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On the Concept of Model in Physics

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Abstract

The concept of model is the basis for theoretical and experimental research in physics. A model is defined as an application of physical theories to the description of a physical system. As a premise to the introduction of models, physical theories are defined as mathematical frameworks with a hypothetical-deductive structure, provided with a mapping on the real world, allowing the description of a class of phenomena. The main characteristics of theories are discussed, in particular the domain of application and the relationships between different theories. The concept of physical system is introduced as a subset of the universe that is defined thanks to the property of separability and is characterized by appropriate boundary conditions. The structure and the main characteristics of models are then discussed and the steps to be taken to construct a model are considered. The role of assumptions is underlined and different kinds of models are illustrated. Finally, four examples of simple physical models are described, that are fit to be used for didactic purpose.

Keywords

Physical Theories, Physical Systems, Physical Models, Model Assumptions

1. Introduction

The concept of *model* is at the heart of scientific method. In physics, models are the basis for theoretical and experimental research (Giere, 1988) and are the main tool of researchers since the beginning of science in the Hellenistic Age (Russo, 2004). The aim of the present paper is to give a simple definition of a model as an application of physical theories to a physical system. The distinctive features of physical models are discussed and some examples are given that may serve as an illustration. In order to reach this goal, the concepts of *physical theory* and *physical system* are first introduced.

Current physics education is drawing significant insight from the model-based view of science, in which models and modeling take a central role in the formation and communication of new scientific knowledge (Sands, 2021). Since many decades, it is sustained that mathematical modeling of the physical world should be the central theme of physics instruction (Hestenes, 1987) and it was shown that the modeling method can produce much larger gains in student understanding than alternative methods of instruction (Wells et al., 1995). Modeling Instruction is a pedagogical approach which has been widely and successfully employed in high school physics instruction (Buffler et al., 2008) as well as at the university level (Brewe, 2008; Brewe & Sawtelle, 2018). A review of the concepts of explanation in physical science teaching is in Geelan (2020).

Models are of common use in many fields, ranging from natural sciences (physics, geology, biology) to social sciences (economics, sociology, psychology). Modeling activity has also drawn much interest in philosophers of science (Hesse, 1970; Weisberg, 2007). An interesting discussion on model construction, function and use is in Morrison & Morgan (1999). However, the concept of model, as it is intended in physics, is far from being obvious outside the circle of researchers and it is often misunderstood by students.

In dictionaries, several definitions of the word may be found: 1) any object or human an artist or a craftsman has in front of him in order to make or to portray it more or less equal; 2) a sketch at smaller scale of something that will be made at larger scale; 3) a rule to follow in doing something or a person that is an example to follow and to imitate; and so on. The word comes from the Latin *modulus* that was used in architecture with a similar meaning.

A definition that could be found in the web and is more pertinent to science is “a conceptual representation of the world or of a part of it that is able to explain its behavior” (Wikipedia, 2015). This definition is correct in a broad sense, but it is too wide for our purposes, because it does not mention the essence of a physical model, that is its mathematical structure and its foundation in physical theories (Chassy & Jones, 2019). For example, geologists usually interpret the concept in a more qualitative sense (Greenwood, 1989).

2. Theories

Physics is often presented as a collection of *physical laws* governing the different aspects of nature. However, it is important to stress that such laws are organized into *theories*. Broadly speaking, a physical theory can be defined as a set of mathematical statements concerning a certain class of phenomena.

By *class of phenomena* we mean a set of phenomena that exhibit similar characteristics and can be traced back to a unique cause. Examples are the elastic deformation of solid bodies, the motion of fluids, thermal processes, electromagnetic phenomena, gravitation, phenomena occurring when the velocities of bodies approach the velocity of light, the behavior of matter at the atomic and subatomic level, and so on.

