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The strategic proximity-concentration trade-off with multiproduct multinational firms[☆]

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ABSTRACT

I study the proximity-concentration trade-off faced by two multiproduct multinational companies (MNCs) that operate in two countries under horizontal product differentiation. In this context, characterized by two-way trade and foreign direct investment, the trade-off regulates the domestic stock of investment (concentration) against the foreign stock (proximity) in a way that is centred around the exploitation of large market shares and market power. I show that MNCs follow pricing-to-market that mirrors the proximity-concentration trade-off, and I characterize how market shares, prices, and markups react to changes in investment and trade frictions. Endogenous variables turn out to be closely interrelated across markets and firms.

1. Introduction

The need of a renewed effort to incorporate oligopolistic firms into trade models can be motivated on the grounds of both the empirical relevance of large multiproduct firms and the plausibility of theoretical predictions generated by oligopolistic models (the latter point being advocated, for example, by Neary, 2003). In their comprehensive review of oligopoly models in international trade, Head and Spencer (2017) argue convincingly in favour of these and other aspects related to the prominence of large firms. Yet, despite the importance for the world economy of multinational companies (MNCs) and the occurrence of inherently strategic interactions among them, little recent research has addressed the integration of MNCs into oligopolistic trade models.

In this paper I present a two-country model centred around the strategic interaction of two multiproduct multinational firms that compete choosing quantities and deciding on the stock of domestic investment (implying domestic concentration of production) and foreign investment (implying proximity of production to the foreign market). The model delivers simple testable implications related to the impact of investment and trade policy on markups, market shares and the domestic and foreign stocks of investment by MNCs. I show that policies discouraging foreign direct investment (FDI) in a host country end up increasing the MNCs' shares in

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both markets where they are headquartered, in addition to reducing foreign market shares.[1] Moreover, liberalizing trade between two countries ends up increasing the domestic market shares of MNCs in both countries, along with a reduction in the foreign ones. Changes in investment and trade policy that bring concentration of investment in the domestic markets do so, in relative terms, at the expenses of proximity of investment to the foreign markets, a result that I label strategic proximity-concentration and that is novel in the international trade literature based on oligopoly.

These results are obtained under the segmented markets' assumption. However, due to the possibility of two-way trade and cannibalization among competing varieties belonging to the same MNC portfolio, there is strategic interaction across markets and MNCs. The model brings to the forefront some key aspects of MNCs' behaviour: *(a)* the interrelatedness of endogenous variables (market shares, markups, investment) across different markets for a given MNC (within MNC dimension); *(b)* the interrelatedness of endogenous variables across different MNCs for a given market (between MNC dimension). As explained below, the interaction of these two elements imply that an exogenous change, for example, in the cost of foreign investment in a certain host country can propagate to the other MNC and the other market through the MNCs' ownership network and strategic interaction.

The trade-off between proximity and concentration has a long-standing tradition in the international trade literature. Traditionally, horizontal FDI takes the form of the duplication of production of an existing variety through a foreign plant and, by serving the foreign market, fully substitutes export. Under appropriate conditions such as high transport costs, a multinational production structure with two production facilities (each serving the local market) emerges. In the model of Brainard (1993), the duplication of production of an existing variety in another plant can happen since there are firm-level increasing returns to scale, in the form of some corporate activity unique to the firm such as R&D, whose fixed costs can be spread over two production facilities instead of one. If corporate costs are zero (no firm-level increasing returns) the equilibrium features only single-plant firms that reach the foreign market through export. In the model of Helpman et al. (2004), the duplication of production of an existing variety in another plant can arise since there is heterogeneity in productivity, with more productive firms engaging in FDI and less productive firms engaging in export, and the relevant cut-off coefficient between export and FDI is influenced by trade costs and plant-level returns to scale. In my framework the incentives to do FDI hinge upon strategic considerations related to market power that are absent in the previous literature. Instead of the exploitation of scale economies (which play no role in my setting) I study a trade-off which is centred around the exploitation of large market shares and, consequently, market power. If the MNC market share in one country goes down, this makes more profitable the investment in the other country. The dependency of a MNC market share on the competitor's share provides the strategic flavour to the proximity-concentration choice.

Baldwin and Ottaviano (1998, 2001) propose models where FDI does not substitute completely export. The coexistence of multiple production facilities at home and abroad that sell to the same markets is enabled by horizontal product differentiation. In Baldwin and Ottaviano (1998) intraindustry trade and intraindustry FDI occur simultaneously. There are two multiproduct multinational firms, each producing a continuum of differentiated varieties at home and abroad. This paper constitutes the starting point of my analysis, since I use very similar assumptions in terms of market structure. I complement their analysis by providing the comparative statics for equilibrium market shares for general values of investment and trade costs. This is the key analytic step, since prices, markups and the stock of domestic and foreign investment can be written in terms of market shares, and I then provide a complete characterization of the behaviour of these endogenous variable in equilibrium. Specifically, it is new in the present paper the finding of a strategic proximity-concentration trade-off. A technical difference with Baldwin and Ottaviano (1998) is that they use calculus of variations to determine the model solution, while I borrow the methodology for multiproduct firms' models of Minniti and Turino (2013).[2]

Baldwin and Ottaviano (2001) present a somehow simplified version of Baldwin and Ottaviano (1998), since the number of varieties per firm is limited to two. FDI in Baldwin and Ottaviano (2001) happens when one of the two varieties is produced abroad, while the firm is said to be national when both varieties are produced in the same country where the firm is headquartered. Although some very interesting results can be derived in a setting with only two varieties per firm, the main channel that I exploit in this paper goes through making endogenous the choice about the number of varieties per firm and, consequently, the stock of domestic and foreign investments.

My paper is also related to the literature that has looked at pricing-to-market in an international trade framework. Atkeson and Burstein (2008) find that the practice of pricing-to-market in a direction consistent with the data arises due to the presence of within-sector cost dispersion and large firms. They study the impact on international relative prices of changes in aggregate productivity at the country level that lead to changes in aggregate costs. In their setting, if there is no cost dispersion across firms and all firms export, there is no pricing-to-market. Without the need to assume cost heterogeneity, in a two-way investment and trade setting, I provide an alternative micro-foundation for the practice of pricing-to-market in the presence of trade and foreign investment frictions. Under strategic interaction of multiproduct multinational firms, there is a trade-off in pricing-to-market that mirrors the proximity-concentration trade-off, because MNCs change prices and markups in one market in opposite direction to the other market. Prices and markups charged in domestic and foreign markets by each MNC correlates positively with their corresponding endogenous domestic and foreign market shares, which, in turn, are negatively correlated within and between MNCs.[3] The relevance of the pricing-to-market mechanisms predicted in this paper should be assessed in future empirical work.

[1] In the paper I will talk interchangeably of MNCs' domestic market, source market, market of origin, or market where they are headquartered.

