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Emanuele Bacchiega*, Elias Carroni and Alessandro Fedele

Quasi-Collusive Equilibria in Two-Sided Hotelling Models

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Abstract: The Hotelling model is widely used to analyze platform competition in two-sided markets. Under standard assumptions, namely, sufficiently high standalone utility and strong product differentiation relative to cross-group externalities, the model yields a competitive equilibrium with two active platforms. Relaxing either assumption typically leads to corner outcomes such as market tipping or uncovered markets. This paper shows that, when the full parameter space is analyzed, an additional class of equilibria emerges: market-sharing *kinked-demand* equilibria (Mérel and Sexton, 2010), at which platforms soften competition and replicate a collusive outcome. These findings extend the traditional two-sided Hotelling framework by highlighting market environments, common in digital settings, in which cross-group externalities outweigh both differentiation and standalone utility.

Keywords: Hotelling model; platform competition; two-sided markets; kinked-demand equilibria; quasi-collusion; digital markets

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***Corresponding author: Emanuele Bacchiega**, Department of Computer Science and Engineering, University of Bologna, Bologna, Italy, E-mail: emanuele.bacchiega@unibo.it. https://orcid.org/0000-0002-7822-6486
Elias Carroni, Department of Economics and Management, University of Cagliari, Cagliari, Italy, E-mail: elias.carroni@unica.it. https://orcid.org/0000-0002-6809-5339
Alessandro Fedele, Faculty of Economics and Management, Free University of Bolzano, Bolzano, Italy, E-mail: alessandro.fedele@unibz.it. https://orcid.org/0000-0002-0528-0131

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1 Introduction

In many markets, competing platforms connect different groups of agents. Since Armstrong (2006)'s seminal work, the Hotelling duopoly has become the workhorse for analyzing platform competition in such two-sided markets. In applying the Hotelling's model to platform competition, two assumptions are commonly retained. First, Armstrong (2006) (condition (8), p. 674) shows that platforms must display a high degree of horizontal differentiation relative to the value agents place on the interaction with agents from the opposite side: this assumption guarantees the existence of a market-sharing Hotelling equilibrium, namely an equilibrium where both platforms attract positive demand. Without sufficient differentiation, the incentive to expand market share by poaching users from the rival platform leads to cutthroat competition and the so-called *tipping equilibria*, in which all agents on both sides converge on a single platform, effectively resulting in a *de facto* monopoly. Second, the *stand-alone value*, namely the utility an agent obtains from joining a platform even if no one else is active there must be sufficiently large to ensure that the market is covered at equilibrium. Once these two assumptions are satisfied, the analysis reduces to the standard "textbook" Hotelling model with fixed locations. We will refer to the equilibrium derived under these assumptions as the "competitive" equilibrium.[1]

However, in many real-world markets, especially digital ones, such as streaming services, online marketplaces, ride-sharing and travel platforms, access to desired content or services (i.e., cross-group benefits) often matters more than individual preferences for specific platforms (i.e., the degree of differentiation); in addition, most platforms have little intrinsic value in the absence of content or services, that is, their stand-alone utility is limited.

Despite this factual evidence, many digital markets are far from monopolized, and the platforms operating therein reap high profit margins; also, the cross-group externalities generally ensure full market coverage. This apparent contradiction with the model's equilibrium predictions suggests that the canonical assumptions of the two-sided Hotelling framework may lead to disregard market-sharing equilibria where platforms earn positive profits and the market is fully covered.

In the present paper we consider a simple two-sided Hotelling setup and characterize its equilibria over the entire parameter space. Our analysis delivers two

1 The present paper borrows most of its terminology from Mérel and Sexton (2010).

novel results. First, we show that a market-sharing equilibrium may exist even if product differentiation is weak relative to the cross-group externalities. This equilibrium is of the *kinked-demand* type (Mérel and Sexton 2010), where platforms aim to serve all the consumers that do not patronize the rival *and only them*, therefore avoiding business stealing. This mimics a collusive outcome that defuses ruthless competition and allows both platforms to reap positive profits. Second, we show that the kinked-demand equilibrium coexists with the competitive equilibrium for low stand-alone values, and it generally delivers higher profits to both platforms: with a payoff-dominance refinement, the competitive equilibrium is not selected. All in all, our analysis predicts that, in two-sided markets where externalities are relatively strong, platform coexistence with muted competition turns out not only to be a possibility, but rather the most likely strategic outcome.

To illustrate how our setting and findings can be applied, consider two ride-sharing platforms competing for both riders and drivers. Also suppose that riders primarily value access to drivers – the cross-group benefit is strong – while the platform itself offers little stand-alone value, and rider preferences across platforms are moderately heterogeneous. The conventional wisdom from the two-sided Hotelling models predict cutthroat competition: if cross-group benefits dominate differentiation, even a small price advantage can trigger a tipping dynamic, leading to a single dominant platform. Yet in practice, across ride-sharing, streaming, online marketplaces, and travel, multiple platforms coexist with healthy margins.

Our paper explains this by showing that when cross-group externalities are sufficiently strong relative to both differentiation and stand-alone utility, a kinked-demand equilibrium emerges in which each platform targets only its own segment of riders. Rather than undercutting rivals, platforms sustain a stable market split, a quasi-collusive outcome that remains a Nash equilibrium, since poaching competitors' users would provoke a price war that erodes profits. Importantly, platforms prefer this outcome to tipping, as winning a tipping race requires aggressive discounting that dissipates profits.

In the remainder of the paper, Section 2 presents the model and briefly summarizes the characterization of Nash equilibria in the unrestricted one-sided Hotelling parameter space. Section 3 examines the two-sided framework and derives the main results. Section 4 concludes. All proofs are provided in Appendix.