Specific theories have been developed during the last centuries in order to describe and understand each class of phenomena. Examples are the mechanics of point masses, thermodynamics, the theory of elasticity, fluid mechanics, the Newtonian theory of gravitation, electromagnetism, special relativity, quantum mechanics, and so on. Euclidean geometry must be also considered a physical theory concerning the properties of space in our surroundings.

More specifically, a theory is organized as follows (Russo, 2004):

1) It defines theoretical quantities that are the object of the theory;
2) It makes basic statements (postulates) about such quantities;
3) It deduces all possible consequences of such postulates;
4) It is applied to the real world by a mapping between theoretical and observable quantities.

Some points deserve to be stressed.

1) The postulates of a theory are not necessarily “evident truths”. On the contrary, they may be far from intuition and even amazing (for example the postulates of quantum mechanics). The only requirement is that postulates permit to deduce logically the observed phenomena (i.e., “to explain” them). The ability of a theory to reproduce phenomena is proportioned to the extent to which its postulates fit the real world.

2) The theory has a rigorous deductive structure: from a certain number of postulates, an unlimited number of possible consequences (theorems) is derived. Thanks to this structure, the correctness of statements of a theory is guaranteed, if one accepts its postulates. On the contrary, the mapping does not have any absolute guarantee. Hence the well-known Einstein’s aphorism: *As far as physical theories are not applied to the world, they are true; if they are applied to the world, they are uncertain* (Einstein, 1922).

3) A theory makes statements about quantities that are defined within the theory itself and are mathematical entities, not objects of the real world. Only the mapping gives physical meaning to the theory. Thanks to the mapping, theoretical statements are transferred to the real world. We can say that a physical theory deprived of the mapping becomes a mathematical theory. Often we are not aware of the mapping, because the names we use for theoretical quantities coincide with those of the observable quantities of real world. For example, when we define displacement or strain in continuum mechanics, we identify them with observable quantities: in a sense, the mapping is implicit in the names. The method to test the validity of the mapping is the experimental method that is the comparison of all possible consequences of the theory with observations.

4) Theories do not achieve immediately the status of well-established theories. At the borderline of physics, there are research fields where such theories are not yet available and their achievement is the aim of research. However, once it is formulated, a theory has an autonomous existence as a deductive formal system. Therefore, it is possible to develop theories that are not yet supported by experimental evidence (such as string theory).

On the basis of what has been said, it should be clear that theories are general frameworks. They do not consider particular phenomena occurring in particular systems. For example, in a textbook of electromagnetism we do not find in general the explanation of lightning; similarly, textbooks of elasticity do not mention earthquakes; and textbooks of fluid mechanics do not consider lava flows. Phenomena like lightning, earthquakes and lava flows can be explained in terms of such theories, but they take place in particular systems (the atmosphere, the solid Earth or its surface), when particular initial and boundary conditions are present. In order to study particular systems and the phenomena occurring there, we employ *models*.

2.1 Application domain

An important characteristic of theories is that each theory has a limited range or application domain. With this term, we mean an interval of values of a theoretical quantity (in general, a region of a space defined by more than one quantity), within which the theory can be applied and reproduces the observed phenomena. Outside this interval, the theory loses its validity.

For example, the theory of elasticity is valid as long as stress (or strain) does not exceed a certain value (called the limit of elasticity) beyond which the behavior of the material is no longer elastic: the material fractures or deforms plastically.

In general, the boundaries of the domain are not sharp, but they are rather fuzzy. For instance, Newton's theory of gravitation is valid as long as we consider masses and velocities that are not too large. However, there are not particular values of mass or velocity beyond which the theory gives wrong results. What happens is that the ability of the theory to explain observations progressively decreases as mass and/or velocity increase, until the theory predictions become completely wrong and Einstein's theory is to be used.

The concept of application domain can be extended also to space and time. There is a general consensus that physical theories apply not only to our region of space, but to the entire universe and not only to present times, but also to the past and the future. This is the reason why we can discuss about the big bang.