[2] Minniti and Turino (2013) maximize profits with respect to prices in a one-country world, while I maximize profits with respect to quantities in a two-country world.

[3] Within each MNC, the market share in the domestic market moves in opposite direction of the market share in the foreign market. Within each market, the market shares between the two MNCs are negatively correlated in a mechanic manner, since each MNC gains market share at the expense of the other MNC.

Another strand of the literature related to my paper is the one assuming oligopolistic market structure in international trade models. Eckel and Neary (2010) present a multiproduct oligopolistic firms model to address the consequences of the emergence of globalization (perfect integration of many countries that freely trade) with many realistic features, such as marginal cost heterogeneity across varieties, but which does not consider the multinational dimension of firms' decisions which is the key issue of my analysis.

Finally, Tintelnot (2017) develops a rich quantitative model of global production with export platforms. His model features, among other things, random location-specific productivities for each firm's products. However, his paper lacks strategic interaction, because each firm faces a CES demand function for each product and, because of the assumption of monopolistic competition, a constant markup pricing is obtained.

The paper is organized as follows. Section 2 presents the model building blocks (the demand side and the production side). The two-stage equilibrium solution is provided in Section 3. Section 4 contains the comparative statics on equilibrium market shares and markups. Section 5 derives the gravity equations for FDI and characterizes the proximity-concentration trade-off in a setting with strategic interaction. In Section 6 I use a numerical analysis to study the behaviour of some endogenous variables. Lastly, there are the conclusions.

2. The model

I consider a framework with two countries, country H (home) and country O (overseas), and a single factor of production, labour. Each consumer/worker is assumed to supply inelastically one unit of labour. Labour endowments in country H and O are, respectively, L_H and L_O. There are two sectors in the economy, a homogeneous good sector and a horizontally differentiated sector. The homogeneous good, A, is the numeraire and is produced under constant returns to scale and perfect competition. I normalize the units of A so that one unit of labour is needed to produce one unit of A. Labour is freely mobile across sectors, but is immobile across countries. All these hypotheses taken together imply that the wage rate equals one in the model.

The differentiated good is produced under increasing returns to scale by two multiproduct multinational firms, whose headquarters are located in countries H and O, respectively. I assume that MNC 1 is headquartered in country H, while MNC 2 is headquartered in country O. I will describe the production structure of the differentiated sector below.

2.1. Preferences and demand

Preferences are defined over the consumption of the homogeneous good, A, and of the horizontally differentiated good, which enters through a CES index of the quantities of each individual variety, and are of the following quasi-linear form[4]:

$$U(A, M) = A + e \log M, \quad \text{with} \quad e < 1$$

where e is a parameter characterizing preferences and M is equal to

$$M = \left[\sum_{j \in \Omega} c(j)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{1}$$

with $c(j)$ the consumption of variety j, Ω the total mass of varieties available to the consumer, and $\sigma > 1$ the elasticity of substitution between varieties. This is the well known Dixit–Stiglitz preference-for-diversity CES index. The individual consumer's budget constraint is equal to

$$A + \sum_{j \in \Omega} p(j)c(j) \leq y$$

where $p(j)$ is the delivered price of variety j and y is the consumer's income.

It can be easily proved that, following maximization of utility, the demand function for A reduces to

$$A = y - e$$

and the constant e is equal to the expenditure on the horizontally differentiated good

$$e = \sum_{j \in \Omega} p(j)c(j).$$

It will be clear that what matters for the solution of the model is total expenditure on the differentiated good in each country. Total expenditure differs in the two countries, being $E_H = eL_H$ in country H and $E_O = eL_O$ in country O. Expenditure levels E_H and E_O are just proportional to the population of the two countries. Then, the total direct demand function from country H for a generic variety i is

$$c(i) = \frac{p(i)^{-\sigma} e L_H}{\sum_{j \in \Omega} p(j)^{-(\sigma-1)}} \tag{2}$$

[4] See, for instance, section 2.A.5 of Baldwin et al. (2003) and Feenstra and Ma (2008).

and the inverse demand function is

$$p(i) = \frac{c(i)^{-1/\sigma} e L_H}{\sum_{j\in\Omega} c(j)^{(\sigma-1)/\sigma}}. \tag{3}$$

I assume that ownership of the two MNCs is equally distributed among residents of both countries. Income of consumers/workers in each country is then equal to $1+(\Pi^1+\Pi^2)/(L_H+L_O)$, with Π^1 and Π^2 the total profits of MNC 1 and 2, respectively.[5] In Appendix I show that the homogeneous and the differentiated sectors are active in both countries.

2.2. Production

The two MNCs produce more than one variety of the differentiated commodity. Hence, the model is a duopoly with multiproduct firms. Each multinational may set up the production of a given variety in any country. There are increasing returns to scale, because a fixed cost F in terms of labour is needed to produce a single variety if the production takes place in the same country where the headquarters are located. Otherwise, if production facilities are located in the foreign country, the fixed cost is $\Gamma_k F$, where k is the index identifying the country where the foreign investment is located, $k = \{H, O\}$, and $\Gamma_k \geq 1$. This is due to barriers that may hinder foreign investment. If the MNC wants to produce an additional variety, it has to bear the fixed cost F again ($\Gamma_k F$ if the production takes place abroad). This implies that MNCs will never find profitable to replicate the production of an existing variety, and they will opt for creating a brand new differentiated product, which guarantees higher profits.[6] By choice of measurement units, the unit labour requirement of the differentiated good is normalized to one.

The game between the two MNCs is modelled in the following manner. The first stage concerns the number of varieties to be produced in each country. The second stage concerns the quantity of each variety to be produced. Finally, shipping goods across countries is costly. To represent this fact I introduce iceberg transport costs: in order to sell one unit abroad, $\tau \geq 1$ units must be shipped.

I now describe the profit functions. Markets H and O are segmented. Each MNC sets the quantity to be sold for each variety in each market, given consumers' demand. I indicate the origin–destination pattern in the subscript, and the ownership of the variety in the superscript. Let us focus on MNC 1. The set of varieties by MNC 1 that are produced in country H is Ω_H^1, and that of varieties produced in country O by the same MNC is Ω_O^1. The objective of the MNC is to maximize profits in the two segmented markets, taking as given the quantities chosen by the other MNC that influence market prices. In order to do so in market H, MNC 1 simultaneously sets, for each $i \in \Omega_H^1$, the quantity produced of variety i in country H and then sold in H, $c_{HH}^1(i)$, and, for each $i \in \Omega_O^1$, the quantity produced in O and then re-imported in H, $c_{OH}^1(i)$. A similar choice concerns quantities sold in market O by MNC 1, $c_{HO}^1(i)$ and $c_{OO}^1(i)$. Total operating profits of MNC 1, π^1, come from the two segmented markets, H and O, and are thus made up by the components π_H^1 and π_O^1, so that $\pi^1 = \pi_H^1 + \pi_O^1$, where

$$\pi_H^1 = \sum_{j\in\Omega_H^1} [p_{HH}^1(j) - 1] c_{HH}^1(j) + \sum_{j\in\Omega_O^1} [p_{OH}^1(j) - \tau] c_{OH}^1(j), \tag{4}$$

$$\pi_O^1 = \sum_{j\in\Omega_H^1} [p_{HO}^1(j) - \tau] c_{HO}^1(j) + \sum_{j\in\Omega_O^1} [p_{OO}^1(j) - 1] c_{OO}^1(j). \tag{5}$$

The model is solved in the following manner. First of all, the equilibrium quantities sold of each variety by each MNC are derived. Then, conditional on the equilibrium quantities, the optimal number of varieties in each market by each MNC is obtained.