2 Model

Two platforms, indexed by $i \in \{0,1\}$ and located at the endpoints of a unit-length Hotelling segment, mediate interactions between two groups of agents, referred to as consumers and producers. Platforms set prices on the consumer side only.[2]

A unit mass of consumers is uniformly distributed along the line and incurs linear transportation costs. Consumers decide whether to stay out of the market or to join one platform; in the former case, their utility is zero; in the latter, the utility function of a consumer located at $x \in [0,1]$ and joining platform i is

$$\mathcal{U}(x, p_i, \mathbb{E}(n_i)) = v + \alpha \cdot \mathbb{E}(n_i) - td_i(x) - p_i. \tag{1}$$

Parameter $v \geq 0$ denotes the stand-alone value, namely the utility the consumer enjoys if joining a platform with no producers; $\alpha\mathbb{E}(n_i)$ captures the cross-group benefits enjoyed by the consumer on platform i, $\alpha \in (0,1]$ being the marginal benefit, and $\mathbb{E}(n_i)$ the mass of producers the consumer expects to be active on platform i; $t > 0$ is the marginal transportation cost and $d_i(x)$ is the Euclidean distance of the consumer from platform i; p_i is the price charged by platform i on each consumer.

On the other market side, a unit mass of producers chooses whether to join one, both or no platform. Producers incur heterogeneous setup costs f to operate on either platform, with f uniformly distributed on the interval $[0, 1]$; production costs are normalized to zero. Each producer enjoys the marginal cross-group benefit $\gamma \in (0,1]$ and pays no fee to join a platform. Overall, a cost-f producer is willing to join platform i iff $\gamma \cdot \mathbb{E}(D_i) \geq f$, where $\mathbb{E}(D_i)$ is the mass of consumers the producer expects to be active on platform i. The resulting mass of producers joining platform i is $n_i = \text{Prob}(\gamma \cdot \mathbb{E}(D_i) \geq f)$, which simplifies to $n_i = \gamma \cdot \mathbb{E}(D_i)$ under the assumption of uniform distribution of setup costs.

Platforms, consumers, and producers play the following game. Platforms simultaneously set prices p_i to maximize profits. Then, consumers and producers simultaneously make their joining decisions. In the literature, this game is solved as follows. Equation $\mathcal{U}(x, p_0, \mathbb{E}(n_0)) = \mathcal{U}(x, p_1, \mathbb{E}(n_1))$ yields the location of the consumer indifferent between joining platform 0 or 1:

$$\hat{x} = \frac{1}{2} + \frac{p_1 - p_0 + \alpha(\mathbb{E}(n_0) - \mathbb{E}(n_1))}{2t}. \tag{2}$$

Producers and consumers have correct expectations about the other side's participation at equilibrium. As a consequence, $\mathbb{E}(D_0) = \hat{x}$, $\mathbb{E}(D_1) = (1 - \hat{x})$, and $\mathbb{E}(n_i) =$

2 A similar framework is adopted by, e.g., Carroni et al. (2024), who, in turn, build on Armstrong (2006) for the consumer side and on Hagiu (2006) for the producer side.

$\gamma\mathbb{E}(D_i)$. Solving the system of producers' and consumers' expectations yields $\mathbb{E}(n_0) = \gamma\hat{x}$ and $\mathbb{E}(n_1) = \gamma(1-\hat{x})$, which are then plugged into (2),

$$x_I = \frac{1}{2} - \frac{p_0 - p_1}{2(t - \alpha\gamma)}. \tag{3}$$

Platforms 0 and 1 simultaneously solve problems $\max_{p_0} p_0 x_I$ and $\max_{p_1} p_1(1 - x_I)$. The symmetric equilibrium price and profits are

$$t - \alpha\gamma \quad \text{and} \quad \frac{t - \alpha\gamma}{2}, \tag{4}$$

respectively. Note that both values are positive if, and only if, $t > \alpha\gamma$: the transportation cost parameter must be larger than the product of marginal cross-group benefits. This condition is akin to (8) in Armstrong (2006) and is necessary and sufficient for the existence of a market-sharing, competitive equilibrium.

In the remainder of the paper, we argue that: (i) a different type of market-sharing equilibrium may exist, even when condition $t > \alpha\gamma$ is violated; (ii) this equilibrium may coexist with the competitive one and Pareto-dominates it from the platforms' standpoint. To this end, we look for kinked-demand equilibria, as in Mérel and Sexton (2010). For the reader's convenience, we briefly summarize their equilibrium characterization for the one-sided Hotelling model and provide some economic intuition regarding such equilibria. We instead refer to Mérel and Sexton (2010)'s Proposition 1 for a complete analysis and to their Figure 1 for a graphical representation of the equilibrium prices.

Three types of Nash equilibria may arise in the one-sided Hotelling game, depending on the relative size of t and v.

(i) *Competitive* (C-)equilibria, which correspond to the textbook Hotelling duopoly equilibria. At this equilibrium the market is fully covered, the firms try to steal business from the rival, and the indifferent consumer enjoys positive utility.

(ii) *Kinked-demand* (K-)equilibria, where firms jointly cover the market, but without attempting to steal business from the rival. The indifferent consumer obtains zero utility.

(iii) *Monopoly* (M-)equilibria, where firms forego competing with each other and behave as monopolists in their own neighborhood. The market is uncovered and the indifferent consumers abstains from consuming to avoid a utility loss.

In the one-sided Hotelling pricing game, these types of equilibria are mutually exclusive: the C-equilibrium arises if, and only if, t is low relative v; (a continuum of) K-equilibria emerges if, and only if, t takes intermediate values relative to v; the M-equilibrium is the unique one if, and only if, t is high relative to v.

