Educational Studies in Mathematics
https://doi.org/10.1007/s10649-025-10394-0

There is more to algebra than meets the eye: the case of blindness

Andrea Maffia[1] · Carola Manolino[2,4] · Elisa Miragliotta[3]

Accepted: 8 February 2025

Abstract
Research literature about visually impaired students' approach to mathematics is still very scarce, especially in the case of algebra, even though mathematical content is becoming increasingly accessible thanks to assistive technologies. This paper presents a case study aimed at describing a blind subject's process of algebraic symbol manipulation while solving equations supported by assistive technology. In particular, we analyze how a subject with blindness manifests their structure sense within algebraic instrumented activity mediated by programming languages (such as LaTeX). The epistemic and pragmatic mediations of the programming language are described in terms of the developed utilization schemes. The results show how hearing can replace the eye in developing algebraic structure sense. Data analysis reveals that the system of screen reader, speech synthesis, and programming language has interesting potentialities in the development of structure sense for subjects with blindness, but, taking an inclusive perspective, we discuss how these potentialities can be applied more generally in mathematics education.

Keywords Visual impairment · Structure sense · Artifact-mediated activity · Instrumental approach · Assistive technologies

1 Introduction

While discussing his discovery of a bijection between the interval $I=[0, 1]$ and the square $[0, 1]\times[0, 1]$, Cantor wrote in a letter to Dedekind "I see it, but don't believe it!" This statement captures the extent to which our conceptualization of mathematics relies on visual metaphors (Arcavi, 2003), often treating sight as an essential sense for engaging with mathematical ideas. Mathematical learning environments frequently emphasize visual modes of access, such as graphs, spatial arrangements, and symbolic notations, shaping a

✉ Andrea Maffia
andrea.maffia@unibo.it

1 Department of Mathematics, University of Bologna, Bologna, Italy

2 Department of Social and Human Sciences, University of Valle d'Aosta, Aosta, Italy

3 Department of Mathematics, University of Pavia, Pavia, Italy

4 Laboratorio "S. Polin", University of Turin, Turin, Italy

Published online: 06 March 2025

perception of mathematics as inherently and fundamentally visual (consider, for instance, the pivotal role of subitizing in the innate understanding of numbers). This focus raises important considerations about how people with visual impairments access and engage with mathematical concepts.

Research literature on the experiences of blind people in mathematics is limited, with most studies focusing on learners who are still developing their understanding (e.g., Alajarmeh et al., 2011; Healy & Fernandes, 2011). Rarely does research explore how individuals who have successfully mastered mathematics navigate and overcome visual barriers, insights from which could significantly inform the design of inclusive educational practices. Unfortunately, many students with visual impairments are often discouraged from studying mathematics due to assumptions about its reliance on sight. This perception is fueled by educational practices but also by the complexity of semiotic systems in mathematics, which are predominantly conveyed through visual representations: mathematics may seem inaccessible to those who cannot see. Some mathematical representations, such as geometrical figures or graphs, may be reasonably approachable through alternative sensory inputs or prior physical experiences. However, the manipulation of other representations like algebraic symbols, that lack any direct connection to sensory experience, poses unique challenges. In this paper, we focus on blind subjects' algebraic manipulation while solving equations supported by assistive technology. By examining their strategies and cognitive processes, we aim to shed light on how mathematical understanding can be constructed independently of visual perception, offering new perspectives on fostering inclusivity in mathematics education.

2 Learning algebra with assistive technologies

Before the advent of assistive digital technologies, there was no easy avenue for the manipulation of mathematical content for students with visual impairments. Classroom interactions with these students were predominantly oral, due to teachers' considerable difficulties implementing active mathematics instruction (Del Campo, 2000). Teachers would often ask blind students just to verbalize their mathematical activity, making verbalization the primary mode of mathematical discourse, the main exception being the manipulation of material models of geometric shapes. The rapid development of digital technologies has significantly advanced the field of assistive technologies (Dabi & Golga, 2023). Today, a vast array of tools is available to make mathematical content accessible, enabling users to independently engage with it (Klingenberg et al., 2020). Prior to these developments, and sometimes even now, students depended on a third party to write or read for them. These technologies now facilitate direct access to mathematical content, although they do not necessarily guarantee inclusion (Piroi et al., 2023); many current studies focus on creating new environments (prototypes) for accessing formulas, but few investigate how this access occurs (Zambrano et al., 2023). The involvement of digital technologies within the process of teaching–learning mathematics has provided new opportunities for students with visual impairments, who can rely on speech synthesis or Braille displays to access text, including algebraic formulas (Alajarmeh et al., 2011; Armano et al., 2018; Herzberg & Rett McBride, 2023) with a substantial difference between audio and haptic access (Kim et al., 2023).

