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Comparison of Soil Water Content Estimation Equations using Ground Penetrating Radar

Anbazhagan P.[1], Marco Bittelli[2,*], Pallepati Raghuveer Rao1 and Puskar Mahajan[1]

[1]Department of Civil Engineering, Indian Institute of Science, Bangaluru, India

[2]Department of Agricultural and Food Sciences, University of Bologna, Italy

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* Corresponding author. Department of Agricultural and Food Sciences, University of Bologna, Italy, Viale Fanin, 44, 40100 Bologna, Italy. Phone:+39-051-2096644, Fax:+39-051-2096641,E-mail: marco.bittelli@unibo.it; (M. Bittelli)

1. Abstract

Soil water content (SWC) has an important impact on many fundamental biophysical processes. The quantification of SWC is necessary for different applications, ranging from large-scale calibration of global-scale climate models to field and catchemnt scale monitoring in hydrology and agriculture. Many techniques are available today for measuring SWC, ranging from point scale soil water content sensors to global scale, active and passive, microwave satellites. Geophysical methods are important methods used for several decades to measure SWC at different scales. Among these methods, Ground Penetrating Radar has been shown to be one the most reliable and promising methods. Soil water content measurement using Ground Penetrating Radar requires the applications of parametric equations that will convert the measured dielectric permittivity to water content. While several tests have been performed to test equations for soil water content sensors such as Time Domain Reflectometry sensors, a few studies have been performed to test different formulae for application to Ground Penetrating Radar. In this study, we compare available formulae for converting dielectric permittivity obtained from detailed laboratory scale measurement of reflected waves using Ground Penetrating Radar. Four soils covering a wide range of textures were used and the measured soil water contents were compared with values obtained from gravimetric measurements. Results showed that the dielectric mixing model of Roth (1990) provided the best fit both for individual soil textural classes and for all soils combined, the latter with RMSE = 0.038 m^3 m^{-3}. Sensitivity analysis was then performed to provide detailed information for the most accurate application of the selected model.

Keywords: Ground Penetrating Radar, Soil Water Content, Dielectric Mixing Models, Em-

pirical Equations, Two-way Travel Time

2. Introduction

Soil water content (SWC) is a fundamental property affecting a large variety of processes relevant to hydrology, agricultural sciences, engineering and soil sciences. Over the last decades many techniques have been developed to measure SWC at different temporal and spatial scales. Bittelli (2001) provided a review describing the most common methods available for measuring SWC. Among these methods, geophysical methods have been widely used. Among geophysical methods, Ground Penetrating Radar (GPR) is a powerful and promising one. GPR has the advantage of covering larger areas with respect to point-based measurements typical of soil moisture sensors such as Time Domain Reflectometry (TDR), filling the gap between point scale and large scale satellite-based measurement. Soil water content can be obtained by performing different types of analysis and methods using GPR. Huisman et al. (2003) and Klotzsche et al. (2018) presented reviews about advances in applications of GPR, for measurement of SWC. In their reviews they discuss the available methods, including continuous multi-offset measurements, off-ground measurements, three-dimensional measurements, vertical radar profiling, modelling and inverse methods.

When the value of soil dielectric permittivity is obtained from GPR, relationships must be employed to convert permittivity to volumetric SWC. Commonly, the relationships used for GPR are the ones derived from the calibration of TDR data against SWC data obtained from gravimetric methods. Since both TDR and GPR are volumetric measurements, during the calibration measurement of bulk density is necessary to convert the mass-based gravimetric measurement to volume-based soil water content. Many equations were derived over the years. One of the most widely used equations is the one by Topp et al. (1980), which is a third order polynomial. The authors used TDR to measure the dielectric permittivity for a range of granular samples placed in a coaxial transmission line. Ledieu et al. (1986) proposed an equation where the calibration of TDR was performed against gamma-ray attenuation, an accurate technique used for measuring water content. The calibration equation accounted for the change in bulk density of the specimen. Later, Roth et al. (1992) proposed calibration functions for mineral, organic and magnetic soils. These are empirical equations.

Roth et al. (1990) proposed a dielectric mixing model based on theoretical considerations. This model includes: 1) the effect of bulk density (by accounting for soil porosity), 2) a geometrical parameter describing the orientation of soil particles with respect to the electric field and 3) the values of dielectric permittivity for the solid, liquid and gas phase. While the gas phase permittivity is constant, the solid phase permittivity changes with soil minerals, while the liquid phase permittivity is temperature dependent (assuming constant or narrow-band frequency).

The dielectric mixing model of Roth et al. (1990) belongs to the family of the electromagnetic mixing models, which are applied to a large variety of media including snow, ice, liquids and biological materials. One of the most exhaustive description and review about the theory of electromagnetic mixing formula was presented by Sivhola (1999). As pointed out by Sivhola, inhomogeneous mixtures (such as a soil) have properties that are somehow dependent and determined by its constituents but different from the original components. Although the dielectric properties of a mixture are somehow an average of the components permittivities, often the whole character of the dielectric is changed by the mixing process.

The relationships currently used for GPR applications were derived from experiments performed with the TDR and applied to various studies. Weihermuller et al. (2007) used the Topp et al. (1980) formula to derive water content from GPR. Gerhards et al. (2008) derived SWC from multiple transmitter-and-receiver GPR, employing the Roth et al. (1990) dielectric mixing model.

However, there are many differences between TDR and GPR, in terms of frequency of operation, sampling volume, data analysis and interpretation. Therefore there is the need to test the current equations applied to GPR. Only a few studies have been performed. Lambot et al. (2004) estimated SWC directly from GPR, using a soil-specific empirical model (third-order polynomial) similar to Topp's equation. However, their experiment was limited to a sand box with only sand sample as testing material. Steelman and Endres (2010) presented a comparison among petro-physical relationships for application to GPR. They concluded that the general empirical equation by Roth et al. (1992) provided the best fit for the sandy loam soil. When the entire data set was analyzed, they found that the Topp et al. (1980) and Roth et al. (1992) relationships provided the most accurate estimates.