While C- and M-equilibria have been extensively analyzed in the Hotelling literature, K-ones have been mostly overlooked, besides Mérel and Sexton (2010) and few other sparse contributions.[3] At the K-equilibria, producers strategically interact to keep the market covered, hence their prices are interdependent, but do not directly compete with one another to steal consumers. The K-equilibrium prices coincide with those of a multi-product monopolist that serves the entire market. This outcome is akin to, but distinct from, collusion because the K-equilibrium prices maximize the industry profits, on one hand, but they are a Nash equilibrium of a one-shot game, on the other hand. Because the prices at the K-equilibrium are observationally undistinguishable from actual collusive ones, we describe this outcome as *quasi-collusion*.[4]

3 Results

We extend the analysis in Mérel and Sexton (2010) to the two-sided framework presented in Section 2, and solve the game played by platforms, consumers, and producers for any positive value of t relative to υ, α, and γ; we let $e \equiv \alpha\gamma$. Before proceeding, we shall remind the reader that a fourth possible type of equilibrium, specific to two-sided frameworks, exists, in which one platform monopolizes the market and the other is inactive; this is the so-called *Tipping* (T-) equilibrium.

In what follows, we derive the parametric conditions on t, υ, and e under which each of the four types of equilibria arises. While the complete proofs are relegated to Appendix, here we clarify our assumptions regarding off-equilibrium expectations that sustain the equilibria and briefly sketch the structure of the proofs.

We let off-equilibrium expectations adjust in a way qualitatively consistent with economic incentives. Specifically, following a unilateral price increase by a platform, consumers expect a mass $\gamma\hat{b} \geq 0$ of producers listed on the deviating platform; such mass is weakly lower than that in the candidate equilibrium. Similarly, following a unilateral price drop by a platform, consumers expect a mass $\gamma\check{b} \geq 0$ of producers listed on that platform; this mass is weakly larger than in the candidate equilibrium one.

More precisely, for each candidate equilibrium configuration, we characterize the range of off-equilibrium expectations $\gamma\hat{b}$ and $\gamma\check{b}$, under which unilateral deviations are unprofitable. Intuitively, equilibrium existence requires off-equilibrium

3 See Salop (1979); Thépot (2007); Cowan and Yin (2008); Rey and Salant (2012); Fedele and Depedri (2016); and Rey and Tirole (2019). More recent contributions are Bos et al. (2025a); Navarra et al. (2024); Bos et al. (2025b).

4 For an explanation of the quasi-collusive behavior's determinants, see Section 3.2 in Bacchiega et al. (2023), who investigate the one-sided Hotelling setup.

expectations to be sufficiently reactive following upward deviations and sufficiently passive following downward deviations, thereby preventing them from generating demand responses that excessively reward deviations; formally, both $\gamma\hat{b}$ and $\gamma\check{b}$ must be sufficiently low. An extreme example within this class of off-equilibrium expectations is the "worst-case" scenario for the deviating platform: after any upward price deviation, consumers expect no producers to join the deviating platform ($\gamma\hat{b} = 0$), whereas following any downward deviation consumers expect the mass of producers not to increase ($\gamma\check{b}$ coincides with the candidate equilibrium one).

3.1 T-equilibrium

At the T-equilibrium, all consumers sort onto one platform, say platform i. Therefore, producers' correct expectations are $\mathbb{E}(D_j) = 0$ and $\mathbb{E}(D_i) = 1$, $j \neq i$, which lead to $n_j = \gamma \times 0$ and $n_i = \gamma \times 1$. The equilibrium prices, given correct expectations, must induce all consumers to (weakly) prefer joining platform i: $\mathcal{U}(x, p_j, \mathbb{E}(n_j)) \leq \mathcal{U}(x, p_i, \mathbb{E}(n_i))$, for any x. This is equivalent to require that the consumer located at $x = j$ weakly prefers platform i: $v + 0 - t \times 0 - p_j \leq v + e - t \times 1 - p_i$ or,

$$p_i \leq p_j + e - t. \tag{5}$$

In Appendix we show that the T-equilibrium prices are $p_i^T \equiv e - t$ and $p_j^T \equiv 0$, with corresponding profits $\pi_i^T \equiv (e - t) \times 1$ and $\pi_j^T \equiv 0 \times 0$. Note all these values are nonnegative if

$$t \leq e. \tag{6}$$

We state

Lemma 1. *The T-equilibrium exists if, and only if, $t \leq e$ and $\hat{b} \leq \check{b}^T$.*[5]

3.2 C-equilibrium

The C-equilibrium is derived in Section 2, where the indifferent consumer obtains positive utility. At the symmetric equilibrium prices $p^C \equiv t - e$, the indifferent consumer utility is $\mathcal{U}(x_I(p^C), p^C, \mathbb{E}(n_i(p^C)) = v + \frac{e}{2} - (t - e) - \frac{t}{2}$. This value is positive if, and only if,

$$(e <)t < e + \frac{2}{3}v. \tag{7}$$

5 The exact value of the off-equilibrium-expectation threshold $\hat{b}^T$ can be found in Appendix, along with all the ensuing ones.

Recall that the bracketed inequality is Armstrong (2006)'s condition, ensuring that platforms realize positive profits in the C-equilibrium.

We state

Lemma 2. *A C-equilibrium exists if, and only if,* $e < t < e + \frac{2}{3}v$ *and* $\hat{b} \leq \hat{b}^C$.

3.3 K-equilibrium

At the K-equilibrium the indifferent consumer obtains zero utility. To characterize the equilibrium prices, two cases shall be considered, depending on whether $t > e$ or $t \leq e$. We start by the former and substitute (3) into (1) to get the indifferent consumer's utility, $v + (e-t)\left[\frac{1}{2} - \frac{p_0 - p_1}{2(t-e)}\right] - p_0$, which is zero if, and only if,

$$p_0 + p_1 = 2v - t + e. \tag{8}$$

As already mentioned, a continuum of asymmetric K-equilibria exists in the one-sided setup. All these equilibria share the same characteristics as the symmetric one, except for the lower total industry profit they generate. In the following, we will restrict our attention to the symmetric K-equilibrium.[6] Letting $p_0 = p_1 = p$ and solving (8) for p yields $p^K \equiv \frac{2v-t+e}{2}$: this is the symmetric K-equilibrium price if $t > e$.