During schooling, mathematical notations become quite soon two-dimensional: written or printed space is used to convey information to sighted readers (e.g., exponents, fractions, square roots, summations). Screen readers (like JAWS[1] or NVDA[2]) do not usually allow a semantically consistent reading of formulas, and text is linearly presented both by Braille displays and speech synthesis. Then, the information which is usually implicit in the spatial arrangement of algebraic notation must be made explicit. This is realized by using programming languages (whether expressed verbally or via Braille), like Lambda (Bernareggi, 2010), LaTeX (Smeureanu & Isăilă, 2011), and others (e.g., EDICO—"Editor Científico ONCE," Moreno Martínez et al., 2022). Dedicated symbols indicate, for example, if an expression is positioned as a numerator or denominator of fractions. Figure 1b shows four examples of equations written in Lambda code which provides dedicated symbols (double slash) for algebraic fractions. The LaTeX code for those equations is shown in Fig. 1a: the command \frac{}{} represents a fraction having as numerator the content of the first curly braces and as denominator the content of the following ones. The examples in Fig. 1 may help to understand that the reading of an algebraic expression requires interpreting additional symbols (than what sighted users have to) which, however, may emphasize certain parts of the expression. Van Leendert et al. (2021) note that it is possible to train Braille-reading students to optimize finger movements on the Braille display by helping them in identifying useful characters (e.g., the equal sign); such training may reduce the time required to solve an equation. Indeed, using algebraic symbols for mathematical problem-solving does not only require reading the symbols but also being able to act upon them. While there are studies focusing on how digital textbooks can aid students' algebraic activity (e.g., Bouck et al., 2016), there is a dearth of research about systems for enabling students to act productively on symbols (Alajarmeh et al., 2011; van Leendert et al., 2021).

Referring to the manipulation of algebraic symbols, we consider that it encompasses much more than just the rote application of procedural rules. It involves a broader competence in using "equivalent structures of an expression flexibly and creatively" (Linchevsky & Livneh, 1999, p. 191), which is called *structure sense*. Perceiving the structure of an algebraic expression means having a broad view understanding of how the expression is made of its parts, analyzing relationships between these parts (Hoch & Dreyfus, 2004). Even if "[a]ny algebraic expression or sentence represents an algebraic structure [whose] internal order is determined by the relationships between the quantities and operations that are the component parts of the structure" (Hoch & Dreyfus, 2004, p. 50), we recognize structure sense being strictly personal, depending both on individual resources (cognitive and meta-cognitive) and on cultural-historical established manipulations (Schüler-Meyer, 2017). Students showing structure sense navigate algebraic expressions with a keen understanding of identifying, handling, and manipulating sub-structures. In this paper, we wonder how blind subjects rely on their structure sense while solving equations if supported by assistive technology. A more operational research question is provided after Sect. 3.

In the context of learning algebra with assistive technologies, the development of blind students' structure sense is strictly intertwined with their knowledge of programming languages (LaTeX, Lambda, etc.). As assumed by Verillon and Rabardel (1995), artificial systems amplify a human's means of action, "[f]or example, they might structure his categories of thought and knowledge or, by developing his ability to act on the environment, they might in return extend his cognitive capacities" (p. 79). To the best of our knowledge, there is no international literature

[1] "Job Access With Speech" https://www.freedomscientific.com/products/software/jaws/

[2] "NonVisual Desktop Access" https://www.nvaccess.org/

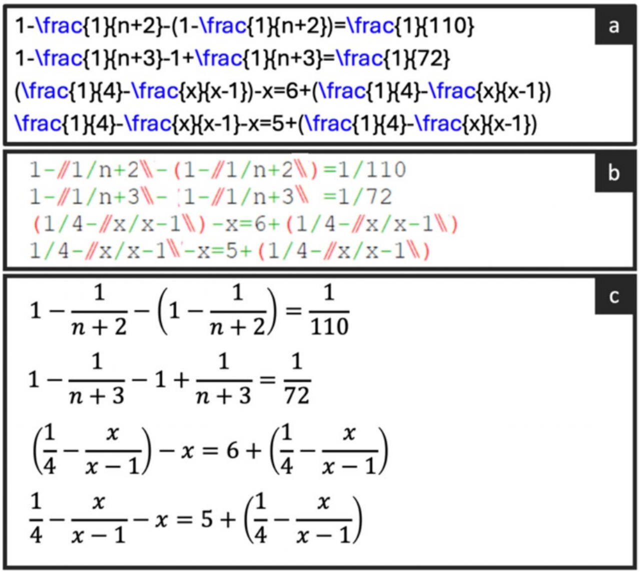

Fig. 1 Example of LaTeX code (**a**), Lambda code (**b**), and corresponding equations in usual notation (**c**)

in the field of mathematics education about those transformations introduced in the relationship between the blind subject and algebraic tasks resulting from the implementation of technologies such as programming languages, speech synthesis, and Braille displays. This study is a first step towards filling this gap by offering a *thick description* (Bell & Kissling, 2019) of the process of algebraic symbol manipulation performed by an experienced blind individual while solving an equation. Following Verillon and Rabardel (1995), in doing this research, we cannot distinguish psychogenesis and anthropogenesis: we assume it is impossible to dissociate the development of cognition from the technological context in which it functions and manifests. As detailed below, we will then resort to the construct of instrumented activity to analyze an individual's activity as mediated by assistive technology.

3 Theoretical framework

Following Rabardel's (2002) framework, an *artifact* is any material or symbolic human creation with a purpose. Instead, we use "instrument" to refer to a mixed unit comprehending an artifact (or a part of an artifact or a system of artifacts) and utilization schemes. Utilization may result either from the subject's own construction or from the appropriation of social utilization schemes (Verillon & Rabardel, 1995). An instrument "is identified by its recognized uses" (Wallon, 1941/2020, p. 165), and then, utilization schemes can be defined as representative invariants corresponding to classes of instrumented activities. Here, we will describe utilization schemes in terms of their *rules of action*, the "if–then" rules allowing the sequencing of subjects' actions to be generated (Vergnaud, 1991, as in Rabardel, 2002).

The two components of an instrument are possibly independent. A utilization scheme can apply to a range of artifacts, and at the same time, an artifact may fit into a range of utilization schemes (Rabardel & Bourmand, 2003). Then, a specific instrument results from "an instrumental relation [of the subject] with an artifact, whether material or not, whether produced by others or himself" (Verillon & Rabardel, 1995, p. 85). When a subject acts on an object, the instrument—material or psychological—acts as mediator, "a new intermediary element situated between the object and the psychic operation directed at it" (Vygotskij, 1930/1997, p. 86).