2.2 Relationships between theories

A characteristic aspect of physical theories is that they are not separated from each other, but they are related in different ways. They form a network that tends to cover all the observable phenomena. In particular:

1) The postulates of a theory can be often explained by another theory (being theorems of the latter theory). For instance, Newton's law of gravitation is a postulate in Newtonian theory, but it can be derived from Einstein's theory of gravitation as an approximation in the case of small velocities and masses.

2) In most theories, there are constant quantities, the values of which are assumed as empirical data: the theory does not require an explanation for them. However, those values can be calculated in the framework of a different theory. For example, in thermodynamics the radiance of a black body is proportional to the fourth power of the absolute temperature. The constant of proportionality is called the Stefan constant and its value is an empirical datum. But the Stefan constant can be expressed in terms of the Boltzmann constant, the Planck constant and the velocity of light in quantum mechanics.

3) Theories are organized in hierarchies, where each theory is included in a more general one. For example, continuum mechanics includes fluid mechanics as a particular case, when the appropriate constitutive equation is assumed. In other cases, a theory is an approximation of a more general one: relativistic mechanics reduces to Newtonian mechanics, when velocities are small with respect to the velocity of light.

4) A theory can be developed from one or more other theories with the addition of postulates. So, the theory of thermoelasticity can be developed from the theory of elasticity and thermodynamics with the addition of appropriate postulates. Analogously, quantum field theory is developed from nonrelativistic quantum mechanics and the theory of special relativity.

3. Systems

Before introducing models, it is necessary to introduce the concept of *physical system*. A physical system can be defined as a subset of the universe, i.e. a particular collection of matter/energy that is chosen to be studied (for instance an atom, a solid body, a volcano, the Earth, the Galaxy or even the whole universe).

The concept of physical system is tightly connected to the property of *separability*. This means that we can understand how a part works even if we do not understand the whole universe.

The possibility of separating a large system into subsystems A, B, C, … is due to the fact that the interaction between them is weak or, in many cases, it is one-way, i.e. A has effect on B, but B does not affect A, that is the two subsystems are uncoupled.

Once these conditions are satisfied, the choice of system boundaries is often arbitrary, depending on the particular phenomena we wish to study. Appropriate boundary conditions must be assigned at the boundaries. In particular, a system is open if it can exchange matter and energy with its surroundings. It is closed if it does not exchange matter with its surroundings (but it can exchange energy). It is isolated if it does not exchange matter nor energy.

We can consider all possible physical systems as grouped into classes of similar systems: so, we can study the behavior of a generic element of a class, for example a volcano, a planet, a galaxy and so on. Or we can study a specific element of such classes, such as Vesuvius, Mars, the Milky Way, and so on. This choice has relevant consequences on the kind of model that is developed.

4. Models

In general, a model can be defined as a representation of a physical system that is accomplished by the use of one or more theories. The model allows us to describe the system, to explain its behavior and possibly to give predictions about its evolution. Hence the possibility of constructing a model is based on the availability of physical theories.

Being a representation of a physical system, a model is by definition an idealization. It is supposed that the behavior of the model approaches that of the physical system, at least for some aspects. Therefore, the study of the evolution of the model may allow (with a certain approximation) to predict the evolution of the system.

Being based on theories, a model shares the same properties. More specifically, a model is a set of mathematical statements concerning a given system. These statements result from the application of one or more theories to the system and concern theoretical quantities. Then a model has a deductive structure based on assumptions and theorems, like a theory. As for theories, a mapping between theoretical and observable quantities allows the model to represent and simulate the system's behavior.

The development of a model requires a number of steps:

- Definition of the system to be modeled.
- Choice of coordinate system.
- Choice of theories to be employed.
- Choice of relevant theoretical quantities and equations.
- Selection of known and unknown quantities.
- Assignment of boundary and initial conditions.
- Formulation of assumptions and approximations.
- Solution of equations.
- Representation of results.