If $t \leq e$ instead, the demand of platform i as in (3) would increase with p_i, which is implausible. To compute the indifferent consumer's location, we then proceed as follows. At the K-(symmetric-)equilibrium, producers' correct expectations must be $\mathbb{E}(D_i) = \frac{1}{2}$, which implies $n_i = \frac{\gamma}{2}$. Therefore, consumers' correct expectations require $\mathbb{E}(n_i) = \frac{\gamma}{2}$. Plugging $\frac{\gamma}{2}$ into (1) and solving $\mathcal{U}(x, p_0, \mathbb{E}(n_0)) = \mathcal{U}(x, p_1, \mathbb{E}(n_1))$ for x gives $x_I = \frac{1}{2} + \frac{p_1 - p_0}{2t}$. This indifferent consumer's utility is $v + \frac{e}{2} - p_0 - t\left(\frac{1}{2} + \frac{p_1 - p_0}{2t}\right)$ that is zero under the same condition as in (8). The symmetric K-equilibrium price is hence $p^K \equiv \frac{2v-t+e}{2}$ in this case as well.

We state

Lemma 3. *A symmetric K-equilibrium exists if, and only if,* $\frac{1}{3}e + \frac{2}{3}v \leq t \leq e + v$, $\check{b} \leq \check{b}^K$ *and* $\hat{b} \leq \hat{b}^K$.

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6 Two considerations motivate our focus on the unique symmetric K-equilibrium. First, platforms are ex ante identical, so the symmetric outcome is the natural one. Second, in asymmetric equilibria the platform with lower profits has a stronger incentive to deviate, making those equilibria more fragile than the symmetric one. Overall, we believe that the extra complexity of analyzing asymmetric K-equilibria does not pay back in terms of additional insights.

3.4 M-equilibrium

At the M-equilibrium, the indifferent consumer abstains from consumption to avoid a utility loss, hence the market is partially covered. To derive the equilibrium price, we exploit symmetry between platforms and consider platform 0. Solving $\mathcal{U}(x, p_0, \mathbb{E}(n_0)) = 0$ for x gives $x_0 = \frac{v+\alpha\mathbb{E}(n_0)-p_0}{t}$: this is the location of the marginal consumer, i.e., the consumer indifferent between joining platform 0 and not joining any platform. With correct equilibrium expectations, platform 0's demand is therefore $D_0 = \frac{v-p_0}{t-e}$. Platform 0 faces problem $\max_{p_0} p_0\left(\frac{v-p_0}{t-e}\right)$, whose solution is $p^M \equiv \frac{v}{2}$, with $x_0(p^M) = \frac{v}{2(t-e)}$. At the M-equilibrium, the utility of the indifferent consumer is $\mathcal{U}(x_I(p^M), p^M, \mathbb{E}(n_0(p^M)) = v + e\frac{v}{2(t-e)} - \frac{v}{2} - \frac{t}{2}$. This value is negative if, and only if,

$$t > e + v. \tag{9}$$

We state the following

Lemma 4. *A M-equilibrium exists if, and only if, $t > e + sv$ and $\check{b} \leq \check{b}^M$.*

The next proposition summarizes the findings of Lemmata 1–4 and characterizes the different equilibrium configurations of the game. Before proceeding, we define $\hat{b}^* \equiv \min\{\hat{b}^T, \hat{b}^C, \hat{b}^K\}$ and $\check{b}^* \equiv \min\{\check{b}^K, \check{b}^M\}$.

Proposition 1. *When $\hat{b} \leq \hat{b}^*$ and $\check{b} \leq \check{b}^*$, the two-sided Hotelling price competition game admits four possible equilibrium configurations:*

(T) If $t \leq e$, T-equilibrium with prices $p_i^T \equiv e - t$ and $p_j^T \equiv 0$, $j \neq i$, and profits $\pi_i^T \equiv e - t$ and $\pi_j^T \equiv 0$.

(C) If $e < t < e + \frac{2v}{3}$, C-equilibrium with symmetric prices $p^C \equiv t - e$, and profits $\pi^C \equiv \frac{t-e}{2}$.

(K) if $\frac{1}{3}e + \frac{2}{3}v \leq t \leq e + v$ K-symmetric equilibrium with prices $p^K \equiv v - \frac{t}{2} + \frac{e}{2}$, and profits $\pi^K \equiv \frac{v}{2} - \frac{t}{4} + \frac{e}{4}$.

(M) if $e + v < t$, M-equilibrium with symmetric prices $p^M \equiv \frac{v}{2}$, and profits $\pi^M \equiv \frac{v^2}{4(t-e)}$.

It is immediate that T- and K-equilibria co-exist if $\frac{1}{3}e + \frac{2}{3}v \leq t \leq e$, while C- and K-equilibria co-exist if $\max\left\{e, \frac{1}{3}e + \frac{2}{3}v\right\} < t \leq e + \frac{2}{3}v$. Also, note that the former interval is non-empty if, and only if, $e > v$. We refine the set of equilibria by adapting to our setup the payoff-dominance criterion (Harsanyi and Selten 1988), which – in its original formulation – selects the equilibrium yielding strictly higher payoffs to all players. In our exercise, we restrict the set of relevant players to the

platforms, so that the refinement selects the equilibrium yielding the largest profits to each firm.