There are different forms of mediation realized by instruments. Here, we are interested in what Rabardel and Bourmand (2003) call "mediation to the object"—the object being algebraic expressions. There are two forms of "mediation to the object" called *epistemic* and *pragmatic mediations*. Epistemic mediation is aimed at getting to know the object, while pragmatic mediation concerns acting on the object. Here, we consider the epistemic mediation of programming languages when they provide access to (they make it possible to read) an equation. The pragmatic mediation of a programming language is manifested when a subject manipulates the algebraic symbols while addressing a task (e.g., solving equations).

Adopting this framework, instrumented acts are our unit of analysis, taking into consideration the features of individuals, artifacts, and tasks. Considering this unit of analysis is necessary to avoid two possible forms of reductionism (Rabardel & Bourmand, 2003): ignoring that the action is shaped by cultural tools and ignoring the individual's activity in favor of a mechanical determinism by tools. The set of instrumented actions related to the same task constitutes instrumented activity. The development of instrumented activity is called *instrumental genesis*.

The development of instrumented activities is understood as:

> The result of a largely artificial process in which the acquisition of instruments plays a leading role. It is not so much the instrument as such which determines evolution but the functional reorganization and redeployment that its acquisition and use impose on […] sensori-motor, perceptive, mnemonic, representational [mechanisms]. (Verillon & Rabardel, 1995, p. 82)

Verillon and Rabardel (1995) postulate that in almost all human activities, the mediation of artifacts intervenes so early in the subjects' development that they constitute contextual factor for cognitive functioning, and following Vygotskij (1930/1997), we assume a transformation of the psychic processes during the instrumented acts. Indeed, instrumented acts "involve a restructuring of the activity directed towards the subject's main goal" (Rabardel, 2002, p. 83). Therefore, during artifact-mediated activities, users might re-enact the previous experiences with the instrument and researchers can observe specific utilization acts that correspond to schemes perceived as relevant to those situations by the users.

We assume the same happens when the activity prompted by algebraic tasks is mediated by a symbolic artifact as a programming language (as LaTeX). Then, the development of structure sense may be strongly interdependent with the instrumental genesis of the programming language. Aiming at exploring this conjecture, we will need to identify blind individuals' structure sense as it is manifested while performing algebraic tasks using programming languages. Hence, we need to operationalize structure sense. For this purpose, we refer to Hoch and Dreyfus (2004) who describe structure sense (SS from here on), for school algebra, as composed of six abilities: (1) seeing an algebraic expression/sentence as an entity, (2) recognizing an algebraic expression or sentence as a previously met structure, (3) dividing an entity into sub-structures, (4) recognizing mutual connections between structures, (5) recognizing which manipulations it is possible to perform, (6) recognizing which manipulations it is useful to perform.

Adopting this theoretical framework, our research question can be stated as: How do blind subjects manifest their structure sense within algebraic instrumented activity mediated by programming languages?

4 Methods

Due to the qualitative nature of our research question and because of the paucity of research literature on the topic, we have chosen to conduct exploratory case studies. This is recommended when research desires “to portray ‘what it is like’ to be in a particular situation, to catch the close-up reality and ‘thick description’ […] of participants’ lived experiences of […] a situation” (Cohen et al., 2007, p. 254). Aiming at describing how subjects draw upon and manifest their SS, we interviewed adult individuals with strong education in mathematics—subjects who had several opportunities of developing SS.

The interview analyzed in this contribution is part of a broader data collection: a convenience sample of five solvers was involved, and the interviews consisted of questions about algebra, geometry, and functions (among the main strands of knowledge in Mathematics curriculum in Italy) and their learning experience. Preliminary results about geometry questions are published elsewhere (Miragliotta et al., 2023).

We opted for task-based interviews: within the algebraic strand, the interviewee was asked to select two equations among those proposed by Hoch and Dreyfus (2004) and to solve them (Fig. 2); such task was adopted since it proved to provide opportunities to elicit SS in previous studies. We present excerpts from the solution process of these equations (C and X in Fig. 2). The task was presented through a PDF file (Fig. 2) implemented with the Axessibility package for LaTeX (Armano et al., 2018). By adding a line of code to the source file, Axessibility automatically inserts a hidden alternative text in the PDF for each formula, making it accessible to screen reading software. The interviewee could decide which hardware and software to use during the interview.

Question 1: You are presented with two groups of three equations each. You are free to choose to solve one equation from the first three (A, B, C) and one from the last three (X, Y, Z). You cannot choose the pairs (A, X), (B, Y), (C, Z).
Explain/illustrate out loud each reasoning you make and how you make it.

A

$$1 - \frac{1}{n+2} - (1 - \frac{1}{n+2}) = \frac{1}{110}$$

B

$$(1 - \frac{1}{n+1}) - (1 - \frac{1}{n+1}) = \frac{1}{132}$$

C

$$1 - \frac{1}{n+3} - 1 + \frac{1}{n+3} = \frac{1}{72}$$

X

$$\frac{1}{4} - \frac{x}{x-1} - x = 5 + (\frac{1}{4} - \frac{x}{x-1})$$

Y

$$(\frac{1}{4} - \frac{x}{x-1}) - x = 6 + (\frac{1}{4} - \frac{x}{x-1})$$

Z

$$\frac{1}{4} - \frac{x}{x-1} - x = 7 + \frac{1}{4} - \frac{x}{x-1}$$

Fig. 2 Snapshot of the PDF version of the task translated in English

The interview presented in this study involves Antonio (a pseudonym) who has a degree in Physics and has worked as a fellow researcher for 2 years. He became blind 5 years before the interview due to a degenerative pathology. He learned to use LaTeX with speech synthesis as a visually impaired undergraduate student 11 years ago and has used LaTeX regularly since then. The screen reader NVDA was installed on his laptop, allowing him to hear the read-aloud of the code. Although Antonio can use Braille and recognizes its usefulness, he prefers to use speech synthesis for simple tasks as those proposed in the interview.