Once the model has been formulated, all possible behaviors of the system can be explored by changing initial and boundary conditions. Data must be available for at least some theoretical quantities of the model. For example, this is the case for the use of fluid mechanics in meteorology or the use of electromagnetism in the study of the geomagnetic field.

As said before, the choice of the system strictly depends on the phenomena under study: only parts which are relevant to them are included in the model. A system can be often separated into different subsystems: for example, a volcano can be separated into source region, magma chamber, volcanic conduit, and so on. For each subsystem, separate models can be developed, that are linked to each other by appropriate boundary conditions. Vice versa, models of different systems can be joined to make a model representing a larger system, if it is useful.

In many cases, systems are not homogeneous, but are made of regions that are different phases of matter (solid, liquid or gas). Therefore, it is necessary to employ different theories in the same model. Hence different assumptions are likely to be adopted for different regions that are matched by appropriate boundary conditions. Moving boundaries are often to be considered (for example, in the presence of phase transitions).

4.1 Role of assumptions

A model is always a simplified description of a physical system: only a few aspects of the system are considered, while many others are neglected purposely, because they are not considered to be relevant to the problem. For example, in many Earth's models the fact that the Earth is not exactly a sphere, but is closer to an ellipsoid, is neglected. Therefore, a crucial role is played by model assumptions.

There is a large freedom in the choice of assumptions: the choice depends on the aim of the model. Like postulates of theories, assumptions must not necessarily be "true" statements, in the sense that they derive from observation. On the contrary, they are often far from being evident and cannot be directly checked: for instance, that the temperature in the Earth's interior is thousands of degrees Celsius.

In many cases, assumptions are adopted without knowing to what extent they are close to reality. They are made because they are believed to be "reasonable" and their validity is checked a posteriori, on the basis of the ability of the model to reproduce phenomena.

In some cases, assumptions may be even contradictory. However, this is accepted under the assumption that small quantities are involved. An example is the following. In studying thermal convection of a viscous fluid, the Navier-Stokes equation is commonly employed, implying that the fluid is incompressible. In fact, it is assumed that the fluid density does not change as a consequence of pressure changes. But it must be admitted that density changes as a consequence of temperature changes, otherwise there would be no convection.

Therefore, it is meaningless to say that an assumption is right or wrong, because all assumptions are “wrong” in a strict sense, being approximations of the real world. However, some assumptions are more crucial than others: if they neglect a fundamental fact, the model results may be in complete disagreement with observations (see examples 2 and 3 below).

4.2 Kinds of models

While all models share the characteristics outlined above, they can be distinguished into different kinds. The chief role of models is to help us to understand phenomena occurring in physical systems and to disclose the relationships existing between the quantities describing the system itself.

In most cases, it is believed that phenomena can be reproduced on the basis of available theories, even if the structure of the system may not be known (as in the case of inaccessible systems, such as the interior of a planet or a star).

When developing a model, one often chooses to “isolate” the aspects that are crucial for the reproduction of a phenomenon and to exclude others that just blur the frame. Simplification may not be a defect, but a modeling strategy (e.g. Redish, 2021). In these cases, the quality of a model cannot be established in general, but must be evaluated with reference to the specific aim of the model itself: sometimes a simpler model may be preferable to a more complex one. Generally, in this approach the model represents a class of physical systems (for instance, a volcano) rather than a specific case (Vesuvius or Etna).

In a different approach, a specific physical system (for example, Vesuvius) is considered, with the aim of reproducing its behavior as faithful as possible and to formulate predictions about it. The model is the best the more details it includes. In this case, the model may become very complicated and the governing equations must be necessarily solved by numerical methods.

A typical example of the latter kind of models is provided by models for weather prediction. They consider the Earth’s atmosphere as a gas layer and employ fluid mechanics and thermodynamics. The aim of research in this field is to develop more and more accurate models, taking into account all factors that may affect the atmospheric system.