However, Steelman and Endres (2010) used permittivity data obtained from GPR using the Common Midpoint (CMP) method. With this method, stacking velocity fields are extracted from multioffset radar soundings at a fixed central location. Yet, CMP-derived velocity estimates are generally characterized by low resolution and high uncertainty (Tillard and Dubois, 1995; Lambot et al, 2004). The success of the measurements depends on the presence of clearly reflecting layers in the soil. For this reason the calibration equations derived from dielectric permittivity obtained from CMP may be affected by low resolution and high uncertainty.

The travel time of the reflected GPR wave depends on the depth of the reflector and the mean dielectric permittivity above the reflector. In general, in field applications the reflectors depth is unknown, therefore alternative techniques are used (Klotzsche et al., 2018). However, for controlled studies on calibration equations, it is more accurate to perform GPR measurements where a reflector is installed at a known depth and derive an accurate travel time, as performed by Lambot et al. (2004), where radar measurements were carried out in controlled laboratory conditions on a tank filled with a disturbed sandy soil.

In this study, the performance of various published physical relationships used to obtain soil water content estimates from GPR obtained from a known-depth reflector, were evaluated. Detailed GPR experiments were setup for soils having different texture, bulk density and water content. These variables were controlled and independent gravimetric measurements were used for testing.

3. Theory

Ground Penetrating Radar reflections occur when there are significant changes in dielectric permittivity. In natural conditions they can ben sedimentation layers, groundwater tables, rocks stratification. One of the most common techniques for measuring SWC is based on derivation of dielectric permittivity from travel time analysis.

The velocity v [m s^{-1}] of an electromagnetic wave, is affected by the dielectric permittivity ϵ, and the magnetic permeability μ, as:

$$v = \frac{c}{\sqrt{\mu\epsilon}} \tag{1}$$

where c is the speed of light, 2.997×10^8 [m s^{-1}]. From a mechanical standpoint, the velocity v of an electromagnetic wave traveling through a space of length d [m], is given by:

$$v = \frac{2d}{t} \tag{2}$$

where t is time [s]. For a reflected wave, the number 2 in front of the length is included because the wave is reflected back to the receiving antenna. For most soils μ_r is equal to 1 (Roth et al., 1992), therefore Eqn. 1 can be written as:

$$v = \frac{c}{\sqrt{\epsilon}} \tag{3}$$

By equating the definitions of velocity:

$$\frac{c}{\sqrt{\epsilon}} = \frac{2d}{t} \tag{4}$$

and solving for ϵ:

$$\epsilon = \left(\frac{ct}{2d}\right)^2 \tag{5}$$

Equation 5 allows for obtaining the relative dielectric permittivity by measuring the travel time t, since the position of the reflecting plane d and the speed of light c are known. When the material is a composite mixture such a soil, we refer it as bulk dielectric permittivity (ϵ_b).

Knowledge of the distance between the antenna and the reflector d, allows for obtaining the travel time and the dielectric permittivity, this method is usually called the $two - way$ travel times analysis (Pereira et al., 2005).

3.1. Soil Water Content relationships

3.1.1. Empiral Equations

The empirical relationship by Topp et al. (1980) is:

$$\theta = -5.3 \times 10^{-2} + 2.92 \times 10^{-2}\epsilon_b - 5.55 \times 10^{-4}\epsilon_b^2 + 4.3 \times 10^{-6}\epsilon_b^3 \tag{6}$$

where θ is the volumetric water content (m^3 m^{-3}) and ϵ_b is soil bulk dielectric permittivity. The authors fitted the third-order polynomial to TDR data collected in a coaxial transmission line for four soils. They estimated an error for their data of 0.013 m^3 m^{-3}.

Ledieu et al. (1986) developed an equation obtained from calibrating TDR against SWC data obtained from gamma-ray attenuation. Since dielectric permittivity is density dependent they also included the bulk density. They stated that their procedure and calibration equation had accuracy of less than 1 %. However the experiment was performed only on one sample of sand. The equation proposed is:

$$\theta = 0.1138\sqrt{\epsilon_b} - 0.1758 \tag{7}$$

Roth et al. (1992) proposed three different empirical equations for mineral, organic and magnetic soils. The equation for mineral soil is also a third-order polynomial similar to Topp's equation, but with different coefficients and a prediction error of 0.015 m^3 m^{-3}:

$$\theta = -7.28 \times 10^{-2} + 4.48 \times 10^{-2}\epsilon_b - 19.5 \times 10^{-4}\epsilon_b^2 + 36.1 \times 10^{-6}\epsilon_b^3 \quad (8)$$

3.1.2. Electromagnetic Mixing Formulas

Electromagnetic mixing formulae relate the value of the individual permittivities of the mixture components to their volumetric fractions. A widely used class of mixing models are called power-law models (see Sivhola, 1999, page 166), where a certain power of the permittivity is averaged over volume weights:

$$\epsilon_b^\beta = f\epsilon_i^\beta + (1-f)\epsilon_j^\beta \quad (9)$$

where ϵ_i and ϵ_j are the generic dielectric permittivities of a two phase systems. In the Birchak et al., (1974) equation, the parameter β is equal to 1/2. Another known model is the Looyenga (1965) formula, where β is equal to 1/3.