3. Equilibrium analysis

In this section I present the solution of the model. The model consists of two stages and is solved by backward induction. In the first stage, firms decide how many varieties are produced in each country. In the second stage, conditional on the investment decisions of the first stage, firms maximize profits with respect to quantities.

3.1. Second stage equilibrium

I now characterize the second stage equilibrium. MNC 1's optimization in each segmented market takes into account the fact that the firm faces a downward sloping demand curve for each variety. The price at which each variety can be sold is inversely related to the quantity sold of that variety, to the quantity of all the other varieties belonging to MNC 1 (which MNC 1 directly controls), and to the quantities sold of the varieties belonging to MNC 2 (which MNC 1 takes as given). The necessary conditions related to the optimality of prices in the context of profit maximization with respect to quantities of each variety are derived as follows.

[5] Given the quasi-linear structure of preferences, the assumption about the allocation of ownership rights of MNCs does not influence the equilibrium outcome, since, under plausible conditions, any other distribution of profits to the residents of a given country is absorbed by a corresponding change in the consumption of the numeraire at the individual level.

[6] Under the assumption that each plant produces only one variety, something which is customary in the proximity-concentration literature, F stands also for the plant-level fixed costs.

Operating profits of MNC 1 in market H are

$$\pi_H^1 = \sum_{j\in\Omega_H^1} [p_{HH}^1(j) - 1]c_{HH}^1(j) + \sum_{j\in\Omega_O^1} [p_{OH}^1(j) - \tau]c_{OH}^1(j). \tag{6}$$

I want to maximize π_H^1 with respect to the quantity of a variety i located in market H and sold in the same market. The first order condition is

$$\frac{\partial \pi_H^1}{\partial c_{HH}^1(i)} = 0. \tag{7}$$

As pointed out by Yang and Heijdra (1993) and d'Aspremont et al. (1996), a central issue with the Dixit–Stiglitz model is to determine what effects of firms' strategic choices should be taken into account in the computation of the equilibrium and what effects should be neglected. From a practical point of view, expanding the first order condition (7) requires the identification of the partial derivatives that are different from zero and of those that are equal to zero. I proceed as follows. While solving the maximization problem, the MNC correctly identifies the effect of the change in $c_{HH}^1(i)$ on the same-variety-price, $p_{HH}^1(i)$, and the effect on the other-variety-prices, $\partial p_{HH}^1(j)/\partial c_{HH}^1(i)$, with $j \neq i$, and $\partial p_{OH}^1(j)/\partial c_{HH}^1(i)$. All these terms are considered to be different from zero. In other terms, the firm takes into account that varying the quantity of a certain variety reduces the price at which that variety can be sold and, simultaneously, reduces also the price at which all the other varieties belonging to MNC 1 can be sold. The condition (7) becomes

$$\frac{\partial p_{HH}^1(i)}{\partial c_{HH}^1(i)} c_{HH}^1(i) + p_{HH}^1(i) + \sum_{j\in\{\Omega_H^1\setminus i\}} \frac{\partial p_{HH}^1(j)}{\partial c_{HH}^1(i)} c_{HH}^1(j) - 1 + \sum_{j\in\Omega_O^1} \frac{\partial p_{OH}^1(j)}{\partial c_{HH}^1(i)} c_{OH}^1(j) = 0. \tag{8}$$

The procedure that I then follow to retrieve the prices and the markups for a multiproduct firm resembles that in Minniti and Turino (2013), although they maximize profits with respect to prices in a one-country world, while I maximize profits with respect to quantities in a two-country world. Further derivations can be found in the Appendix. They yield the following expressions for prices by MNC 1:

$$p_{HH}^1 = \frac{\sigma}{(\sigma-1)(1-S_H^1)}, \quad p_{OH}^1 = \tau\frac{\sigma}{(\sigma-1)(1-S_H^1)}, \tag{9}$$

$$p_{OO}^1 = \frac{\sigma}{(\sigma-1)(1-S_O^1)}, \quad p_{HO}^1 = \tau\frac{\sigma}{(\sigma-1)(1-S_O^1)}. \tag{10}$$

The prices depend on three elements: the substitutability parameter σ, the total market shares of MNC 1 in markets H and O, S_H^1 and S_O^1 respectively, and the marginal cost, which is equal to 1 in the case of local varieties and τ in the case of varieties shipped from one market to the other. From the definition of total market shares, we get

$$S_H^1 = n_H^1 s_{HH}^1 + n_O^1 s_{OH}^1, \tag{11}$$

$$S_O^1 = n_H^1 s_{HO}^1 + n_O^1 s_{OO}^1, \tag{12}$$

where $s_{HH}^1 \equiv p_{HH}^1 c_{HH}^1 / eL_H$ is the share out of total expenditure eL_H of a single variety produced locally by MNC 1 and sold locally (with similar notations for the other market shares of individual varieties).

The markup of MNC 1 on varieties sold in market H, in percentage terms, is

$$\frac{p_{HH}^1 - 1}{p_{HH}^1} = \frac{p_{OH}^1 - \tau}{p_{OH}^1} = \frac{1}{\sigma} + \frac{\sigma-1}{\sigma} S_H^1, \tag{13}$$

a quantity rising in the total market share S_H^1 as long as $\sigma > 1$. The markup of MNC 1 on varieties sold in market O is

$$\frac{p_{OO}^1 - 1}{p_{OO}^1} = \frac{p_{HO}^1 - \tau}{p_{HO}^1} = \frac{1}{\sigma} + \frac{\sigma-1}{\sigma} S_O^1. \tag{14}$$

Eqs. (13) and (14) show an important property; that is, markups are positively correlated to total market shares in a given segmented market. The economic rationale for this result goes through the decrease in the own and cross elasticities of price with respect to quantity that a higher market share guarantees. In Appendix I show that the higher it is the market share commanded by a certain variety, the steeper it is the inverse demand curve.[7] Since, by (11) and (12), total market shares, S_H^1 and S_O^1, are positively associated with the market shares of individual varieties, *ceteris paribus*, when the MNC enjoys a large total market share, this will be mirrored by large market shares enjoyed by individual varieties, by unelastic inverse demand curves, and, consequently, by high markups. This implies that any change in the total market share in a given market is met by a corresponding variation in the markups charged by a certain MNC. In other terms, by looking at total market shares, the behaviour of markups can be inferred.

