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Spatio-temporal Modeling of Elderly Mortality in Italy with Design and Structure Dependent Priors

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Abstract. This paper examines the spatio-temporal evolution of individuals aged 80 and over across Italian provinces from 2010 to 2022. From a methodological point of view, we employ a well-established Bayesian model for spatio-temporal disease mapping, with particular attention to model parameterization, the application of appropriate constraints, and the specification of priors. For the latter, we use design- and structure-dependent priors that account for both the design and structure matrices involved in defining the prior distributions of the random effects.

Keywords: disease mapping, Gaussian Markov Random Fields, Poisson regression, prior elicitation

1 Introduction

Modeling the evolution of disease or mortality rates across time and space is a challenging task that requires the development of specialized statistical models able to distinguish between the contributions of marginal effects and their interactions. A unified approach for tackling these models was proposed by Knorr-Held in 2000 [1], discussing how the spatial and temporal effects can be combined accounting for possible correlation structures.

Usually, spatio-temporal disease mapping is performed specifying a Poisson mixed model within the Bayesian Framework. The particular form of the considered linear poses different challenges related to model parameterization and prior specification for variance components. In this paper we adopt a parameterization that takes fully account of the null space characterizing each model component, while prior specification is carried out following the Design and Structure Dependent (DSD) approach outlined in [2].

Analyses refer to publicly available data from the Italian Institute of Statistics (ISTAT) concerning elderly mortality (people aged 80 and over) for all death causes in Italy from 2010 to 2022 at a provincial level (NUTS-3). As a starting point, we consider yearly data, planning to extend the analyses to monthly data in the future. Sardinia is dropped from the analyses because of some data inconsistencies in the study period.

2 The Model

Consider a count of cases Y_{st} observed in area $s = 1, \dots, S$ and time $t = 1, \dots, T$, along with the corresponding number of expected cases E_{st} , derived from an indirect standardization procedure that accounts for the age structure of the area's population. We specify the following Poisson mixed model for all s, t :

$$Y_{st} \mid E_{st}, \eta_{st} \sim \mathcal{P}(E_{st} \exp\{\eta_{st}\}), \quad \eta_{st} = \mu + \theta_s + \zeta_s + \gamma_t + \delta_{st}.$$

The linear predictor can also be written in matrix form as:

$$\boldsymbol{\eta} = \mu \mathbf{1}_N + \mathbf{Z}_{\text{space}} \boldsymbol{\theta} + \mathbf{Z}_{\text{space}} \boldsymbol{\zeta} + \mathbf{Z}_{\text{time}} \boldsymbol{\gamma} + \boldsymbol{\delta},$$

where $N = S \times T$, $\mathbf{1}$ is the unit vector, $\mathbf{Z}_{\text{space}} = \mathbf{I}_S \otimes \mathbf{1}_T$, and $\mathbf{Z}_{\text{time}} = \mathbf{1}_S \otimes \mathbf{I}_T$.

The predictor includes an overall intercept μ , for which a non-informative Gaussian prior is specified, along with two spatial effects: an unstructured effect $\boldsymbol{\theta}$ and a structured effect $\boldsymbol{\zeta}$. Additionally, the model includes a temporally structured random effect $\boldsymbol{\gamma}$ and an interaction term $\boldsymbol{\delta}$. Their prior distributions are:

$$\begin{aligned} \boldsymbol{\theta} \mid \sigma_{\theta} &\sim \mathcal{N}_I(\mathbf{0}, \sigma_{\theta}^2 \mathbf{I}_I), \quad \boldsymbol{\zeta} \mid \sigma_{\zeta} \sim \mathcal{N}_I(\mathbf{0}, \sigma_{\zeta}^2 \mathbf{K}_{\zeta}^{-}), \\ \boldsymbol{\gamma} \mid \sigma_{\gamma} &\sim \mathcal{N}_T(\mathbf{0}, \sigma_{\gamma}^2 \mathbf{K}_{\gamma}^{-}), \quad \boldsymbol{\delta} \mid \sigma_{\delta} \sim \mathcal{N}_N(\mathbf{0}, \sigma_{\delta}^2 \mathbf{K}_{\delta}^{-}), \end{aligned} \quad (1)$$

where $\mathbf{K}_{\zeta}$ is a rank-deficient precision matrix describing an intrinsic conditional autoregressive (ICAR) prior, while $\mathbf{K}_{\gamma}$ corresponds to a random walk prior of order 1 (RW1). Following [1], the precision matrix for the interaction term $\mathbf{K}_{\delta}$ can take one of four forms: $\mathbf{K}_{\delta} \in \{\mathbf{I}_T \otimes \mathbf{I}_I, \mathbf{K}_{\gamma} \otimes \mathbf{I}_I, \mathbf{I}_T \otimes \mathbf{K}_{\zeta}, \mathbf{K}_{\gamma} \otimes \mathbf{K}_{\zeta}\}$, which are usually referred to as Type I, II, III and IV respectively.

