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Simplified Pushover Analysis of Moment Resisting Frame Structures

Timothy J. Sullivan, Daniel Saborio-Romano, Gerard J. O'Reilly, David P. Welch, Luca Landi

Abstract

Seismic assessment of a building will typically require consideration of its non-linear force-displacement response. Such information can be estimated from pushover analysis, also referred to as non-linear static analysis, in which the structure is analysed for incrementally increasing lateral loads and the non-linear structural behaviour is accounted for during the analysis by updating the stiffness matrix at each load increment. A number of computer programs are now available to permit the application of pushover analysis in practice. However, it is argued that there is a need for simplified pushover analysis methods to permit independent checks of computer outputs and also to inform engineers of the key characteristics of the structural system being assessed. This work builds on previous contributions in the literature to provide a simplified pushover analysis approach for RC frame structures. A novel procedure for the assessment of the displacement profile of RC frames is provided, with guidelines to account for different types of yielding mechanisms. By comparing force-displacement response predictions with those obtained from rigorous non-linear static analyses for a range of frame configurations and mechanisms, it is shown that the proposed approach offers an effective means of undertaking simplified pushover analysis.

Keywords: pushover analysis; simplified pushover analysis; non-linear static analysis; reinforced concrete frames; seismic assessment

1. Introduction

Non-linear static analysis, often referred to as pushover analysis, is being used increasingly for the seismic assessment of structures, possibly because it offers a useful means of identifying weak links in a structure [Hall 2017] and because a number of codes and guidelines have indicated it as a preferred assessment procedure [EN 1998-3:2005 2005; NZS 1170.5:2004 2004]. In order to undertake pushover analysis of a multi-storey frame structure, a set of lateral loads is increased in small steps with internal member forces and deformations computed at each load increment. Different options exist for the definition of the lateral load distribution, with triangular and uniform lateral load patterns prescribed within Eurocode 8 [EN 1998-1:2004 2004] in addition to loading profiles that are updated incrementally as part of a more accurate adaptive pushover analysis [Antoniou and Pinho 2004]. Non-linear structural behaviour, such as local yielding of beams or columns, can be accounted for during the analysis by updating the stiffness matrix at each load increment. In addition to obtaining useful information on internal forces and displacements, the results of the analysis can be transformed into an equivalent single-degree-of-freedom (SDOF) force-displacement plot that enables more simplified seismic assessment using procedures such as the capacity spectrum method [Freeman 1978], the N2 method [Fajfar 2000] or displacement-based assessment (DBA) [Priestley et al. 2007]. Detailed descriptions and evaluations of different options for pushover analyses are provided in [FEMA 440 2005; Papanikolaou et al. 2006; Pinho et al. 2006].

A number of computer programs, such as SAP2000 [CSI 2000], Seismostruct [Seismosoft 2014], Ruaumoko [Carr 2007] and OpenSees [McKenna et al. 2010], are now available to permit the application of pushover analysis in practice. However, engineers should always check the results obtained from a computer analysis model. An apparent difficulty with checking the non-linear structural analysis of a multi-degree-of-freedom (MDOF) system is that one should arguably be prepared to form

a large stiffness matrix and undertake numerous calculation steps, making this process impractical in the design office.

In light of the above, the objective of this paper is to further develop and test a simplified pushover analysis procedure for moment resisting frame (MRF) structures that can be used to validate the results of a computer-based pushover analysis. It is also argued that by developing a simplified mechanics-based method for pushover analysis, engineers may gain a better understanding of the key characteristics of a structure than they do from computer analysis alone. The method will build on the findings and recommendations from a number of previous contributions to the subject, where the work by Priestley and Calvi [1991] for the assessment of reinforced concrete (RC) frames is of particular relevance. Priestley and Calvi [1991] proposed a two-level approach in which capacity design principles [Park and Paulay 1975] were first applied in reverse so as to identify the likely collapse mechanism. Strength and ductility capacity estimates for the system were then combined to provide an equivalent elastic response force level that was then used together with design response spectra to identify the annual probability of exceeding the structural capacity. Priestley [1997] developed this procedure further, emphasising the importance of assessing the displacement response rather than force levels, and developing a so-called displacement-based seismic assessment approach. In doing so, Priestley [1997] proposed a number of simplified displacement profiles with which to establish an equivalent SDOF system for RC frames, illustrated in Figure 1(b), to be selected according to the expected mechanism (Figure 1(a)) and number of storeys, n , in the building.

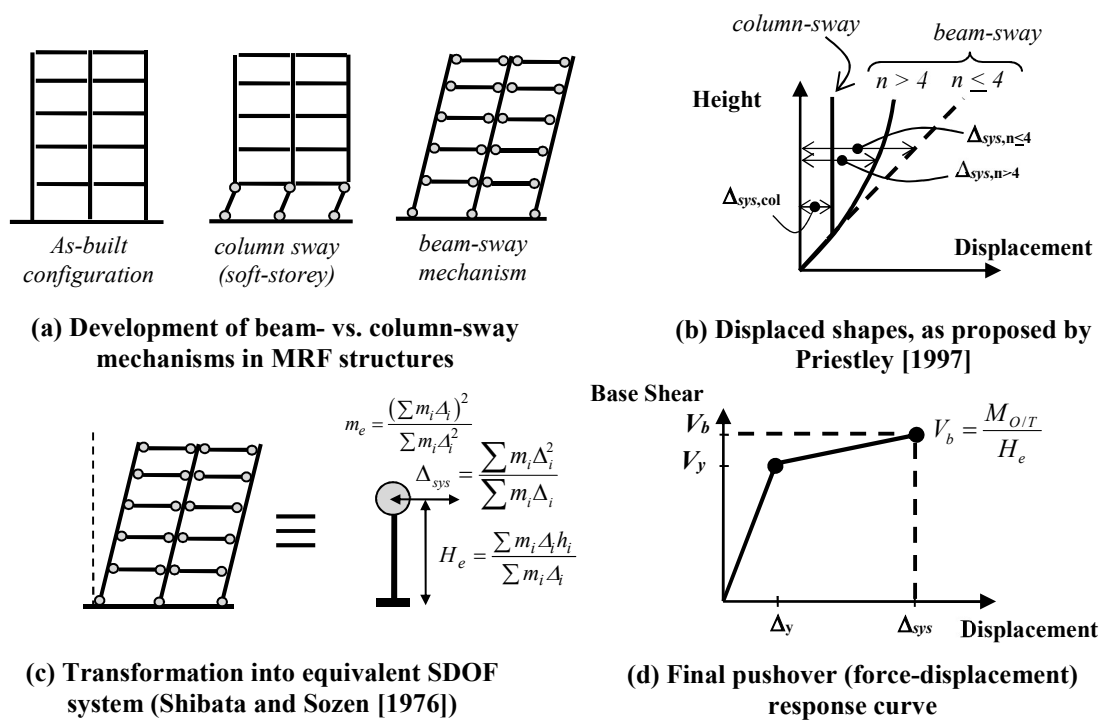


Figure 1. Overview of the displacement-based assessment procedure of Priestley et al. [2007].

As seen in Figure 1, for an RC frame with a column sway (soft-story) mechanism, the deformations are assumed to be concentrated at the soft-story level, with the storeys above translating like a rigid body for a total system displacement capacity of $\Delta_{sys,col}$. For a beam sway mechanism, more linear displacement profiles were suggested and Priestley [1997] provided equations for an equivalent SDOF system displacement, Δ_{sys} , as a function of the storey drift demand at the base. The following equations for the full displacement profile, Δ_i , were later advocated for assessment by Priestley et al. [2007]:

$$\text{for } n \leq 4: \quad \Delta_i = \theta_c h_i \quad (1a)$$

$$\text{for } n > 4: \quad \Delta_i = \theta_c h_i \left(\frac{4H_n - h_i}{4H_n - h_1} \right) \quad (1b)$$

where θ_c is the critical (maximum) storey drift (at ground level), h_i is the height of level i above the base, H_n is the total height of the structure and h_1 is the first-floor height. This is an empirical expression verified through non-linear dynamic analyses of RC frame structures by Pettinga and Priestley [2005], amongst others. Note that the precise form of Eq. (1b) is taken from the model design code proposed by Sullivan et al. [2012] but it gives the same displacement profile as the expression presented within Priestley et al. [2007].

With the displacement profile of a MDOF system known, an equivalent SDOF system displacement, Δ_{sys} , effective mass, m_e , and effective height, H_e , can be computed according to the substitute structure approach [Gulkan and Sozen 1974; Shibata and Sozen 1976], illustrated in Figure 1(c). Subsequently, by plotting V_b versus Δ_{sys} , the pushover force-displacement curve for the equivalent SDOF system is obtained, such as that shown in Figure 1(d). The pushover curve could be used to validate results of a pushover analysis conducted in a computer or for simplified seismic assessment in accordance with the detailed seismic assessment guidelines of the New Zealand Society for Earthquake Engineering [NZSEE 2017].

The overview provided above shows that some useful guidance for simplified pushover analysis of RC frames already exists. However, displacement profiles for RC frames are estimated using a limited number of expressions that are based either on simplistic reasoning or results of non-linear dynamic analyses of a set of case study buildings. Unfortunately, as will be demonstrated later in Section 3.2, the existing expressions for displaced shape can lead to significant errors in the estimation of system displacement capacity and hence seismic risk. For example, it will be demonstrated later in Section 3.2 that adoption of Eq.(1) overestimates the displacement capacity of a real 6-storey RC frame building by as much as 34%. Consequently, the main focus of this paper is to develop and test an improved mechanics-based procedure for estimating the lateral displacement profile of moment-resisting frame structures, providing guidelines that cover a range of mechanisms that may potentially develop. To do this, the means of identifying the likely yield mechanism and resistance of beam-column sub-assemblages will first be reviewed. After then presenting a new procedure for estimation of shear and displacement profiles that should be comparable to those that would be obtained from pushover analyses, a number of case study buildings are used to illustrate the methods potential and gauge the likely benefits it offers over the existing simplified guidelines.

2. Proposed procedure for simplified pushover analysis of RC frames

2.1. *Assessing the likely mechanism and lateral resistance of each storey*

In the assessment procedure proposed by Priestley and Calvi [1991], the relative strengths of adjoining members are compared in order to identify the weakest element and action, which indicates the likely mechanism. To illustrate this, consider the 2-storey RC frame structure shown in Figure 2.

