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# Innovation and product market concentration: Schumpeter, Arrow and the inverted-U shape curve\*

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## Abstract

We investigate the relationship between market concentration and industry innovative effort within a familiar two-stage model of R&D race in which firms compete *à la* Cournot in the product market. With the help of numerical simulations, we show that such a setting is rich enough to generate Arrovian, Schumpeterian and inverted-U curves. We interpret these different patterns on the basis of the relative strength of the technological incentive and the strategic incentive. We then bridge our theoretical results and some recent empirical research.

**Keywords:** innovation; market structure; aggregate investment;

**JEL Codes:** L13, O31

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# 1 Introduction

In a seminal paper, Aghion *et al.* (2005) collect empirical evidence about the relationship between product market concentration and the intensity of innovative activity at the *aggregate economy* level. They show that such relationship follows an inverted-U shape pattern with respect to market concentration as measured by a modified Lerner index. They then go on to rationalize such curve by means of a model in which technologically asymmetric firms strive for improving their cost gap in R&D races under uncertainty. Innovations occur step-by-step and the effect of market competition on R&D investment results from the balance between what they call the “Schumpeterian” effect and the “escaping competition” incentive. Their panel of firms comes from UK industries, 1973-94. As for the indicator of innovation intensity, they select the average number of patents (weighted by citations) taken out in the U.S. patent office. Incidentally, it is worth noting that the number of patents underestimates the R&D effort, as it records only successful and patented inventions. While derived in an elegant general equilibrium model, their conclusions rely upon severe assumptions: all industries are homogeneous duopolies, no entry, spillovers are such that the leader has no benefit from innovating. Hence, “it is not entirely clear why even lagging firms cannot catch up to or even leapfrog the current best technology through their innovative efforts” (Gilbert 2006, p. 199).

Aghion *et al.* (2005) have then revitalized the old debate about the relationship between market structure and innovation. Such debate<sup>1</sup> was mostly focussed on a binary menu contrasting the arguments behind Schumpeter (1942) well-known alleged superiority of monopoly in driving innovative ac-

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<sup>1</sup>See Reinganum (1989), for an excellent survey of the early literature. More recent surveys tackling also competition policies are Gilbert (2006) and Shapiro (2012). Whether competitive pressure fosters innovation depends also on competition modes. On this, see the interesting results in Vives (2008).

tivity and the opposite conclusion by Arrow (1962). By showing that the relationship may exhibit an inverted-U shape, they provide an important empirical contribution; moreover, their model offers a theoretical frame accommodating such pattern.<sup>2</sup> An exhaustive view of the debate about the inverted-U pattern in industrial organization and growth theory is in Aghion *et al.* (2015a,b, 2019).

The traditional debate on innovation races that started towards the end of the 1970s looked for monotone relationships between industry structure (or the intensity of market competition) and aggregate innovation efforts (or incentives), since the issue at stake was whether the Schumpeterian view or the Arrovian one was correct, with controversial results (cf. Reinganum, 1989, *inter alia*). The very fact that the attention of researchers focussed on monotone patterns might have caused the possible arising of non-monotone ones to remain overlooked.

While “to the best of our knowledge, no existing model of product market competition and innovation predicts an inverted-U pattern” (Aghion *et al.*, 2005, p. 722), our aim is to illustrate that models belonging to the backbone of the literature on this matter may indeed generate concave and single-peaked patterns under plausible conditions. In this paper, we revisit those models of product market competition and innovation to show that they

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<sup>2</sup>On the inverted-U shape relationship, see the empirical evidence collected by Mansfield *et al.* (1968). To the best of our knowledge, the first scholars hinting at such shape within a theoretical model are Kamien and Schwartz (1976). They “address the reported empirical finding that the rate of innovative activity increases with the intensity of rivalry up to a point, peaks, and declines thereafter with further increase in the competitiveness of the industry” (Kamien and Schwartz, 1976, p. 247). However, they do not explicitly model the R&D race as a game, and the prize to the winner is independent of the intensity of rivalry. Also Spence (1984, p. 110) shows numerically that the market incentive for cost reductions, for low levels of spillover in the R&D activity, initially rises and then falls w.r.t. the number of firms.

may also predict the presence of an inverted-U shape curve. We borrow from the game-theoretic literature utilized in the '90s and follow Lee and Wilde (1980). However, we do not blackbox - as they do - the nature of market competition, but we model it explicitly as a homogeneous oligopoly à la Cournot. In such a static two-stage game of R&D, firms participate in an uncertain race to get a non-drastic cost-reducing innovation which will allow the winner to compete with a cost advantage in the market game. There is no spillover and Cournot competition in the product market allows all (initially identical) firms to be active also in the asymmetric post-innovation non-cooperative equilibrium. For sake of tractability, we adopt a linear-quadratic specification of the R&D technology and the market game of Lee and Wilde (1980) as in Delbono and Denicolò (1991), where it is shown that, under Cournot competition in the product market,<sup>3</sup> aggregate R&D may respond both ways to increases in market concentration. However, the large number of parameters prevents us from deriving clear-cut analytical conclusions as for the existence of an inverted-U shape relationship. Hence, we resort to numerical simulations and show the emergence of such a shape. Moreover, albeit simple, our model is rich enough to generate also an Arrovian as well as a Schumpeterian behaviour in the relationship between aggregate R&D effort and the numerosity of firms.

