

# Dealing with Significant Noise Levels in Vibration-based Bridge Health Monitoring? A novel ARMA+Noise algorithm in the Frisch Scheme Context

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## Abstract

System Identification (SysId) refers to an ensemble of methodologies, as the ones built on statistical autoregressive models, which are among the most effective tools for spectral analysis and, by extension, for vibration-based assessment. However, the application of standard SysId strategies might be hampered by the non-negligible levels of noise dominating in harsh environments (or those intrinsic to electronic devices). This is especially true in bridge-related applications, where faint modal components at low frequency are very common. To this end, the ARMA+Noise algorithm is proposed in this work, which is built on a novel frequency-domain adaptation of the Autoregressive with Moving Average (ARMA) model in the Frisch scheme context: the technique is superior in that it can ensure the best trade-off between the frequency resolution and the hidden signal noise to be identified. A dedicated workflow has been developed, that extracts the ARMA model parameters by combining the advantages of the AR+Noise identification method with the Graupe's algorithm. The validity of the proposed technique has been tested on the Z24 bridge dataset, showing that the ARMA+Noise solution can properly identify the first four modes of the structure, even when the signal-to-noise ratio is low.

## Keywords

Autoregressive with Moving Average plus Noise, Frisch Scheme Context in the Frequency Domain, System Identification, Vibration-based Bridge Health Monitoring

## 1 Introduction

Nowadays, assessing the integrity condition of structures is regarded as one of the main priorities all over the world, thus preventing dreadful catastrophes and ensuring safer environments. Among the very manifold application domains, this necessity is particularly claimed in the case of civil infrastructures, where natural and man-made ageing factors are frequent. This condition specifically applies to bridges, due to the longevity of the structures and their importance for transports, connections between countries and human travels. Vibration-based signal processing techniques, such as those aimed at extracting frequency-related features basing on Operational Modal Analysis (OMA) precepts, are considered as the most effective strategies for evaluating the integrity of these infrastructures.

Indeed, as per report of EU Science Hub on "Research and innovation in bridge maintenance, inspection and monitoring" [1], there are many bridges in the Trans-European

Transport Network that were built after 1945, with an estimated life span of 50 to 100 years. Evidently, they are now at the end of their life cycle since more than 75 years have been completed being them still in operation today. Moreover, according to the BRIME project [2], which was one of first attempt made by EU towards the condition assessment of bridges on the European road network, 39% of bridges in France, 37% of bridges in Germany, 26% of bridges in Norway and 30% bridges in UK were identified early in 2000 as defective; the reasons were various, comprising corrosion of reinforcement, overloading, design/construction faults, etc.

Given these premises, monitoring the health condition of structures in their operative settings, which might largely be influenced by environmental and operational sources of noise, has become of the utmost importance. To this end, it is worth mentioning that the nature of noise hidden in vibration signals could be very different [3], encompassing (i) high-frequency noise caused by electric and electromagnetic disturbances associated with spikes, harmonics

or short impulses; (ii) white stationary noise, as the one primarily originated by the intrinsic noise density of inertial accelerometers or, more generally, of electronic components; (iii) non stationary or random noise caused by the surrounding environment.

Consequently, the above-mentioned and noise-related uncertainties can significantly corrupt the acquired vibration signals, e.g., by hindering some spectral and low-energy components, or by generating spurious peaks at non-physical frequencies. Hence, it is paramount to adopt processing frameworks which can properly deal with these problems. The current work explores, for the first time, the feasibility of a novel algorithm, the ARMA+Noise approach belonging to the family of system identification (SysId) methods, to provide a tangible solution in response to this need.

The paper is organized as follows. In Section 2, a formal description of the proposed ARMA+Noise algorithm is enclosed, together with a literature review of previous works pursuing the same goal. Section 3 is dedicated to the experimental validation: in Section 3.2, results for the Z24 bridge dataset are presented. Additionally, to further corroborate the applicability of the devised solution, another experimental benchmark taken from a wind turbine blade prototype is shown in Section 3.3. Conclusions are finally drawn in Section 4.