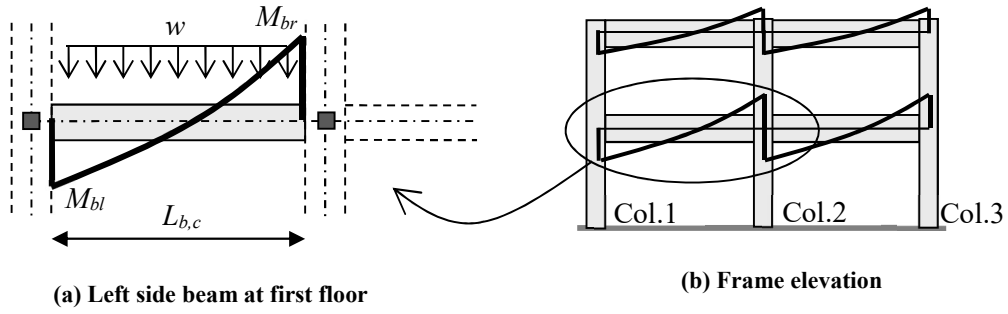


Figure 2. Investigating the likely mechanism in a 2-storey RC frame.

Beginning, for this example, with the left beam at the first floor of the frame, one could first hypothesise that beam flexural hinges could form at either end of the beam. In line with this, the bending moments, M_{bl} and M_{br} , are computed, corresponding to the section resistances at the column faces of the left and right end of the beams, respectively, as indicated in Figure 2(a). The bending moments at any point along the beam should then be calculated, making due allowance for the uniformly distributed gravity load, w , and any anticipated vertical earthquake acceleration demands. If the beam flexural resistance were to be inferior to the flexural demands at any intermediate point along the beam, then the location of the expected hinges would need to be revised and the bending moment profile recomputed.

The development of a flexural mechanism in the beam will only be possible if the beam is able to sustain the shear forces associated with the flexural hinging. This is usually assured through capacity design but for existing RC frames, there is a possibility of premature shear failure and so it should be checked. This is done by computing the shear resistance of the beam, V_{Rb} and comparing this with the shear demand, V_{Db} , expected in the beam if a beam flexural mechanism were to form; calculated for the right-hand side of the beams shown in Figure 2(a) as follows:

$$V_{Db} = \frac{M_{bl} + M_{br}}{L_{b,c}} + \frac{wL_{b,c}}{2} + V_{vEQ} \quad (2)$$

where $L_{b,c}$ is the clear length of the beam between columns as indicated in Figure 2(a), V_{vEQ} is the shear associated with vertical excitation component of ground motion (if deemed significant) and all the other symbols have been defined above. The magnitude of vertical accelerations will be uncertain, owing to uncertainty in the ratio of peak horizontal and vertical acceleration demands as well as the uncertain dynamic amplification of vertical acceleration demands up the height of a building. Clearly, if the purpose of these calculations is to assess the likely mechanism under lateral loading only, then V_{vEQ} could be set to zero. If V_{Db} is equal to or greater than V_{Rb} (the beam shear resistance), then a shear mechanism in the beam is likely to occur prior to a flexural mechanism. If instead V_{Db} is less than V_{Rb} , then the development of a flexural mechanism in the beam is more likely.

The process described above could be repeated for all beams in the frame such that the maximum beam end moments at all locations in the frame are identified. Subsequently, the column bending and shear capacities should be checked in a similar fashion. When computing section resistances for beams and columns, the impact of poor detailing, such as the presence of smooth reinforcement or inadequate lap splice details, should be accounted for following guidelines [Calvi et al. 2002a; Priestley et al. 1996]. When computing the column section strengths, an important uncertainty will be the axial load acting on the column during seismic loading, recognising that, in addition to varying vertical acceleration demands, the beam shears induced by seismic loading will tend to increase axial loads on some columns (such as Col. 3 in Figure 2(b)) and reduce them on others (such as Col.1 in Figure 2(b)). It is recommended that the columns strengths be computed initially assuming gravity-induced axial loads only. This is because, as argued by Priestley et al. [2007], the difference in total base shear resistance

obtained with account for varying column axial loads will typically be negligible. However, local mechanisms (e.g. premature joint failures) can be affected and so once the base shear resistance has been computed, the axial forces due to seismic loading can be estimated, member strengths recalculated and an iterative assessment process followed. Whether such a refined iterative assessment process is necessary will depend on the case at hand considering the potential impact on overall base shear estimates and displacement capacity.

Having computed both the maximum end-moments for columns and beams, one then needs to establish whether beam or column hinging is most likely. This again refers to the application of capacity design principles in reverse, recognising that in capacity design, column flexural strengths are set as a function of beam flexural resistances in order to force a beam-sway mechanism to occur. As such, the sum of end moments of columns framing into a joint should be compared with those from beams (taking care to compare equivalent moments at joint centrelines as opposed to moments at plastic hinge locations) and the lower will indicate whether column or beam hinging can be expected. For example, Figure 3(a) presents beam and column end-moments at the centrelines of two joints in the 2-storey frame structure. It can be seen that the sum of column end moments (480 kNm) is greater than the beam end moment (200 kNm) for column 1. Hence, beam hinging is expected for this beam-column sub-assembly. In contrast, the sum of column end moments (480 kNm) is less than the beam end moments (500 kNm) for column 2 and hence, column hinging is expected for this beam-column sub-assembly.

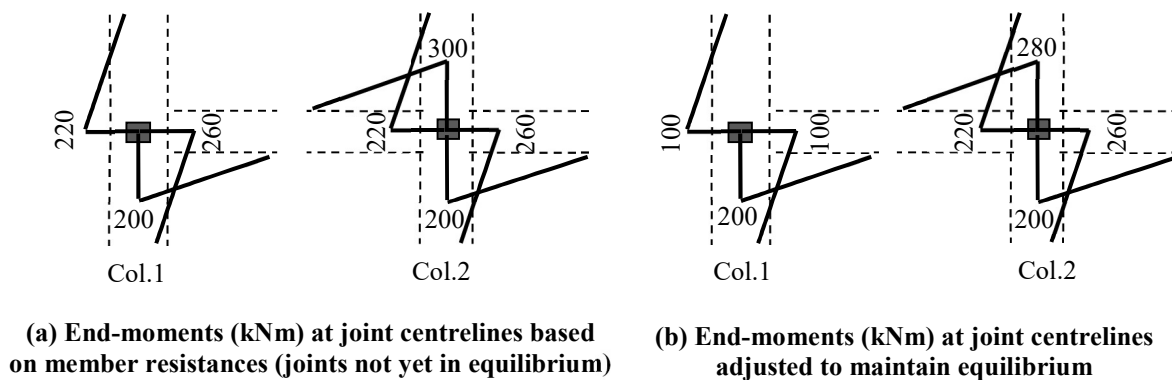


Figure 3. Adjusting beam and column end-moments at first floor of 2-storey frame example.

In addition to comparing beam and column resistances, however, the beam-column joints themselves should be checked, following recommendations such as those provided in Priestley et al. [2007] and [Tasligedik et al. 2018] as they may be weaker than the adjoining elements and hence would dictate the local mechanism. A final set of end moments in beams and columns, which accounts for beam, column and joint resistances, should then be established that respects equilibrium. This will require redistribution of the mechanism joint moment to the adjoining members. The proportions of joint moment that will be distributed to adjoining elements will actually depend on their stiffness and strength. However, in order to avoid the need to compute member stiffnesses in this process, it is recommended that joint moments are initially split equally to adjoining members. In line with this, Figure 3(b) shows that for column 1, the beam moment resistance of 200 kNm is split equally to the columns above and below the joint, which both remain elastic. For column 2, it can be seen that if the resisting moment of 480 kNm at the beam-column joint were split equally to the adjoining members (i.e. 240 kNm to each beam) then the beam on the right would be overloaded since it has a resistance of only 200 kNm. In such cases, the recommendation is to set the end moment in the right beam equal to its resistance (i.e. 200 kNm) and then redistribute the remaining moment demand to the other beam (such that it carries a total demand of 280 kNm).

Upon completion of the process described above, column end-moments will have been identified for all beam-column joint regions in the frame. These end-moments can then be used to compute values of storey shear resistance, by summing up the column shears for each storey, as follows:

$$V_{R,i} = \frac{\sum M_{col,b,i} + \sum M_{col,a,i-1}}{(h_i - h_{i-1})} \quad (3)$$

where $V_{R,i}$ is the shear resistance of storey i and, h_i and h_{i-1} are the heights above the base foundation of levels i and $i-1$ respectively. The terms $\sum M_{col,b,i}$ and $\sum M_{col,a,i-1}$ are the sum of the column end moments immediately below the joint centrelines at level i and above the joints at level $i-1$ respectively. This equation thus assumes that the joint centrelines at a given level are all at the same height. For the first storey it is clear that h_{i-1} should be set to zero whereas the term $\sum M_{col,a,i-1}$ should be taken equal to the column base moment capacities (which may need to be set as a function of column shear capacity if a column were not to have sufficient shear strength to support the development of flexural hinges).

2.2. Assessing the drift required to yield each storey

Recognising that the pushover procedure will provide information on both the forces and displacements that develop in a structure, attention is now turned to the drift that is required to cause an RC frame to yield. Eqs. (8) to (12) provide a series of approximate mechanics-based expressions for the drift at yield of level i in an RC frame as a function of the expected mechanism.

For beam-sway flexural mechanisms:
$$\theta_{y,i} = 0.5 \frac{\varepsilon_y L_{b,i}}{h_{b,i}} = \theta_{y,bs,i} \quad (4)$$

For column-sway flexural mechanisms:
$$\theta_{y,i} = 0.43 \frac{\varepsilon_y h_{s,i}}{D_{col,i}} = \theta_{y,cs,i} \quad (5)$$

For beam-column joint mechanisms:
$$\theta_{y,i} = \frac{M_{j,jo\ int,i}}{M_{j,beams,i}} \theta_{y,bs,i} \quad (6)$$

For beam shear mechanisms:
$$\theta_{y,i} = \frac{L_{b,c,i} (V_{Rb,i} - 0.75wL_{b,c,i})}{M_{bl,i} + M_{br,i}} \theta_{y,bs,i} \quad (7)$$

For column shear mechanisms (no infills):
$$\theta_{y,i} = \frac{V_{R,i}}{V_{R,i,cs}} \theta_{y,cs,i} \quad (8)$$

where ε_y is the yield strain of the longitudinal reinforcement in the beams, $L_{b,i}$ is the length of the beams between column centres, $h_{b,i}$ is the section depth of the beams, $h_{s,i}$ is the storey height (between floor centrelines), $D_{col,i}$ is the column section depth, $M_{j,jo\ int,i}$ is the beam-column joint equivalent moment capacity, $M_{j,beams,i}$ is the total flexural resistance of beams framing into a beam-column joint (see earlier discussion with reference to Figure 3), $L_{b,c,i}$ is the length of the beams between column faces, $M_{bl,i}$ and $M_{br,i}$ correspond to the beam section resistances at the left and right end of the beams (see earlier discussion with reference to Figure 2), $V_{Rb,i}$ is the beam shear resistance, $0.75wL_{b,c,i}$ represents the beam shear demand associated with gravity loading and vertical excitation (see earlier discussion with respect to Eq. (2)), $V_{R,i}$ is the storey shear resistance associated with a column shear failure and $V_{R,i,cs}$ is the storey shear resistance computed assuming that a column-sway flexural mechanism could form (see earlier discussion with respect to Eq.(3)).