Specifically, it turns out that, for a given market size, if the innovation is non-drastic:

1. A low productivity of the R&D technology (and/or a high level of the discount rate) yields a Schumpeterian relationship, e.g. the equilibrium aggregate R&D effort is maximised under monopoly and then monotonically

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<sup>3</sup>The first attempt of modelling the market game as a Cournot one to investigate the relationship between innovation and concentration is Horowitz (1963). Stewart (1983) drops the “winner-takes-all” assumption in Lee and Wilde’s model, but he does not model explicitly the market game.

decreases with the number of firms. This holds irrespective of the size of the innovation.

2. When the productivity of the R&D technology is high (and/or the level of the discount factor is low), then two scenarios emerge, depending on the magnitude of the cost reduction reached by the winning firm.

2a. If such a reduction is small w.r.t. the given market size, then we detect an inverted-U shape curve between aggregate R&D and the number of firms.

2b. If the cost reduction is large - making the innovation almost drastic - then we observe an Arrovian pattern, e.g., the aggregate investment monotonically grows with the number of firms.

The shapes of aggregate R&D emerging from the model find their counterparts in recent empirical analysis to which we will come back later. While simpler than Aghion *et al.* (2005)'s, our model, which is partial equilibrium in nature, succeeds to generate a richer set of patterns. Moreover, as in the vast majority of the literature, our conclusions hold at the *industry* level, and not at the aggregate one as in their general equilibrium framework.

The ensuing analysis also has some bearings for horizontal merger policy, an issue which has been lively debated in recent literature.<sup>4</sup> This discussion has been spurred by the Dow-DuPont merger case faced by the European Commission, in which the impact of the merger on the pace of technological progress played a central role in the assessment of the Commission.

The paper is organized as follows. In section 2 we set the background. In section 3 we specialise the general model and summarize the findings from a large number of numerical simulations by providing some intuition behind different patterns. Section 4 aims at assessing concordance between theory and some empirical research. Section 5 concludes.

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<sup>4</sup>For a detailed account of the debate started by Federico *et al.* (2017), see see Haucap *et al.* (2019) and Marshall and Parra (2019).

## 2 The background

Consider  $n$  identical firms investing in R&D to be first in getting a technological improvement.<sup>5</sup> Firms act noncooperatively and choose an investment expenditure  $x$  to maximise the discounted stream of expected profits. Technological uncertainty is of the exponential type, i.e., the discovery time is described by an exponential (or Poisson, or ‘memoryless’) distribution function. Firm  $i = 1, 2, \dots, n$  then maximises the following payoff

$$\Omega_i = \int_0^\infty e^{-(r+H)t} [h(x_i) V_i + H_i v_i + \pi_i - x_i] dt \quad (1)$$

where  $r > 0$  is the common discount rate,  $h(x_i)$  is  $i$ ’s hazard rate (i.e., the instantaneous probability of innovating conditional upon not having innovated before),  $H = \sum_{i=1}^n h(x_i)$ ,  $H_i = H - h(x_i)$ ,  $\pi_i$  are  $i$ ’s current gross profits,  $V_i$  ( $v_i$ ) is the discounted continuation value of the game if  $i$  wins (loses) the race. This is the formulation of Lee and Wilde (1980) which modifies Loury’s (1979) as for the specification of the R&D cost. Here, they are non-contractual, that is, a fixed rate of spending  $x_i$  until a firm succeeds.<sup>6</sup>

As for the hazard function, as in Loury (1979) and its follow-ups, it is assumed that it is strictly concave, with  $h'(x_i) > 0$ ,  $h''(x_i) < 0$ ,  $h(0) = 0$ ,  $\lim_{x_i \rightarrow 0} h'(x_i) = \infty$  and  $\lim_{x_i \rightarrow \infty} h'(x_i) = 0$ . These are the so-called Inada conditions ensuring the existence of an interior solution and the satisfaction of the second order condition.

The specification of the nature of the R&D cost matters as for the comparative statics properties of the model. Indeed, while Loury (1979) proves

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<sup>5</sup>Should firms differ in terms of their innovation technologies, credit constraints would enter the picture and might contribute to the arising of an inverted-U pattern, as shown in Bonfatti and Pisano (2020).

<sup>6</sup>In Marshall and Parra (2019), the reference setup is similar, except that firms are heterogeneous, as some of them are innovators without productive facilities and therefore must auction their inventions.



that, in the Nash equilibrium, the optimal individual R&D effort decreases in the number of firms, Lee and Wilde (1980) prove the opposite.<sup>7</sup>

Slightly later, a parallel debate started on the relationship between market power and the incentive to get an exogenous innovation, with Gilbert and Newbery (1982, 1984) and Reinganum (1983), reaching opposite conclusions about the persistence of monopoly. This discussion echoes the old dichotomy between Schumpeter (1942) and Arrow (1962). The subsequent literature focusses on the impact of industry structure or the intensity of competition (e.g., Bertrand vs Cournot) for a given market structure on the aggregate investment in R&D, and is accurately accounted for in Tirole (1988) and Reinganum (1989), *inter alia*. On the basis of the original contraposition between Schumpeterian and the Arrovian views, the main concern dealt with the sign of the monotonicity of the aggregate innovative effort w.r.t. industry structure. Aghion *et al.* (2005), instead, show the emergence of a concave and single-peaked relationship from the data and rationalise it with a theoretical model.