## 2 The ARMA+Noise model in the Frisch scheme context

### 2.1 Motivations

SysId refers to an ensemble of statistical signal processing strategies that postulate a mathematical model on the observed time series, that can be used as a proxy of the underlying physical dynamics [5]. In this sense, the objective of SysId techniques is to find an optimal set of parameters, termed as *model parameters*, that can replicate the measured input-output relationship. Hence, they are widely used by the civil and electrical engineering community for the purpose of spectral analysis. However, notwithstanding the advantages over classical non-parametric approaches, the spectral smoothing effect being the most important one, standard SysId solutions based on Autoregressive models (AR) (which are among the most widely known approaches in the field) are, regrettably, ineffective in real scenarios. Moreover, it must be underlined that, when the signal-to-noise ratio (SNR) dramatically decreases, the number of required model parameters (i.e., the model order) rapidly increases and, in turn, this might provoke several false positive peak spectral components which must be taken into account during the diagnostic phase.

The aim of the novel strategy developed here is to incorporate noise understanding at the modelling step, as ensured by the Frisch scheme context, such that it is possible to compute high-quality spectral profiles even in presence of unfavourable noise levels.

### 2.2 Related works

The application of the Frisch scheme for the identification of noisy sequences, a procedure which is also referred to

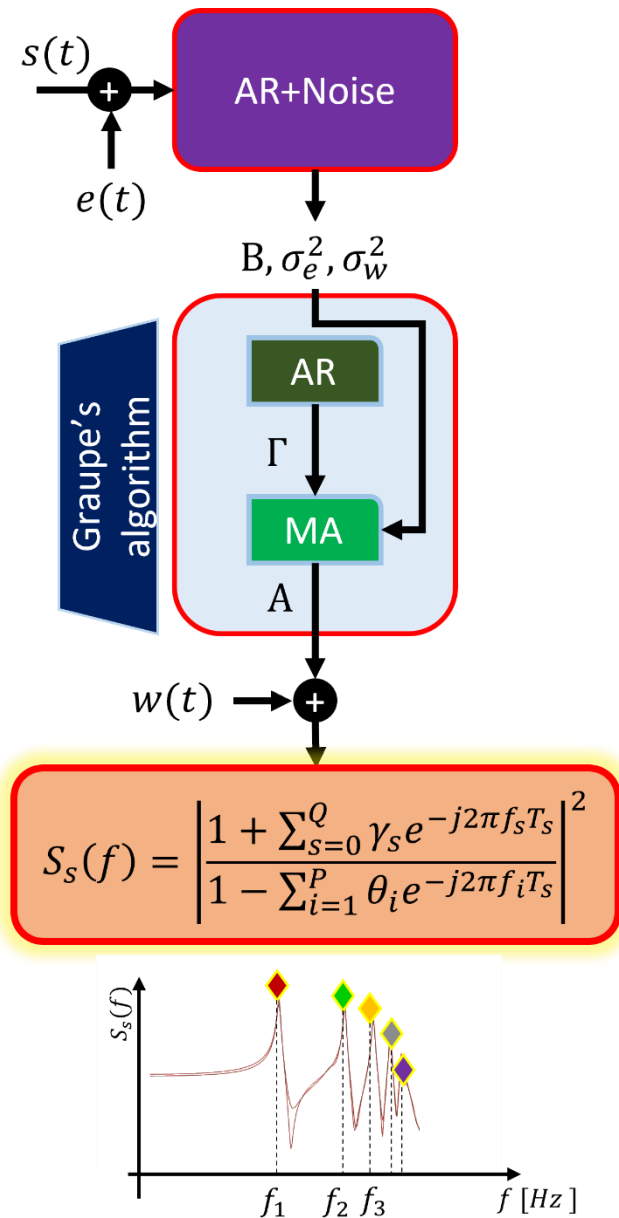
as *errors-in-variables* (EIV) problems, has firstly been studied within the control system society [6]. Despite being the SysId taxonomy quite various, attention has been primarily focused on the definition of the AR+Noise model, serving as the EIV counterpart of the standard AR one. This was due to its output-only nature, namely the fact that it is applicable even when the input stimulus is not known. The latter represents a basic requirement for on-condition maintenance, given the non-predictability/non-measurability of the force exciting the structural system. The literature reports some successful applications of the AR+Noise algorithm for the condition monitoring of industrial targets, e.g., gas turbine prototypes [7]. More importantly, the method showed superior performances for the identification of frequency shifts in two different Structural Health Monitoring (SHM) use cases: a heavy seismic event and the diagnostic of a medieval tower under vehicle passing excitation [8]. Besides these successful initial attempts, in which the AR+Noise algorithm has been tested following the pristine formulation in the time domain, a more recent alternative postulated in the frequency domain has started to grow interest for vibration diagnostics. The reason is that vibration analysis is intrinsically dominated by spectral-related quantities; thus, shifting the computing paradigm in the frequency domain allows for the immediate retrieval of the sought modal quantities without adding additional processing. This is the case of the work in [9], which explores the frequency-driven variant of the AR+Noise algorithm for the monitoring of a wind turbine prototype.