Priestley et al. [2007] proposed Eq. (4) as a means of estimating the yield drift of a well detailed RC frame in which a flexural mechanism is expected to form. The equation was set by making estimates of the different contributions to yield drift of a frame and was verified using experimental results collected from the literature. Eq. (5) was formulated by Glaister and Pinho [2003] along similar lines, making estimates of beam, joint and column-shear deformations relative to the column flexural deformations. Eq. (6) proposes that the drift at formation of a beam-column joint mechanism can be found by simply

scaling the yield drift associated with a beam-sway flexural mechanism (i.e. Eq. (4)) by the ratio of the force required to form a beam-column joint mechanism to the force that would have been required to form a beam-sway flexural mechanism. Similarly, Eqs. (7) and (8) are scaling the yield drifts associated with flexural sway mechanisms by the ratio of the shear resistance available against seismic loading to the shear demand that would be expected if the associated flexural sway mechanism were to develop. Note that the drift predicted by Eq. (8) presumes that masonry infills are not present as these will tend to increase both stiffness and shear demands on columns in RC frames, subsequently affecting yield drifts. To the authors' knowledge, Eqs. (5) to (8) have not been verified experimentally and may need to be revised as part of future research. However, they are recommended for the purposes of simplified pushover analysis until more refined expressions become available. Also, note that no allowance has been made for the effects of smooth reinforcement or lap-splice problems, but this is considered outside the scope of the current paper.

At the base of an RC frame, the drift required to yield the cantilever RC columns should also be estimated. For columns yielding in flexure, this can be done via the relevant form of Eq. (9). Note that in the event that a shear mechanism is expected in the columns at the ground storey, the yield drift should instead be computed according to Eq. (8).

For rectangular ground-storey columns:

$$\theta_{y,0} = 0.70\varepsilon_y \frac{h_{cf}}{D_{col}} \quad (9a)$$

For circular ground-storey columns:

$$\theta_{y,0} = 0.75\varepsilon_y \frac{h_{cf}}{D_{col}} \quad (9b)$$

where D_{col} is the column section depth (outside diameter in the case of circular sections) in the direction of loading and h_{cf} is the height of contraflexure expected in the ground storey column that can be estimated as follows:

$$h_{cf} = \frac{h_1}{\left(\frac{M_{col,b,1}}{M_{col,base}} + 1 \right)} \quad (10)$$

where, as shown in Figure 4, h_1 is the first storey height, $M_{col,b,1}$ is the moment expected to develop at the top of the column (just below the beam-column joint centreline) and $M_{col,base}$ is the base flexural strength of the column. Figure 4 also illustrates the manner with which Eq. (9) has been derived, integrating the linear curvatures (approximated as varying linearly in proportion to the moment demands) and adopting the nominal yield curvature expressions for RC columns provided by Priestley et al. [2007]. No allowance has been made for foundation flexibility and if this were deemed significant, it should be added.

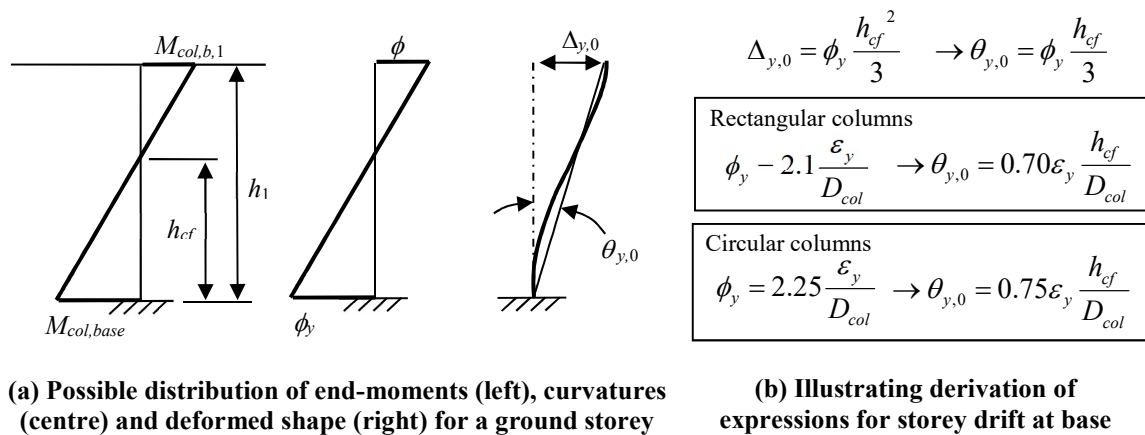


Figure 4. Yield drift considerations for ground storey cantilever columns.

Up until this point in the paper, the term “storey” and “level” could be seen as being somewhat equivalent. However, when undertaking structural analyses, the “storey” drift demand is typically computed as the relative lateral displacements of adjacent “levels”, divided by the storey height [O’Reilly et al. 2017]. This is relevant because the yield drift expression given by Eq. (4) refers to beams at a certain level and Eq. (9) refers to the bases of ground storey columns. Clearly, the yield drift of a “storey” as opposed to a “floor” level, will require that resistance has been developed at both the top and bottom of the storey. In the case of the ground storey, this could imply that flexural yielding needs to occur at the base of the columns and also at the ends of the beams of the first floor. At upper storeys, it could imply yielding of beams at the levels above and below the storey in question. To account for this, the yield drifts at each adjacent level can be weighted according to strain-energy proportions, as per Eq. (11):

$$\theta_{y,sys,i} = \frac{\sum M_{j,i} \theta_{y,i} + \sum M_{j,i-1} \theta_{y,i-1}}{\sum M_{j,i} + \sum M_{j,i-1}} \quad (11)$$

where $\theta_{y,sys,i}$ is the storey drift required to yield storey i , $M_{j,i}$ and $M_{j,i-1}$ are the total flexural resistances (for the governing mechanism) provided at joint centres and $\theta_{y,i}$ and $\theta_{y,i-1}$ are the drifts at yield at levels i and $i-1$ respectively. Note that the summation terms are indicated since it is expected that at a given level there will be a number of beam-column joints (in multiple frames) providing resistance against lateral movement of that level. At the ground storey $i=1$, $M_{j,0}$ would correspond to the flexural strength and $\theta_{y,0}$ the yield drift of the column bases, found in line with guidelines provided above.

Typically, the above distinction between “storey” and “floor” level drift is not made (as, for instance, in Priestley et al. [2007]), since the differences that result in design are not significant. However, the authors have found that for pushover assessment of existing frames, such a rigorous evaluation of the storey drift can be useful in understanding the mechanism that forms and correctly predicting the non-linear response that develops.

2.3. Assessing values of storey stiffness

Having found means of identifying the storey shear resistance (Eq. (3)) and storey drift at yield (Eq. (11)), the secant stiffness to yield, $k_{y,i}$, at each storey i can now be computed as follows:

$$k_{y,i} = \frac{V_{R,i}}{\theta_{y,i} h_{s,i}} \quad (12)$$

where all symbols have been defined in previous sections. The storey stiffness values so defined are useful for calculation of elastic inter-storey displacement components along the height of the frame, as will be explained in the next section.

2.4. Assessing displacement and storey shear profiles

The purpose of this paper is to provide a simplified means of undertaking pushover analyses of RC frames, which essentially requires the ability to identify the lateral displaced shape and storey shear demands for various levels of roof displacement or base shear. The information derived in the preceding sections can be used for such pushover analyses, by following the general procedure outlined in Figure 5. The details of each step will be explained in the paragraphs that follow.

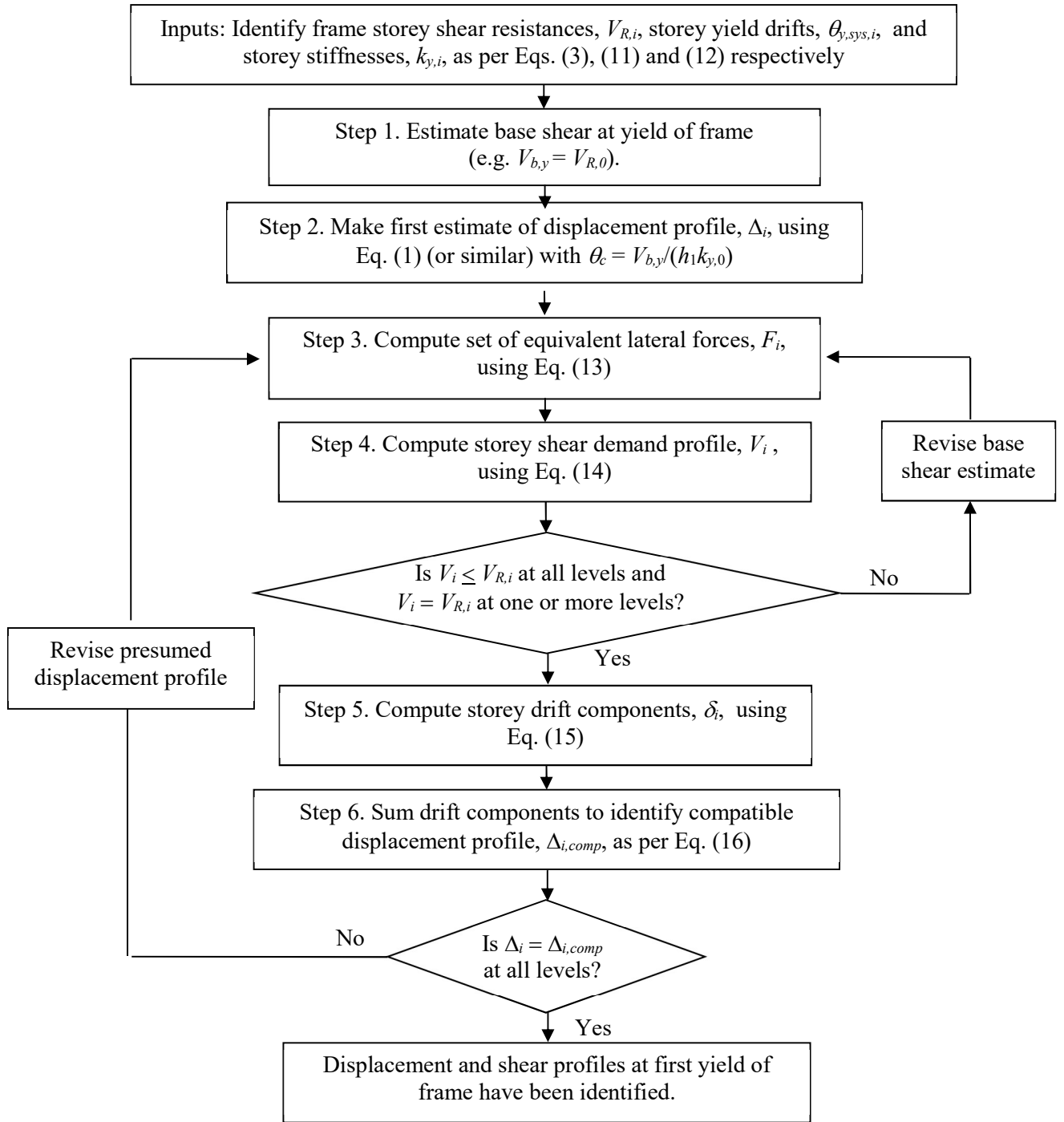


Figure 5. Procedure to identify displacement and shear profiles at yield of RC frame.