In other cases, the structure and the behavior of a system may not be known and the model contains conjectures that must be tested by observations (Randall, 2005). The starting point is usually the observation of a phenomenon that is not explained by available theories (for instance, the stability of atoms at the beginning of the 20th century). In these cases, the model is still based on available theories, but it includes some assumptions that are foreign to such theories or even contradict them. The model is a preliminary attempt to set the phenomenon into a conceptual scheme, as a premise to the construction of a new theory. In this sense, models are heuristic tools. A classic example is Bohr’s model of the hydrogen atom (example 4).

Therefore, the difference between various kinds of models is mainly in the aim of the model and in the kind of assumptions, whether they are consistent with applied theories or they are extraneous or even contradictory with them.

5. Examples

To illustrate typical characteristics of models, four examples are considered, drawn from different fields of physics. The system considered in the first two models is the Earth. They were not devised to reproduce observed phenomena, but to obtain values of unknown quantities: the length of the circumference and the age of the Earth, respectively. However, the Earth is modeled in very different ways in the two cases: as a sphere in the first and as a body with a flat surface in the second one.

The other two models regard systems with extremely different sizes: the entire universe and an atom, respectively. In these cases, models were devised to reproduce observed phenomena: the darkness of sky at night and the spectrum of the hydrogen atom. These simple models are fit to be used for didactic purpose.

5.1 Eratosthenes’ Model

As a first example, let us consider the model employed by Eratosthenes of Cyrene (276-194 BC) to calculate the length of the Earth’s circumference. His work *On the measurement of the Earth* was lost, but the model was reported by the astronomer Cleomedes and is one of the first physical models that we know (Russo, 2004).

The system considered is the Earth, the model geometry is a sphere, lit by solar light. A spherical coordinate system is employed. The relevant theories are Euclidean geometry, optics and an astronomical theory.

The astronomical theory contains the following information:

1) Earth and Sun are two bodies separated by void space;
2) The Earth-Sun distance is much greater than the Earth's radius;
3) At noon of summer solstice, sunbeams are vertical at the tropic of Cancer.

Because of (1), optics and Euclidean geometry imply that light rays are straight lines. From (2) it can be assumed that sunbeams arriving at the Earth are parallel to each other. Eratosthenes considered two points located on the same meridian: point A at Alexandria and point B on the tropic (close to the town of Syene). Since they are on the same meridian, noon is simultaneous at the two points.

Quantities that must be considered are the Earth's radius, the distance between points A and B, measured at the Earth's surface, and the angle between the vertical lines at the two points. The angle can be obtained from the height of the Sun over the horizon at A, because sunbeams are vertical at B, so that the required angle is that between sunbeams and the vertical at A. The simple relationship between the three quantities allows calculation of the Earth's radius, hence of the circumference. According to Cleomedes, who outlined the procedure with rounded figures, the length of Earth's circumference resulted to be 250,000 stades. Eratosthenes' value was 252,000 stades, with a stade equal to 157.5 m, yielding 39,690 km, very close to the real value.

The model is extremely simple. However, it includes the essential elements of a model: simplification of the physical system, application of theories, assumptions, theorems, data from observation and an obvious mapping on the real Earth.

5.2 Kelvin's model

In the mid-19th century, William Thompson, later Lord Kelvin, devised a model to estimate the age of the Earth (Turcotte & Schubert, 2014). The choice of the model was based on the following assumptions. Kelvin supposed that the Earth was formed at a uniform high temperature and that later cooled down, while the surface temperature was kept at a lower constant value. He also assumed that, since rocks have a small thermal conductivity, cooling was controlled by the uppermost layer of the Earth (the thermal boundary layer), where there is a high temperature gradient. Since this layer is thin with respect to the Earth's radius, the cooling process can be studied by considering a small portion of the Earth in the proximity of its surface and neglecting the surface curvature. If the Earth is uniform, conclusions drawn from the flat model should be valid for all the thermal boundary layer, hence for the whole planet.