Later Roth et al. (1990), extended the power-law model to compute the bulk dielectric permittivity as a weighted sum of the dielectric permittivity of each soil phase:

$$\epsilon_b = (\phi_s\epsilon_s^\alpha + \theta\epsilon_l^\alpha + \phi_g\epsilon_g^\alpha)^{1/\alpha} \quad (10)$$

where ϕ_s, θ and ϕ_g are the solid, liquid and gas phase volumetric fractions. The corresponding dielectric permittivities are ϵ_s, ϵ_l and ϵ_g, while α is the parameter describing the geometry of the medium with relation to the applied electric field (Roth et al., 1990). The volumetric solid fraction can be also written as $\phi_s = (1 - \phi_f)$, where ϕ_f is the porosity and the volumetric fraction of the gas phase as $\phi_g = (\phi_f - \theta)$. Using these relationships and substituting into eqn. 10, leads to:

$$\theta = \frac{\epsilon_b^\alpha - \left[(1-\phi_f)\epsilon_s^\alpha + \phi_f \epsilon_g^\alpha\right]}{\epsilon_l^\alpha - \epsilon_g^\alpha} \tag{11}$$

The liquid phase dielectric permittivity is temperature dependent with:

$$\epsilon_l = 78.54 \times (1 - (4.579 \times 10^{-3} \times \Delta T)) \tag{12}$$

where T is temperature in Celsius and $\Delta T = T - 25$. To use this equations, knowledge of porosity (which can be obtained from measurement of bulk density) and dielectric permittivity of the solid phase is needed. Porosity is obtained from measured bulk density by:

$$\phi_f = 1 - \frac{\rho_b}{\rho_s} \tag{13}$$

where the density of the solid phase (ρ_s) was assumed to be equal to 2.65 g cm^{-3}.

The sum of the different volume-weighted permittivities can be extended to include the contribution of organic matter in organic soils, or ice in partially frozen soils (Bittelli et al., 2004). Table 1 provides dielectric permittivity values for different materials (Daniels, 2004). In this study we used the following values: $\epsilon_s = 4$, ϵ_l was computed with eqn. 12 at 28 °C, $\epsilon_g = 1.005$ and $\alpha = 0.5$.

Overall, the selection of these four models was based on previous results obtained by Steelman and Endres (2010). They found that the general empirical equation by Roth et al. (1992) provided the best fit for the sandy loam soil and when the entire data set was analyzed

Table 1: Dielectric permittivity of materials at 100 MHz. From Daniels, 2004.

Material	Dielectric permittivity
Vacuum	1
Air	1.0005
Fresh water	$78.54 \times (1 - 4.579 \times 10^{-3}(T - 25))$
Fresh water ice	3.2
Quartz	4−6
Concrete dry	4−10
Sand Dry	2−6
Sandstone dry	2−5
Soil Dry Clay	4−10
Granite Dry	5
Limestone dry	7

using the general empirical expressions, they found that the Topp et al. (1980) and Roth et al. (1992) relationships provided the most accurate. Regarding the electromagnetic mixing formulae, Steelman and Endres (2010) found that the Roth et al. (1990) dielectric mixing model produced better results for the entire data set, but performed only slightly better than the general empirical relationships.

4. Material and Methods

Five different soils were used in this study, namely sand, sandy loam, loamy sand and kaolinite clay. Samples were collected from the Tumkur district, Karnataka, India. The soil samples were collected from the top 25 cm of soil. The experiments were conducted at laboratory temperature of 28 °C. This value was used for correcting the dielectric permittivity of the liquid phase in the dielectric mixing models (eqn.12), which provided a value of $\epsilon_l = 77.46$. The tested soils were cleaned for presence of organic material like grass, leaves etc. and sieved with a 2.5 mm size sieve. Figure 1 shows a schematic of the experimental setup, while Figure

2 shows two photographs of the experimental setup.

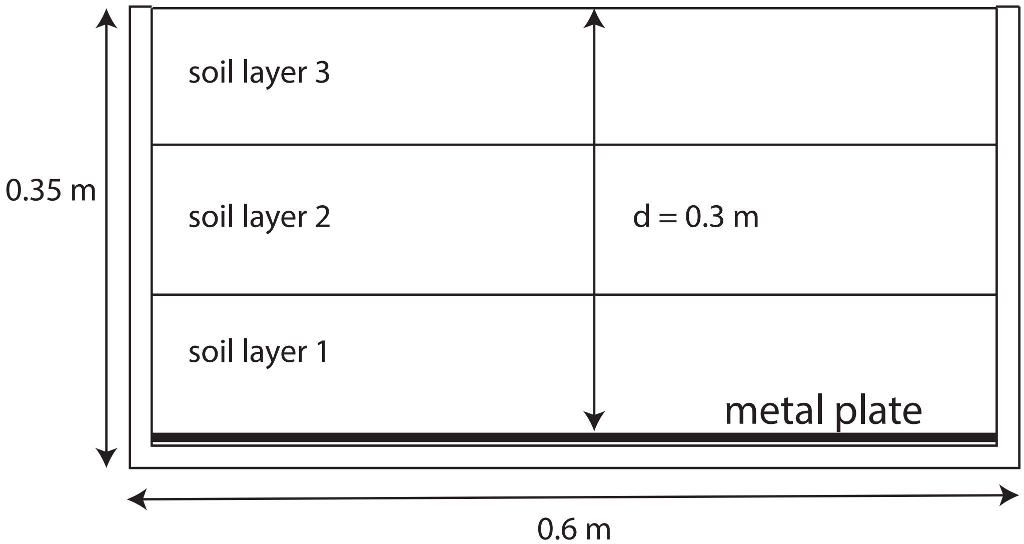

Figure 1: Schematic of the soil plastic tank

Figure 2: Picture of the soil plastic tank and GPR (Mala Inc., 800 MHz).