Nevertheless, it is important to observe that smoother spectral profiles are yielded by the Autoregressive with Moving Average (ARMA) model, which similarly belongs to the family of output-only SysId methods, thanks to the introduction of the moving average (MA) term. However, beside this very advantageous trait, an extension of the standard ARMA model to the class of EIV problems treated in the frequency domain is still missing. Indeed, only one work [10] can be found in the literature in which a time domain definition of the ARMA+Noise algorithm has been introduced and tested on simulated time series data. In the current work, we specifically address this gap by:

- i. Providing a frequency domain adaptation of the ARMA+Noise method, a result which is attained by combining the AR+Noise workflow in [9] with the Graupe's algorithm.
- ii. Verifying the superior performances of the approach for SHM bridge identification, thus extending the method to one representative industrial setting.

### 2.3 The algorithm

The processing flow of the ARMA+Noise model is based on a two-step procedure (see Figure 1) which exploits the cascade of a first step, in which the AR+Noise algorithm in [10] is regressed on the acquired vibration signal, followed by the application of the Graupe's algorithm returning the



**Figure 1** Processing flow of the ARMA+Noise algorithm in the frequency domain.

set of AR and MA coefficients.

To justify the two-stage nature of the processing scheme adopted here, it is worth recalling the mathematical formulation of a generic ARMA model, provided in Equation (1) ( $T_s$  corresponds to the sampling interval):

$$s(t) = e(t) + \sum_{s=0}^Q \gamma_s e(k - iT_s) - \sum_{i=1}^P \theta_i s(k - iT_s) \quad (1)$$

from which it is easy to notice how the dynamics of the system is regressed from both the previous  $P$  instances of the output itself (the autoregressive AR component) and the last  $Q$  input samples (the MA part) by means of the  $\theta_i$  and  $\gamma_s$  coefficients, respectively. As such, since no prior is available concerning the driving source  $e(t)$  except from the common assumption that it belongs to a Gaussian normal distribution with zero-mean and prescribed (unknown) variance, the problem comes to be ill-posed and it

is impossible to estimate, in a single-shot, the sought model parameters. To account for this issue, Hannan and Rissanen (HR) [11] were the first to propose a simple yet effective solution in which a high-order AR model is firstly fitted to the signal, such that an estimate of the noisy input term  $\hat{e}(t)$  can be computed. Then, the same quantity is used in a second step in which a low-order input-output model (such as the Autoregressive with eXogenous Input (ARX) one) is matched to the same time series for the estimation of the final batch of desired parameters.

In the specific implementation discussed here, we propose to employ a different and less computational demanding strategy, in which the second regression stage is substituted with simple algebraic equations. The latter allow for an equivalent estimation of the AR and MA parameters directly from the AR parameters extracted at the end of step one.

The entire ARMA+Noise identification can be formalized as follows.