In the first step of the procedure, an estimate of the base shear at yield of the frame is made. This estimate will be checked and revised during the assessment procedure and hence the exact value is not too important, but a useful starting point may be to assume that the base shear at yield of the frame would be equal to the storey shear resistance at the ground floor, found from Eq. (3) with $i=0$.

A trial displacement profile for the frame is set at step 2, and for this purpose it is suggested that Eq. (1) be used, with the drift at ground storey, θ_c , obtained by dividing the yield base shear by the ground storey stiffness, $k_{y,0}$, (from Eq. (12)) and height, h_1 .

In step 3 a set of equivalent lateral forces, F_i , is found according to Eq. (13) from Priestley et al. [2007]:

$$F_i = \frac{m_i \Delta_i}{\sum m_i \Delta_i} V_b \quad (13)$$

where V_b is the frame base shear force (noting that $V_b = V_{b,y}$ at first yield of the frame), m_i is the seismic mass and Δ_i the lateral displacement of level i .

In step 4 the storey shear demands associated with V_b are then found by summing the equivalent lateral forces from Eq. (13) down the height of the n -storey frame, from levels n to i , according to:

$$V_i = \sum_{j=i}^n F_j \quad (14)$$

At this point in the procedure, the shear demands should be compared with the shear resistances for each storey. Ideally, the shear demand will be equal to the shear resistance at one or more storeys and should nowhere exceed the shear resistance. If the demand exceeds the resistance at any storey, then the estimated base shear at yield of the frame was too high, a reduced base shear estimate should be made and the procedure repeated from step 3. On the other hand, if the shear demands are less than the shear capacities at all levels, this suggests that the yield base shear was too low and hence, an increased estimate should be made and the procedure repeated from step 3.

At step 5, elastic storey drift components are found by dividing the storey shear demands by the storey shear stiffness values, $k_{y,i}$, (found from Eq. (12)), according to:

$$\delta_i = \frac{V_i}{k_{y,i}} \quad (15)$$

In step 6, the lateral displacement profile, $\Delta_{i,comp}$, that is expected to be compatible with the base shear is computed by summing the drift components up the height of the frame, according to:

$$\Delta_{i,comp} = \sum_{j=1}^i \delta_j \quad (16)$$

The displacement profile computed according to Eq. (16) should then be compared with the displacement profile assumed at step 2. If the displacements do not match, a revised estimate of the displacement profile should be made (and it is recommended that the revised estimate be set equal to the displacements computed by Eq. (16)) and the assessment procedure repeated from step 3. If the displacement profiles match, the procedure is complete and the storey shear and lateral displacements at yield of the frame have been found.

2.5. Assessing the global post-yield mechanism

Beyond yield, the deformed shape of the frame structure will depend on the mechanism that develops. The process described in section 2.1 will have indicated the sort of mechanism that is expected at a beam-column sub-assembly level, recalling that one might identify either flexural beam- or column-hinging, joint-hinging or shear failures of beam or column elements. Knowing also the storey at which yielding is first expected to develop (from the procedure just described in the previous section) one may be in a position to anticipate the global mechanism too. For example, if column flexural hinging is assessed for the sub-assemblies of the storey at which the shear resistance is first reached (i.e. the storey where $V_i = V_{R,i}$ according to the process described in Figure 6), then a global soft-storey mechanism at this level will develop and plastic deformations will concentrate at the soft-storey level (see Figure 2a). If, however, flexural beam-hinging is assessed at the subassembly level then one could anticipate that a beam-sway mechanism will begin to develop, in which deformations increase gradually at all levels (again see Figure 2a).

In some cases, a mixed mechanism might be expected, whereby some beam-column sub-assemblies at a level will have been assessed as, say, column-sway critical whereas other sub-assemblies may be beam-sway. In such cases one might expect the same result as a beam-sway mechanism, with deformation demands spreading over the height of the building. However, some judgement may be required since the capacity to redistribute loads will depend on the number and relative resistance of those columns not expected to yield. Priestley et al. [1991] and Priestley et al. [2007] recommend that a sway potential index, S_i , be calculated for each floor to indicate the likelihood of a column sway mechanism.

$$S_i = \frac{\text{sum of beam strength at level } i}{\text{sum of column strengths at level } i} \quad (17)$$

When S_i is greater than 1.0 a column-sway mechanism is expected. When S_i is less than 1.0 a beam-sway mechanism could be expected, but Priestley et al. (2007) point out that if S_i is greater than 0.85 then it would be reasonable to assume (in an effort to be conservative) that a column-sway mechanism may develop. However, this expression is not likely to capture those cases with unusual distributions of beam and column strengths. In addition, Sullivan and Calvi (2011) point out that for older buildings one might obtain sway potential indices greater than 1.0 at more than one level and hence be unsure about where the soft-storey should be expected. To overcome this, Sullivan and Calvi (2011) proposed a sway-demand index, SD_i , as follows:

$$SD_i = \frac{\text{storey } i \text{ shear demand}}{\text{storey } i \text{ shear resistance}} = \frac{V_i}{V_{R,i}} \quad (18)$$

The storey with the greatest value of SD_i for which column- (or joint) hinging is expected, will indicate the storey most likely to form a soft-storey mechanism. Note that by following through the process described in Figure 6 one will have evaluated both V_i and $V_{R,i}$ and hence will be able to calculate sway-demand indices easily.

Another complex scenario arises when one expects a beam-sway mechanism to begin at one level but then a column sway mechanism is expected at another. In such cases the drift profile found from the process described in Figure 6 can be uniformly amplified until the drift reaches yield at a column-sway level, from which point a column-sway deformed shape could be assumed. This sort of scenario is difficult to predict via simplified methods as there is likely to be some redistribution of moments going to columns above and below joint centrelines, such that the shear resistance of different storeys changes with increasing drifts and this in turn means that a soft-storey can eventually develop even if the shear demands expected from the first-yield assessment process in Figure 6 might not have indicated this.

2.6. *Assessing the displacement and shear profiles post-yield*

With knowledge of the displacement and shear profiles at yield of the RC frame, displacement and shear profiles at other post-yield states can also be derived. The means of doing this will be different for different sway mechanisms; for a beam-sway mechanism it is proposed that the displaced shape obtained from the procedure described in Figure 5 be uniformly scaled until a target value of storey drift is achieved. For a column-sway mechanism it is instead expected that post-yield, deformations tend to concentrate at the soft-storey level as shown in Figure 6. As such, in this case the drift component, θ_i , at the soft-storey level should be increased until a target drift level of interest is achieved, and then a revised displacement profile established according to Eq. (16). The changing displaced shape associated with the soft-storey mechanism will change the distribution of equivalent lateral forces (as per Eq. (13)), with the tendency being for the levels above the soft-storey to deform less. An iterative approach could be used to account for this changing displaced shape, but it will typically be sufficient to presume that the levels above the soft-storey maintain drifts similar to those at formation of the mechanism, as indicated in Figure 6.

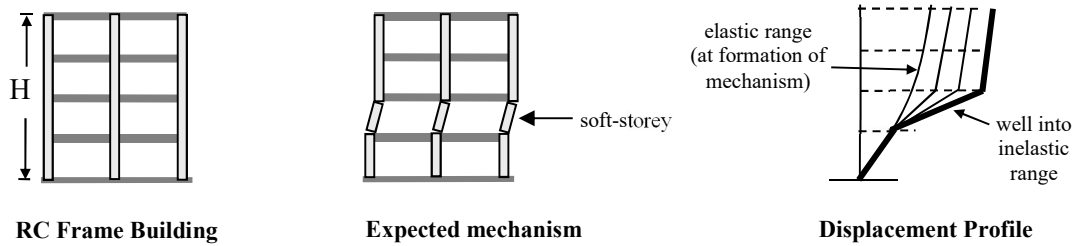


Figure 6. Illustrating potential impact of soft-storey mechanism on lateral displacement profiles.

For the case that an RC frame is expected to develop beam-column joint hinging there is uncertainty as to the likely displaced shape. This is because, as pointed out by Calvi et al. [2002b], and others, joint shear hinges may form across two storeys, rather than top and bottom of a single storey. A conservative approach in this case might be to assume that the soft-storey does form at a single storey (since the global deformation capacity will be less in this case) but it is acknowledged that this is an area requiring further research.

3. Validation of the simplified approach

To gauge the validity of the simplified procedure described in the previous section, a series of case study applications have been made and the results obtained from accurate non-linear static analysis procedures are compared with those obtained from the simplified approach and previous recommendations in the literature. The potential benefits of the new approach will become clear in the subsections that follow.

3.1. Assessing lateral force-displacement profiles for a series of 2D RC frames

The first of these case study buildings consist of a series of 2D RC frames with different storey number and are shown in Figure 7. The designs are adapted from a previous study by Galli [2006] and have been designed for gravity load-only using allowable stress and other such design provisions specified in Regio Decreto 2229/39 [Regio Decreto 1939], along with other common construction conventions prior to the introduction of seismic design provisions in Italy in the 1970's, which are summarised in Vona and Masi [2004]. A common feature of these older frames is the absence of capacity design considerations in the beam and column members meaning that non-ductile mechanisms such as column-sway mechanisms are quite typical. Column members in these older frames typically possess low ratios of longitudinal reinforcement (<1.3%) as they were sized for axial loading and the use of smooth reinforcing bars was also common. The use of these bars in place of modern ribbed bars affects the bonding of the reinforcement to the concrete paste resulting in a modified hysteretic behaviour, as discussed in O'Reilly and Sullivan [2017]. Experimental testing by Pampanin et al. [2002] also showed how the small percentage of shear reinforcement stirrups in the members and no reinforcement in the beam-column joints meant that they are quite vulnerable to a non-ductile mechanism when combined with the use of smooth end-hooked bars.