The soil was placed into a plastic tank (with base 0.6 m $\times$ 0.4 and 0.35 m height) for a total volume of 0,072 m^3, with a reflecting metal plate at the bottom. The distance for travel time calculations between the antennas and the reflecting metal plate was $d \simeq 0.3$ m. According to the manufacturer (Mala Inc.) the antennas are positioned at the bottom of the GPR, where a plastic lower case of a few mm thickness separate the antennas from the soil. Therefore a value of $d \simeq 0.3$ is the correct physical distance between the antennas and the metal reflector. The distance between the transmitting and receiving antennas is 0.1 m. Materials underneath the metal sheet have no influence on the measured backscattered signal (Lambot et al, 2004). The soil was prepared by adding a fixed amount of water to a specific mass of soil, and mixed to obtain uniform distribution of water. The mixed sample was then placed into the tank and packed to a specific density in three layers of 0.1 meters each, of equal mass. The layers in the figure do not represent different soil types, but the layers used for packing. GPR antenna was then placed on the top of box and readings were taken in time-triggering mode.

Subsequently, the soil was removed from the tank and fixed amounts of water were added to increase water content. The same packing procedure was then repeated, therefore everytime the soil was prepared and repacked into the tank, for each SWC measurement. This procedure was followed since it was not possible to increment water content within the tank by either percolation or capillary rise. At the bottom of the tank a reflecting metal plate was positioned for GPR analysis, therefore we could not control the lower boundary condition for percolation or capillary rise with installation of either ceramic or porous plates. Moreover, percolation of water into a tank often results in preferential flows of water along the walls and preferential pathways, resulting in non-homogeneous distributions. For these reasons, the soil was repacked each time for each individual SWC measurement.

To verify water content and bulk density values and to test SWC equations, after the GPR measurement was performed, soils samples were collected in metal rings from the center of the tank and independent gravimetric SWC and bulk densities were measured. Although special care was payed to pack the soil at the same density, since the volume of the tank was fairly large, it was not possible to repack the soil at the same densities, therefore variations in bulk densities were recorded during the measurement. These values were used in the equations for estimation of water content, where bulk density (or porosity) was required. Specifically the variations in bulk density ranged from 1.33 to 1.8 g cm^{-3} for sand, from 1.21 to 1.71 g cm^{-3} for sandy loam, from 1.6 to 2.1 g cm^{-3} for loamy sand and from 1.04 to 1.24 g cm^{-3} for kaolinite clay.

4.1. Ground Penetrating Radar measurements

The GPR was a Mala Inc., with an 800 MHz shielded antennas. The setup was the following: time window = 38.8 ns, depth = 0.3 m, sampling frequency = 8230.951172 MHz and antenna separation = 0.1 m. The data were analysed using the software Prism 2 (Radar System Inc.) and Reflex (Sandmeier, 2019). The acquisition was performed in time-based trace triggering mode.

Figure 3 shows an example of radargrams showing the reflector depth. The reflection in the upper part of the signal are the typical air and ground wave as shown in Figure 4. The transmitting antenna propagates waves giving rise to an air wave that travel directly from the transmitting to the receiving antenna. Similarly, the propagating wave give rise to the ground wave. The upper part of the radargram shows the air and ground wave.

As indicated in Figure 3, the lower change in amplitude indicated by the arrows is the reflection due to the metal plate, determining the travel distance (d) of the wave. Note that in the figure on the left plate the reflection is attenuated (the shade of gray is less intense) with respect to the figure on the right. This is due to a higher water content, determining a higher value of dielectric permittivity, and higher attenuation of the reflected wave. Since the reflector was

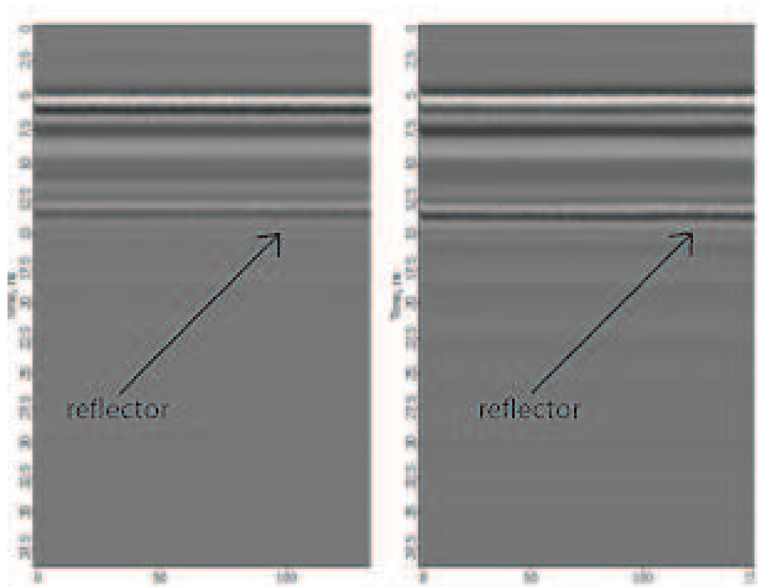

Figure 3: Example of radargrams for two different water contents. The left plate is for a sample at higher water content.

fairly close to the antennas (0.3 m), in this study the reflection of the metal plate was always visible, even at high water content in the kaolinite clay sample.

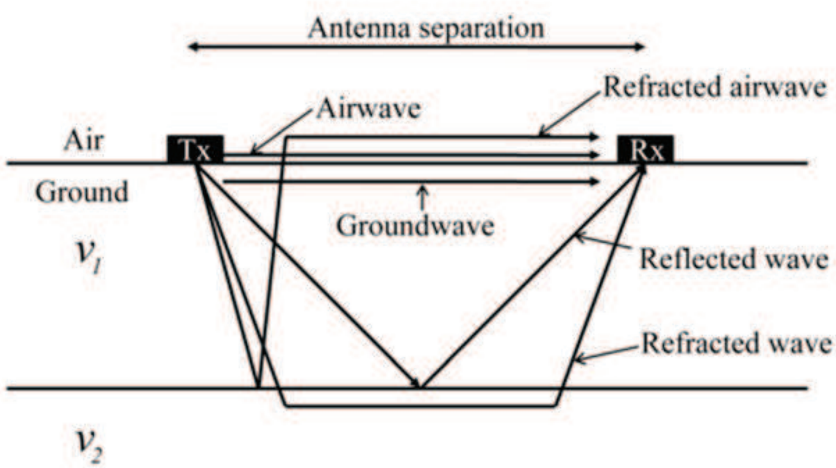

Figure 4: Propagation paths of electromagnetic waves in soils with different layers (Huisman et al., 2003)

Indeed, depth penetration is controlled by the dielectric permittivity and electrical conductivity of the sample. In fine textured soils, in particular in clay soils, the signal can be highly attenuated. Moreover, in fine textured samples relaxation processes (such as Maxwell-Wagner or double layer polarization) may determine additional dissipation processes (Schwing et al., 2013) and further attenuation of the signal.