*Stage 1: Fitting a high-order AR+Noise model to the signal*

The basic idea behind EIV models is that the measured signal is affected by additive measurement errors in the form of white and mutually uncorrelated sources of noise. When applied to the class of AR models processing a generic time series  $s(t)$ , this leads to the subsequent system formulation:

$$\begin{cases} s(t) = -\sum_{i=1}^N \beta_i s(t - i\Delta t) + e(t) \\ y(t) = s(t) + w(t) \end{cases}$$

in which  $e(t)$  is defined as above and  $w(t) \sim \mathcal{N}(0, \sigma_w^2)$  corresponds to a zero-mean white ergodic process belonging to a Gaussian distribution:  $e(t)$  acts as the noise source driving the noise-free measured vibration signal  $s(t)$ , while the second term identifies the additive measurement noise corrupting the noisy observations  $y(t)$ .

Starting from the system of equations above, the frequency domain formulation of the AR+Noise strategy is peculiar in that it aims at estimating, directly from the Discrete Fourier Transform (DFT) of  $y(t)$ , these defining quantities:

- the set of AR parameters  $B = [\beta_1 \dots \beta_N]$  associated with the  $N$ -long (with  $N = P + Q$ ) polynomial  $B(z^{-1}) = 1 + \sum_{i=1}^N \beta_i z^{-i}$  ( $z^{-1}$  being the backward shift operator<sup>1</sup>) representative of the noise-free system transfer function
- the noise variances  $\sigma_e^2, \sigma_w^2$

*Stage 2: Obtaining ARMA( $P, Q$ ) parameters via the Graupe's approach*

Upon obtaining  $B$ , the Graupe's algorithm [12] can be implemented. Its mathematical formulation is obtained by expressing Equation (1) as a function of the backward shift operator, turning into

$$A(z^{-1})s(t) = C(z^{-1})e(t) \quad (2)$$

$x(t)$  returns its one-time lagged version, i.e.,  $z^{-1}x(t) = x(t - 1)$

<sup>1</sup> The backward shift operator  $z^{-1}$  applied to a time sample

which can be further simplified as

$$s(t) = \frac{C(z^{-1})}{A(z^{-1})} e(t) \approx \frac{1}{B(z^{-1})} e(t) \quad (3)$$

From the approximate relation in Equation (3), one might write this system of linear equations:

$$\begin{bmatrix} b_p & b_{p-1} & \dots & b_{p-Q+1} \\ b_{p+1} & b_p & \dots & b_{p-Q+2} \\ \vdots & \vdots & \ddots & \vdots \\ b_{p+1} & b_p & \dots & b_p \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_Q \end{bmatrix} = - \begin{bmatrix} b_{p+1} \\ b_{p+2} \\ \vdots \\ b_{p+Q} \end{bmatrix} \quad (4)$$

Equation (4) can easily be solved yielding an estimate for the MA parameters  $\Gamma = [\gamma_1 \dots \gamma_Q]$ ; this quantity can then be used in Equation (5) to get the AR counterpart  $A = [\theta_1 \dots \theta_p]$ .

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_Q \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{bmatrix} + \begin{bmatrix} 1 & 0 & \dots & 0 \\ b_1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_{p-1} & b_{p-2} & \dots & b_{p-Q} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_Q \end{bmatrix} \quad (5)$$

Finally, the power spectrum  $S_s(f)$  of the noise-free input signal can be extracted by taking the square power of the (noise-free) system frequency transfer function in Equation (1):

$$S_s(f) = \frac{|1 + \sum_{s=0}^Q \gamma_s e^{-j2\pi f_s T_s}|^2}{|1 - \sum_{i=1}^p \theta_i e^{-j2\pi f_i T_s}|^2} \quad (6)$$

whose peaks corresponds to the modal frequencies of vibration characterizing the dynamics of the inspected structure.

### 3 Experimental validation

Two different SHM use-cases have been considered for validation purposes. The first one tackles a strongly bridge-related application scenario: the dynamic identification of the Z24 bridge. The second one is, instead, taken from an industrial setting, being a wind turbine blade prototype the target structure.