[7] Both the direct and indirect inverse demand curves are affected by the market share of a certain variety i. The direct inverse demand is the price of variety i as a function of the quantity of variety i. The indirect inverse demand is the price of a variety $j \neq i$ as a function of the quantity of variety i. In the Appendix I show that both the direct and indirect prices are steeper (lower elasticity) with respect to a change in the quantity of i when variety i commands a larger market share.

Another noticeable aspect of Eqs. (13) and (14) is that the dependence of markups on market shares goes through the substitutability parameter σ. When σ is close to one, each variety enjoys an almost perfect monopoly power and its demand is almost independent from the other varieties (substitutability is negligible). In this case, the own and cross elasticities of price with respect to the quantity of variety i tend to a constant (they are -1 and 0, respectively), and they are not affected by the market share enjoyed by i.[8] As σ increases, varieties become increasingly better substitutes, and the own and cross price elasticities become increasingly more sensitive to the market share. This happens because, with some substitutability, the demand for a certain variety is influenced by the other varieties' price, and so the size of the market share commanded by the MNC matters: only with a large market share the MNC can charge a high price (and markup) for its varieties, because only with a large share consumers have less opportunity to switch to varieties of the other MNC to escape uncompetitive pricing.

The following equations are also true (see the Appendix for the derivation):

$$S_H^1 = \frac{(n_H^1 + \phi n_O^1)^{1/\sigma}}{(n_H^1 + \phi n_O^1)^{1/\sigma} + (n_H^2 + \phi n_O^2)^{1/\sigma}}, \tag{15}$$

$$S_H^2 = 1 - S_H^1, \tag{16}$$

$$S_O^1 = \frac{(\phi n_H^1 + n_O^1)^{1/\sigma}}{(\phi n_H^1 + n_O^1)^{1/\sigma} + (\phi n_H^2 + n_O^2)^{1/\sigma}}, \tag{17}$$

$$S_O^2 = 1 - S_O^1, \tag{18}$$

where $\phi \equiv \tau^{-(\sigma-1)}$ is an inverse measure of transport costs (ϕ is usually called in the literature freeness of trade). The total market share of each MNC in the two markets is a function of the number of varieties established in each market. It is convenient to express the market shares with the following notation:

$$S_H^1 = \frac{A}{A+C},$$

$$S_H^2 = \frac{C}{A+C},$$

$$S_O^1 = \frac{B}{B+D},$$

$$S_O^2 = \frac{D}{B+D},$$

where

$$A \equiv (n_H^1 + \phi n_O^1)^{1/\sigma},$$

$$B \equiv (\phi n_H^1 + n_O^1)^{1/\sigma},$$

$$C \equiv (n_H^2 + \phi n_O^2)^{1/\sigma},$$

$$D \equiv (\phi n_H^2 + n_O^2)^{1/\sigma}.$$

I can write, for example, $S_H^1/S_H^2 = A/C$. The equations above will be very useful to solve analytically the model.

3.2. First stage equilibrium

In the first stage MNCs choose the optimal products' scope (number of varieties) in each market. In doing so, they correctly identify the equilibrium that is prevalent in the second stage of the game. I indicate total profits for MNC 1 as Π^1. They are equal to total operating profits, $\pi^1 = \pi_H^1 + \pi_O^1$, minus the fixed costs. It can be proved (see the Appendix) that

$$\Pi^1 = \frac{e}{\sigma}\left\{L_H S_H^1[1 + (\sigma-1)S_H^1] + L_O S_O^1[1 + (\sigma-1)S_O^1]\right\} - (n_H^1 + n_O^1 \Gamma_O)F. \tag{19}$$

The first order condition for the optimal number of varieties located in market H is $\partial \Pi^1/\partial n_H^1 = 0$, and leads to

$$\frac{e}{\sigma}\left\{L_H[1 + 2(\sigma-1)S_H^1]\frac{\partial S_H^1}{\partial n_H^1} + L_O[1 + 2(\sigma-1)S_O^1]\frac{\partial S_O^1}{\partial n_H^1}\right\} = F, \tag{20}$$

[8] Again, see the Appendix.

while the condition for the optimal number of varieties located in market O is $\partial \Pi^1/\partial n^1_O = 0$, yielding

$$\frac{e}{\sigma}\left\{ L_H[1+2(\sigma-1)S^1_H]\frac{\partial S^1_H}{\partial n^1_O} + L_O[1+2(\sigma-1)S^1_O]\frac{\partial S^1_O}{\partial n^1_O}\right\} = \Gamma_O F. \tag{21}$$

Given (15) and (17), the marginal effect of an increase in the number of varieties belonging to MNC 1 on total market shares is:

$$\frac{\partial S^1_H}{\partial n^1_H} = \frac{1}{\sigma}\frac{(n^1_H+\phi n^1_O)^{1/\sigma-1}(n^2_H+\phi n^2_O)^{1/\sigma}}{[(n^1_H+\phi n^1_O)^{1/\sigma}+(n^2_H+\phi n^2_O)^{1/\sigma}]^2} = \frac{S^1_H S^2_H}{\sigma(n^1_H+\phi n^1_O)} \tag{22}$$

$$\frac{\partial S^1_O}{\partial n^1_H} = \frac{\phi}{\sigma}\frac{(\phi n^1_H+n^1_O)^{1/\sigma-1}(\phi n^2_H+n^2_O)^{1/\sigma}}{[(\phi n^1_H+n^1_O)^{1/\sigma}+(\phi n^2_H+n^2_O)^{1/\sigma}]^2} = \frac{\phi S^1_O S^2_O}{\sigma(\phi n^1_H+n^1_O)} \tag{23}$$

$$\frac{\partial S^1_H}{\partial n^1_O} = \frac{\phi}{\sigma}\frac{(n^1_H+\phi n^1_O)^{1/\sigma-1}(n^2_H+\phi n^2_O)^{1/\sigma}}{[(n^1_H+\phi n^1_O)^{1/\sigma}+(n^2_H+\phi n^2_O)^{1/\sigma}]^2} = \frac{\phi S^1_H S^2_H}{\sigma(n^1_H+\phi n^1_O)} \tag{24}$$

$$\frac{\partial S^1_O}{\partial n^1_O} = \frac{1}{\sigma}\frac{(\phi n^1_H+n^1_O)^{1/\sigma-1}(\phi n^2_H+n^2_O)^{1/\sigma}}{[(\phi n^1_H+n^1_O)^{1/\sigma}+(\phi n^2_H+n^2_O)^{1/\sigma}]^2} = \frac{S^1_O S^2_O}{\sigma(\phi n^1_H+n^1_O)} \tag{25}$$