The procedure to identify the reflections was based on the calibration procedure presented by Pereira et al. (2005). The authors pointed out that one of the main problems related to GPR technology is that the technical information provided by the different companies is practically inexistent. The lack of information for the different parameters for antenna emissions and emitted signal is a serious difficulty for data interpretation. For instance, the authors showed that the rate of drift of the signal was not exactly the same for the three antennas under test, operating at 500, 800 and 1000 MHz. Indeed, the time base of GPR measurements is also not exactly defined and it may exhibit a significant drift due to a temperature difference between the instrument electronics and the air temperature. Accordingly, we increased the warming time of the GPR to 30 minutes to equilibrate with the laboratory temperature. Since the authors used the same GPR manufacturer used in this study (Mala Inc.), we employ their procedure to identify the time zero parameter.

An exact definition of time zero is nearly impossible. It is not a constant value but depends on the surface material type and the antenna set up configuration (Sandmeier, 2019). However, when the physical distance of the reflector and the distance between the antenna are known, it is possible to determine the time zero for the investigated material. An automatic and stable static correction (definition of time zero) may be done either on the first negative, first zero crossing or first positive peak (Sandmeier, 2019). Pereira et al. (2005) suggested to use the first positive peak (Fig. 4 in Pereira et al. (2005)) for the 800 MHz antenna.

Figure 5 shows an example of a trace and the identification of the reflection for computation of travel time. The lower plate shows the complete trace acquired during the experiment and the upper plate a zoom over the relevant section. The origin was fixed by starting off at the greatest amplitude value from the first positive semiperiod peak. After obtaining the travel time, the bulk dielectric permittivity was then computed as detailed above.

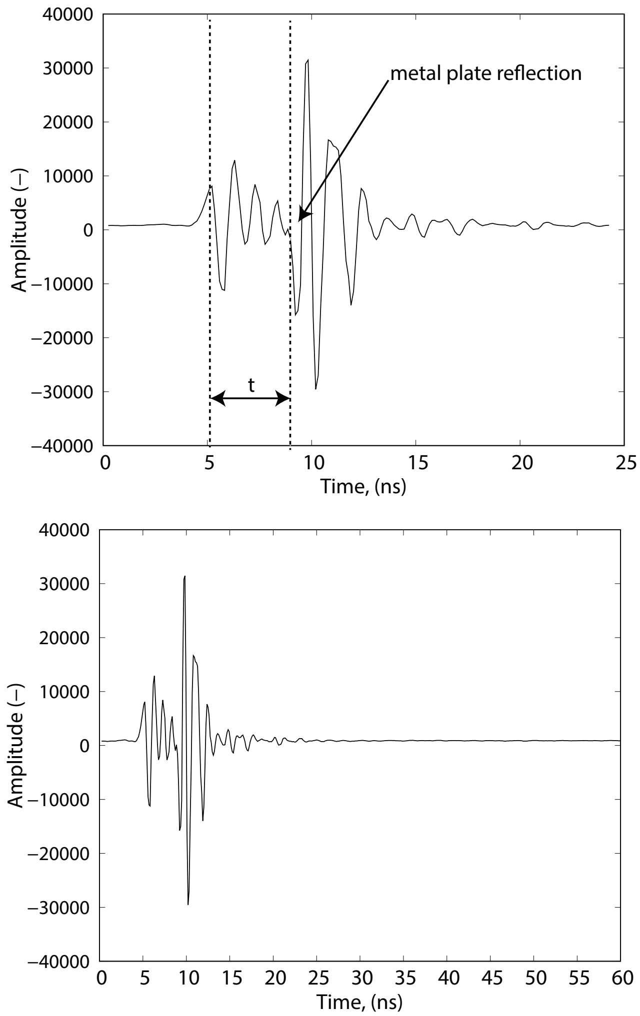

Figure 5: Example of travel time determination on a representative trace

4.2. Error Analysis

The accuracy of the volumetric soil water content estimates was estimated using the Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{\sum_{i=N}^{N}(\theta_{meas} - \theta_{pred})^2}{N}} \tag{14}$$

where N is the total number of samples, θ_{meas} [m^3 m^{-3}] is the volumetric water content obtained from gravimetric measurements and θ_{pred} [m^3 m^{-3}] is the volumetric water content predicted by the different equations, and obtained from GPR measurement of bulk dielectric permittivity.

5. Results and Discussion

The estimated volumetric water contents, θ, obtained from the different equations are presented in Fig. 6 for the four different textural classes and the RMSE results are presented in Table 2. The dielectric mixing model of Roth et al. (1990) provided the best fit for the tested soils, except for the sandy loam. For the sandy loam, the Topp's and Ledieu's equations provided the best fit. When all the data were combined the dielectric mixing model of Roth et al. (1990) provided the best fit, with RMSE of 0.038 m^3 m^{-3}.

Table 2: Root Mean Square Error (RMSE) [m^3 m^{-3}] for the four different soil types and all data. DMM stands for dielectric mixing model.