#### 3.1 Methods

The enhanced spectral insight delivered by the ARMA+Noise methodology has been appraised by comparing the quality of the computed spectral profiles with those yielded by 1) more conventional SysId approaches (i.e., AR and ARMA) keeping equal the model order, and 2) the well-known Welch's periodogram, under different noise conditions. To this purpose, white additive Gaussian noise was added to the time waveforms.

Additionally, it is worth mentioning that, to make the SysId strategies effective, the optimal model order has to be determined, too. The pursuit of this goal, which is out of the scope of this manuscript, has been achieved via the Akaike Information Criterion applied to a batch of data, during an initial pre-processing phase. This quantity corresponds to  $P + Q = N = 20$  for the Z24 bridge, while a model order of 8 has been deemed sufficient in the case of the turbine blade.

#### 3.2 Application of the ARMA+Noise algorithm to the Z24 bridge dataset

#### Dataset and bridge description

The Z24 bridge represents one of the very first and most successful attempts towards the SHM of ancient concrete bridge, under the cap of the Brite-EuRam project SIMCES (System Identification to Monitor Civil Engineering Structures [13]). It was located in Switzerland between the villages of Zürich and Koppigen and demolished in 1999 since at the end of its life cycle. Before reaching the complete destruction, an extensive experimental campaign has been conducted, during which a network of piezoelectric accelerometers and several environmental sensors have been deployed all over the bridge span and pillars. Data were collected over a time period of one year, at the end of which a campaign of short-term and purposely provoked progressive damage tests has been performed. For more details about the sensor network, the testing procedure, and the sensor installation plan, interested readers are referred to [14].

For our purpose, only a small subset of signals related to the initial days of the experimental campaign has been processed, being our interest in qualifying the trend of the spectral signature independently from seasonal variations.