In this paper, we aim at showing that the early approach using stochastic race models along the lines of Loury (1979), Dasgupta and Stiglitz (1980), Lee and Wilde (1980) and Reinganum (1983) may indeed generate both monotone patterns as well as the inverted-U shape one.

To do so, we make a further step by specifying the nature of prizes in

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<sup>7</sup>In Loury's (1979) formulation, the firm's maximand is:

$$\Omega_i = \int_0^{\infty} e^{-(r+H)t} [h(x_i) V_i + H_i v_i + \pi_i] dt - x_i$$

where  $x_i$  is a lump-sum paid by firm  $i$  at the outset. "The intuition behind these conclusions is simple. In the Dasgupta and Stiglitz (1980) and Loury model, an increase in the number of firms reduces the expected benefit to investment... leaving expected costs unchanged. The firm responds by reducing investment. In the Lee and Wilde model, both expected benefits and expected costs are reduced by the addition of another firm... and the net effect is to enhance incentives to invest" (Reinganum, 1984, p. 62).

the race, following Delbono and Denicolò (1991) who consider firms striving for a non drastic cost-reducing innovation and Cournot competition in the market game. The expected stream of discounted profits for firm  $i$  becomes:

$$\Omega_i = \frac{h(x_i) \pi_W^*/r + H_i \pi_L^*/r + \pi^C - x_i}{r + H_i + h(x_i)} \quad (2)$$

where  $\pi_W^*$  is the instantaneous profit accruing forever to the winner of the R&D race,  $\pi_L^*$  is the instantaneous profit accruing forever to each loser, and  $\pi^C$  is the instantaneous profit in the pre-innovation symmetric Cournot equilibrium. In the symmetric equilibrium, the following condition must hold:

$$\frac{(\pi_W^* - \pi_L^*)(n-1)h(x_i)h'(x_i)}{r} + (\pi_W^* - \pi^C)h'(x_i) - r - nh(x_i) + xh'(x_i) = 0 \quad (3)$$

It can be shown (Beath *et al.*, 1989; and Delbono and Denicolò, 1990) that the equilibrium R&D effort is increasing in both  $\pi_W^* - \pi^C$  and  $\pi_W^* - \pi_L^*$ . Let us label the former as *technological incentive* and the latter as *strategic incentive*. Notice that  $\pi_W^* - \pi^C$  is the difference between the profit of the winner and the current profit. Such a difference captures what has been called the ‘profit incentive’ by Beath *et al.* (1989), the ‘stand alone incentive’ by Katz and Shapiro (1987) and it is related to - but it doesn’t coincide with - the ‘replacement effect’ in Fudenberg and Tirole (1986) who follow Arrow’s (1962) expression.

On the other hand,  $\pi_W^* - \pi_L^*$  is the difference in profits between winning and losing the race, and it captures what has been named as the ‘competitive threat’ by Beath *et al.* (1989), the ‘incentive to pre-empt’ by Katz and Shapiro (1987) and the ‘efficiency effect’ by Fudenberg and Tirole (1986) and Tirole (1988).

From the standpoint of the debate inaugurated by Aghion *et al.* (2005), we can single out an elementary property of the aggregate R&D effort which was first underlined in Delbono and Denicolò (1991, p. 959). Writing the

individual symmetric equilibrium effort as  $x^*(n)$ , we clearly have<sup>8</sup>

$$\frac{\partial (nx^*(n))}{\partial n} = x^*(n) + n \cdot \frac{\partial x^*(n)}{\partial n} = x^*(n) \left[ 1 + \frac{n}{x^*(n)} \cdot \frac{\partial x^*(n)}{\partial n} \right] \quad (4)$$

which, if  $\partial x^*(n)/\partial n < 0$ , may be nil for some  $n$  (possibly more than once, as the expression in square brackets will not be linear w.r.t.  $n$ , in general). This amounts to saying that  $\partial x^*(n)/\partial n < 0$  is a necessary (but not sufficient) condition for the arising of inverted U's. This fact could have triggered a deeper investigation of the relationship between aggregate effort and market structure in the vein of the debate between Schumpeter (1942) and Arrow (1962), possibly spotting a non-monotone behaviour as in Aghion *et al.* (2005). If this idea had emerged at the time, one should have tried to sign the following expression:

$$\frac{\partial^2 (nx^*(n))}{\partial n^2} = 2 \cdot \frac{\partial x^*(n)}{\partial n} + n \cdot \frac{\partial^2 x^*(n)}{\partial n^2} \quad (5)$$

In (5), the sign of  $\partial x^*(n)/\partial n$  was established, on the basis of various specifications of the model.<sup>9</sup> Conversely, the sign of  $\partial^2 x^*(n)/\partial n^2$  has never been discussed, as (5), in itself, was not considered.

What we are setting out to do in the remainder of the paper is to specify all of the components of (2) as in Delbono and Denicolò (1991) to show the

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<sup>8</sup>Wherever useful, we follow this literature by treating  $n$  as a continuous variable.