Relationships	sand	sandy loam	loamy sand	kaolinite clay	all data
Topp et al. (1980)	0,024	0,035	0,022	0,033	0,051
Ledieu et al., (1986)	0,023	0,035	0,025	0,030	0,052
Roth et al. (1992)	0,049	0,054	0,015	0,012	0,051
Roth et al. (199O)-DMM	0,022	0,040	0,010	0,010	0,038

Figure 6 shows the SWC predicted by the four different equations against the independent gravimetric SWC. The gravimetric measurements were converted into volume based measurement by multiplying them by the bulk densities. As confirmed by the values of RMSE, it is also possible to visually see the best fitting of the dielectric mixing model of Roth et al. (1990) for the indicated textures.

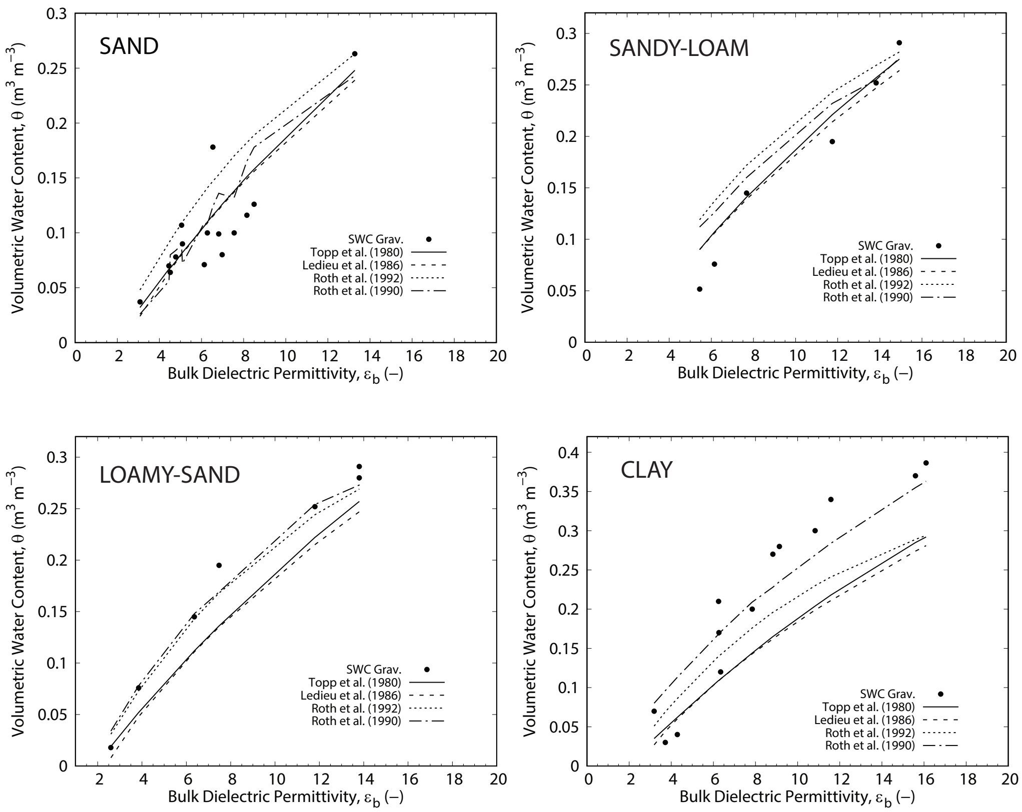

Figure 6: Ground penetrating radar (GPR) permittivity with corresponding volumetric water contents collected for the sand, sandy loam, loamy sand and kaolinite clay textural classes. Points are gravimetric water contents and lines are estimated values for the four different models.

Considering the difficulties in achieving uniform packing of wetted soil into a large tank and the multiple repetitions of the procedure, the scatter of the experimental data is fairly small, confirming the accuracy of the experimental procedure.

Figure 7 shows a scatter plot between the measured volumetric water contents and the estimated ones for the different models. The performance of the dielectric mixing model may be further improved by including different values of solid phase dielectric permittivity based on mineralogical measurements of the samples.

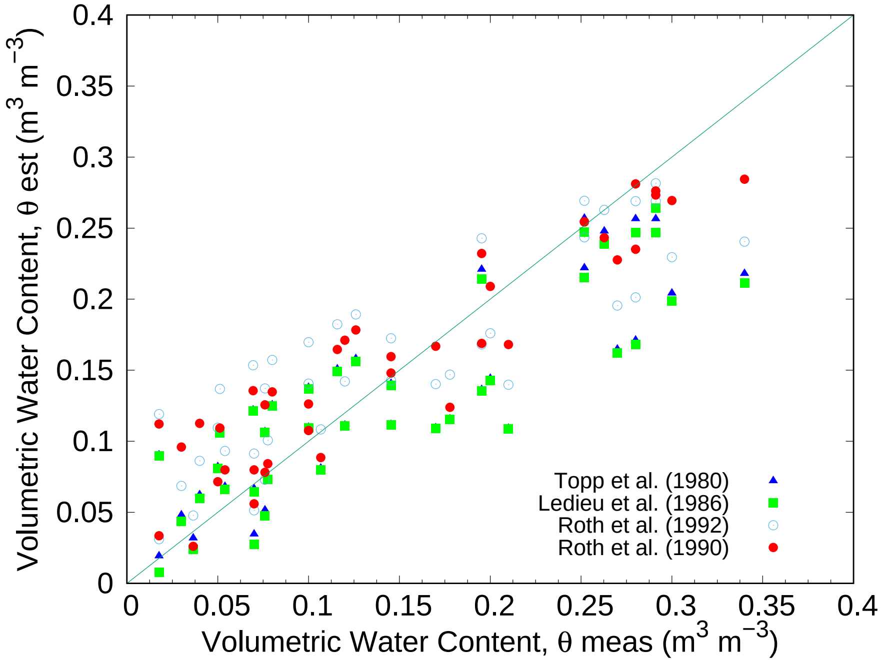

Figure 7: Scatter plot of measured and estimated data for the four soil types

Note that the equations that use the value of porosity (or bulk density), such as the dielectric mixing model of Roth et al. (1990) are not always smooth lines and in particular for sand. This is due to the varying values of bulk density measured for each independent measurement of gravimetric SWC.