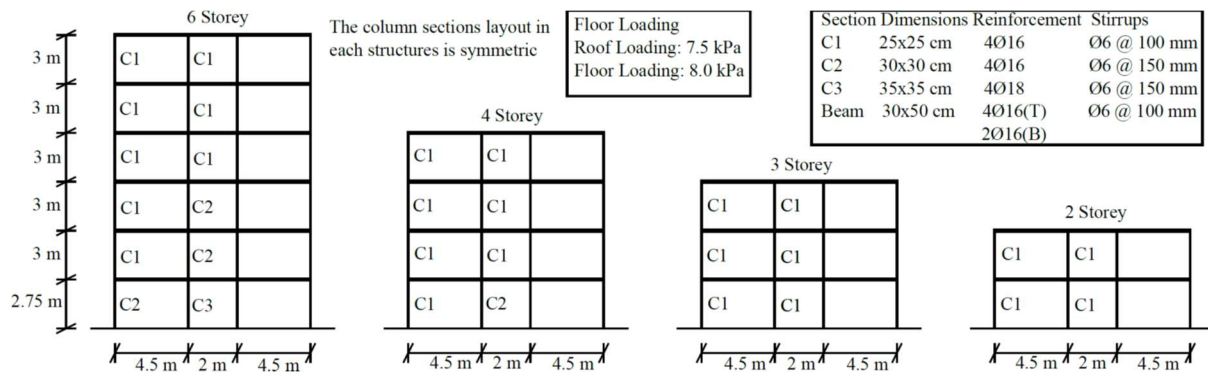


Figure 7. Layout of 2D case study structures adopted from Galli [2006].

The layout and various details regarding the member cross-sections of the case study structures are shown in Figure 7 where the column section sizes remain constant for the two and three storey frames and the lower levels have slightly larger sections in the four and six storey frames. The strength of the reinforcing steel and concrete were 3800kg/cm^2 (372MPa) and 200kg/cm^2 (19.6MPa), respectively, as per typical design manuals in use at the time of construction. In terms of numerical modelling the case study frames, the developments of O'Reilly and Sullivan [2017b] were utilised herein for the structures and further details regarding the characterisation of the response of these frames can be found in O'Reilly and Sullivan [2017a].

The static pushover analyses were conducted and four limit states corresponding to those described in the Italian National code [NTC 2008] were identified for each structure in O'Reilly and Sullivan [2017b]. These limit states can be qualitatively described as follows along with their respective abbreviations listed in NTC 2008: operational (SLO), damage control (SLD), life safety (SLV) and collapse prevention (SLC). Figure 8 shows the comparison between the four case study frames at each of these four limit states normalised to the roof displacement. As is evident, the simplified procedure outlined here provides an excellent match for each limit state. The initial elastic behaviour of the frames described via the elastic first mode shape in Figure 8 is seen to be well represented by the proposed procedure followed by the gradual evolution of the plastic mechanism in the non-linear range of response. This is especially evident in the case of the six-storey building in how the stiffness of the lower floors is captured well in the elastic range of response and how the displaced shape moves gradually from the elastic first mode pattern to the concentration of damage on the fourth floor for the SLC limit state. The critical storey predicted using the sway demand index in Eq. (18) is hatched in grey for each case study building in Figure 8, where it can be seen that the critical storey was predicted correctly in each case. This correct prediction of the storey mechanism in each case provides further validation to the use of the sway demand index described in Section 2.5.

One instance where it is clear that more research is required, in terms of considering the beam-column joint mechanism's influence on the distribution of drift between adjacent storeys, is evident in the case of the three-storey frame at the SLC limit state shown in Figure 9. The procedure highlighted above determines that a soft storey mechanism forms at the second storey of the building and when considering the comparison at the lower limit state SLV, it appears the method captures this well. However, at the final limit state SLC, the static pushover (SPO) has formed column base hinging together with a joint mechanism at the second floors, resulting in a spread in drift demand between these two floors.

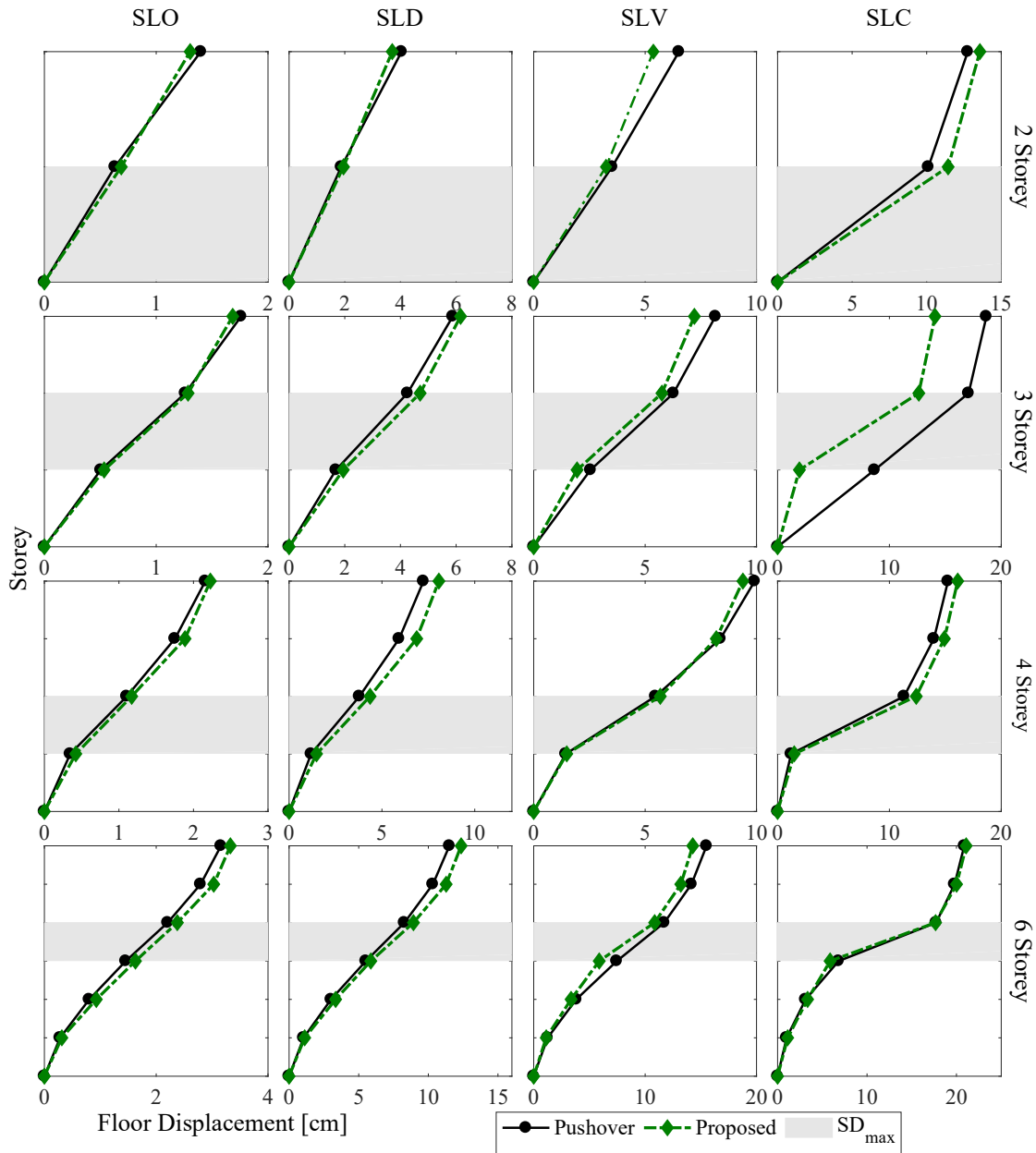


Figure 8. Comparison of the normalised displaced shape expression with the SPO analyses at various limit states.

Having determined the likely mechanism and associated displaced shape, the base shear associated with the anticipated mechanism at different levels of response can be computed to construct the overall backbone response curve of the building. That is, knowing that a column sway mechanism will form, for example, the total base shear can be computed by estimating the storey shear from the critical storey along with the other relevant contributions to the total base shear from the other storeys. The contributions from the other floors that are expected to remain elastic are determined as a ratio of the expected ductility demand, whereby using the yield drift approximation formula for a beam or column sway, respectively, the expected ductility can be computed from the ratio of the limit state demand drift profile. Following this approach, the static pushover curves for each of the case study frames are compared to the results of the proposed simplified approach at each of the four limit states and are illustrated in Figure 9. This is performed by taking the critical drift associated with each limit state identified in O'Reilly [2016] and computing the base shear corresponding to the identified mechanism and overall displaced shape. Using the predicted base shear at each of the limit states and comparing them with those marked on the static pushover curve, it can be seen that a good match is obtained. This

is notable for the case of the operational limit state (SLO) in the initial elastic range and also in the drop in the base shear capacity at the collapse prevention limit state (SLC) of the structure where both strength degradation and the influence of P-Delta effects become more pronounced.

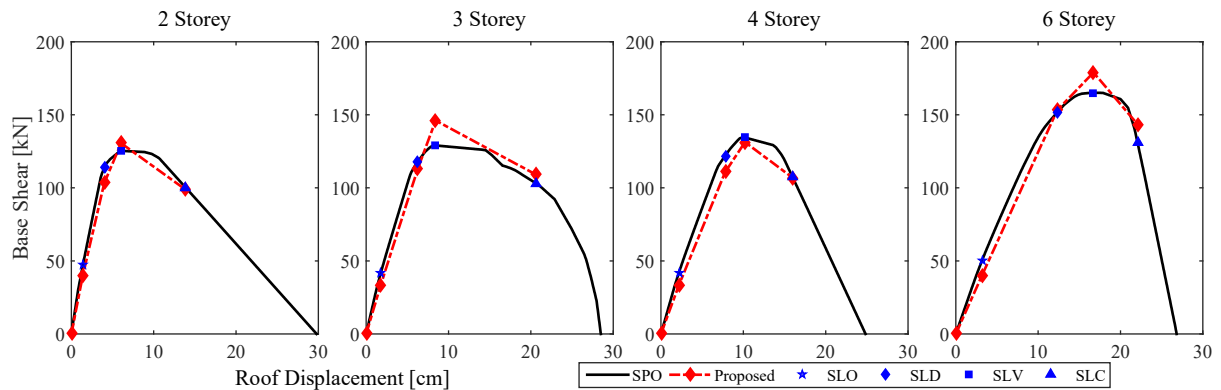


Figure 9. Comparison of the backbone response computed via the proposed procedure to that of static pushover (SPO) analysis.