2.1 Prior for Scales: the DSD Approach

To complete the model specification, prior specification for the random effects scale parameters introduced in (1) is required. This step is particularly delicate, as the design and the presence of correlation structures are known to influence the prior variability of the effects [2, 3]. Moreover, this choice is closely linked to another critical aspect of spatio-temporal disease mapping models: the imposition of identifiability constraints [4, 5].

To ensure coherent calibration of the prior variability associated with each model component, we adopt DSD priors [2]. To illustrate the underlying strategy, consider a generic term of the linear predictor, denoted as $\boldsymbol{\phi} \in \mathbb{R}^N$. The key quantity used for prior calibration is the sampling variance, given by:

$$V_{\phi} \mid \sigma_{\phi}^2 = \frac{\sigma_{\phi}^2}{N-1} \boldsymbol{\phi}^{\top} \mathbf{M} \boldsymbol{\phi}, \quad (2)$$

where $\mathbf{M}$ is the centering matrix. The prior for σ_{ϕ}^2 is then derived by considering a Beta distribution of the second kind (B2) as the base distribution, adapting it to ensure that the marginal distribution of the sampling variance, V_{ϕ} , aligns

with the distribution of the sampling variance in the specific case of completely independent and unstructured random effects. This is achieved by solving an integral equation and resulting in a prior distribution expressed in terms of the ${}_2F_1$ hypergeometric function: such prior depends on the eigenvalues of the covariance matrix of ϕ , which incorporate both the design and structure involved in ϕ . As an example, if one considers $\phi = \mathbf{Z}_{\text{space}}\boldsymbol{\zeta}$, the covariance matrix of ϕ depends both on $\mathbf{Z}_{\text{space}}$ (the design) and $\mathbf{K}_{\boldsymbol{\zeta}}$ (the structure). Following the indications in [2], we set the base prior as a $B2(b = f(y, 0.5), p = 0.5, q = 1.5)$ distribution, obtaining a prior similar to a half- t distribution on σ_ϕ with 3 degrees of freedom. The scale parameter b is set to match the median of the component sampling variance with the data pseudo-variance under the Poisson model.

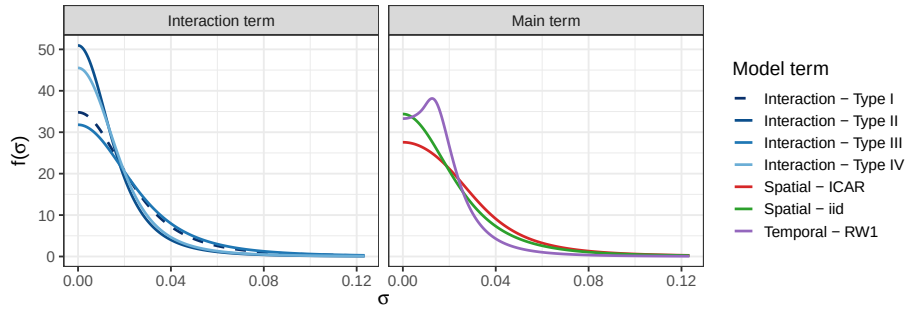
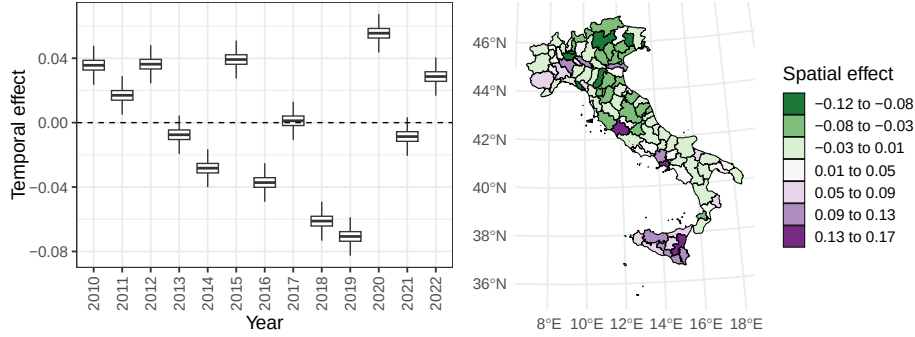


Fig. 1. Density of the DSD priors on σ for each model component.

An important consideration is that, in some cases, the precision matrices of the effects included in the model are rank-deficient. To ensure a proper prior and finite prior variability, we constrain the effects according to their null space [4], subsequently incorporating into the predictor any components not already included [5]. When dealing with the Italian map considered in this paper, the presence of an island (Sicily) generates two disconnected subgraphs: this needs to be taken into account by means of appropriate linear constraints, as the rank deficiency of the spatial precision matrix equals the number of disconnected subgraphs. Note that, as part of the model architecture, all adopted constraints contribute to the determination of the DSD prior.

The DSD prior density for scale parameters of all model components is shown in **Fig. 1**. Even though they differ, these priors imply the same prior distribution on the variance (2) for each model component. We anticipate that **Fig. 3** will present the posterior counterparts of these variances (relative to the total explained variance), evaluating the posterior contribution of each model component to the spatio-temporal variability of mortality risk in the Italian provinces.