One can easily check that the K-equilibrium profits, π^K, exceed the positive T-equilibrium ones, π_i^T, if either $(v <)e < 2v$ or $e \geq 2v \cap e - \frac{2}{3}v < t (\leq e)$. In these cases, both platforms strictly prefer the K-equilibrium to the T-equilibrium. The intuition is twofold. First, unlike the positive profits in the T-equilibrium, K-equilibrium profits are increasing in v because they are pinned down by the indifferent consumer's zero-utility condition (8). Second, the positive T-equilibrium profits are more strongly negatively affected by t than those in the K-equilibrium, since under tipping the sole active platform attracts higher demand than each platform in the kinked-demand equilibrium. Taken together, these two features explain why platforms prefer the K equilibrium when v is not too low or, alternatively, t is relatively high.

Consider now the K- and C-equilibrium profits and recall that the two equilibria co-exist when $\max\left\{e, \frac{1}{3}e + \frac{2}{3}v\right\} < t \leq e + \frac{2}{3}v$. One can verify that, within this interval, $\pi^K > \pi^C$: intuitively, in the C-equilibrium each platform sets prices to steal business from its rival, whereas in the K-equilibrium platforms avoid business stealing, which explains why they reap larger profits.

We summarize the above in the following proposition.

Proposition 2. *After applying the payoff-dominance criterion, the two-sided Hotelling price competition game admits the following equilibrium configurations.*

(i) *If $e \leq v$: (a) T-equilibrium for $t \leq e$; (b) C-equilibrium for $e < t < \frac{1}{3}e + \frac{2}{3}v$; (c) K-equilibrium for $\frac{1}{3}e + \frac{2}{3}v \leq t \leq e + v$; (d) M-equilibrium if $t > e + v$.*

(ii) *If $v < e < 2v$: (a) T-equilibrium for $t \leq \frac{1}{3}e + \frac{2}{3}v (< e)$; (b) K-equilibrium for $\frac{1}{3}e + \frac{2}{3}v < t \leq e + v$; (c) M-equilibrium if $t > e + v$.*

(iii) *If $e \geq 2v$: (a) T-equilibrium for $t \leq \frac{1}{3}e + \frac{2}{3}v$; (b) T- and K-equilibria for $\frac{1}{3}e + \frac{2}{3}v \leq t \leq e - \frac{2}{3}v$; (c) K-equilibrium for $e - \frac{2}{3}v < t \leq e + v$; (d) M-equilibrium if $t > e + v$.*

In regions (*i-a*), (*ii-a*), and (*iii-a*), transportation costs are negligible relative to the other parameters, so that even minimal price differences trigger full tipping: the T-equilibrium naturally emerges.

At the opposite extreme, namely in regions (*i-d*), (*ii-c*), and (*iii-d*), transportation costs are sizeable, and local monopolies arise, with some consumers not subscribing to any platform. Both these outcomes align with the extant literature. Two novel results arise instead in the intermediate regions, in particular if cross-group benefits e are large relative to both v and t, as is typically the case in digital markets.

First, if $e \geq t$, the sole outcome acknowledged in the literature is the T-equilibrium, where the competitive pressure due to relatively low differentiation causes the market to tip toward a single surviving platform. However, Proposition 2 shows that a market-sharing kinked-demand equilibrium also exists in regions (*ii-b*) and (*iii-c*), namely, if t takes intermediate values and e is larger than v. At this equilibrium, platforms avoid the potentially destructive price war characteristic of the T-equilibrium by behaving in a quasi-collusive manner. Rather than undercutting their rival, each platform sets prices to attract only those consumers who do not patronize the rival, thereby preserving profits.

The same reasoning explains a second insightful results: in regions *(ii)-(iii)*, where $e > v$, the C-equilibrium is the only full-coverage market-sharing equilibrium usually considered in the literature. Yet, this equilibrium is Pareto-dominated by the K-equilibrium; as such, it should be eliminated from the set of predicted outcomes under this refinement. This is a striking result, as it suggests that in markets where cross-side externalities outweigh stand-alone values, platforms may prefer to quasi-collude rather than compete actively.[7]

4 Conclusions

In this paper, we have fully characterized the Nash equilibria of a simple two-sided Hotelling framework. In particular, we have derived the parametric conditions under which a kinked-demand (K-)equilibrium à la Mérel and Sexton (2010) arises. This has allowed us to unveil two novel results. At the K-equilibrium, platforms adopt a quasi-collusive behavior that (i) may result in higher profits for each platforms compared to the highly competitive T-equilibrium, if cross-group externalities are large relative to platform differentiation ($e \geq t$); and (ii) results in higher profits for both platforms compared to the competitive (C-)equilibrium, if cross-group externalities are large relative to the stand-alone value ($e > v$). Notably, these two conditions are likely to hold in most digital markets, where externalities are typically strong.

7 As a last remark, note that in region *(iii-b)* T- and K-equilibria coexist. This is because $\pi^T \geq \pi^K > 0$, hence the only active platform under the T-equilibrium is better-off than under the K-equilibrium, whereas the opposite holds for the inactive platform. The intuition for this result is as follows. In this region, the externality e is relatively large, which gives platforms an incentive to tip the market. At the same time, t takes intermediate values: on the one hand, t is high enough to make firms potentially willing to share the market in the K-equilibrium; on the other hand, it is not high enough to make platforms unambiguously prefer this market-sharing outcome over the tipping one.

Appendix

As mentioned in the text, we assume that off-equilibrium expectations adjust in a way qualitatively consistent with economic incentives (henceforth, we refer to them as consistent expectations). Formally, we assume that, when each consumer observes an unilateral deviation by one platform, she expects γb producers to be listed on that platform. In the case of deviation to a covered-market configuration, we impose (further) consistency by requiring that $\gamma(1-b)$ producers are expected on the non-deviating platform.

In the following, we use the shorthand "deviation to M", or "M-deviation" to denote a deviation to a local monopolies market configuration, and we adopt analogous shorthand for the other possible market configurations.