Aiming at a thick description (Bell & Kissling, 2019), we collected several sources of data including "speech acts; non-verbal communication; descriptions in low-inference vocabulary; […] recording of the time and timing of events; the observer's comments […]; detailed contextual data" as prescribed by Cohen et al., (2007, p. 405). This was realized by recording the interview, including the audio- and videorecording of the interviewee (through a webcam) and his computer screen. The second author of this report acted as the interviewer and took notes during the interview. When needed, the interviewer prompted the interviewee to verbalize his solving process. The video was transcribed verbatim by the authors integrating the transcription of "what is said" with descriptions of "what is done" (e.g., Table 1) and with screenshots. The authors analyzed this enriched transcript by describing the observed utilization schemes and coding each line with the six components of SS. The coding process was discussed among the researchers until they achieved consensus.

5 Results

At the beginning of the interview, the programming language exploits epistemic mediation. In the phase of reading and selecting the equations to solve, there is an interaction between the subject's SS and the mediation provided by the programming language. For instance, Antonio is using LaTeX, which has a dedicated command (\frac) for representing even simple fractions. He uses the functionalities of the Axessibility package (Armano et al., 2018) to hear the reading of the LaTeX code behind the PDF provided to him. The NVDA software reads the code aloud while he navigates the PDF file. For instance, a code as \frac{1}{2} is read by the speech synthesis as "backslash frac, curly brace open, one, curly brace closed, curly brace open, two, curly brace closed." Despite not being familiar with the theoretical framework of this paper, from the very beginning, he explicitly refers to the attempt of "seeing" the equation as an entity. He states: "I am reading them, let's say, one after the other, […] with a continuous reading to understand the general structure of these equations." Below, we report the transcript of this phase.

17 Antonio: Let's start with the first one [10 s pause] There is a first fraction: one over n plus y. Plus two! Over n plus two, no, minus.
18 Interviewer: Ok.
19 Antonio: and there is a brac… brackets start. Then: here there is one minus a fraction, like the one that was outside of the [brackets] before. Then, equal the second part of the expression. We can say "of the equation" since there is n as unknown. So: there is another fraction, the fraction one over one hundred and ten.
20 Interviewer: Ok.
21 Antonio: And the first one is finished.
22 Interviewer: Yes, right. But it starts with "one minus".
23 Antonio: Minus. Yes, yes.

24 Interviewer: You didn't say that before.

25 Antonio: That's because, initially, I do a quick reading and then, usually, to get a better fruition, I copy the text on a notepad so that I can work it better.

Antonio states that he wants to perform a "continuous" reading to understand "the general structure" of the equation and it is realized here by dividing the equation into sub-entities. Apparently, he navigates the LaTeX code looking for specific indicators: fractions, brackets, and the equal sign (e.g., line 19). Specific symbols within the code allow him to find those indicators, and furthermore, he recognizes a previously met structure (line 19: "a fraction, like the one that was outside the brackets before"). Here, the code is accomplishing its epistemic mediation, but it is the subject's SS guiding the reading. Antonio highlights that, in the initial approach, what matters is to have "a quick reading" (line 25) before acting on details, emphasizing the importance of having a first glance at the algebraic expression—as an entity. Moreover, in understanding the structure of an equation, finding the position of the equal sign is relevant, as it might be for a sighted solver. Particular importance is also given to fractions: we wonder if this depends on how they are highlighted by the presence of a dedicated command in LaTeX—which is read by the NVDA screen reader. This will be analyzed further later.

After this episode, he decides to solve equation C (Fig. 2) and copy/pastes the corresponding LaTeX code in Microsoft Notepad. The NVDA software reads the code aloud while his cursor navigates through the Notepad; we can observe it as captured in the video. This shift to another software highlights how the PDF reader could not exploit the pragmatic mediation of LaTeX. A text editor is needed. The successive manipulation of the equation is described in Table 1 where each line corresponds to an instrumented action.

Table 1 Enriched transcript of the second excerpt

Line	What is said	What is done
47		He deletes the first 1 and then he moves the cursor after the first occurrence of the \frac command. Then, the cursor goes on after the – 1, which is deleted backwa4rd, so obtaining the equation in Fig. 3
48	Ok, then… let's look at these fractions with precision	The cursor moves back to the beginning of the equation
49	Because it's one…	The cursor moves forward till before the closing curly brace of the \frac command
50	One over n plus three	The cursor moves forward, along the second \frac command, and stops after the equal sign
51	Here, these are… these are opposite. It should be zero, because minus one over n plus three plus one over n plus three	The cursor is not moving
52	I check again	The cursor goes back to the beginning of the equation
53		The cursor moves along the left side of the equation and stops right after the equal sign. Then, it moves along the right side
54		The cursor moves to the following line and "0 = \frac{ }{ }" is typed
55	Then, I copy the other one	He types 1 and 72 within curly braces
56	So, the results should be like that, zero equals one over seventy-two	