As described above, experimentally was not possible to repack the soil at the exact same values of bulk density, therefore bulk density was measured every time the soil was repacked. The ability of estimating SWC as function of porosity is one of the reasons the dielectric mixing model performed better than the other models.

Moreover, the varying bulk densities stress the experimental difficulties of preparing large amount of soil material at uniform water content and density.

Using empirical equations, such as the Topp's equation, where estimation of SWC is not density dependent, will lead to inaccurate estimation of SWC since density, in natural conditions, changes with depth. In agricultural conditions, where soil is subject to compaction and softening due to machines and tillage, the changes in bulk density over the growing season are significant, requiring equations that include the possibility of time and space dependent bulk density. For these reasons, there are active lines of research, where direct measurement of bulk density is derived from TDR waveforms, such to obtain both SWC and bulk density from the same waveform measurement (Jung et al., 2013a; Jung et al., 2013b; Curioni et al., 2018).

Overall, the dielectric mixing model provides better estimates but it requires knowlegde of soil porosity (or bulk density). We assumed that the liquid phase was in thermal equilibrium with the soil and therefore we used soil temperature for the temperature of the liquid dielectric permittivity. If temperature is not available, default values for ϵ_l at 25°C can be used. However, as discussed below the effects of temperature variations are fairly small. Adjusting the value of ϵ_s would have improved the estimation for the loamy sand soil as well, however we did not want to use ϵ_s as an fitting parameter.

These results are consistent with the work of Gerhards et al. (2008), where they derived accurate SWC from GPR using a multiple transmitter-and-receiver setup, and employing the dielectrid mixing model of Roth et al. (1990). As pointed out by Sivhola (1999) the use of dielectric mixing models is preferable with respect to the use of empirical equations since they allow for incorporating dielectric properties of constituent materials and theirvtemperature and frequency dependence. While the major dipole relaxation for water occurs at higher frequency (19 GHz), additional relaxations in soils, such as double layer or Maxwell-Wagner relaxations, may occur in the operational frequencies of GPR, depending on the selected antenna (Olmi and Bittelli, 2015).

Another parameter that significanly changes soil water content estimation is the parameter α, which is discussed in the next section.

5.1. Sensitivity Analysis of the dielectric mixing model

To employ the dielectric mixing model for different media it is important to quantify the effect of the individual parameters on the estimation of water content. As described above the permittivity of the gas phase is constant, the porosity depends on bulk density, the permittivity of the liquid phase is temperature dependent (assuming a constant or narrow band frequency) and the permittivity of the solid phase depends on mineralogy.

Figure 8 depicts the variations of volumetric water content as function of permittivity for different values of α. The other parameters are kept fixed with $\epsilon_s = 2$, ϵ_l= 77,46 (at 28 °C), $\epsilon_g = 1.005$ and $\phi_f = 0.547$ (with ρ_b= 1.2 g cm^{-3}).

The parameter α depends on the shape and orientation of the inclusions affecting the depolarisation factors, as detailed by Sivhola, (1999). The value of 1/2 was used by Birchak et al. (1974) or 1/3 by Looyenga, (1965). Other values can also be selected for the power-law relationship. The domain is $-1 \leq \alpha \leq 1$, where $\alpha = 1$ for plates or other inclusions for which no depolarisation is induced, or when the electric field is parallel to the layering. $\alpha = -1$ if the field is perpendicular to the layering and $\alpha = 0.5$ for isotropic two-phase medium (Roth et al., 1990). Using a non-linear least square minimization algorithm, Roth et al., (1990) found an optimal value of $\alpha = 0.46$ for their experimental data, which is close to 0.5, the value

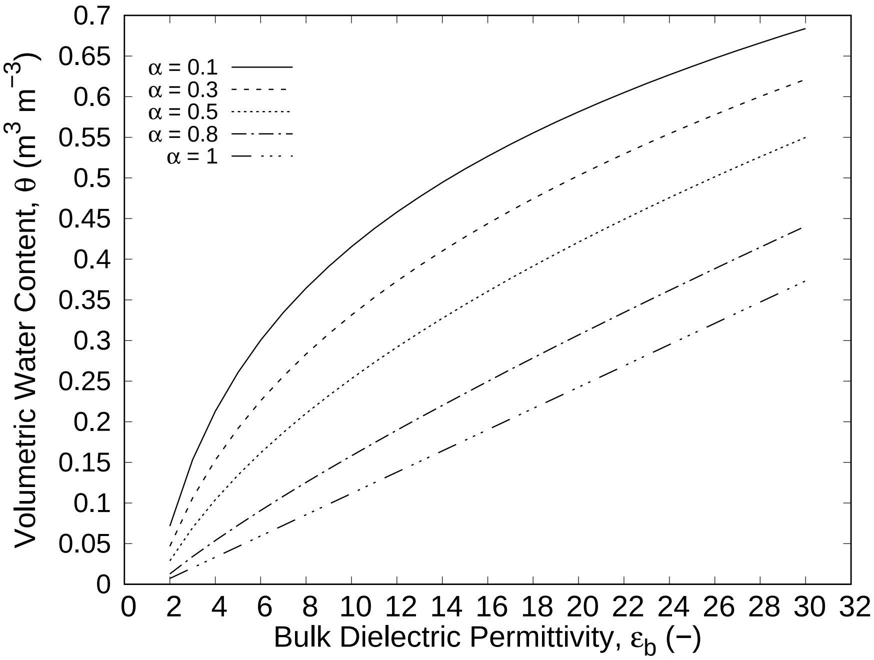

Figure 8: Sensitivity analsyis for the parameter α

obtained by Birchak et al. (1974) from theoretical reasons. While in this study the dielectric model was not calibrated and a fixed value of 0.5 was used, α can be modified if information about the soil layering is available, such as stratifications, sedimentation layers and others. Alternatively, α can also be used as fitting parameter. At decreasing values of α corresponds significantly increasing values of θ. Being the relationship non linear the variation depends on the corresponding values of permittivity.