A.1 Proof of Lemma 1: T-equilibrium

At the candidate T-equilibrium, platform i's profits, $p_i \times 1$, are increasing in pi, so condition (5) must hold with equality: $p_i = p_j + e - t$. At prices $p_i = p_j + e - t$ and p_j, the utility enjoyed by the consumer located at j, if joining either platform is $v - p_j$. As a result, all consumers (weakly) prefer joining platform i: platform j's profits are $p_j \times 0 = 0$ and platform i's are $\left(p_j + e - t\right) \times 1$. Note however that p_i cannot exceed $e - t$ at equilibrium because (5) would imply $p_j = p_i - (e - t) > 0$ and firm j could profitably deviate by cutting p_j. We then consider prices $p_i = e - t$ and $p_j = 0$, and check whether they are a T-equilibrium. Note that $p_i = e - t$ is nonnegative iff (6) holds. We first rule out possible deviations by platform j by observing that: (i) any downward deviation requires negative prices and, therefore, yields negative profits; (ii) any upward deviation yields zero demand, and therefore, zero profits. Next, we consider possible deviations by platform i. (i) Any downward deviation does not increase the demand, bounded at 1, but yields lower profits. (ii) By contrast, an upward deviation, $p_i = e - t + \eta$, with $\eta > 0$, induces consumers to expect $\gamma\hat{b}$ producers on the deviating platform and $\gamma(1-\hat{b})$ on the other one, provided that the deviation is to a covered market configuration. These off-equilibrium expectations are consistent if $\hat{b} \in [0,1]$ and lead to full coverage if the utility of all consumers joining platform j is positive: $v + \left(1-\hat{b}\right)e - t \times 1 - 0 \geq 0 \Leftrightarrow \hat{b} \leq \frac{v+e-t}{t} \equiv \check{b}_1^T$. The solution for x to $v + \left(1-\hat{b}\right)e - td_j(x) - 0 = v + \hat{b}e - td_i(x) - (e - t + \eta)$ gives the location of the indifferent consumer. The resulting deviation profits are weakly lower than the equilibrium one if, and only if,

$$(e-t+\eta)\left[1 - \frac{\eta + 2\left(1-\hat{b}\right)e}{2t}\right] \leq (e-t) \times 1 \Leftrightarrow \hat{b} \leq 1 - \frac{\eta}{2e}\frac{3t-e-\eta}{e-t+\eta} \equiv \check{b}_2^T(\eta).$$

One can check that under (6): (i) $1-\frac{\eta}{2e}\frac{3t-e-\eta}{e-t+\eta}>0$ for any $\eta>0$; (ii) $\frac{v+e-t}{t}>0$. To rule out profitable deviations it is then sufficient to assume $\hat{b}\leq \min\left\{\check{b}_1^T,\check{b}_2^T(\eta)\right\}\equiv \check{b}^T$, i.e., that the consumers' off-equilibrium expectations are sufficiently reactive to the price increase.

A.2 Proof of Lemma 2: C-equilibrium

To check that $p^C\equiv t-e$ is the C-equilibrium price if (7) holds, we derive the conditions under which no unilateral profitable deviations exist to T, K, and M. Exploiting symmetry, we consider deviations by platform 0.

Deviation to T

Consider a downward deviation $p_0^D=p^C-\eta<p^C$, for given $p_1=p^C$, which induces consumers to expect $\gamma\check{b}$ producers on the deviating platform and $\gamma(1-\check{b})$ on the other one. The expectations are consistent if $\check{b}\in\left[\frac{1}{2},1\right]$ and resulting location of the indifferent consumer is $\frac{1}{2}+\frac{\eta+\left(2\check{b}-1\right)e}{2t}$. This must be equal to 1 at a T-deviation. Solving for η yields $\eta=t-\left(2\check{b}-1\right)e$, which implies that the deviation price is $t-e-\left[t-\left(2\check{b}-1\right)e\right]=-2e\left(1-\check{b}\right)$. This value is weakly negative, hence there is no profitable deviation to T.

Deviation to K

Consider an upward deviation $p_0^D=p^C+\eta>p^C$, for given $p_1=p^C$, which induces consumers to expect $\gamma\hat{b}$ producers on the deviating platform and $\gamma(1-\hat{b})$ on the other one. Such off-equilibrium expectations are consistent if $\hat{b}\in\left[0,\frac{1}{2}\right]$. At a K-deviation, the indifferent consumer has zero utility, which implies that the locations of the two marginal consumers, $x_0=\frac{v+\hat{b}e-p_0^D}{t}$ and $x_1=1-\frac{v-t+(2-\hat{b})e}{t}$, coincide. Solving $x_0=x_1$ for p_0^D yields $p_0^D=2v-2t+2e$. Plugging this value into $x_0=\frac{v+be-p_0^D}{t}$ gives $x_0=\frac{2t-v-(2-\hat{b})e}{t}$. The resulting deviation profits are weakly lower than the equilibrium ones iff

$$(2v-2t+2e)\left[\frac{2t-v-(2-\hat{b})e}{t}\right]\leq\frac{t-e}{2}\Leftrightarrow\hat{b}$$
$$\leq\frac{1}{2}\frac{8e^2-17te+12ve-12tv+9t^2+4v^2}{2e(e+v-t)}\equiv\hat{b}_1^C(\eta).$$

One can check that the RHS of the second inequality is positive under (7). It is then sufficient to assume that such inequality is fulfilled, i.e., that the consumers' off-equilibrium expectations are sufficiently reactive, to rule out profitable deviations to K.