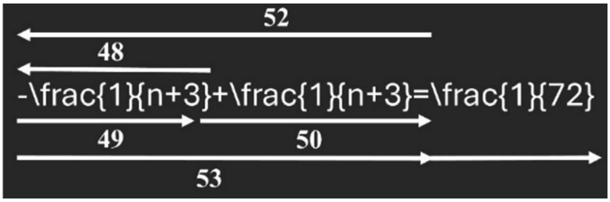

Fig. 3 Cursor's movements during the second excerpt. Numbers are keyed to Table 1. The screenshot has been elaborated adding arrows representing the movements of the cursor

In this excerpt, we see how the epistemic and pragmatic mediations of the code are exploited. The initial reading of the equation led Antonio to cancel the opposite terms $+1$ and -1 (components 5 and 6 of SS). There is a particular sequence of actions for the cancelation of terms: the deletion is always realized backwards, after the reading of the term by the screen reader. Then, he "divides an entity into sub-structures" (component 3). Indeed, the movements of his cursor suggest he initially reads the first fraction (line 49) and only then the second one (line 50). The successive reading of these two structures (recognized as opposite, component 4 of SS) brings him to the conclusion that they should be deleted (line 51), but he is not sure (component 5 of SS). Then, he realizes a second reading; this time it is a continuous reading of the whole left side of the equation (line 53) leading to the decision of moving to a second line of text to type 0=\frac{}{}. This behavior exemplifies the pragmatic mediation of LaTeX in structuring the following step: having understood the general structure of the following manipulation of the equation, Antonio prepares an "empty structure" using the code with no numbers typed in. Numbers are added only later (line 56).

Antonio's behavior appears different when he solves the other equation (Table 2). In this case, he does not recognize the equal sub-structures on the two sides of the equation, as shown in the following excerpt.

Again, Antonio expresses the intent to understand "the structure" (line 69). As before, this corresponds to reading the different sub-structures (the fractions, the parenthesis) of the equation: he is relying on the third component of SS. By analyzing the movements of his cursor, we see that his attention is initially caught by the first fraction (lines 69–70): the \frac command plays an important role in exploiting the epistemic mediation of the code. Then, he analyzes the second fraction and stops at the end of the left side of the equation (line 72). As the \frac command, also the bracket seems to structure Antonio's reading (line 73): he goes back and forth over the two fractions within the brackets (lines 74–77). Then, we can notice that—even if the screen readers allow only a left to right reading—Antonio analyzes the structures of specific parts of the equation by realizing multiple readings of the sub-structures identified.

After this excerpt, Antonio copies the whole equation on a second line in the Notepad (first line in Fig. 5). Now, LaTeX pragmatic mediation is exploited. He decides to work inside the brackets first (component 5 of SS), and so he prepares the environment for such purpose.

A different sequence of actions is observable: several blank spaces are added before the closing bracket (second line in Fig. 5), then he states that he should calculate the least common multiple of the denominators (component 5 of SS). He writes the denominator in the obtained blank space within braces (third line in Fig. 5) and then adds a couple of braces before (fourth line)—so preparing the space for the numerator.

Table 2 Enriched transcript of the third excerpt

Line	What is said	What is done
69	Antonio: Let's see the structure precisely. One fourth	The cursor moves till the denominator of the first fraction and stops before the closing curly brace (see Fig. 4)
70	Interviewer: While you are understanding the structure, would you like to tell it?	Antonio moves the cursor forward and stops after the minus sign
71	Antonio: One fourth, yes	The cursor moves back, before the minus sign
72	Minus x over… Over x minus one. Minus x equals… Then there is the second member of the equation	The cursor moves forward and stops right before the equal sign
73	Five plus, open bracket. Then there is a bracket	The cursor reaches the opening bracket
74	One fourth inside the bracket	The cursor moves back and forth over the \frac command
75		The cursor reaches the end of the last fraction
76	Minus x over x minus one	The cursor moves back and forth over the denominator of the last fraction, then it goes back to the minus sign
77	Closed bracket and that's it	The cursor moves forward till the end of the equation
78	Here I would start by working on the brackets	

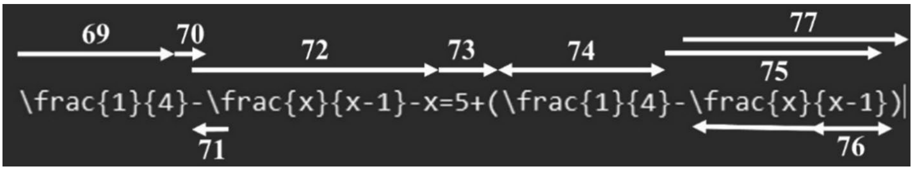

Fig. 4 Cursor's movements during the first excerpt. Numbers are keyed to Table 2

He performs his calculations between these braces (lines 5 and 6 in Fig. 5). Only after, he adds the command \frac before the braces (seventh line). Finally, he replaces the content of the brackets with the calculated fraction (last line in Fig. 5). We can see that

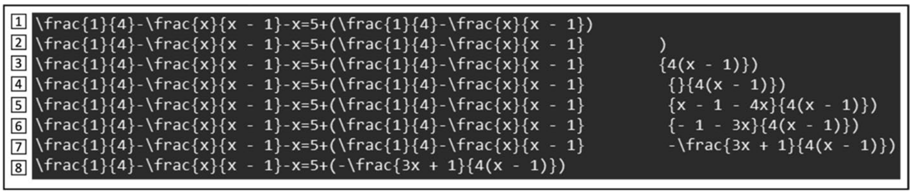

Fig. 5 Different phases of Antonio's manipulations of the fractions in brackets; each line represents a manipulation which is realized (on the actual Notepad) within the same line

he recognizes which manipulation he can realize and uses the braces for structuring the result of such manipulations, as in the previous excerpt. With a different sequence of actions (adding blank space), we can observe again the pragmatic mediation of LaTeX. After these manipulations, Antonio works on the left side of the equation, as shown in Table 3.