The effect of the solid phase permittivity was also evaluated (Figure 9). The parameters were kept fixed as for the previous analysis, with $\alpha = 0.5$, and ϵ_s was changed from 2 to 10. These values are the ones reported in Table 1, for different earth materials. Lower values are associated to dry sandstone and sand, while higher values are associated to dry clay. The increase of the solid phase dielectric permittivity determines a decrease in the estimated SWC. For this parameters set, a change from 2 to 10, determines a decrease in θ of 0.1 m^3 m^{-3}.

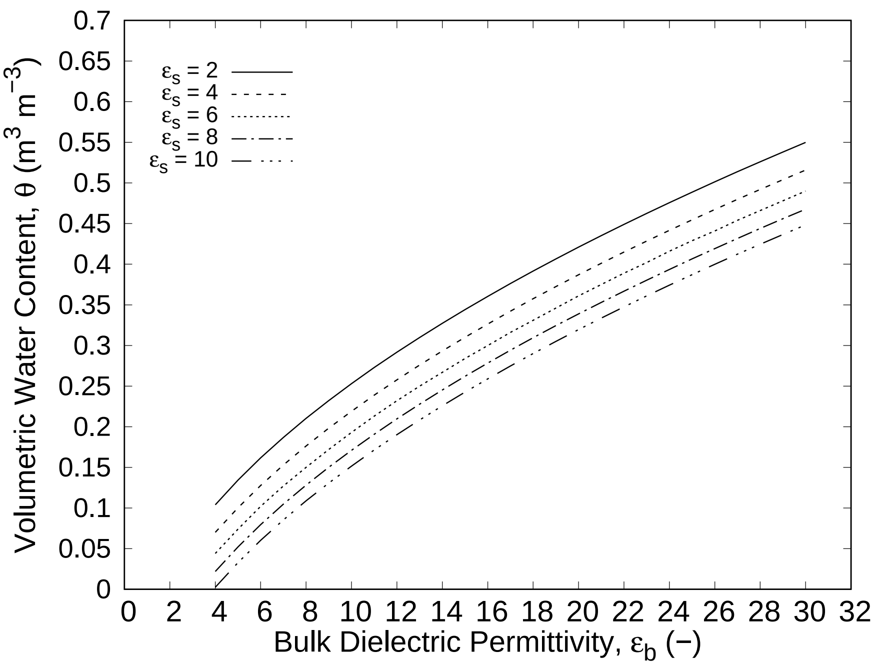

Figure 9: Sensitivity analsyis for the parameter ϵ_s

This beahavior is due to the higher weight given to the the solid phase by an increased ϵ_s in the weighted volumetric sum, and therefore less weight to the volumetric contribution of the liquid phase. Also in this case, information regarding the mineralogical composition of the analyzed media allows for modification of this parameter.

The effect of temperature on the liquid phase permittivity, and therefore on θ, is fairly small with estimated variation in volumetric water contents of about 0.03 m^3 m^{-3} over a temperature range between 4 and 20 °C. Finally, the effect of porosity on soil water content is about 0.06 m^3 m^{-3} over a variation of ϕ_f between 0.7 and 0.1, with increasing θ with increasing porosity. Considering that in field conditions bulk density can easily range, for instance, between 0.8 and 2.4 g cm^{-3} (corresponding to variations in porosity between 0.7 to 0.09 m^3 m^{-3}), the effect of bulk density is significant on SWC estimation.

Overall, the parameters that have a larger effect on estimated SWC with the dielectric mixing model are the exponent α, the solid phase permittivity ϵ_s and porosity ϕ_f. The first two can be used as fixed parameters with values of 0.5 and 4 respectively or used as fitting parameters. Porosity should be measured or obtained from bulk density. In absence of porosity or bulk density data, density can be obtained from TDR waveforms (Jung et al., 2013a,Jung et al., 2013b, Curioni et al., 2018) or from pedotranfer functions by knowledge of textural composition (Rodriguez Lado et al., 2015).

6. Conclusions

Different relationships to estimate SWC derived from soil permittivities obtained from a two-way GPR analysis data were compared. The GPR data were obtained in a controlled laboratory setting using a soil tank with a metal reflector positioned at a known depth, allowing for accurate determination of the soil bulk dielectric permittivity. The data were obtained for four distinct soil textural classes (sand, sandy loam, loamy sand and kaolinite clay) covering a wide range of soil moisture conditions. The physical relationships were empirical and dielectric mixing models. Results showed that the dielectric mixing model of Roth et al. (1990) provided the most accurate estimate of volumetric soil water content, expect for sandy loam. The estimation of the dielectric could have been further improved by using the geometric parameter and the dielectric permittivity of the solid phase as fitting parameters. Sensitivity analysis of the dielectric mixing model was performed showing that the geometric parameter α and the dielectric permittivity of the solid phase ϵ_s are the two most sensitive parameters, determining important variations in the estimation of SWC. Based on these results, these two parameters are suggested as fitting parameters to be selected if the model is fitted to data. However, the model can successfully be used without calibration as presented in this study, by using $\alpha = 0.5$ (as also suggested by the authors) and $\epsilon_s = 4$, which is an average value for soil minerals. We suggest to employ the dielectric mixing model of Roth et al. (1990) for estimation of SWC from dielectric permittivity obtained with GPR.

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