<sup>9</sup>If the winner takes all, under contractual R&D costs as in Loury (1979) and Dasgupta and Stiglitz (1980), the sign is negative. Therefore, in their setting, one might have envisaged a peak in industry effort w.r.t. concentration, because (4) may vanish for some values of  $n$ . Under non-contractual costs, as in Lee and Wilde (1980), it is positive. Under non-contractual R&D costs and Cournot competition, as in Delbono and Denicolò (1991), the sign may change, because both pre- and post-innovation profits are all decreasing in  $n$ . Hence the sign of  $\partial x^*(n)/\partial n$  is ambiguous - and so is the sign of  $\partial (nx^*(n))/\partial n$  without further specification of cost and demand. Of course, this ambiguity would persist also outside Cournot rules as long as profits are decreasing in  $n$ . This was already noted in Delbono and Denicolò (1991, p. 955).

arising of both Schumpeterian and Arrovian patterns of aggregate R&D as well as an inverted-U shaped curve as in Aghion *et al.* (2005).

### 3 The specialised model

We consider the homogeneous Cournot model with a linear demand  $p = a - Q$  and a constant marginal production cost initially equal to  $c \in (0, a)$ . If one defines the initial market size as  $s \equiv a - c$  and the cost reduction as  $d \equiv c - c^*$ , where  $c^*$  is the new marginal cost patented by the winner of the R&D race ( $s > d$  because we focus on non-drastic innovation), then the relevant profits to be substituted into (2) are

$$\pi_W^* = \frac{(s + nd)^2}{(n + 1)^2}; \pi_L^* = \frac{(s - d)^2}{(n + 1)^2}; \pi^C = \frac{s^2}{(n + 1)^2} \quad (6)$$

As for the hazard function, we stipulate that  $h(x_i) = 2\mu\sqrt{x_i}$ , where  $\mu$  is a positive parameter measuring the efficiency of R&D expenditure. In what follows, we consistently use  $\theta \equiv \mu/r$  to save on notation.

Given the triple of profits in (6) and the above specification of the hazard function, the first order condition (FOC) taken on (2) w.r.t.  $x_i$ , under symmetry, is

$$\frac{-\theta(2n - 1)x + [2\theta^2(n - 1)(\pi_W^* - \pi_L^*) - 1]\sqrt{x} + \theta(\pi_W^* - \pi^C)}{[2n\theta\sqrt{x} + 1]^2\sqrt{x}} = 0 \quad (7)$$

which yields:

$$x_{\pm} = \frac{\Phi + 2\theta^2[\pi_W^* + 2(n - 1)\pi_L^* - \Psi] \pm \Xi\sqrt{\Phi + 4\theta^2[\Psi + n\pi_W^* + (n - 1)\pi_L^*]}}{2\theta^2(2n - 1)^2} \quad (8)$$

where  $\Psi \equiv (2n - 1)\pi^C$ ,  $\Phi \equiv 1 + 4\theta^4(n - 1)^2(\pi_W^* - \pi_L^*)^2$  and  $\Xi \equiv 1 - 2\theta^2(n - 1)(\pi_W^* - \pi_L^*)$ . Notice that

$$\text{sign}(x_+ - x_-) = \text{sign}(\Xi) \quad (9)$$

and  $\Xi = 0$  at

$$\theta_{\pm} = \pm \sqrt{\frac{n+1}{2d(n-1)[2s+d(n-1)]}} \quad (10)$$

which, considering that  $\theta > 0$ , implies

$$\begin{aligned} \Xi > 0 \forall \theta \in (0, \theta_+ \equiv \sqrt{\frac{n+1}{2d(n-1)[2s+d(n-1)]}}) \\ \Xi < 0 \forall \theta > \theta_+ \end{aligned} \quad (11)$$

Before proceeding, it is worth noting that  $\lim_{n \rightarrow 1} \theta_+ = \infty$ ,  $\lim_{n \rightarrow \infty} \theta_+ = 0$ ,  $\partial \theta_+ / \partial n < 0$  and  $\partial^2 \theta_+ / \partial n^2 > 0$ . That is, (i)  $\theta_+$  is decreasing and convex in  $n$ ; (ii) in monopoly,  $\Xi > 0$  surely; and finally (iii) under perfect competition,  $\Xi < 0$  surely.

Since the numerator of the expression on the l.h.s. in (7) is concave in  $x$ , the foregoing analysis proves

**Proposition 1** *The equilibrium individual R&D effort is  $x^* = \max \{x_-, x_+\}$ , with*

$$\begin{aligned} \max \{x_-, x_+\} &= x_+ \forall \theta \in (0, \theta_+) \\ \max \{x_-, x_+\} &= x_- \forall \theta > \theta_+ \end{aligned}$$

This Proposition, in combination with the limit properties of  $\theta_+$ , entails that when  $n = 1$ , the relevant R&D effort is  $x_M^* = x_+|_{n=1}$ ; if instead  $n$  becomes infinitely large, the equilibrium R&D effort is  $\lim_{n \rightarrow \infty} x_-$ .

We are now in a position to assess the impact of industry structure on the aggregate R&D effort.