Deviation to M

Consider an upward deviation $p_0^D = p^C + \eta > p^C$, for given $p_1 = p^C$. This deviation leads to M iff $x_0 < x_1$. With consistent off-equilibrium expectations, we have $x_0 = \frac{v+\hat{b}e-p_0^D}{t}$ and $x_1 = 1 - \frac{v-t+(2-\hat{b})e}{t}$. Solving $x_0 < x_1$ for η yields $\eta > 2v - 3t + 3e$. The deviation profits are weakly lower than the equilibrium ones iff

$$(t-e+\eta)\left[\frac{v+\hat{b}e-(t-e+\eta)}{t}\right] \leq \frac{t-e}{2} \Leftrightarrow$$

$$\hat{b} \leq \frac{2e^2 - 4\eta e + 4t\eta - 2v\eta + 2\eta^2 - 5te + 2ve - 2tv + 3t^2}{2e(\eta + t - e)} \equiv \hat{b}_2^C(\eta).$$

One can check that the RHS of the second inequality is positive because $\eta > 2v - 3t + 3e$. It is then sufficient to assume that such inequality is fulfilled, i.e., that the consumers' off-equilibrium expectations are sufficiently reactive, to rule out profitable deviations to M. In conclusion, we let $\hat{b}^C = \min\left\{\hat{b}_1^C(\eta), \hat{b}_2^C(\eta)\right\}$.

A.3 Proof of Lemma 3: K-equilibrium

To check that $p^K \equiv \frac{2v-t+e}{2}$ is the K-(symmetric)equilibrium price, we proceed as above and simply require that $p^K > 0 \Leftrightarrow t < e + 2v$.

Deviation to T

With consistent off-equilibrium expectations, a downward deviation $p_0^D = p^K - \eta < p^K$, for given $p_1 = p^K$, leads to the following location of the indifferent consumer: $\frac{1}{2} + \frac{\eta+\left(2\check{b}-1\right)e}{2t}$, with $\check{b} \in \left[\frac{1}{2}, 1\right]$. Such deviation is to T if the indifferent consumer locates at 1: $\frac{1}{2} + \frac{\eta+\left(2\check{b}-1\right)e}{2t} = 1 \Leftrightarrow \check{b} = \frac{1}{2} + \frac{t-\eta}{2e}$. If $\check{b} < \frac{1}{2} + \frac{t-\eta}{2e} \equiv \check{b}_1^K(\eta)$, T-deviations are unfeasible because the indifferent consumer's location is to the left of 1. It is then sufficient to assume that$\check{b} < \check{b}_1^K(\eta)$, i.e., that the consumers' off-equilibrium expectations are sufficiently passive, to rule out feasible deviations to T.

Deviation to C

With consistent off-equilibrium expectations, a downward deviation $p_0^D = p^K - \eta < p^K$, for given $p_1 = p^K$, leads to C if the indifferent consumer's location $\frac{1}{2} + \frac{\eta+(2b-1)e}{2t}$, with $\check{b} \in \left[\frac{1}{2}, 1\right]$, is to the left of 1. Note that $p_0^D > 0 \Leftrightarrow \eta < \frac{2v-t+e}{2}$, which is necessary for the deviation to be profitable. The resulting deviation profits are not larger than the equilibrium ones if, and only if,

$$\left(\frac{2v-t+e}{2} - \eta\right)\left[\frac{1}{2} + \frac{\eta + \left(2\check{b}-1\right)e}{2t}\right] \leq \frac{2v-t+e}{4} \Leftrightarrow \check{b} \leq \frac{e^2 - te + 2ve - 3\eta e + 3t\eta - 2v\eta + 2\eta^2}{2e(e-2\eta-t+2v)} \equiv \check{b}_2^K(\eta).$$

One can check that the RHS of the second inequality is larger than $\frac{1}{2}$ for any $\eta < \frac{2v-t+e}{2}$, provided that $t \geq \frac{1}{3}e + \frac{2}{3}v$; under the latter condition, it is then sufficient to assume that such inequality is fulfilled to rule out profitable deviations to C. If instead $t < \frac{1}{3}e + \frac{2}{3}v$, the RHS of the second inequality is lower than $\frac{1}{2}$ for sufficiently low η, hence a profitable deviation to C exists. In conclusion, we let $\check{b}^K = \min\left\{\check{b}_1^K(\eta), \check{b}_2^K(\eta)\right\}$.

Deviation to M

With consistent off-equilibrium expectations, an upward deviation $p_0^D = p^K + \eta > p^K$, for given $p_1 = p^K$, leads to the following location of platform 0's marginal consumer: $x_0 = \frac{1}{2} - \frac{\eta}{t} - \frac{e(1-2b)}{2t}$, with $\hat{b} \in \left[0, \frac{1}{2}\right]$. The deviation leads to zero demand, and is therefore not profitable, if $\frac{1}{2} - \frac{\eta}{t} - \frac{e\left(1-2\hat{b}\right)}{2t} < 0 \Leftrightarrow \hat{b} < \frac{1}{2} - \frac{t-2\eta}{2e}$. Note $\frac{1}{2} - \frac{t-2\eta}{2e} > 0$ for any $\eta > 0$ if $t < e$, in which case it is sufficient to let $\hat{b} < \frac{1}{2} - \frac{t-2\eta}{2e} \equiv \hat{b}_1^K(\eta)$ to rule out profitable deviations. If instead, $t \geq e$, the deviation leads to positive demand for any $\eta < \frac{t-e}{2}$. In this case, the deviation is not profitable iff,

$$\left(\frac{2v-t+e}{2} + \eta\right)\left[\frac{1}{2} - \frac{\eta}{t} - \frac{\left(1-2\hat{b}\right)}{2t}e\right] \leq \frac{2v-t+e}{4} \Leftrightarrow \hat{b} \leq \frac{e^2 + 4\eta e - 4t\eta + 4v\eta + 4\eta^2 - te + 2ve}{2e(e+2\eta-t+2v)} \equiv \hat{b}_2^K(\eta).$$

One can check that the RHS of the second inequality is positive for any $\eta < \frac{t-e}{2}$, provided that $(e \leq)t \leq e + v$; under this condition it is then sufficient to assume

that such inequality is fulfilled to rule out profitable deviations to M. We let $\hat{b}^K = \min\left\{\hat{b}_1^K(\eta), \hat{b}_2^K(\eta)\right\}$.

