the native language for most (but not all) children.

Australian children were interviewed during their Foundation year, the first year of school. Within Victoria children must turn 5 before April 30 of the year they start school. Multiplication is introduced in Year 1 (Grade 1) with a focus on equal group situations (Victorian Curriculum and Assessment Authority [VCAA], 2024). In Foundation the focus is on quantifying and comparing collections to at least 20, partitioning and combining collections up to 10 using part whole relationships and subitizing skills, and practical situations involving partition division. While multiplication is not formally taught in Foundation, children become familiar with the language of ‘doubles’ in the context of addition and may experience the language of ‘times as many’ in measurement contexts when comparing lengths.

Children in Israel were interviewed during the last months of Kindergarten. In Israel, there is a compulsory mathematics curriculum for children aged 4-6 years old (Israel National Mathematics Preschool Curriculum, 2010). While learning multiplication is not an explicit aim for kindergarten, the curriculum states that other numerical skills, such as skip counting and creating equal groups of objects, are a prelude to multiplication which will be learned later. The language of teaching is Hebrew, the first language of all children participating in this study. In Hebrew, the word ‘times,’ as in ‘five times three,’ is ‘kaful.’ However, in a different context, this same term could mean ‘double,’ or ‘twice as many,’ as in Esther has double (‘kaful’) what Ann has. There is no one term for triple or quadruple, and instead, those meanings must be said as ‘three times (or four times) as many,’ but here, the term for “times” is not the same ‘kaful’ as used above. Instead, there is another term which literally translates as ‘bigger by 3’. Finally, when children are first introduced to multiplication in first or second grade, they learn to count the number of equal groups (harking back to kindergarten) and thus instead of immediately using the expression ‘kaful’ for ‘three time five,’ they use another term ‘pa’amim,’ which loosely translates as ‘three repetitions of five’.

Italian children were interviewed during their last months of Kindergarten, which is not compulsory schooling in Italy. There is no subject-related teaching and the national curriculum for Kindergarten does not refer to Mathematics directly (Italian National Standards for K-8, 2012). There are recommendations for teachers to work on the procedures of counting and measuring as well on recognizing geometrical shapes. The language of teaching is Italian even if children from migrant families (mainly second generation) are present in almost all class-groups. The use of multiplicative numeral adjectives as ‘doppio’, ‘triplo’, etc. (‘double’, ‘triple’ in English) is common in the Italian language. In Italian grammar, the use of ‘volte’ (meaning ‘times’ as in ‘two times’ or ‘five times’) is considered more common than the corresponding multiplicative adjective only when the number is higher than five. Even in those cases, the phrase ‘times-as-many’ is still uncommon. The Italian word ‘volte’ has several meanings. In the most popular Italian dictionaries, the meaning as ‘repetition’ is never the first or second one presented and when examples using the word are provided, they include phrases such as “l’ho detto tre volte” (“I said that three times” in English), but the equivalent of the phrase ‘times-as-many’ (which would be ‘tante volte quanto’) does not appear in any of the dictionaries consulted by the third author.

During the interviews in all countries, a research assistant presented verbally the situation and asked the child to provide an answer. For each task half of the children were requested to represent the situation using manipulatives, while the other children were provided with paper and pencil and

prompted to draw the situation. In this way, we could avoid the effect of the means of representation (being a manipulative or a drawing, cf. Downton & Maffia, 2023).

Results

Results from the interviews are summarized in Table 2 indicating the number of correct answers for each task in the three samples.

Table 2: Correct answers in the three samples (corresponding percentages are indicated in brackets)

	Australian sample	Israeli sample	Italian sample
Apples task	14 (36%)	5 (13%)	3 (7%)
Towers task	16 (41%)	2 (5%)	1 (2%)

It is apparent, there is a strong difference: Australian children were able to answer correctly to both tasks much more frequently than Israeli and Italian children. Furthermore, the Apples task appeared more difficult for the Australian children than the Towers task, while this was not the case for the Israeli and Italian children. This is possibly because discrete quantities are involved and the word ‘double’ may be less familiar to the Australian children than the phrase ‘times-as-tall’. As noted in the Methods section, the Israeli and Italian languages differ from English in how the phrase ‘times-as-many’ is presented, and those phrases may be less familiar for children aged 5-6 years old than the words equivalent to ‘double’. Indeed, we had examples of children asking about the meaning of the words; in those cases, the interviewer asked the children what they thought. In some cases, the children replied that they just did not know the word.

For a deeper interpretation of these results, we analysed the most common answers provided by children when the given answer was incorrect. Table 3 summarizes the percentage of selection for the most common wrong answers to both tasks across the three samples. Again, we can observe different phenomena in the three samples: the most common wrong answer is not always the same. In the case of the Apple task, the answer 6 (corresponding to the sum of the involved numbers) was the most common for Australian children, but Italian and Israeli children opt more often for 4 (the first number mentioned in the task). Several children were able to represent the numbers 4 and 2 involved in the Apple task, but then they were not able to answer the question as expected; some of them merely counted all.