This time, Antonio recognizes the same structure of the expression within the brackets (line 86, component 2 of SS). Now, the mentioned expression is not present in the current form: Antonio refers to a part of the original expression, demonstrating how the SS plays a leading role, allowing him to compare the current sub-structure with another previously perceived. Then, he understands that, instead of performing again all the manipulations shown in Fig. 5, he can replace the fractions on the left side with the fraction calculated on the right side (lines 88–89, components 4 and 6 of SS), which was into the round brackets (Fig. 2). Having two identical sub-structures on the two sides of the equality, he decides to cancel them (lines 92–93, components 4 and 6 of SS): a different recognition of the structure of the equation (a different *if* in the "if–then" rule) leads to a different course of actions (the same of line 51).

Table 3 Enriched transcript of the fourth excerpt

Line	What is said	What is done
84	Antonio: Ok. Now let's see what was here	The cursor moves back to the beginning
85	There was one fourth	The cursor moves till the end of the second fraction
86	And then there was the same thing as in the brackets, but outside	Then the cursor moves back to the beginning
87	Thus… Thus, the result is the same of the other side because it's equivalent	The cursor moves till the end of the left side of the equation
88	Then I can copy this	The cursor moves to the fraction on the right side of the equation, which is then selected
89		The fraction is copied in a new following line on the Notepad
90	Then I copy 'minus x equals'	He types "–x = "
91	Well, I can copy and paste the second member. Yes, I copy and paste it	He selects, copies, and pastes the right side of the equation
92	Thus, since they are… the fractions are equal but opposite in sign… because if… then, I bring at the second member what is in the first member, they are equal and opposite. The result should be…	The cursor goes back to the beginning and then moves till the end
93	x equals minus five	He moves to a new line and types "x = –5"
94	Let me check…	The cursor goes back to the previous line and moves through the whole line, from left to right
95	Yes, that should be the result	

6 Discussion and conclusion

Building on the scarce literature about mathematics teaching and learning for the visually impaired, we began to investigate how solvers who cannot visually perceive the representations of mathematical objects deal with algebraic tasks. We intend to turn a spotlight on an underrepresented part of the students' community that is too often excluded from participating in mathematical discourse. Starting from the assumption that we need to know more about how visually impaired people do mathematics before we can design any educational interventions, we conducted an in-depth analysis of interviews with solvers with a strong mathematical background. To show the richness and complexity of data analysis and consider the specificity of each solver, in this paper, we focused on a single solver and on algebraic tasks.

Our analysis described how a blind subject, Antonio, manifests his structure sense while solving equations using Microsoft Notepad and NVDA screen-reader to manipulate the equations in LaTeX code. We analyzed Antonio's instrumented acts while using LaTeX for a particular task, highlighting the mediation role played by the artifact and describing the related utilization schemes, which are the result of a long process of instrumental genesis since Antonio has used this code for many years.

LaTeX programming language provides epistemic mediation to Antonio using the speech synthesis to read (and re-read) self-selected portions of the equation instead of simply reading from left to right. Initially, Antonio declares the need for a "continuous" and "quick" reading, to get the general structure of the equation (components 1 and 2 of SS, Hoch & Dreyfus, 2004), then the equation is divided into sub-structures (component 3 of SS), and then Antonio recognizes possible manipulations (component 5) on sub-structures. We have noticed that Antonio was able to recognize previously met sub-structures and use them to shortcut his manipulation (Table 1), so mobilizing many components (2–4–6) of SS. This recognition is realized after hearing the reading of the first part of the equation (line 13).

As noted in another study involving Braille readers (van Leendert et al., 2021), also in the case of speech synthesis, we noted that the blind solver navigates the code looking for indicators: fractions, brackets, and equal sign. As observed for sighted subjects, brackets play a relevant role in structuring the equation (Hoch & Dreyfus, 2004). However, in this case, we can notice that the use of the LaTeX code may play an important role as well in structuring the equation, since the \frac command is often a place where the cursor stops and where a "back and forth" reading acts are enacted (e.g., line 74, Fig. 4).

Furthermore, the LaTeX code becomes not only a tool for *reading* mathematics, but a tool for *doing* mathematics as well (Alajarmeh et al., 2011)—it serves as pragmatic mediator. This is observable when the code appears before the numbers (line 54), so structuring the result, or when the curly braces are used to organize the space of manipulation, distinguishing numerator and denominator of the algebraic fraction (Fig. 5). We described different utilization schemes: the notepad is used by Antonio to manipulate the equation in LaTeX both within the same line (differently than what is doable with paper and pencil, Fig. 5) or connecting different lines (e.g., lines 86–91). The activation of the different schemes depends on whether he succeeds in identifying certain sub-structures (different rules of action): *if* he recognizes the possibility of canceling opposite terms, *then* he quickly reaches the solution of the equation enacting an inter-line scheme; *if* this possibility is not seen, *then* he manipulates the equation in a more routinely fashion, enacting an intra-line scheme.