The theories employed are Euclidean geometry and thermodynamics. A Cartesian coordinate system is adopted and model assumptions are the following:

1) The surface thermal boundary layer is representative of the whole Earth;
2) The boundary layer is represented as a solid half-space, where heat is transferred by conduction;
3) The half-space is uniform, i.e. physical quantities describing its properties have the same values everywhere;
4) The initial temperature of the Earth's interior is uniform and equal to the melting temperature of rocks, while the surface temperature is constant and equal to present average temperature;
5) There is no internal heat production.

The ability of the medium to conduct heat is described by its thermal diffusivity. Thanks to assumptions, the problem is one-dimensional, i.e. temperature depends only on depth. The evolution of temperature in the Earth model is provided by a simple differential equation provided by thermodynamics. With appropriate initial and boundary conditions, the equation can be solved and gives temperature as a function of time and depth. From this function, the geothermal gradient, that is the rate of increase of temperature with depth, can be easily calculated. Hence the model can predict the present value of geothermal gradient as a function of the time elapsed from the beginning of cooling and of other quantities whose values are assumed or can be measured. With appropriate values for these quantities, Kelvin obtained that the age of the Earth was about 65 million years, a much smaller value than the real one that is about 4.5 billion years (Burchfield, 1975).

To find out the weak point of the model, let us examine the assumptions. Assumptions 1 and 4 are rough, but acceptable for an order-of-magnitude estimate. At a first sight, assumption 2 (modeling the Earth as a half-space) seems very far from reality. However, this assumption is not crucial: if a spherical model were considered, the result would not change substantially. Assumption 3 can be questioned, because the Earth's interior is far from being uniform, but is not crucial. The weak point is assumption 5: Kelvin neglected the inner heat production due to radioactivity that was unknown at his times. This assumption neglects an important process that strongly reduced the Earth's cooling rate and is crucial for an estimate of its age.

5.3 Olbers' model

In 1826 the German astronomer Heinrich Wilhelm Olbers (1758-1840) wondered why the night sky is dark. He argued that the number of farthest stars, even if they are not very bright, should be so large that they should give the sky a uniform light background. To investigate the problem, he devised a model including the entire universe. The employed theories were Euclidean geometry, optics and an astronomical theory.

For the present purpose, a simplified version of his model is considered (Bondi, 1960), based on the following assumptions:

1) The universe is an infinite space;
2) It exists since an infinite time;
3) It contains a uniform distribution of stars;
4) Stars are point-like light sources all equal to each other;
5) The universe is static: there are no large and systematic motions of stars.

The model considers the Earth as a point in a 3-dimensional Euclidean space. A system of spherical coordinates with origin in the Earth is introduced. Relevant quantities are the number of stars per unit volume and the energy of light emitted from each star per unit time.

If we consider a thin spherical shell with radius r with its center in the Earth, the number of stars in the shell is proportional to the square of r, but the light intensity from each star is inversely proportional to the same quantity. The two effects cancel and it results that the light intensity due to the whole universe is infinite. This is known as the Olbers' paradox.

This is a case in which the model gives an answer that is in complete disagreement with observation. In fact, all model assumptions are questionable. Nowadays we know that the universe has a finite lifetime and probably a finite extension, although very large. The assumption that there is a uniform distribution of stars is far from reality. The universe is not static, but it is expanding and light coming from distant layers is strongly diluted and redshifted.

However, careful examination of all assumptions (Harrison, 1987) shows that the failure of the model to give the correct answer comes from just one of them: that the universe exists since an infinite time. If the age of the universe is finite, we can see only a part of it, due to the finite velocity of light. If the number of visible stars is finite, calculation shows that they are incapable of lighting up the night sky. Therefore, the darkness of sky can be considered evidence of big bang (Al-Khalili, 2012).