Substituting the partial derivatives back in (20) and (21) I get:

$$\begin{cases} eL_H[1+2(\sigma-1)S^1_H]\dfrac{S^1_H S^2_H}{(n^1_H+\phi n^1_O)} + eL_O[1+2(\sigma-1)S^1_O]\dfrac{\phi S^1_O S^2_O}{(\phi n^1_H+n^1_O)} = \sigma^2 F \\ eL_H[1+2(\sigma-1)S^1_H]\dfrac{\phi S^1_H S^2_H}{(n^1_H+\phi n^1_O)} + eL_O[1+2(\sigma-1)S^1_O]\dfrac{S^1_O S^2_O}{(\phi n^1_H+n^1_O)} = \sigma^2 \Gamma_O F \end{cases} \tag{26}$$

Another similar system can be derived from the optimization problem of MNC 2. Baldwin and Ottaviano (1998) do not pursue further results for the general case $0 \le \phi < 1$. In what follows I extend their analysis, to show that new analytical results can be derived. I first implicitly characterize equilibrium foreign market shares, S^1_O and S^2_H (which pin down also domestic shares, S^1_H and S^2_O), and then I provide comparative statics results for both market shares and markups with respect to exogenous parameters. Subsequently, I provide analytical expressions that relate the number of varieties to the equilibrium market shares.

The system (26) can be solved applying Cramer's rule, considering as two separate variables the quantities $1/(n^1_H+\phi n^1_O)$ and $1/(\phi n^1_H+n^1_O)$. In other words, the system (26) can be written as a linear system through an appropriate choice of variables. I obtain:

$$\begin{cases} n^1_H+\phi n^1_O = \dfrac{eL_H[1+2(\sigma-1)S^1_H]S^1_H S^2_H(1-\phi^2)}{\sigma^2 F(1-\Gamma_O\phi)} \\ \phi n^1_H+n^1_O = \dfrac{eL_O[1+2(\sigma-1)S^1_O]S^1_O S^2_O(1-\phi^2)}{\sigma^2 F(\Gamma_O-\phi)} \end{cases} \tag{27}$$

where the system has a positive interior solution if FDI is sufficiently free, $\Gamma_O < 1/\phi$. This condition becomes more binding as ϕ goes up; that is, as trade becomes less inhibited. The economic interpretation is that, for an MNC to be profitable to invest abroad, investment costs should be not too high. Moreover, the likelihood of FDI raises with the level of transport costs, because higher transport costs provide a more effective shield against mutual cannibalization between domestic and foreign varieties.

Assumption 1. In order to have a strictly positive finite solution, $n^1_H+\phi n^1_O > 0$, the conditions $\Gamma_O < 1/\phi$ and $0 \le \phi < 1$ have to be satisfied.

I can derive a set of conditions similar to (27), but related to MNC 2, working on the corresponding first order conditions:

$$\begin{cases} n^2_O+\phi n^2_H = \dfrac{eL_O[1+2(\sigma-1)S^2_O]S^2_O S^1_O(1-\phi^2)}{\sigma^2 F(1-\Gamma_H\phi)} \\ \phi n^2_O+n^2_H = \dfrac{eL_H[1+2(\sigma-1)S^2_H]S^2_H S^1_H(1-\phi^2)}{\sigma^2 F(\Gamma_H-\phi)} \end{cases} \tag{28}$$

Similarly to before, I assume the following.

Assumption 2. In order to have a strictly positive finite solution, $n^2_O+\phi n^2_H > 0$, the conditions $\Gamma_H < 1/\phi$ and $0 \le \phi < 1$ have to be satisfied.

The equations in (27) and (28) taken together lead to the following set of conditions:

$$\begin{cases} A^{\sigma} = eL_H \left[1 + 2(\sigma-1)\dfrac{A}{A+C}\right] \dfrac{AC}{(A+C)^2} \dfrac{1-\phi^2}{\sigma^2 F(1-\Gamma_O\phi)} \\ C^{\sigma} = eL_H \left[1 + 2(\sigma-1)\dfrac{C}{A+C}\right] \dfrac{AC}{(A+C)^2} \dfrac{1-\phi^2}{\sigma^2 F(\Gamma_H-\phi)} \\ B^{\sigma} = eL_O \left[1 + 2(\sigma-1)\dfrac{B}{B+D}\right] \dfrac{BD}{(B+D)^2} \dfrac{1-\phi^2}{\sigma^2 F(\Gamma_O-\phi)} \\ D^{\sigma} = eL_O \left[1 + 2(\sigma-1)\dfrac{D}{B+D}\right] \dfrac{BD}{(B+D)^2} \dfrac{1-\phi^2}{\sigma^2 F(1-\Gamma_H\phi)} \end{cases} \tag{29}$$

Taking the ratio between the fourth and the third equation in (29), and expressing it in terms of the total market share of MNC 1 in country O, S^1_O, I get

$$\left(\frac{1-S^1_O}{S^1_O}\right)^{\sigma} = \frac{1+2(\sigma-1)(1-S^1_O)}{1+2(\sigma-1)S^1_O}\,\frac{\Gamma_O-\phi}{1-\Gamma_H\phi}. \tag{30}$$

This equation implicitly defines the equilibrium market share $\overline{S}^1_O$. A symmetric equation can be derived for S^2_H, working on the first two equations of system (29). I can prove the following.

Lemma 1. *In equilibrium, the value of the foreign market shares $\overline{S}^1_O$ and $\overline{S}^2_H$ is the following.*

(1) *When $\phi = 0$, $\overline{S}^1_O$ is strictly less than 1/2 if and only if $\Gamma_O > 1$; $\overline{S}^2_H$ is strictly less than 1/2 if and only if $\Gamma_H > 1$.*
(2) *When $0 < \phi < 1$, both equilibrium foreign market shares $\overline{S}^1_O$ and $\overline{S}^2_H$ are strictly less than 1/2 if either $\Gamma_H > 1$, or $\Gamma_O > 1$, or both Γ_H and Γ_O are larger than one; they are equal to 1/2 if both Γ_H and Γ_O are equal to 1.*