3.2. Assessing the lateral force-displacement profile for a real building in L'Aquila Italy

3.2.1. Overview

The assessment procedure has also been tested on an existing six-storey building, consisting of multiple reinforced concrete (RC) frames with unreinforced masonry partitions and infill walls located in the Italian city of L'Aquila. The first storey consists of a basement for storage and is separated from the perimeter retaining walls by a gap of 80cm. The second storey consists of offices, shops and garages and the last four storeys are residential. The structure has seven bays in the East-West direction (X-direction) and three in the North-South direction (Y-direction), with a total area per storey of 340.5m², as shown in Figure 10. The building is regular in plan, with the RC columns continuous along the height, while the masonry infill walls and partitions vary along height due to differences in architectural configurations. The frame sections (beams and columns) are gradually reduced from the lower to the upper storeys in a somewhat conventional fashion. However, the storey heights increase from the lower to the upper storeys, causing a storey stiffness irregularity along the height, with stiffer storeys in the lower levels and more flexible in the upper levels.

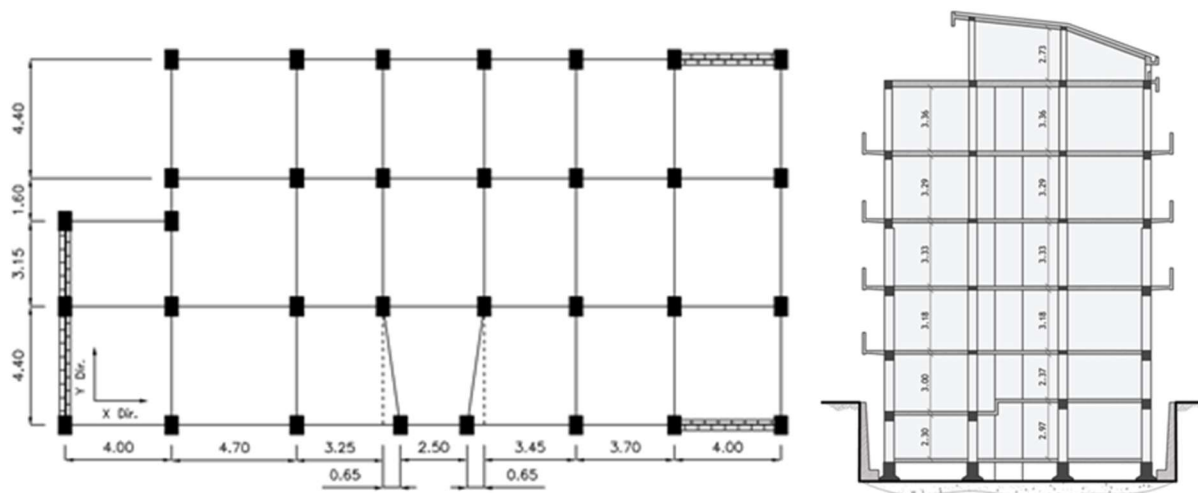


Figure 10. Plan view of the building (left) and transverse section (right) in the Y direction, respectively, taken from Saborío-Romano [2016] .

In order to assess the appropriate displaced shape of RC frames only, the building was analysed as bare frames, neglecting the contribution of the masonry infill walls in the structure capacity in this paper. To see the contribution of the masonry walls into the building capacity, see the results of Saborío-Romano [2016]. A three-dimensional structural model was created using the Ruaumoko 3D [2007] software, where the RC columns and beams were modelled as lumped plasticity elements, allowing plastic hinges formation on both ends of the elements. The material properties and moment capacities used for this model were established and are detailed further in Saborío-Romano [2016].

3.2.2. Numerical Analysis

The structural model was subjected to both elastic modal and non-linear static pushover analysis. While the modal analysis provides the elastic dynamic properties of the structure, the pushover analysis characterises the inelastic mechanism formation sequence and force-displacement relationship under increasing lateral forces. The results of the modal analysis are shown in the Table 1, with mode shapes plotted in Figure 12(a). The first mode in each direction has the highest percentage of effective mass and so is taken as the fundamental natural period and mode shape. In addition, for this mode it can be seen that the structure seems to be stiffer in the first two storeys of the structure, which is a reasonable result due to the increase in the storey height and the reduction of the column section moving from the lower storeys to the higher levels.

The pushover analysis was then performed using an inverse triangular load distribution for both principal directions of the building and results are shown in Figure 11. The limit states were defined according to the Italian National Code [NTC 2008] and correspond to: Operational (SLO), Damage Control (SLD), Life Safety (SLV) and Collapse Prevention (SLC). The corresponding storey drift capacities at the SLO, SLD, SLV and SLC limit states were 0.33%, 0.50%, 2.0% and 2.6% respectively for the X-direction and .33%, 0.50%, 2.0% and 2.7% respectively for the Y-direction.

Table 1. Modal analysis results for the structural model, modified from Saborío-Romano [2016].

		X-direction		Y-direction	
		Mode 1	Mode 2	Mode 1	Mode 2
Period, T (s)		1.83	0.64	1.43	0.56
% Participating Mass		66.1%	15.4%	64.1%	13.7%
Mode Shape	Level 6	1.000	1.000	1.000	1.000
	Level 5	0.862	0.130	0.836	0.100
	Level 4	0.622	-0.703	0.584	-0.681
	Level 3	0.351	-0.865	0.374	-0.796
	Level 2	0.155	-0.525	0.187	-0.531
	Level 1	0.039	-0.153	0.048	-0.146

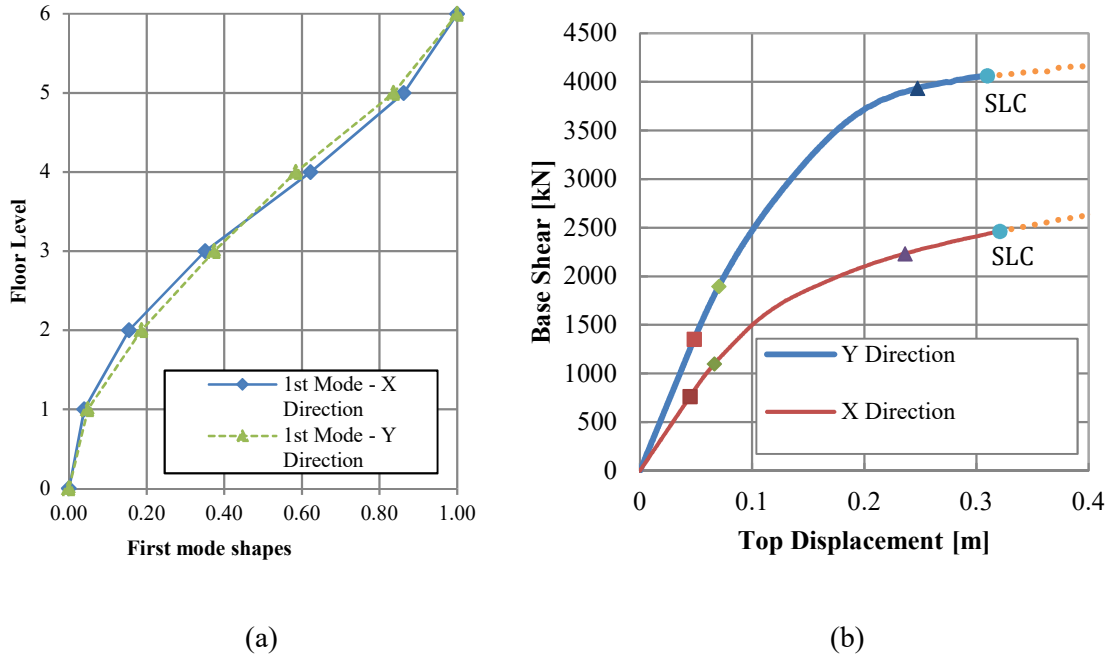


Figure 11. (a) First mode shapes, and (b) force-displacement capacity curves for both analysis directions up to the collapse prevention (SLC) limit state, modified from Saborío-Romano [2016].

Comparing the capacity curves of Figure 11, the structure is stiffer and stronger in the Y-direction for both models with respect to the X-direction, which is an expected result since all of the frames in the Y-direction have deep beams whereas in the X-direction, the outer frames have deep beams and the interior frames have flat, shallow beams that were typical of older RC frame construction in Italy.

3.2.3. Application of Simplified Pushover Analysis Method

The procedure described in section 2 was applied to the case study building in order to estimate the displacement profiles at the four limit states assessed during pushover analyses. A detailed description of the process followed for this building is provided in Saborio-Romano [2016]. In evaluating the likely mechanism, a beam-sway mechanism was predicted to develop, with sway indices all less and 1.0 (except at roof level, where column hinging would be expected) as indicated in Tables 2 and 3 for the X- and Y-directions respectively. As such, the relative stiffnesses of different levels was first assessed and used to establish the likely displaced shape at yield and this shape was then scaled until the limit state drift was reached at the critical storey. For this case study building, storey drifts at the limit states SLO, SLD, SLV and SLC were again taken as 0.33%, 0.50%, 2.0% and 2.6% respectively for the X-direction and .33%, 0.50%, 2.0% and 2.7% respectively for the Y-direction. The resulting displacement profiles for the various limit states are presented in Figure 13 and are compared with the displacement profiles obtained from pushover analyses, as well as those predicted from Eq.(1) from Priestley et al. (2007).

Table 2. Sway indices, displacements and shear resistance profiles for the overall structure in the X direction of the L'Aquila building, using the proposed procedure.

Storey	Sway Potential Indices, S_i	Storey Drift Average $\delta_{i,aver}$ [m]	Displ. Profile Δ_i [m]	Norm. Displ. Profile Δ_i	Shear Resistance Profile V_i [kN]
6	1.03	0.0067	0.0981	1.000	1334.2
5	0.44	0.0232	0.0915	0.932	936.6
4	0.32	0.0317	0.0683	0.696	1013.7
3	0.27	0.0231	0.0366	0.373	1608.3
2	0.36	0.0106	0.0135	0.138	2713.9
1	0.46	0.0030	0.0030	0.030	6379.3

Table 3. Sway indices, displacements and shear resistance profiles for the overall structure in the Y-direction of the L'Aquila building, using the proposed procedure.