#### Results

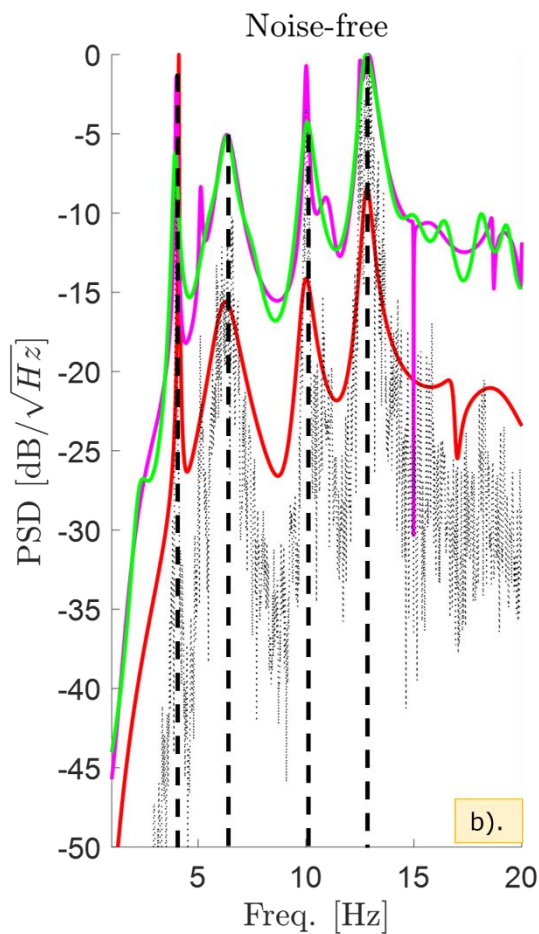
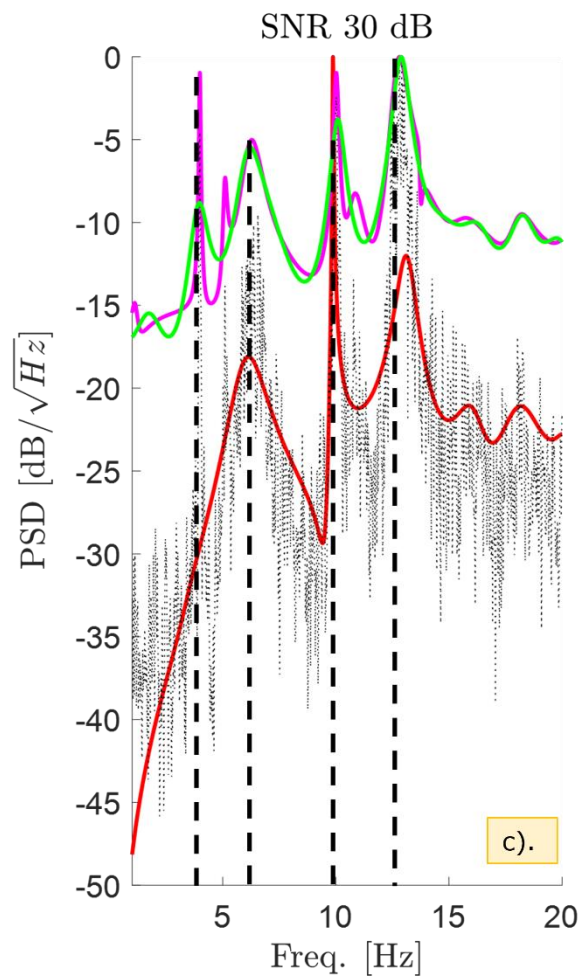
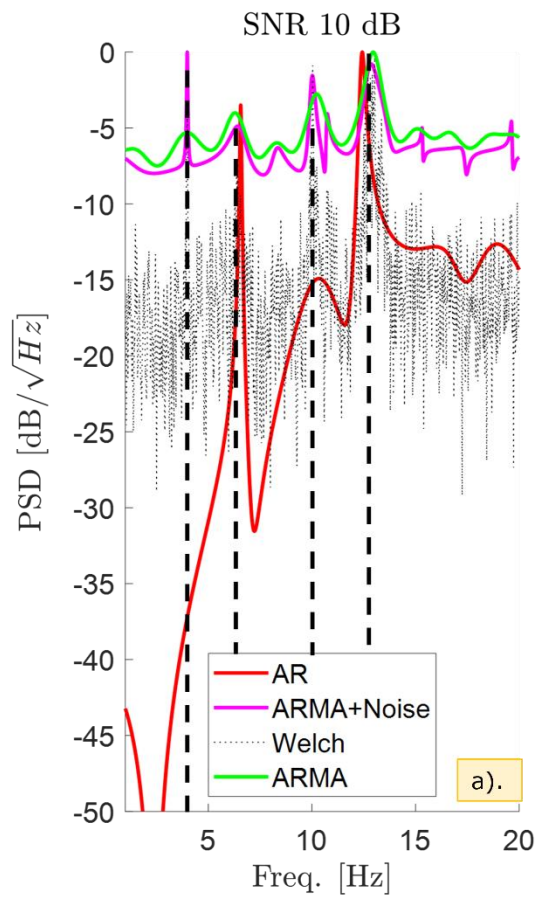
Results are depicted in Figure 2, which displays the computed spectra in case of severe noisy conditions (10 dB, Figure 2.a), noise-free data (Figure 2.b) and favourable SNR (30 dB, Figure 2.c). As can be seen, the ARMA+Noise solution (magenta line) is always capable of properly identifying the first four dominating modes of the structure (dashed, black vertical lines), even when the signal-to-noise ratio (SNR) reduces to 10 dB. Conversely, the detection of the peak around 4.2 Hz is completely lost by the AR counterpart (red line) even in presence of minor additive noise (i.e., 30 dB). Similarly, it must be observed that, while being the spectral peaks well aligned, the standard ARMA alternative (green line) shows a smoother profile yielding to a worse frequency resolution, especially in the low spectral region.