### 3.1 Schumpeter, Arrow, and inverted-U's

Define the aggregate equilibrium investment as  $X^* = nx^*$ . Despite the use of an extremely simplified specification of the model,  $X^*$  remains highly non linear in  $n$ , which prevents the analytical treatment of the problem under

scrutiny and calls for numerical simulations.<sup>10</sup> We distinguish two cases, depending on the size of  $\theta = \mu/r$ .<sup>11</sup> Hence, what follows lends itself to a twofold interpretation, which can focus either on the productivity of R&D for a given level of impatience, or the opposite. In both scenarios, we set parameter values so that (2) be positive.

**Scenario I:**  $\theta \leq \theta_+$  In this case,  $\mu$  and  $r$  are set so as to identify values of  $\theta \in (0, \theta_+]$ . Hence, by Proposition 1, aggregate effort is  $X_+^* = nx_+$ .

First of all, we evaluate the behaviour of  $X_+^*$  w.r.t.  $n$  in  $n = 1$ . Were the aggregate effort be increasing in  $n$  under monopoly, this would exclude a Schumpeterian pattern. To see that this is not the case, note that the following derivative:

$$\left. \frac{\partial X_+^*}{\partial n} \right|_{n=1} = - \frac{6(1+\Lambda) + d\theta^2 [4d^2s\theta^2 + d(8s^2\theta^2 + 3) + 2s(8 + 5\Lambda)]}{4\theta^2\Lambda} \quad (12)$$

where  $\Lambda \equiv \sqrt{1 + d(d+2s)\theta^2}$ , is clearly negative. Moreover, the limit values of  $X_+^*$  are:

$$\begin{aligned} X_+^* \Big|_{n=1} &= \frac{d(d+2s)\theta^2 + 2(1+\Lambda)}{4\theta^2} > 0 \\ \lim_{n \rightarrow \infty} X_+^* &= 0 \end{aligned} \quad (13)$$

These properties, of course, do not exclude a non-monotone behaviour of industry investment in some range of  $n$  greater than one but not arbitrarily large. For this reason we switch to numerical simulations, fixing once and for all  $s = 1$ . We have performed simulations using the following parameter constellations:

$$n \in [1, 100]; d \in [1/100, 10]; \theta \in [1/100, 1] \quad (14)$$

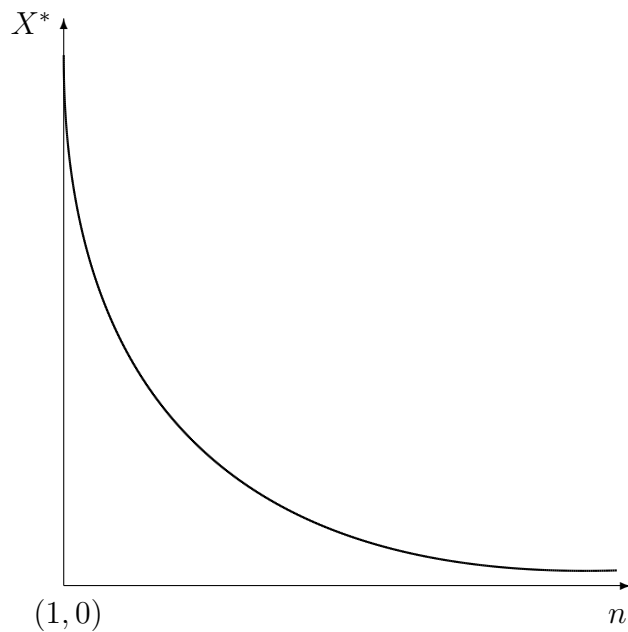
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<sup>10</sup>We have performed simulations using the ManipulatePlot device in Wolfram's Mathematica 10.1.

<sup>11</sup>In Delbono and Denicolò (1991), only one solution is considered because attention is focussed on low values of  $\theta$ , in particular so low that the second-order effects of R&D efficiency (or impatience, as measured by  $r$ ) can be neglected.

focussing on cases where  $s > d$ ,  $x_+^* > 0$  and  $\Omega_i(x_+^*) \geq 0$ . The qualitative properties of the pattern emerging from this simulation are depicted in Figure 1, displaying a Schumpeterian behaviour of aggregate investment w.r.t. industry structure, as  $X_+^*$  consistently looks decreasing and convex in  $n$ .<sup>12</sup>

**Figure 1** The Schumpeterian case



The curve appearing in Figure 1 has been obtained by setting  $d = 1/50$  and  $\theta = 3/2$ . These values describe a situation in which the cost reduction is very small and the efficiency of R&D (time discounting)

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<sup>12</sup>If  $\theta = \theta_+$ , the aggregate industry effort is  $nx_+^* = nx_-^*$  and its expression is

$$X^* = \frac{dn^2(2s + nd)}{(n + 1)^2(2n - 1)}$$

which is decreasing and convex in  $n$  for all  $s > d$ .

is very low (high). This amounts to saying that the winner gains a very small profit increase and the remaining  $n - 1$  firms lose very little as compared to the *ex ante* symmetric equilibrium. This drives the Schumpeterian outcome.