The proofs of all lemmas and propositions are provided in the Appendix. Lemma 1 shows that when $0 < \phi < 1$, it is enough that at least one FDI friction parameter is strictly larger than one to have *both* foreign market shares strictly less than 1/2. The reason is the following. Let us imagine that the FDI friction parameter becomes strictly greater than one in country O, $\Gamma_O > 1$, while it is equal to one in country H, $\Gamma_H = 1$ (this amounts to having no frictions to foreign investment in country H). This will put at a disadvantage the MNC 1 with respect to the MNC 2 in market O, and so it will induce S^1_O to be strictly less than 1/2 and S^2_O to be strictly larger than 1/2. But this is not the only effect. A priori, in market H the two MNCs are on an equal footing, because $\Gamma_H = 1$. However, the optimal number of varieties in market H to be located by both MNCs depends on the *relative* magnitude of the market shares enjoyed by them in the two markets. For example, looking at the first order condition (20) and at the related first equation in (26) shows that, for the maximization of Π^1 with respect to n^1_H, both the domestic and the foreign market shares are relevant. Even if a priori there are no particular advantages for MNC 1 to invest more in market H compared to MNC 2, and hence to end up with a larger market share there, an incentive exists for both MNCs to invest more in the market where their total market share is larger in relative terms, because in this market MNCs benefit from higher markups and higher profitability. Since $\Gamma_O > 1$ implies that $S^1_O < 1/2$ and $S^2_O > 1/2$, this makes in relative terms more attractive market H to MNC 1 and market O to MNC 2, even if both MNCs were initially experiencing equal market shares in market H, $S^1_H = 1/2$ and $S^2_H = 1/2$, due to the absence of FDI frictions there, i.e. $\Gamma_H = 1$. The last step needed to convey a precise intuition of what is going on is to acknowledge that the benefits from a larger share in a given market are reaped more effectively by increasing the amount of investment in that market, more than by increasing the amount of investment in the other market. So, if MNC 1 enjoys a relatively larger market share in country H, i.e. $S^1_H > 1/2 > S^1_O$, in order to raise the total profits Π^1 it is more effective to increase n^1_H up to some point, than to increase n^1_O.[9] The increase in n^1_H eventually makes S^1_H strictly larger than 1/2. All the above reasoning explains why, even with $\Gamma_H = 1$, it is enough that $\Gamma_O > 1$ in order to have *both* domestic market shares S^1_H and S^2_O strictly larger than 1/2, and consequently both foreign market share S^1_O and S^2_H strictly less than 1/2. The crucial issue is that MNCs balance their relative investment stock in the two markets according to the relative total shares in each market, hence an impediment to FDI in market O for MNC 1 leads to a relative advantage to investment in market H.

The lemma also implies that MNCs will charge a higher markup on the units shipped to the markets where they are headquartered (market H for MNC 1 and market O for MNC 2) than on those shipped to the foreign markets, since markups are increasing in market shares, as indicated by Eqs. (13) and (14). Hence, factors affecting market shares will also impact on markups. Moreover, there is dumping in trade in the sense of Brander and Krugman (1983); that is, the f.o.b. price on domestic sales is larger than the f.o.b. price on exports ($p^1_{HH} > p^1_{HO}/\tau$).[10]

[9] This point is important because, ceteris paribus, increasing n^1_O would also make it possible to take advantage of the relatively larger market share in country H, since the markup on exports from market O to market H for MNC 1 is the same of the markup on local sales from market H to market H and both depend on S^1_H, see Eq. (13). However, my working hypothesis about FDI frictions implies that it is more profitable to increase n^1_H than n^1_O, because the fixed cost of an additional variety in market H is lower for MNC 1.

[10] Brander and Krugman (1983) analyze the case of a duopoly with one identical product without FDI, while here I analyze a duopoly with multiproduct firms, horizontal product differentiation, and FDI. However, the definition of dumping in trade as the case where f.o.b. price is higher in the domestic market than in the foreign one applies to both settings.

Proposition 1 (*Dumping in Trade*). *For $0 \le \phi < 1$ there is not dumping in trade if investment is free in both countries ($\Gamma_H = 1$ and $\Gamma_O = 1$). To generate dumping in trade by some MNC it is enough to assume that there are some investment frictions in one of the two countries ($\Gamma_H > 1$ or $\Gamma_O > 1$). In particular, when $0 < \phi < 1$, it is enough that a single friction parameter is strictly larger than one to generate dumping for both MNCs. When $\phi = 0$, a MNC shows a dumping behaviour if it is experiencing FDI frictions in the foreign market, independently of whether the other MNC faces FDI frictions.*

In this paper with intra-industry FDI, the only way to restore the equality of market shares in the foreign and domestic markets (thus inhibiting dumping in trade) is to assume that the investment frictions are absent in both countries. In Brander and Krugman (1983) dumping disappears when the trade frictions are absent, because their model is without FDI, and the only way to secure equal market shares to the firms in the two countries is to assume that trade is free.

4. Comparative statics on equilibrium market shares and markups

I now focus on the effects of investment and trade liberalization on market shares and markups of each MNC.

Proposition 2 (*Investment and Trade Liberalization*). *A lower FDI friction parameter in one single country (either Γ_H or Γ_O) affects negatively market shares and markups in both domestic markets, and positively market shares and markups in both foreign markets. A larger freeness of trade parameter, ϕ, affects positively market shares and markups in both domestic markets, and negatively market shares and markups in both foreign markets.*

The intuition behind these results is the following. Let us consider the impact of a rise in Γ_O, which is the cost for adding a variety in country O by MNC 1. The negative impact on S_O^1 and the positive impact on S_O^2 are straightforward to understand, since higher FDI frictions in country O discourage setting up foreign varieties in country O, and this has also a beneficial impact on MNC 2's market share in its domestic economy. However, this change in market shares leads also to a change in the relative profitability of market H and market O for the two MNCs, and this creates additional reallocations in market shares. The fall in S_O^1 makes market H relatively more profitable for MNC 1, so new varieties are established in that market, and S_H^1 raises accordingly. By the same token, the rise in S_O^2 makes relatively more profitable to invest in market O for MNC 2, and so S_H^2 goes down.[11] The impact of investment frictions in this multiproduct MNCs model with an endogenous number of varieties parallels the result in Baldwin and Ottaviano (2001) obtained when the number of varieties is fixed. They find that markups on domestic sales are lower in the equilibrium with FDI than in the equilibrium with purely national multiproduct firms, while markups on foreign sales are higher in the equilibrium with FDI. Their economic explanation based on the decrease in domestic sales and the rise in foreign sales due to FDI parallels the mechanism of this paper.

Let us now consider the economic intuition behind the comparative statics for trade liberalization. It is convenient to start from a situation where trade is completely inhibited ($\phi = 0$) and there are some investment frictions ($\Gamma_H > 1$ or $\Gamma_O > 1$). With $\phi = 0$, the domestic and foreign market shares depend exclusively on local varieties (n_H^1 and n_O^2, respectively, for MNC 1 and MNC 2 in their domestic markets, and n_O^1 and n_H^2 in their foreign markets). When trade is allowed, the domestic market share of MNCs becomes larger: since the domestic market is more profitable because both MNCs enjoy a larger market share there (see Lemma 1) there is an increment of the domestic investment, which in turn further increases the domestic market shares. This is facilitated by the fact that MNCs can reach the foreign market through exports, thanks to the decrease in trade costs.

I conclude the comparative statics analyzing how market shares vary according to the degree of product substitutability embedded in consumers' preferences.