As in the Apples task, the first number involved in the Towers task (3) was a common incorrect answer (especially in the Israeli sample), but it was not the most common. For this task, the sum of the two involved numbers ($3+3=6$) was the most common answer not only for Australian children, but for Israeli and Italian ones as well. The language in the Hebrew and Italian translations of the expression ‘times-as-many’ directly implies that the second tower is taller than the first one. The necessity to increase the tower’s height may have prompted the children to sum the two given numbers. This interpretation may explain also the presence of answers such as 4 and 5. Indeed, we have examples of children that, when asked to justify their answers, replied “because the tower is big” or “because 3 and another 1”. Another explained, “I think it is 4 because it is more than 3”.

Table 3: Relative frequency (over the total of each sample) for the most frequent wrong answers

	Apples task (correct answer $4 \times 2 = 8$)			Towers task (correct answer $3 \times 3 = 9$)		
	Australian	Israeli	Italian	Australian	Israeli	Italian
Answer “3”	0%	11%	7%	5%	16%	7%
Answer “4”	18%	19%	26%	8%	11%	9%
Answer “5”	10%	5%	9%	0%	11%	14%
Answer “6”	23%	14%	9%	36%	19%	14%

Discussion and Conclusion

We acknowledge that this task may be difficult for young children of this age, but the aim of this study was to investigate the extent to which linguistic aspects influence young children’s answers to MC word problems. We compared the answers provided to two different word problems across three samples of 5-6 years old children from three countries with different languages: English, Hebrew, and Italian. The three languages differ in how multiplicative numeral adjectives are used. Furthermore, the three countries differ in how mathematics is taught to 5-6 years old children. These differences are reflected in the results.

Overall, the Australian sample performed much better than the other two. Furthermore, the analysis of most common incorrect answers provides hints about differences in children’s interpretation of MC as presented in the two tasks. In particular, we noticed that Australian children tended to sum the two numbers involved in the tasks; this phenomenon may stem from the fact that addition is already introduced in Foundation year. In the case of Israeli and Italian children, adding was more common in the Towers task. This may be related to the fact that the presence of two numbers is more transparent in that task, whereas in the Apple task the number 2 is represented by the word ‘double’ in a less obvious way. Another explanation is that in the Italian and Israeli translation of ‘times-as-tall’ there is an explicit reference to the second tower being taller or bigger, as noted by some of the children when justifying their answers.

The general difficulty for Italian and Israeli students to provide a correct answer to these tasks may be explained by their lack of understanding of some words (e.g., ‘double’ for some Israeli children) or lack of familiarity with phrases such as ‘times-as-many’ (uncommon in Italian everyday language). Having an understanding of meaning of terms such as these is critical to interpreting a word problem (Nesher, 1988). We believe that linguistic aspects (as an expression of the more general cultural contexts) can explain most of the observed differences. Indeed, different languages have dissimilar features in addressing mathematical concepts that may, as a consequence, result in being more or less familiar to young children. For instance, in Hebrew there is a suffix that can be added to words to express the idea of two-ness (as being in a pair or couple) in the same way that in Latin the prefix ‘bi’ was used to make explicit the presence of two elements (as the word

‘binoculars’ which is composed of ‘bini’ and ‘oculus’, the Latin words for ‘two’ and ‘eye’). This kind of suffix is common in Hebrew everyday language, which is not the case for ‘bi’ in English or Italian. Furthermore, there are everyday words (like ‘pair’ or ‘couple’), which may express similar meanings but may be more or less familiar to young children according to their linguistic and cultural background.

While preparing the tools for this study, the authors had to discuss thoroughly both language and cultural issues. For instance, we discussed which objects could be referred to in the texts of the tasks so that they would be familiar to children of all three samples. We believe that our study can contribute to the body of literature about comparative studies; in particular, some of our observations point to the difficulty of translating tasks with fidelity from one language to another, especially for very young children. This is also an important limitation of our study: comparability of the results from the different samples is partial and highly dependent on the different linguistic, cultural, and social contexts in which the data have been collected.

The interpretation provided for the results suggest that Kindergarten teachers can work on different linguistic aspects to promote children’s familiarity with mathematical language, which will appear later in schooling (Downton & Maffia, 2023; Vanluydt et al., 2020). For instance, reading story books may help in presenting different words (both ‘two times’ and ‘double’) in contexts that make sense to children.

The analysis presented in this contribution has referred only to correct/incorrect answers to the task and qualitative data have only been used to provide a few examples. A further qualitative analysis may provide additional information about the influence of linguistic background on early representations of multiplicative comparison. In particular, we must recognize that there are cases where the child was able to produce a coherent representation (by drawing or using manipulatives) of the situation even if the provided answer was not correct. Furthermore, we did not distinguish between the answers provided by drawing or manipulatives; we believe that those cases deserve attention in future research.

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