Some intuition behind the Schumpeterian pattern may rely upon the technological incentive *vis à vis* the strategic one. A necessary condition for the aggregate effort to be decreasing in  $n$  is that  $x_M^* > x_+^*$ , and this inequality certainly holds if both incentives are greater for the monopolist than for the generic oligopolist. Straightforward calculations show that the strategic incentive is always greater for the monopolist, whereas the technological incentive is greater for the monopolist when the cost reduction is small.

The same argument can be spelled out, perhaps more explicitly, in the following terms. This is a situation in which the efficiency of the innovative process is low and/or firms are highly impatient: the two factors have the same effect, as - for any given  $\mu$ , even a high one - an extremely high discount rate shrinks any future gain; conversely, given  $r$ , a negligible efficiency level of the R&D technology delivers equally negligible cost advantages. Furthermore, the size of the cost advantage driven by the innovation is small. Hence, the monopolist invests more than oligopolists because the latter (very impatient, for any given R&D productivity level) have little to gain from winning the race and becoming the lowest cost firm in the asymmetric post-innovation Cournot-Nash equilibrium. As competition intensifies, each individual investment in R&D shrinks and so does the aggregate investment. On the opposite, even for a small cost reduction, the (equally impatient) monopolist will fully appropriate the extra profit caused by the cost reduction and will find it profitable to invest so as to replace itself in the monopolistic position.



**Scenario II:**  $\theta > \theta_+$  In this case,  $\mu$  and  $r$  are set so as to identify values of  $\theta > \theta_+$ . Hence, by Proposition 1, aggregate effort is now  $X_-^* = nx_-$  for  $n \geq 2$ , while  $x_M^* = x_+|_{n=1}$ .

To begin with, observe that

$$\text{sign} \left. \frac{\partial X_-^*}{\partial n} \right|_{n=1} = \text{sign} \Upsilon \quad (15)$$

where

$$\begin{aligned} \Upsilon \equiv & 6 + d\theta^2 \left[ 3d + 16s + 4ds(d + 2s)\theta^2 \right] - \\ & 2\sqrt{1 + d\theta^2(d + 2s)} \left( 3 + 5ds\theta^2 \right) > 0 \end{aligned} \quad (16)$$

This clearly rules out a Schumpeterian pattern, while leaving open both possibilities for an Arrovian behaviour or an inverted-U shape curve. Making ourselves sure again that  $s > d$ ,  $x_-^* > 0$  and  $\Omega_i(x_-^*) \geq 0$ , our numerical simulations illustrate that an Arrovian pattern emerges when the cost reduction is large *vis à vis* market size, whereby the model is close to a ‘winner-takes-all’ setup, while a concave and single-peaked pattern may obtain if the innovation is small.

The fact that  $X_-^*$  is monotonically increasing in  $n$  when  $\theta$  is large and the innovation is *almost drastic* is intuitively due to the fact that, in such a case, the prize to the winner is very close to the pure monopoly profits associated with the new technology. Notice that a sufficient condition to obtain an Arrovian pattern is that both the technological and the strategic incentives are greater for the oligopolist than for the monopolist. Suppose the innovation is drastic ( $d = s$ ); then, the technological incentive is greater for the oligopolist, whereas the strategic incentive is identical across firms. By continuity, if  $d$  is lower than  $s$  but close to it, also the strategic incentive is greater for the oligopolist.<sup>13</sup>

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<sup>13</sup>Incidentally, this is precisely the setting considered by Reinganum (1983) in her reply to Gilbert and Newbery (1982).

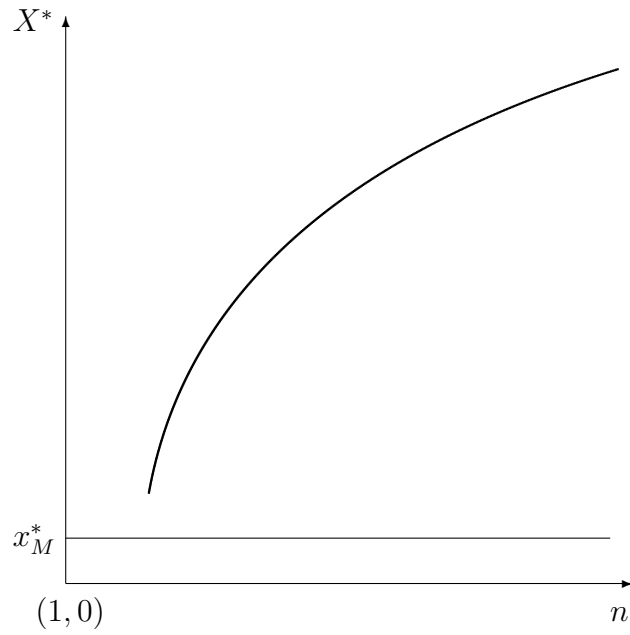
For instance, keeping  $s$  at 1 and taking as a reference set of intervals the following:

$$n \in [1, 100]; d \in [1/100, 10]; \theta \in [1/100, 100], \quad (17)$$

one has to take into account the constraint  $\theta > \theta_+$ , which depends on  $\{d, n\}$ . A pair which surely satisfies this constraint is  $\{d = 1/2, \theta = 50\}$  and this generates the Arrovian graph appearing in Figure 2, where the curve starts at  $n = 2$  and the optimal monopoly effort  $x_+|_{n=1}$  is identified by the flat line.