Proposition 3 (*Product Substitutability*). *A larger substitutability parameter, σ, affects negatively market shares in the domestic markets and positively market shares in the foreign markets. This implies that, ceteris paribus, in industries where substitutability is higher, MNCs foreign market shares and markups are higher.*

The effect of the substitutability parameter, σ, can be explained in the following manner. A rise in σ reduces the profitability in both domestic and foreign markets. However, the reduction is larger for the market where profitability is higher (the domestic), so the MNC finds optimal to increase the foreign capital stock with respect to the domestic one. This in turn raises the foreign market share with respect to the domestic one.

I summarize the effects of a change in the exogenous parameters on the MNCs' market shares in Table 1. It is interesting to note that F (the fixed cost for a variety which is a measure of the intensity of scale economies) has no effect whatsoever on equilibrium market shares and, hence, on prices and markups. In terms of domestic and foreign investment, the expressions derived in the sections below show that what is identified in the model are the equilibrium stocks measured by the product $n_j^i F$, where $i = \{1, 2\}$ and $j = \{H, O\}$. Any change in F only brings an inverse change in n_j^i that leaves the equilibrium value of $n_j^i F$ unaltered. This confirms that scale economies play no role in this model.

[11] These reciprocal adjustments in market shares are further analyzed in the sections below.

Table 1
Comparative statics results for domestic and foreign market shares of a change in the parameters of the model.

	Domestic market shares (S^1_H, S^2_O) Domestic markups	Foreign market shares (S^1_O, S^2_H) Foreign markups
Γ_H	+	–
Γ_O	+	–
ϕ	+	–
σ	–	+
L_H	0	0
L_O	0	0
F	0	0

5. Gravity equations and strategic proximity-concentration

5.1. Derivation of gravity equations

I now derive explicit equilibrium relations between market shares and the number of domestic and foreign varieties. Since I characterized market shares in equilibrium, the system (27) can be expressed with the following notation

$$\begin{cases} n^1_H + \phi n^1_O = \eta \\ \phi n^1_H + n^1_O = \eta^* \end{cases} \tag{31}$$

where η and η^* are constant equilibrium values, and (28) as

$$\begin{cases} n^2_O + \phi n^2_H = \theta \\ \phi n^2_O + n^2_H = \theta^* \end{cases} \tag{32}$$

with θ and θ^* being other constants. Solving system (31), the solution is

$$n^1_H = \frac{\eta - \phi\eta^*}{1-\phi^2}, \quad n^1_O = \frac{\eta^* - \phi\eta}{1-\phi^2}.$$

If I substitute back market shares, I get an explicit solution for the number of varieties in each market:

$$n^1_H = \frac{eL_H[1+2(\sigma-1)\overline{S}^1_H]\overline{S}^1_H(1-\overline{S}^1_H)}{\sigma^2 F(1-\Gamma_O\phi)} - \frac{\phi}{F(\Gamma_O-\phi)}\frac{eL_O[1+2(\sigma-1)\overline{S}^1_O]\overline{S}^1_O(1-\overline{S}^1_O)}{\sigma^2}, \tag{33}$$

$$n^1_O = \frac{eL_O[1+2(\sigma-1)\overline{S}^1_O]\overline{S}^1_O(1-\overline{S}^1_O)}{\sigma^2 F(\Gamma_O-\phi)} - \frac{\phi}{F(1-\Gamma_O\phi)}\frac{eL_H[1+2(\sigma-1)\overline{S}^1_H]\overline{S}^1_H(1-\overline{S}^1_H)}{\sigma^2}. \tag{34}$$

The corresponding expressions for MNC 2 are:

$$n^2_O = \frac{eL_O[1+2(\sigma-1)\overline{S}^2_O]\overline{S}^2_O(1-\overline{S}^2_O)}{\sigma^2 F(1-\Gamma_H\phi)} - \frac{\phi}{F(\Gamma_H-\phi)}\frac{eL_H[1+2(\sigma-1)\overline{S}^2_H]\overline{S}^2_H(1-\overline{S}^2_H)}{\sigma^2}, \tag{35}$$

$$n^2_H = \frac{eL_H[1+2(\sigma-1)\overline{S}^2_H]\overline{S}^2_H(1-\overline{S}^2_H)}{\sigma^2 F(\Gamma_H-\phi)} - \frac{\phi}{F(1-\Gamma_H\phi)}\frac{eL_O[1+2(\sigma-1)\overline{S}^2_O]\overline{S}^2_O(1-\overline{S}^2_O)}{\sigma^2}. \tag{36}$$

Let us concentrate on (34). It is a *gravity* equation since the number of foreign varieties located by MNC 1 in country O depends on the size of the foreign economy, L_O, on the size of the domestic one, L_H, and on freeness of trade, ϕ. This number is the outcome of two contrasting forces: on the one side there is the positive effect on the FDI stock exerted by the attraction of the foreign market (first right-hand side term), on the other side there is the negative effect on FDI exerted by the attraction of the domestic market (second right-hand side term). Since cannibalization among varieties entails that the MNC finds optimal to have a finite number of varieties, the equilibrium FDI stock results from the relative attractiveness of the two markets. The symmetric expression for the equilibrium stock of domestic investment of MNC 1 is given by (33). The attraction exerted by market H simultaneously boosts domestic investment and inhibits FDI, while that of market O favours FDI and hinders domestic investment. Imposing that both n^1_H and n^1_O are strictly positive leads to the condition $k\phi < L_O/L_H < k/\phi$, where k is a constant which depends on the parameters of the model and it is defined as follows:

$$k \equiv \frac{\Gamma_O-\phi}{1-\Gamma_O\phi}\frac{[1+2(\sigma-1)\overline{S}^1_H]\overline{S}^1_H(1-\overline{S}^1_H)}{[1+2(\sigma-1)\overline{S}^1_O]\overline{S}^1_O(1-\overline{S}^1_O)}.$$

Similarly, imposing that both n^2_O and n^2_H are strictly positive implies that $z\phi < L_O/L_H < z/\phi$, where z is a constant defined as follows:

$$z \equiv \frac{1-\Gamma_H\phi}{\Gamma_H-\phi}\frac{[1+2(\sigma-1)\overline{S}^2_H]\overline{S}^2_H(1-\overline{S}^2_H)}{[1+2(\sigma-1)\overline{S}^2_O]\overline{S}^2_O(1-\overline{S}^2_O)}.$$

The values of market shares are those implicitly defined at equilibrium, which are function themselves of the parameters of the model. The conditions amount to say that, in order to have positive investments, market size L_O should not differ too much from market size L_H.