Antonio demonstrates the end of long processes of SS development and instrumental geneses that allow him to perceive, recognize, select, and act on interesting components and single elements of an algebraic expression. Here, perception of interesting or useful elements which prompt specific instrumented acts is not provided by sight, but by hearing; in this sense, we are extending the original notion of SS elicited by a different perception than sight. The reading of the equation and the memorized "soundtrack" acted as a semiotic means to represent the equation—being anything but visual. This fact corroborates that other senses than sight can successfully help not only in the rote manipulations of algebraic symbols but in developing SS as well. This suggests that autonomous verbalization of algebraic expressions may be important in the development of SS for blind people and possibly for other students as well (Maffei & Mariotti, 2011). Thus, adopting inclusive approaches (in the sense of Piroi et al., 2023) to algebra teaching may be fruitful not only for impaired students but for the whole class-group.

Moreover, Antonio demonstrates an efficient use of different artifacts that he has become familiar with, which can be skillfully chosen as instruments to solve a specific task. In particular, he provides a telling example of how the LaTeX programming language can serve as a tool for symbolic manipulation with the interesting "side effect" of being transformable in a PDF which can be then read both by sighted and visually impaired students. As noted by Ahmetovic et al. (2021), LaTeX is a writing system used in all STEM disciplines; its learning may be both useful for academic achievement and for inclusivity. Our contribution suggests that the learning of LaTeX could support SS especially for visually impaired students, but potentially not only. We have observed the utilization schemes developed, over the years, by an expert user of LaTeX to support his own SS. Possibly, some of these schemes may be included in teaching sequences (as proposed by van Leendert et al., 2021) to foster the development of visually impaired subjects' structure sense—and maybe not just theirs.

We must be cautious about the generalizability of the conclusions drawn from a case study; we must consider that visual impairments are very different among them. For instance, Antonio was not completely blind during high school studies, and then, he might rely on visual memories of algebraic expressions. He began his learning of LaTeX when he could still rely partially on sight. To consider the variety of individual differences, further case studies are important. Different results might be obtained in the case of students born blind and/or Braille readers. Also, there might be important individual differences in the process of instrumental genesis of programming languages such as LaTeX. This is likely since the process of instrumental genesis depends on the artifact, and in the case of visually impaired subjects, the programming language is not the only artifact involved: in Antonio's case, many applications are used like NVDA, the speech synthesis, the Notepad, and the Axessibility package for PDF files. Braille readers may use specific keyboards. This observation leads to the importance of widening this kind of study to the whole system of instruments (Rabardel & Bourmaud, 2003) used by different subjects. Future developments of our research project will include subjects with different past histories about their disabilities and about their learning of mathematics. Nevertheless, we believe that this work offers a step forward in unveiling the (many) ways in which visually impaired people can successfully tackle algebra.

Acknowledgements We would like to express our gratitude to Dr. Adriano Sofia for generously sharing his time and thoughts with us, contributing significantly to the ideas behind this paper.

Funding Open access funding provided by Alma Mater Studiorum - Università di Bologna within the CRUI-CARE Agreement. The study was partially funded by the Istituto Nazionale di Alta

Matematica INdAM (GNSAGA project "Mathematics and visual impairment: an inclusive perspective" – CUP_E55F22000270001).

Data availability The raw data that support the findings of this study are protected and are not available due to data privacy laws. The processed data sets are available from "S. Polin" Laboratory (University of Turin) and used under licence for the current study and so are not publicly available. The data are, however, available from the authors upon reasonable request and with the permission of "S. Polin" Laboratory (University of Turin).

Declarations

Ethics approval Approval was obtained from the local ethics committee of the University of Turin (Italy).

Competing interest The authors declare no competing interests.

References

Ahmetovic, D., Bernareggi, C., Bracco, M., Murru, N., Armano, T., & Capietto, A. (2021). LaTeX as an inclusive accessibility instrument for high school mathematical education. In S. Rodriguez Vazquez, T. Drake, D. Ahmetovic, & V. Yaneva (Eds.) *Proceedings of the 18th International Web for All Conference* (pp. 1–9). https://doi.org/10.1145/3430263.3452444

Alajarmeh, N., Pontelli, E., & Son, T. (2011). From "reading" math to "doing" math: A new direction in non-visual math accessibility. In C. Stephanidis (Ed.), *Universal Access in Human-Computer Interaction. Applications and Services* (pp. 501–510). Springer Berlin. https://doi.org/10.1007/978-3-642-21657-2_54

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241. https://doi.org/10.1023/a:1024312321077

Armano, T., Capietto, A., Coriasco, S., Murru, N., Ruighi, A., Taranto, E. (2018). An automatized method based on LaTeX for the realization of accessible PDF documents containing formulae. In K. Miesenberger & G. Kouroupetroglou, (Eds.), *Computers Helping People with Special Needs* (pp. 583–589). Springer. https://doi.org/10.1007/978-3-319-94277-3_91

Bell, J. T., & Kissling, M. T. (2019). Thick description as pedagogical tool: Considering Bowers' inspiration for our work. *Educational Studies in Mathematics, 55*(5), 531–547. https://doi.org/10.1080/00131946.2019.1628762

Bernareggi, C. (2010). Non-sequential mathematical notations in the LAMBDA system. In K. Miesenberger, J. Klaus, W. Zagler, & A. Karshmer (Eds.), *Computers Helping People with Special Needs (ICCHP)* (LNISA, 6180:389–395). https://doi.org/10.1007/978-3-642-14100-3_58

Bouck, E. C., Weng, P.-L., & Satsangi, R. (2016). Digital versus traditional: Secondary students with visual impairments' perceptions of a digital algebra textbook. *Journal of Visual Impairment & Blindness, 110*(1), 41–52. https://doi.org/10.1177/0145482X1611000105

Cohen, L., Manion, L., & Morrison, K. (2007). Research methods in education. *Routledge*. https://doi.org/10.4324/9780203224342

Dabi, G. K., & Golga, D. N. (2023). The role of assistive technology in supporting the engagement of students with visual impairment in learning mathematics: An integrative literature review. *British Journal of Visual Impairment, 42*(3), 674–687. https://doi.org/10.1177/02646196231158922

Del Campo, J. E. F. (2000). *La enseñanza de la matematica a los ciegos* (A. García Martín, Trans.). ONCE and Biblioteca italiana per ciechi "Regina Margherita" ONLUS.