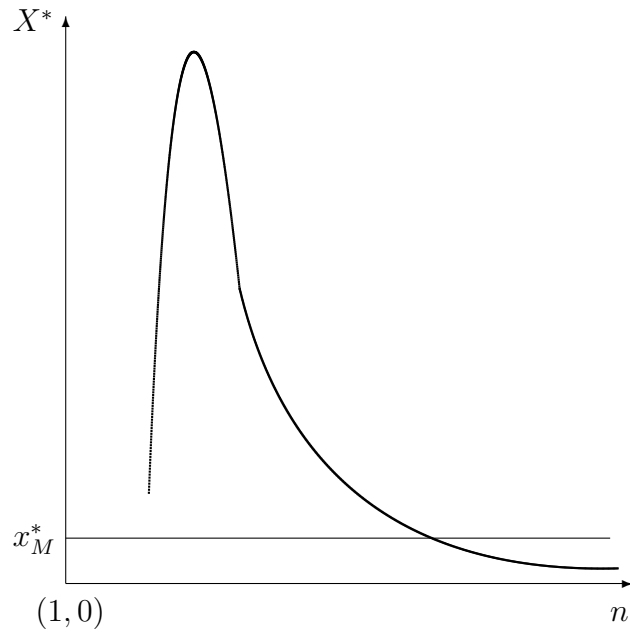
To clarify the details of what is behind the curve in Figure 2, remember that  $\theta$  now is “large”, meaning that the productivity of R&D is high as relative to any given level of the discount factor. Increasing market competition, as measured by  $n$ , spurs the individual effort because in this case the cost reduction is large enough to grant a substantial profit increase to any oligopolist, whereas the monopolist is penalised from being already reaping high profits (this is the so-called replacement effect). The resulting pattern, unsurprisingly, echoes Arrow’s original conclusions, obtained by looking at perfect (or Bertrand) competition under which, in the asymmetric post-innovation equilibrium, each oligopolist would gain a substantial increase in profit, as compared to the pre-innovation equilibrium. Hence, an increase in the number of firms increases also the aggregate R&D effort of the industry.

**Figure 2** The Arroviaian case



The remaining pattern, which is in Figure 3, reflects the inverted-U shape we know from Aghion *et al.* (2005). In our setting, such a curve emerges when the cost reduction is very small as compared to market size. Taking as a general reference (17), then fixing  $\{d = 1/100, \theta = 50\}$ , and accounting for the integer constraint on  $n$ , the peak of  $X_-^*$  is in correspondence of  $n = 3$ , with  $X_-^* \simeq 0.122$ , while  $x_M \simeq 0.007$ .

**Figure 3** The inverted-U case



The intuition behind this non monotonic pattern relies upon the contrast between the small technological gain and the price effect driven by the numerosity of rivals. The situation is like the one in Scenario 2, except for the size of the cost reduction which is now small ( $d = 1/100$ ). Hence, the efficiency of the R&D productivity (and/or the high patience) pushes firms to behave *à la* Arrow as long as the number of rivals is not too large. However, there is a critical threshold of  $n$  beyond which the effect of the limited size of the innovation is more than offset by the impact of a greater number of firms on the market equilibrium price in the asymmetric post-innovation Cournot-Nash equilibrium. Hence, as market competition intensifies and erodes profits, the curve slopes downwards as in the Schumpeterian setting.

## 4 Bridging theory and evidence

The conflicting conclusions emerged in the theoretical literature has not helped the empirical research aimed at detecting the relationship between the intensity of competition, somehow measured,<sup>14</sup> and firms' innovative effort. In the pessimistic words of Cohen (1995, p. 234), "game-theoretic models of R&D rivalry do not provide clear, testable empirical implications". However, usually without a formal theoretical model, a growing number of empirical papers has tackled the issue at stake.

The contribution by Peneder and Woerter (2014), which contains also a rich survey of the relevant empirical research, seems among the most relevant ones to our purposes. We might claim that our model could provide a theoretical support to their empirical findings, especially as for the inverted-U shaped curve. Using a large panel of Swiss firms observed in the periods between 1999 and 2008, they estimate a simultaneous system of three equations and obtain indeed a robust non linear inverted-U relationship between the impact of competition and firms' R&D investment. Moreover, by modelling the patterns of dynamic adjustment, they identify three stable outcomes (equilibria) corresponding to: (i) an uncontestable monopoly with negligible R&D effort; (ii) an Arrovian pattern in which increasing competition increases the innovative effort, and (iii) low innovative effort with intensive market competition. The sequence of these patterns correspond empirically

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<sup>14</sup>Some authors choose the HHI, others the Lerner index; results are sensible to the choice, as reported by Peneder and Woerter (2014), who choose the number of competitors to capture the intensity of competition. In their estimates, the number of firms is endogenous, as they identify two different mechanisms. The first going from the intensity of competition to firms' incentive to invest in R&D; the second going from successful innovation to the intensity of market competition. The system is then closed by a third mechanism linking the research investment and the innovative outcome. In our model, the assumption of Cournot competition allows one to consider the number of firms as a reasonable statistics for the intensity of competition.

to the inverted-U shape curve investigated in the third case considered in our model (high productivity of the R&D technology and small cost reduction).