	Type I	Type II	Type III	Type IV
LOOIC	15746	16049	15529	15652
DIC	15432	15548	15252	15295

Table 1. Model selection criteria for the four interaction types.**Fig. 2.** Posterior distributions of temporal effect (left) and posterior mean of spatial effect (right).

3 Some Results

Models with all interaction types have been estimated via MCMC sampling by using an augmented variable algorithm [6].

Table 1 shows that the best-performing model, both in terms of the Deviance Information Criterion (DIC) and the leave-one-out cross-validation information criterion (LOOIC), is the one that includes the Type III interaction term. According to this finding, we highlight that the preferred model has an interaction term that is spatially structured and independent over time. The results presented in the following sections refer to the study of the relative risk of elderly mortality in the Italian provinces from 2010 to 2022, obtained using this model.

The left panel of **Fig. 2** displays the posterior distributions of the main temporal effects, highlighting the absence of a clear time trend. The highest relative risk is observed in 2020, while the lowest occurs in 2019. The right panel presents the main spatial effect (the sum of structured and independent components), revealing a spatially correlated pattern along with some hotspot provinces.

Fig. 3 displays the posterior distribution of the proportion of explained variance attributed to each model component. As shown in the left panel, the main spatial effect accounts for more than 50% of the explained spatio-temporal variability, while the main temporal effect contributes approximately 23%. The interaction term, which can be interpreted as a residual from the main effects, explains about 18% of the variance. The right panel of **Fig. 3**, where the interaction term is disaggregated by year, reveals that the majority of this contribution

is concentrated in 2020, when the peak of the COVID-19 pandemic occurred, followed by 2021 (also affected by the pandemic) and 2017.

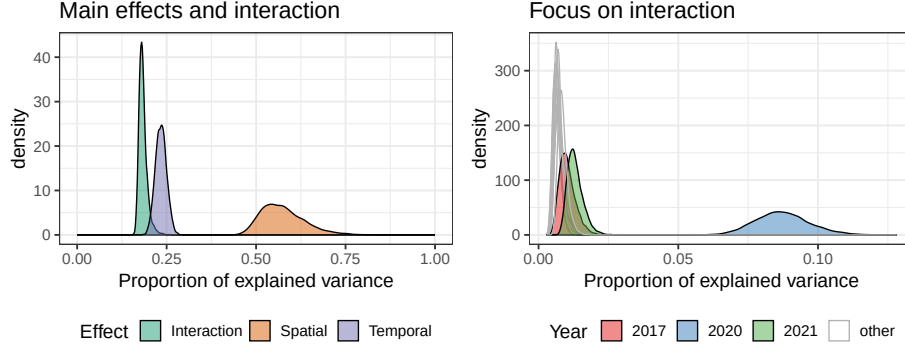


Fig. 3. Proportion of the linear predictor variance attributable to each model component.

Fig. 4 presents the posterior means of the interaction term for the years exhibiting the highest heterogeneity relative to the main effects. The impact of the COVID-19 pandemic in 2020 is particularly evident, with northern areas being most affected. Notably, 2020 is the only year in the study period where positive posterior estimates are concentrated in the north. Results for 2021 suggest a stronger impact of the COVID-19 pandemic in southern regions. Additionally, a possible harvesting effect can be hypothesized for northern provinces. Lastly, the trend observed in 2017 likely reflects a particularly severe influenza outbreak that significantly impacted southern regions.

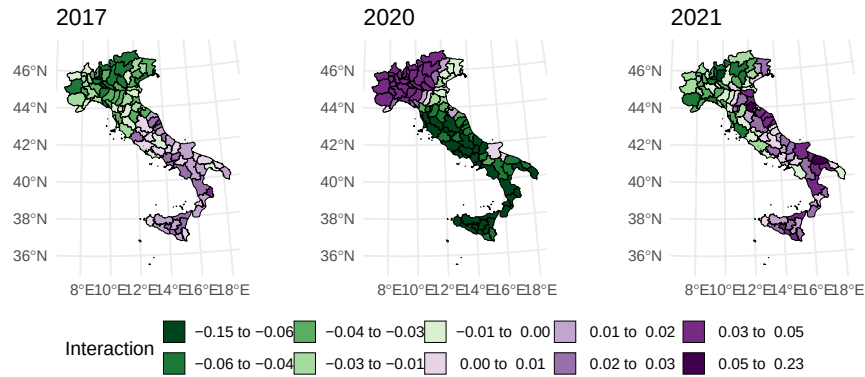


Fig. 4. Posterior mean of the interaction term in selected years.

4 Concluding Remarks

In this paper, we studied the spatio-temporal evolution of elderly mortality in Italy from 2010 to 2022. Particular care is devoted to model parameterization and prior specification. Future developments will consider the modeling of monthly data, as it is well known that mortality exhibits seasonal trends that are worth investigating, particularly regarding their peculiarities across different years characterized by pandemics or summer heat waves. Extending the adopted models requires the construction of a structured matrix capable of capturing seasonality and will increase computational complexity due to both the greater number of required linear constraints and the larger dimension of the random effects.

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