#### 3.3 Application of the ARMA+Noise algorithm to the Sonkyo wind turbine blade

##### Dataset and blade description

A wind turbine blade prototype has been instrumented with a network of accelerometers, humidity and temperature sensors, and subject to multiple defect of various size and entity. The blade, fabricated by Sonkyo energy and hosted at the D-BAUG laboratories of ETH Zürich, is meant at replicating a small-scale version of larger facilities and industrial elements classically adopted in wind industrial applications.

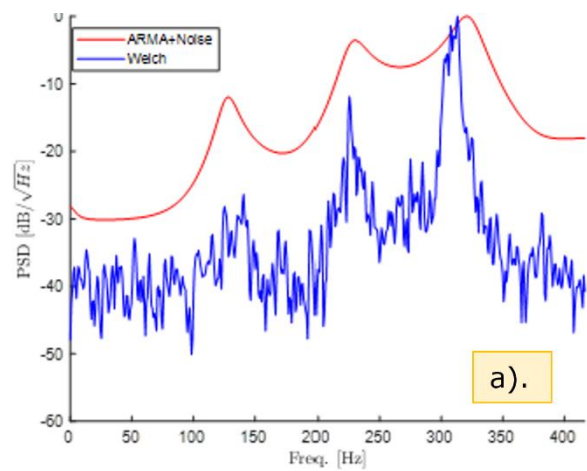
A repository of vibration data (accessible at <https://github.com/ETH-WindMil/Sonkyo-Benchmark>) has been made available for this structure, which might serve as representative dataset for vibration-based diagnostic frameworks.

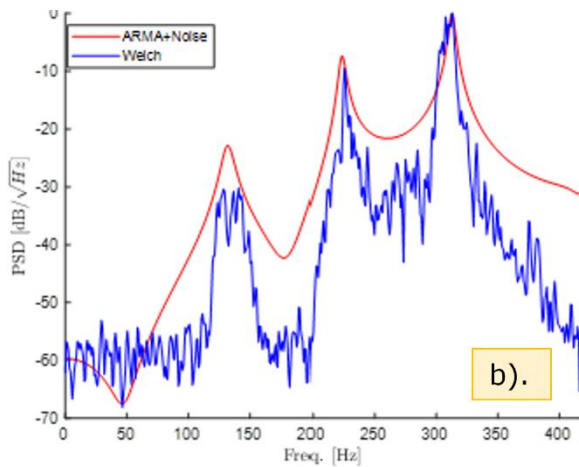


**Figure 2** Z24 bridge use case: spectra for different noise values: 10 dB (a), noise-free (b) and 30 dB (c) comparing different estimators.

### Results

Exploiting the outcome presented in Section 3.2, the comparison has only been limited to the ARMA+Noise algorithm (red line) and the Welch estimator (blue line).





**Figure 3** Wind turbine blade dataset: spectra in different noisy conditions: 5 dB (a) and 15 dB (b) comparing different estimators.

The spectral signatures are plotted in Figure 3, in which the first chart (Figure 3.a) is related to a huge noise-corrupted scenario with 5 dB SNR, whereas a configuration with 15 dB is described in Figure 3.b. These trends reveal that even under very consistent noise levels, the ARMA+Noise algorithm is capable of properly identifying the low-frequency and low-energy peaks at around 120 Hz, with a higher peak SNR if compared to the Welch alternative. This observation holds for both the enclosed figures, which are related to acceleration data collected in nominal conditions by a sensor installed at the centre of the blade.

#### 4 Conclusions

In this work, a novel strategy implementing the ARMA+Noise algorithm in the frequency domain under the Frisch scheme formulation has been presented. The theoretical workflow has been extensively described, together with motivations supporting the extension of the standard ARMA model to the type of EIV problems. The validity of the approach has been tested on two rather different use cases: the former relates to the Z24 bridge, which is one of the most appealing datasets available in the literature for bridge vibration analysis, while the second one pertains to a wind turbine blade prototype representative of more industrial settings.

Results confirm the superiority of the method while dealing with very noisy working conditions, both when compared with standard parametric and non-parametric strategies.

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