Using a detailed dataset from a yearly survey of the Bank of Italy, Bontempi *et al.* (2020) analyse a large number of observations stemming from 3,138 firms operating in different industries in the period 2003-2012. While the focus of their paper mostly differs from ours, their working sample allows them to investigate also the relationship between aggregate R&D investment and different measures of market power.

The inverted-U shape curve emerges for a subset of the Italian panel data, for example in industries as Computer, Pharmaceutical, Shipbuilding and most of Low Tech sectors as Textile. For other industries, their study indicates different patterns, including the U shape, the Arrovian and the Schumpeterian ones. Importantly, relying upon data at firms' level, the econometric analysis of Bontempi *et al.* (2020) indicates a route to estimate the nature and shape of aggregate innovative effort w.r.t. different measures of competitive pressure. Therefore, such estimates could provide crucial information whenever horizontal merger proposals have to be assessed within an industry. Indeed, "For merger enforcement, we need a framework to evaluate the effects of a proposed merger on innovation. In practice, the relevant mergers are those between two of a small number of firms who are best placed to innovate in a given area ... I argue here that we do not need a universal theory of the relationship between competition and innovation ..." (Shapiro, 2012, p. 363). Clearly, if aggregate R&D monotonically decreases in  $n$  as in the Schumpeterian case, then any horizontal merger fosters industry-wide innovative effort, while the opposite applies under an Arrovian pattern. In the case of inverted-U pattern, the evaluation of the merger needs estimating the relative position of actual aggregate R&D w.r.t. the peak of the curve. Our simple model, then, allowing the emergence of all three patterns in correspondence of different combinations of parameters constellations, may

provide a framework for econometric estimates to be used by competition policy makers.

While Aghion *et al.* (2005) illustrate the emergence of an inverted-U curve at the *overall economy* level, other contributions detect an analogous pattern on the basis of data at *firm* level, sometimes complemented by data at the *industry* level. Among those belonging to the first group, Gustavsson Tingvall and Poldahl (2006) use firm-level with data from Swedish industries (1990-2000) to detect a concave and single-peaked relation between R&D expenditure and the HHI; while Negassi *et al.* (2019), using a French database ranging from 1990 to 2006, find an analogous relationship between the number of patents and the Lerner index, but only in *industries* hosting public enterprises. Interesting examples of studies belonging to the second group are those of Polder and Veldhuizen (2012) and Michiyuky and Shunsuke (2013). The first paper relies on data from the Dutch National Accounts to detect the inverted-U pattern at both industry and the firm level, while the second uses Japanese firm level and industry average data from 1964 to 2006 to find out an analogous relationship between patents and the Lerner index. Additionally, the same shape emerges from numerical simulations carried out by Bento (2014), in an endogenous growth model under perfect competition, between firm-level innovations and the number of firms.

To the best of our knowledge, the only attempt at taking the same route as in Aghion *et al.* (2005) is in Hashmi (2013), modifying two assumptions. First, he adopts a partial equilibrium approach, focussing on a single *industry*; second, he allows the maximum technological gap between the laggard and the leader to be more than one step. Then, using US patent data, he obtains an inverted-U curve that would not emerge in the original model.

All of this may suggest that non-monotone patterns could hide themselves in the large theoretical literature starting from the late 1970s, which could have been overlooked because the debate pivoted around the exact nature of

monotonicity implied by either the Schumpeterian position or the Arrovian one. This is precisely what we have done in the present paper.

## 5 Concluding remarks

The long-standing debate about the impact of industry structure on aggregate innovative activity has been revitalised by Aghion *et al.* (2005) putting in evidence a concave non-monotone behaviour in sharp contrast with Schumpeter (1942) and Arrow (1962). This has triggered a new stream of research aimed at modelling this inverted-U shape relationship emerging from empirical evidence.

We have participated in this research by revisiting the model in Delbono and Denicolò (1991), where a hint in this direction was already suggested. Although the complexity of the model requires resorting to numerical simulation, it is nonetheless true that the conclusions we reach are robust to the specification of parameters and, more importantly, lend themselves to an interpretation in line with the intuition inherited from a well established tradition.

We have sketched some implications of our results. In summary, our findings can be spelled out as follows. If innovations are non-drastic, (i) a Schumpeterian pattern is generated by a low productivity of the R&D technology, or, equivalently, high discounting, regardless of the innovation size; (ii) the Arrovian and non-monotone patterns arise when R&D efficiency is high, or discounting is low. The Arrovian (respectively, concave) case is driven by large (respectively, small) innovations.

Finally, we have linked our conclusions and a number of empirical studies, especially the econometric analysis of Peneder and Woerter (2014) and Bontempì *et al.* (2020). Of course, we are aware that the testable predictions delivered by our model depend on a number of exogenous variables often



different from those considered in the literature in which, unsurprisingly, results rely upon alternative measures of market power and innovative activity, in addition to the econometric methods employed in the various studies. However, it remains true that our simple model may provide a preliminary support to many empirical findings, including the inverted-U pattern between the intensity of product market competition and the innovative activity at the industry level.

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