Storey	Sway Potential Indices, S_i	Storey Drift Average $\delta_{i,aver}$ [m]	Displ. Profile Δ_i [m]	Norm. Displ. Profile Δ_i	Shear Resistance Profile V_i [kN]
6	1.55	0.0140	0.1109	1.000	1619.1
5	0.77	0.0262	0.0969	0.874	1825.1
4	0.62	0.0248	0.0707	0.637	2519.5
3	0.64	0.0219	0.0459	0.414	3231.4
2	0.60	0.0184	0.0239	0.216	3745.6
1	0.53	0.0055	0.0055	0.050	7891.3

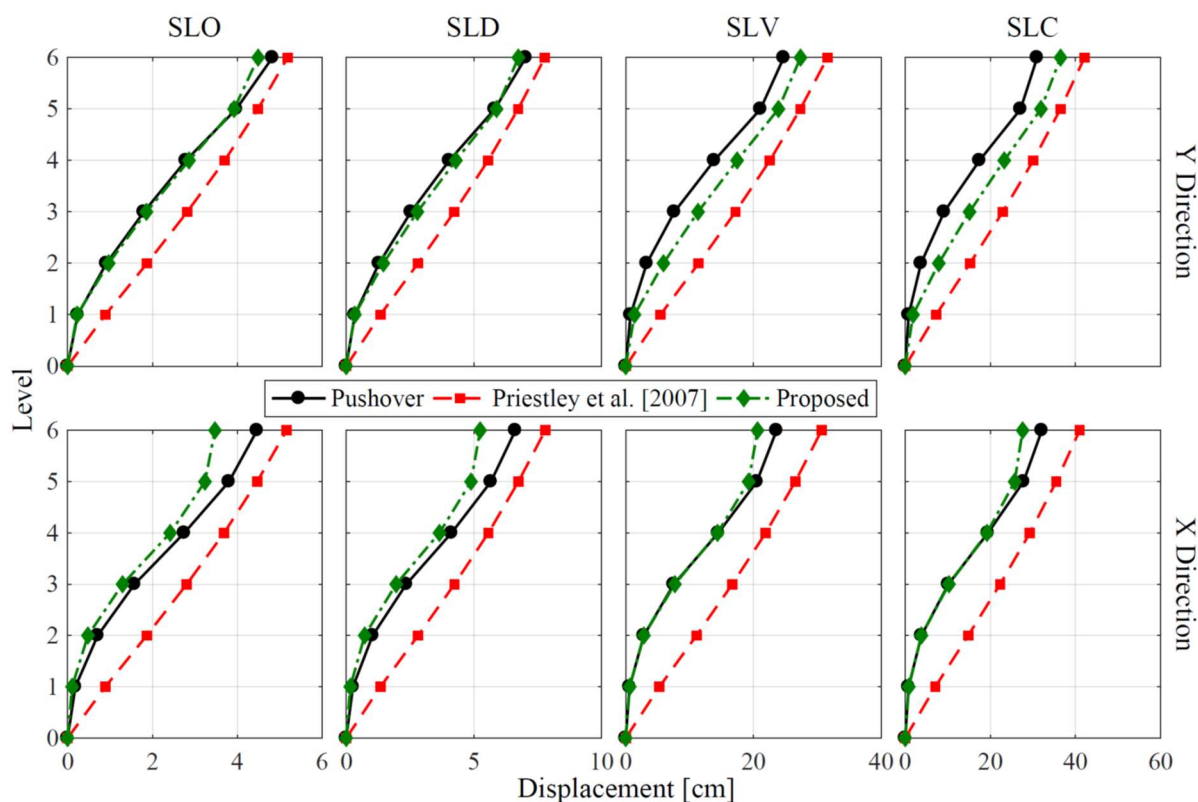


Figure 12. Comparison of the normalised displacement profiles, with different methodologies, in X (left) and Y (right) direction.

As seen in Figure 12, the displacement profile obtained with the proposed procedure accurately predicts the behaviour of the structure, having stiffer lower storeys and having a change in the form of the displacement profile between the 3rd and the 5th storey. The initial displaced shape obtained with Eq. (1.b) that assumes a more traditional beam-sway mechanism typically found in newly designed structures does not give a good estimation on how this structure displaces. This is because it does not account for the relative changes in stiffness and strength along the height of the building.

In order to highlight the potential impact of adopting the different displacement profiles in Figure 14, the equivalent SDOF displacement capacity, Δ_{sys} , was computed for each limit state and the results are reported in Table 4. The expression for Δ_{sys} , was presented earlier in Figure 1 and requires knowledge of only the mass, m_i , and the displacement at each level, Δ_i (taken from the profiles in Figure 14). Comparing the assessed displacement capacities for the building, it can be seen that the approach of Priestley et al. (2007) overestimates the displacement capacity for this building by up to 34%. In contrast, the new procedure provides considerable improvement, with smaller differences in

displacement capacity estimates and the tendency to be conservative relative to the results of pushover analysis. As such, it is concluded that the new procedure offers considerable improvement over the existing simplified approach of Priestley et al. (2007). It is also observed that the simplified pushover analysis approach permits the likely mechanism, resistance and displacement capacity to be estimated with reasonable accuracy. There are, however, some limitations with the method, as will be underlined in the final conclusions.

Table 4. Equivalent SDOF displacement capacities assessed for the L'Aquila building using the displacement profiles assessed via different approaches.

	X-direction				Y-direction			
	SLO	SLD	SLV	SLC	SLO	SLD	SLV	SLC
Δ_{sys} from Pushover Analyses	0.032	0.048	0.174	0.235	0.034	0.049	0.178	0.226
Δ_{sys} from Eq.(1) (% Diff. Pushover)	0.037 (15.7%)	0.056 (17.2%)	0.221 (27.0%)	0.295 (25.2%)	0.037 (10.9%)	0.056 (14.0%)	0.227 (28.0%)	0.303 (33.8%)
Δ_{sys} New Procedure (% Diff. Pushover)	0.027 (-16.7%)	0.040 (-15.6%)	0.159 (-8.6%)	0.212 (-9.8%)	0.032 (-3.5%)	0.049 (-0.8%)	0.198 (11.4%)	0.264 (16.4%)

4. Conclusions

A simplified pushover analysis method that engineers can apply with either hand calculations, or via use of a simple spreadsheet, has been proposed for RC frame structures. The simplified approach permits independent checks of outputs from structural analysis software and also helps inform engineers of the key characteristics of the structural system being assessed. The procedure described in this work builds on previous contributions in the literature. The main development to the state-of-the-art is to provide an improved means of quantifying the displacement profile of RC frames, with account for different types of yielding mechanisms and structural configurations. Previous guidelines for the simplified assessment of RC frames relied on the use of an empirical expression for the displaced shape. The new approach put forward here starts by first estimating the initial storey stiffness as the storey shear resistance divided by the storey yield drift, and then uses this information to compute a displacement profile at formation of a mechanism. Subsequently, the non-linear displacement profile is attained by summing a plastic deformation profile, set as a function of the assessed mechanism, with the yield displacement profile. A step-by-step worked example is included in Appendix 1. Considering that force-displacement response predictions obtained from the simplified approach align closely with those obtained from rigorous non-linear static analyses for a range of frame configurations and mechanisms, it is concluded that the proposed approach offers an effective means of undertaking simplified pushover analysis.

The methodology has been presented for low to medium-rise RC frames prone to the development of beam-sway mechanisms, column-sway mechanisms, joint mechanisms, or combinations of these. However, the approach has not been developed or tested for other important cases, such as frames with masonry infills or frames prone to soil-structure interaction effects. In addition, there are limitations in the ability of the method to account for coupled modes of vibration or torsional sensitivity, similar to a normal pushover analysis. Nevertheless, it is considered that the approach presented in this work will generally aid engineers as a tool for checking a computer-based pushover analysis or for assessment of simple structures without the use of computer software.

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Appendix 1: Example application of the proposed procedure to a 3-storey RC frame with beam-sway mechanism

The proposed simplified procedure is applied to the RC frame structure illustrated in Figure A1. The flexural strengths (referred to joint centrelines) of the structural elements are shown in Fig. A1 (on the right). It is assumed that beams and columns have enough shear strength to allow the development of plastic hinges. The flexural strengths of columns are based on gravity induced axial loads. Variations in axial load on the columns due to seismic loading can be accounted for via an iterative assessment process but this is not shown here for brevity.

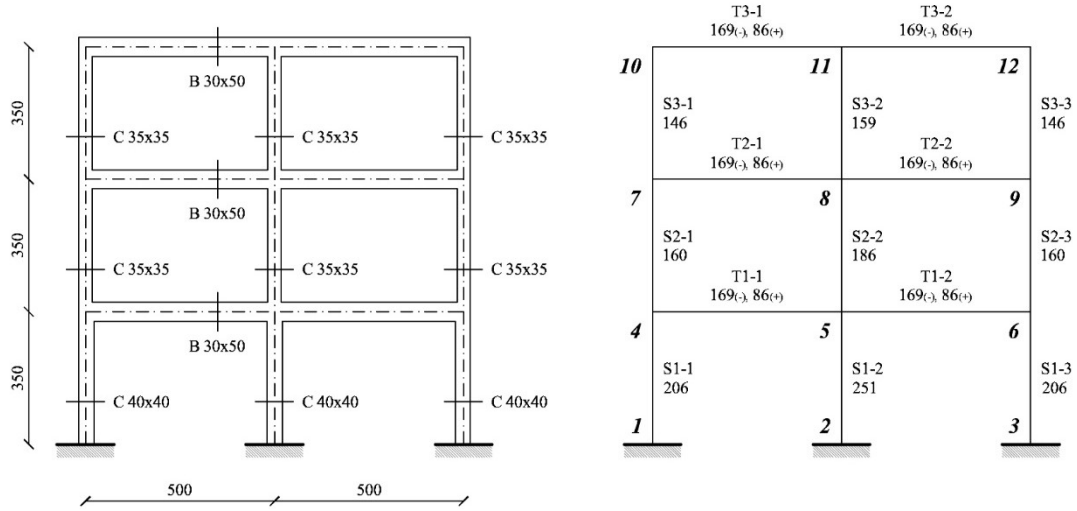


Figure A1. Geometrical dimensions in [cm] and element flexural strengths [kNm].

Assessing the likely mechanism and lateral resistance of each storey: in order to assign the moments at the top and bottom part of a column, it is necessary to consider the flexural strengths that can be developed after reaching joint equilibrium. Example for node 8 from Figure A1:

$$M_{bl} + M_{br} = 169 + 86 = 255 \text{ kNm}$$

$$M_{col,b} + M_{col,a} = 186 + 159 = 345 \text{ kNm}$$

In this case (weak beam-strong column) the flexural capacity of each column (186 kNm for the column below and 159 kNm for the column above) is higher than the half of the sum of the flexural strengths of the beams (128 kNm). Hence the moment of each column is set equal to the half of the sum of the flexural strengths of the beams (128 kNm). The values of moment obtained at the end sections of columns by evaluating the equilibrium of all nodes are shown in Table A1.

Table A1. Values of moment obtained at the end sections of columns.

Storey	1				2				3			
Column	S1-1	S1-2	S1-3	Sum	S2-1	S2-2	S2-3	Sum	S3-1	S3-2	S3-3	Sum
$M_{col,b,i}$ [kNm]	43	128	85	255	43	128	85	255	86	159	146	391
$M_{col,a,i-1}$ [kNm]	206	251	206	663	43	128	85	255	43	128	85	225

The shear resistance at each storey is calculated with Eq. (3) considering that the storey height is the same at each storey and equal to 3.50 m. For example, the second storey is computed as:

$$V_{R,i} = \left(\sum M_{col,b,i} + \sum M_{col,a,i-1} \right) / (h_i - h_{i-1}) = (255 + 255) / 3.50 = 146 \text{ kN}$$

The $V_{R,i}$ obtained at the third storey is 185 kN, at the second storey is 146 kN, at the first storey 262 kN.