Healy, L., & Fernandes, S. H. A. A. (2011). The role of gestures in the mathematical practices of those who do not see with their eyes. *Educational Studies in Mathematics, 77*(2), 157–174. https://doi.org/10.1007/s10649-010-9290-1

Herzberg, T. S., & Rett McBride, C. (2023). Experiences of teachers of students with visual impairments in learning and teaching a new Braille code for mathematics and science. *Journal of Visual Impairment & Blindness, 117*(6), 429–439. https://doi.org/10.1177/0145482X231216472

Hoch, M., & Dreyfus, T. (2004). Structure sense in high school algebra: The effect of brackets. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 49–56). IGPME.

Kim, S., Cherian, J., Ray, S., Lacy, A., Taele, P., Koh, J. I., & Hammond, T. (2023). A wearable haptic interface for assisting blind and visually impaired students in learning algebraic equations. *CHI Conference on Human Factors in Computing Systems, 17*, 1–7. https://doi.org/10.1145/3544549.3585815

Klingenberg, O. G., Holkesvik, A. H., & Augestad, L. B. (2020). Digital learning in mathematics for students with severe visual impairment: A systematic review. *British Journal of Visual Impairment, 38*(1), 38–57. https://doi.org/10.1177/0264619619876975

Linchevski, L., & Livneh, D. (1999). Structure sense: The relationship between algebraic and numerical contexts. *Educational Studies in Mathematics, 40*(2), 173–196. https://doi.org/10.1023/A:1003606308064

Maffei, L., & Mariotti, M.A. (2011). The role of discursive artefacts in making the structure of an algebraic expression emerge. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of the 7th Conference of the European society for Research in Mathematics Education* (pp. 511–520). ERME.

Miragliotta, E., Manolino, C. & Maffia, A. (2023). Figural component in geometrical reasoning: The case of a blind solver. In P. Drijvers, C. Csapodi, H. Palmér, K. Gosztonyi, & E. Kónya (Eds.), *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education* (pp. 4467–4474). Rényi Institute of Mathematics and ERME. https://hal.science/hal-04408161. Accessed 13 Feb 2025.

Moreno Martínez, C., Piorno, J. R., Escribano Otero, J. J., & Guijarro Mata-García, M. (2022). Responsive inclusive design (RiD): A new model for inclusive software development. *Universal Access in the Information Society, 22*(3), 893–902. https://doi.org/10.1007/s10209-022-00893-9

Piroi, M., Manolino, C., Armano, T., Taranto, E., & Capietto, A. (2023). Teacher professional development via a MOOC on assistive technology for visually impaired students learning mathematics. *Journal of Mathematics Education, 16*(1), 59–78. https://doi.org/10.1177/02646196231158922

Rabardel, P. (2002). *People and technology: A cognitive approach to contemporary instruments*. Université Paris. Retrieved from https://hal.archives-ouvertes.fr/hal-01020705. Accessed 13 Feb 2025.

Rabardel, P., & Bourmaud, G. (2003). From computer to instrument system: A developmental perspective. *Interacting with Computers, 15*(5), 665–691. https://doi.org/10.1016/S0953-5438(03)00058-4

Schüler-Meyer, A. (2017). Students' development of structure sense for the distributive law. *Educational Studies in Mathematics, 96*, 17–32. https://doi.org/10.1007/s10649-017-9765-4

Smeureanu, I., & Isăilă, N. (2011). Aspects of mathematics learning objects creation for persons with visual disabilities. *Academy of Economic Studies-Economy Informatics, 11*(1), 37–44.

van Leendert, A., Boonstra, L., Doorman, M., Drijvers, P., van der Steen, J., & Pel, J. (2021). An exploratory study to improve reading and comprehending mathematical expressions in Braille. *British Journal of Visual Impairment, 41*(2), 312–327. https://doi.org/10.1177/02646196211044972

Vergnaud, G. (1990). La théorie des champs conceptuels. *Recherches en didactique des mathématiques, 10*(2-3).

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of though in relation to instrumented activity. *European Journal of Psychology of Education, 10*(1), 77–101. https://www.jstor.org/stable/23420087. Accessed 13 Feb 2025.

Vygotskij, L.S. (1930/1997). The instrumental methods in psychology. In R. W. Rieber, & J. Wollock (Eds.) *The collected works of L. S. Vygotsky: Problems of the theory and history of psychology* (pp. 85–59). Springer. https://doi.org/10.1007/978-1-4615-5893-4_7

Wallon, H. (1941/2020). *L'évolution psychologique de l'enfant - douzième édition*. Dunod.

Zambrano, A. M., Pilacuán, D. I., Salvador, M. N., Grijalva, F., Garzón, N. O., & Carvajal Mora, H. (2023). IrisMath: A blind-friendly web-based computer algebra system. *IEEE Access, 11*, 71766–71776. https://doi.org/10.1109/ACCESS.2023.3281761