By combining the foregoing conditions, the result of Lemma 3 follows.

A.4 Proof of Lemma 4: M-equilibrium

To check that $p^M \equiv \frac{v}{2}$ is the M-equilibrium price if (9) holds, we proceed as follows. With consistent off-equilibrium expectations, we know that a downward deviation $p_0^D = p^M - \eta < p^M$, for given $p_1 = p^M$, leads to the following location of the indifferent consumer: $\frac{1}{2} + \frac{\eta + \left(2\check{b}-1\right)e}{2t}$, with $\check{b} \in \left[\frac{1}{2}, 1\right]$. Note that:

(i) $p_0^D > 0 \Leftrightarrow \eta < \frac{2v-t+e}{2}$, which is a necessary condition for a profitable deviation; (ii) the indifferent consumer must obtain nonnegative utility, so that the deviation actually leads to a covered market configuration: $v + \check{b}e - t\left[\frac{1}{2} + \frac{\eta + \left(2\check{b}-1\right)e}{2t}\right] - \left(\frac{v}{2} - \eta\right) \geq 0 \Leftrightarrow \eta \geq t - (e + v)$. Combining these two conditions yields $t - (e + v) \leq \eta < \frac{v}{2}$. This interval is empty iff $t \geq e + \frac{3}{2}v$, in which case there is no profitable deviation because the deviation price should be negative to cover the market. Accordingly, in what follows we restrict our attention to the case $(e + v <)t < e + \frac{3}{2}v$.

Deviation to T

A T-deviation requires $b = \frac{1}{2} + \frac{t-\eta}{2e}$, with $b \in \left[\frac{1}{2}, 1\right]$. In this case, the resulting deviation profits are larger than the candidate equilibrium ones iff,

$$\left(\frac{v}{2} - \eta\right) \times 1 > \frac{v^2}{4(t-e)} \Leftrightarrow \eta < \frac{v}{2} - \frac{v^2}{4(t-e)}.$$

Recall that deviations to T are unfeasible if $b < \frac{1}{2} + \frac{t-\eta}{2e}$ and that $\frac{1}{2} + \frac{t-\eta}{2e} > \frac{1}{2} \Leftrightarrow \eta < t$. Note also that (9) implies $t > \frac{v}{2} - \frac{v^2}{4(t-e)} (> 0)$, which means that a profitable deviation, $\eta < \frac{v}{2} - \frac{v^2}{4(t-e)}$, implies $\eta < t$ and, accordingly, $\frac{1}{2} + \frac{t-\eta}{2e} > \frac{1}{2}$. It is then sufficient to assume that $b < \frac{1}{2} + \frac{t-\eta}{2e} \equiv \check{b}_1^M(\eta)$ holds true, i.e., that the consumers' off-equilibrium expectations are sufficiently passive, to rule out feasible T-deviations.

Deviation to C

A deviation to C is not profitable iff

$$\left(\frac{v}{2} - \eta\right)\left[\frac{1}{2} + \frac{\eta + \left(2\check{b} - 1\right)e}{2t}\right] \leq \frac{v^2}{4(t-e)} \Leftrightarrow$$

$$\check{b} \leq \frac{2\eta e^2 + t\upsilon^2 - t^2\upsilon - 2\eta^2 e + 2t\eta^2 + 2t^2\eta - \upsilon e^2 - 4t\eta e + \upsilon\eta e - t\upsilon\eta + 2t\upsilon e}{2e(\upsilon - 2\eta)(t - e)} \equiv \check{b}_2^M(\eta).$$

One can check that the RHS of the second inequality is larger than $\frac{1}{2}$ given the full market coverage condition $\eta \geq t - (e + \upsilon)$ and condition (9). Recalling $\check{b} \in \left[\frac{1}{2}, 1\right]$, it is then sufficient to assume that such inequality is fulfilled to rule out profitable C-deviations.

Deviation to K

A deviation is to K if the indifferent consumer is left with zero utility: $\upsilon + \check{b}e - t\left[\frac{1}{2} + \frac{\eta + (2b-1)e}{2t}\right] - \left(\frac{\upsilon}{2} - \eta\right) = 0 \Leftrightarrow \eta = t - (e + \upsilon)$, hence the deviation price is $p_0^D = \frac{\upsilon}{2} - [t - (e + \upsilon)] = e + \frac{3}{2}\upsilon - t$. The resulting deviation profits are not larger than the equilibrium one if, and only if,

$$\left(e + \frac{3}{2}\upsilon - t\right)\left[\frac{1}{2} + \frac{t - e - \upsilon + \left(2\check{b} - 1\right)e}{2t}\right] \leq \frac{\upsilon^2}{4(t - e)} \Leftrightarrow$$

$$\check{b} \leq \frac{(\upsilon + 2e - 2t)(t - e)(3\upsilon - 2t + 2e) + t\upsilon^2}{2e(t - e)(3\upsilon - 2t + 2e)} \equiv \check{b}_3^M(\eta).$$

One can check that the RHS of the second inequality is larger than $\frac{1}{2}$ in interval $(e + \upsilon <)t < e + \frac{3}{2}\upsilon$. It is then sufficient to assume that such inequality is fulfilled to rule out profitable K-deviations. In conclusion, we let $\check{b}^M = \min\left\{\check{b}_1^M(\eta), \check{b}_2^M(\eta), \check{b}_3^M(\eta)\right\}$.

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