5.4 Bohr's model

At the beginning of the 20th century, the atomic structure was unknown. Ernest Rutherford proposed a model that was coherent with the results of his scattering experiments (Rutherford, 1914), suggesting that the light and negatively charged electrons were orbiting in space around a heavy and positively charged nucleus. But such a model was impossible according to the classic theory of electromagnetism: due to radiation of electromagnetic waves, the electrons would lose their kinetic energy and fall onto the nucleus in a fraction of second.

On these grounds, Niels Bohr proposed his model (Bohr, 1913). The physical system considered was the hydrogen atom. As a model, he considered a point-like particle with negative electric charge (the electron) moving in a circular orbit around a point-like particle with a positive charge of the same intensity (the nucleus). He assumed that the nucleus had a much greater mass than the electron, so that the nucleus was motionless.

The theories upon which he constructed his model were Euclidean geometry, classical mechanics and electromagnetism. However, he decided to introduce some innovations, in the form of the following assumptions (Gamow, 1966):

1) Only certain orbits are allowed to the electron, those for which a quantity called *action* is an integer multiple of the Planck constant h;
2) The electron does not radiate electromagnetic waves during its motion around the nucleus on these orbits.

These assumptions were in contradiction with classical mechanics and electromagnetism. For assumption 1, Bohr drew inspiration from quantization of radiation, where the ratio between energy and frequency is an integer multiple of h.

The relevant quantities of the model are the mass and the electric charge of electron; the electric charge of nucleus; the radii of allowed orbits and the associated orbital velocities of electron. The unknown quantities are the energy levels of electron. A spherical coordinate system is considered. Based on classical mechanics, the electron velocity is obtained for each of the allowed orbits, as a function of the mass, electric charge and radius of the orbit.

The action associated with the n-th orbit is calculated from the mass of the electron, the radius of the orbit and the orbital velocity (Goldstein et al., 2001). Allowed values for the orbital radius are easily obtained from assumption 1. The total energy of the electron in the *n*-th orbit can be calculated as sum of kinetic and potential energy and one obtains an equation providing the allowed energy levels as functions of known quantities. From them, energy differences between couples of orbits can be calculated and compared with those given by Balmer's formula, an empirical formula giving the spectral lines of radiation emitted by the hydrogen atom (Gamow, 1966).

The calculated energy differences coincide with those given by Balmer's formula and this decrees the success of Bohr's model. In this case, the introduction of assumptions extraneous to the available theories allowed to reproduce the observed phenomena and set the basis for the new theory of quantum mechanics.

6. Conclusions

Models are essential tools in physics, allowing us to understand the behavior of physical systems and to extend our knowledge of the world. A model has been defined as the representation of a physical system that is accomplished by the application of one or more theories. A model has a deductive structure, based on assumptions and theorems concerning theoretical quantities. A mapping between theoretical and observable quantities allows the model to represent the real world. Only comparison between theoretical and observed values allows to draw conclusions on the validity of the model.

Models are very versatile tools, since there is a large freedom in the choice of assumptions, depending on the aim of the model. In fact, the development of a model may have different targets. A model can be employed for the calculation of quantities that cannot be measured directly or it can be used to investigate structures or processes that are inaccessible and cannot be observed directly. In the latter case, models consent to unveil the mechanisms that originate observed phenomena. In many cases, models are used for the prediction of the evolution of a physical system. Finally, a model can be used to seek an order or a structure in systems that are not yet understood.

It must be borne in mind that all models have limits that are implicit in the model assumptions as well as in the theories employed. In general, models can give answers only to some questions regarding the system and not to all of them. Models can produce wrong answers, even they are based on well-established theories, if they do not include essential elements of the real world.

As shown in the examples illustrated above, any model is based on many assumptions, part of them coming from the involved physical theories and part from the model itself. Assumptions may be more or less severe and their influence on the model may be more or less strong. For a correct appreciation of a model's outcome, it is therefore essential that all model assumptions are made explicit and their importance is carefully evaluated.

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