Assessing the drift required to yield each storey: the yield drift is calculated for each beam or column using Eqs. (8) to (14). Example for the inner column at the base level (Eqs. (13a) and (14)):

$$h_{cf} = h_1 / \left(M_{col,b,1} / M_{col,base} + 1 \right) = 3.50 / \left(128 / 251 + 1 \right) = 2.32 \text{ m}$$

$$\theta_{y,0} = 0.70 \varepsilon_y h_{cf} / D_{col} = 0.70 \times 0.0024 \times 2.32 / 0.40 = 0.0097 \text{ rad}$$

The values of yield drift and flexural capacity of the structural elements are shown for each level in Table A2, where the length is the one used in the expression of the yield drift (for level 0, the contraflexure height h_{cf}).

Table A2. Yield drift and the flexural capacities for each level.

Level	Mechanism	Node	Element	Length [m]	Depth [m]	$\theta_{y,i}$ [rad]	$M_{bl,i}$ [kNm]	$M_{br,i}$ [kNm]	$M_{col,i}$ [kNm]
3	Column	10	S3-1	3.50	0.35	0.0103	-	-	146
		11	S3-2	3.50	0.35	0.0103	-	-	159
		12	S3-3	3.50	0.35	0.0103	-	-	146
2	Beam	7-8	T2-1	5.00	0.50	0.0120	86	169	-
		8-9	T2-2	5.00	0.50	0.0120	86	169	-
1	Beam	4-5	T1-1	5.00	0.50	0.0120	86	169	-
		5-6	T1-2	5.00	0.50	0.0120	86	169	-
0	Column	1	S1-1	2.90	0.40	0.0122	-	-	206
		2	S1-2	2.32	0.40	0.0097	-	-	251
		3	S1-3	2.48	0.40	0.0104	-	-	206

Once the yield drift is known for beams and columns, according to the type of mechanism at each level, it is possible to determine the system yield drift at each storey with Eq. (11). An example is shown below for the second storey. The obtained values for the three storeys are reported in Table A3.

$$\theta_{y,sys,i} = \frac{\sum M_{j,i} \theta_{y,i} + \sum M_{j,i-1} \theta_{y,i-1}}{\sum M_{j,i} + \sum M_{j,i-1}} = \frac{[2 \times (169 + 86) \times 0.0120] + [2 \times (169 + 86) \times 0.0120]}{[2 \times (169 + 86)] + [2 \times (169 + 86)]} = 0.0120 \text{ rad}$$

Assessing values of storey stiffness: the storey stiffness is estimated with Eq. (12). An example is shown below for the second storey. The obtained values for the three storeys are reported in Table A3.

$$k_{y,i} = \frac{V_{R,i}}{\theta_{y,sys,i} h_{s,i}} = \frac{146}{0.0120 \times 3.50} = 3476 \text{ kN/m}$$

Table A3. Values of shear resistance, yield drift and stiffness for each storey.

Storey	Sway mechanism at storey i	$V_{R,i}$ [kN]	$h_{s,i}$ [m]	$\theta_{y,sys,i}$ [rad]	$k_{y,i}$ [kN/m]
3	Mixed (level 3: column; level 2: beam)	185	3.50	0.0112	4719
2	Beam (level 2: beam; level 1: beam)	146	3.50	0.0120	3476
1	Mixed (level 1: beam; level 0: column)	262	3.50	0.0113	6625

Assessing displacement and storey shear profiles:

- Step 1. Estimate base shear at yield of frame $V_b = V_{R,1} = 262 \text{ kN}$
- Step 2. Make first estimate of displacement profile Δ_i , using Eq. (1) with $\theta_c = \theta_{y,sys,1} = 0.0113 \text{ rad}$
- Step 3. Compute set of equivalent lateral forces F_i , using Eq. (13)
- Step 4. Compute storey shear demand profile V_i , using Eq. (15). Compare at each storey the shear demand with the shear capacity and revise the base shear. Evaluate the ratio $V_i/V_{R,i}$ and repeat the calculation from step 3 considering a new estimate of base shear equal to the first one divided by the maximum ratio $V_i/V_{R,i}$. The previous calculations are shown in Table A4.

Table A4. Application of Steps 1 to 4.

Storey	θ_c [rad]	h_i [m]	Δ_i [m] Step 2	w_i [kN]	F_i [kN] Step 3	V_i [kN] Step 4	$V_i/V_{R,i}$	F_i [kN]	V_i [kN] Step 4 final	$V_i/V_{R,i}$
3	0.0113	10.50	0.1187	400	131	131	0.71	88	88	0.47
2	0.0113	7.00	0.0791	400	87	218	1.50	58	146	1.00
1	0.0113	3.50	0.0396	400	44	262	1.00	29	175	0.67

- Step 5. Compute storey drift components δ_i , using Eq. (15)
- Step 6. Sum drift components to identify compatible displacement profile, $\Delta_{i,comp}$, as per Eq. (16). The values from steps 5 and 6 are shown in Table A5. Compare the compatible displacements with

the displacements estimated in Step 2 (see Table A4). If the difference is significant, repeat the procedure from Step 3 using the displacements just derived, otherwise the obtained values of displacement and storey shear at first yield have been identified. The values obtained at convergence are also shown in Table A5.

Table A5. Application of Steps 5 to 6 and final values at convergence.

Storey	V_i [kN]	$k_{y,i}$ [kN/m]	δ_i [m]		V_i [kN]	$\Delta_{i,comp}$ [m]	
			Step 5	Step 6		Step 5 final	Step 6 final
3	88	4719	0.0186	0.0870	81	0.0172	0.0850
2	146	3476	0.0420	0.0684	146	0.0420	0.0678
1	175	6625	0.0264	0.0264	171	0.0258	0.0258

Assessing the global post-yield mechanism: the sway potential index S_i is calculated to identify the most likely inelastic mechanism, using Eq. (17). An example is shown below for the second level. The obtained values for the three levels are reported in Table A6:

$$S_i = \frac{\text{sum of beam strengths at level } i}{\text{sum of column strengths at level } i} = \frac{(2 \times 169) + (2 \times 86)}{(2 \times 160 + 186) + (2 \times 146 + 159)} = 0.533$$

Table A6. Sway potential index for each level.

Level	Sum of beam strengths [kNm]	Sum of column strength [kNm]	S_i	Sway mechanism
3	510	451	1.13	Column
2	510	957	0.53	Beam
1	510	1169	0.44	Beam

The obtained values of sway potential index indicate the formation of a beam-sway mechanism, with plastic hinges at the base of columns (level 0), at the ends of the beams of the levels 1 and 2 and at the top of columns (level 3). This is compatible with the results obtained in the phase ‘‘Assessing the likely mechanism and lateral resistance of each storey’’. In this condition, the post-yield response is established by assuming a fixed displaced shape derived at the first storey yield (first yield point) and subsequently increasing the values of displacement until all storeys have yielded.

Once the displacement profile is fixed, it is possible to calculate the drift demand θ_i at each storey as $\theta_i = \delta_i / h_{s,i}$ and the storey displacement ductility μ_i as $\mu_i = \theta_i / \theta_{y,sys,i}$. The storey shear V_i can then be calculated using the storey displacement ductility as $V_i = \mu_i V_{i,y}$. With reference to the first yield point the ductility and storey shear values are shown in Table A7.

Table A7. Determination of first point of pushover curve (yielding of the second storey).

Storey	Δ_i [m]	θ_i [rad]	$\theta_{y,sys,i}$ [rad]	μ_i	V_i [kN]	$M_{OT,i}$ [kNm]
3	0.0850	0.0049	0.0112	0.439	81	284
2	0.0678	0.0120	0.0120	1.000	146	795
1	0.0258	0.0074	0.0113	0.651	171	1392

From the values of storey shear it is possible to calculate the base overturning moment M_{OT} :

$$M_{OT} = 3.50 \times 81 + 3.50 \times 146 + 3.50 \times 171 = 1392 \text{ kNm}$$

Once the effective height H_e of the equivalent SDOF system is determined (refer Figure 1), whose result is $H_e = 8.16$ m, the base shear V_b is derived from the base overturning moment M_{OT} :

$$V_b = M_{OT} / H_e = 1392 / 8.16 = 171 \text{ kNm}$$

The subsequent yield points are obtained with the same procedure illustrated above for the first yield point. By increasing the values of displacement, when the ductility demand becomes larger than 1.00,

the storey shear remains set to the storey shear resistance. The mechanism is reached when the ductility is equal or larger than 1.00 at all storeys.

A comparison between the results from application of the simplified procedure (labelled DBA) and the pushover curve from SAP 2000 is illustrated in Figure A2(a). Furthermore, the normalised displacement shape obtained by applying the simplified procedure at the first yield is compared in Figure A2(b) with the first mode displacement profile (1stMS) and the displacement shape at the final step of pushover analysis. Finally, Figure A2(c) illustrates the distribution of plastic hinges at the final step of the pushover analysis, which corresponds with that identified previously in Table A6.

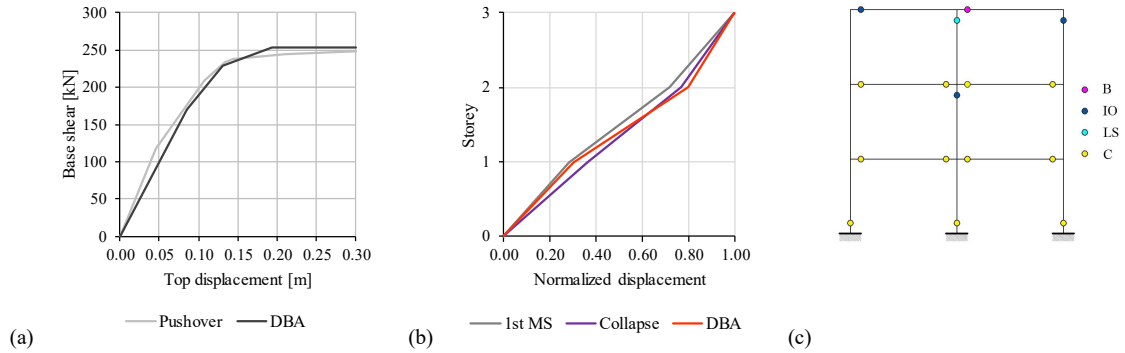


Figure A2. Comparison between pushover curve from simplified procedure and computer program (a); comparison between normalized displacement profiles (b); final configuration of plastic hinges (c).