Focusing on the first right-hand side term of (34), the FDI stock by MNC 1 in the overseas country is positively related to the size of that market, L_O. In addition, it is unambiguously positively related to the total market share in O by MNC 1, $\overline{S}^1_O$, since in equilibrium $\overline{S}^1_O < 1/2$. There are two terms that operate here in the same direction. First, market share $\overline{S}^1_O$ affects investment abroad through the term $[1+2(\sigma-1)\overline{S}^1_O]$. This term influences n^1_O only if varieties are substitutes ($\sigma > 1$); it vanishes as the demand becomes almost independent from the other varieties (i.e., as σ tends to one). The economic interpretation is straightforward, provided that, when $\sigma > 1$, the MNC faces an increasingly inelastic demand the higher it is the market share. This effect gets stronger the higher it is σ; that is, the higher it is the substitutability among varieties. Then, a larger foreign market share raises the markup, and the higher it is the markup, the higher it is profitability in that market, and the higher it is the stock of varieties located abroad, n^1_O. Second, there is the term $\overline{S}^1_O(1-\overline{S}^1_O)$, that can be traced back to the partial derivative (21), where the partial derivative of profits with respect to n^1_O depends on $\partial S^1_O/\partial n^1_O$. Expression (25) provides the value of this partial derivative.

Considering the second right-hand side term in (34), the FDI stock by MNC 1 is inversely related to the expenditure in the home market, eL_H. The overall impact of the equilibrium market share by MNC 1 in country H, $\overline{S}^1_H$, is a priori ambiguous, since it depends on two terms that go in opposite directions. On the one side, through the term $[1+2(\sigma-1)\overline{S}^1_H]$, a larger $\overline{S}^1_H$ makes more inelastic the demand in country H, raises the profitability and hence investment in market H, and promotes a smaller FDI stock, n^1_O, since a limited amount of varieties can exist due to cannibalization (cannibalization being particularly intense when σ is high). On the other side, provided that $\overline{S}^1_H > 1/2$, as $\overline{S}^1_H$ goes up n^1_O could also get bigger, through a decrease in the term $\overline{S}^1_H(1-\overline{S}^1_H)$. This term can be traced back to the partial derivative (21). For MNC 1 it can be stated the following (a similar proposition holds for MNC 2).

Proposition 4 (*Foreign and Domestic Investment and the Domestic Market Share*)**.** *Foreign investment, n^1_O, is inversely related to the domestic market share $\overline{S}^1_H$, $\partial n^1_O/\partial\overline{S}^1_H < 0$, and domestic investment, n^1_H, is positively related to the domestic market share $\overline{S}^1_H$, $\partial n^1_H/\partial\overline{S}^1_H > 0$, if and only if $\overline{S}^1_H < \overline{S}^+$, where $\overline{S}^+$ is a threshold that depends on σ. The threshold verifies that $\overline{S}^+ \in (1/2, 2/3)$, $\partial\overline{S}^+/\partial\sigma > 0$, $\lim_{\sigma\to 1}\overline{S}^+ = 1/2$, and $\lim_{\sigma\to\infty}\overline{S}^+ = 2/3$.*

The proposition tells that, as σ gets larger, it is more likely that the number of foreign varieties, n^1_O, is negatively correlated to the domestic market share, $\overline{S}^1_H$, because it is more likely that the condition $\overline{S}^1_H < \overline{S}^+$ is met ($\overline{S}^1_H$ is decreasing in σ, while $\overline{S}^+$ is increasing in σ). When $\overline{S}^1_H$ is above the threshold there is indeed a positive correlation between n^1_O and $\overline{S}^1_H$. The relationship between n^1_H and $\overline{S}^1_H$ is the inverse, since $\partial n^1_H/\partial\overline{S}^1_H > 0$ below the threshold, and $\partial n^1_H/\partial\overline{S}^1_H < 0$ above it. The threshold $\overline{S}^+$ applies with exactly the same analytic properties to $\overline{S}^2_O$ and its relationship with n^2_H and n^2_O.

Comparative statics on the link between the number of varieties and FDI frictions leads to the following proposition.

Proposition 5 (*Investment Liberalization When $\overline{S}^1_H$ and $\overline{S}^2_O$ are Small*)**.** *Provided that $\overline{S}^1_H$ and $\overline{S}^2_O$ are below $\overline{S}^+$, a lower FDI frictions parameter in a single country (either Γ_H or Γ_O) affects positively foreign investment in both markets, and negatively domestic investment in both markets.*

Comparative statics on the link between the number of varieties and trade frictions leads to the following proposition.

Proposition 6 (*Trade Liberalization When $\overline{S}^1_H$ and $\overline{S}^2_O$ are Small*)**.** *Provided that $\overline{S}^1_H$ and $\overline{S}^2_O$ are below $\overline{S}^+$, a larger freeness of trade parameter affects negatively foreign investment in both markets, and positively domestic investment in both markets.*

The proof of this proposition (not shown in the Appendix) follows closely that for Proposition 5. In this section I have set out when the amount of FDI by MNCs in a host country can be expected to be inversely related to the market share held in their source country, and I have related FDI to policy parameters through the impact on market shares. The mechanism goes through MNCs reacting to a decrease in their domestic market share by increasing the foreign investment in host countries (alternatively, they decrease FDI when they experience a domestic market share surge). This is another result whose relevance should be assessed in future empirical work.

5.2. *Strategic proximity-concentration trade-off*

The total number of foreign varieties in the economy predicted by the model is:

$$n^1_O + n^2_H = \frac{eL_O\overline{S}^1_O(1-\overline{S}^1_O)}{\sigma^2 F}\underbrace{\frac{1-(\Gamma_H+\Gamma_O)\phi+\phi^2}{(\Gamma_O-\phi)(1-\Gamma_H\phi)}}_{\text{Trade and FDI frictions}}\left\{1+2(\sigma-1)\underbrace{\left[\frac{1}{2}-\frac{1-(\Gamma_H-\Gamma_O)\phi-\phi^2}{1-(\Gamma_H+\Gamma_O)\phi+\phi^2}\left(\frac{1}{2}-\overline{S}^1_O\right)\right]}_{\text{Market share effect}}\right\}$$

$$+\frac{eL_H\overline{S}_H^2(1-\overline{S}_H^2)}{\sigma^2 F}\underbrace{\frac{1-(\Gamma_H+\Gamma_O)\phi+\phi^2}{(\Gamma_H-\phi)(1-\Gamma_O\phi)}}_{\text{Trade and FDI frictions}}\left\{1+2(\sigma-1)\underbrace{\left[\frac{1}{2}-\frac{1-(\Gamma_O-\Gamma_H)\phi-\phi^2}{1-(\Gamma_H+\Gamma_O)\phi+\phi^2}\left(\frac{1}{2}-\overline{S}_H^2\right)\right]}_{\text{Market share effect}}\right\}. \tag{37}$$

The world stock of FDI is related to market shares through the terms $\overline{S}_O^1(1-\overline{S}_O^1)$ and $\overline{S}_H^2(1-\overline{S}_H^2)$, which are linked to overall MNCs' profitability (see before), and through the terms $\left(\frac{1}{2}-\overline{S}_O^1\right)$ and $\left(\frac{1}{2}-\overline{S}_H^2\right)$. I label the terms

$$\Delta_H(\Gamma_H,\Gamma_O,\phi)\equiv\frac{1-(\Gamma_H+\Gamma_O)\phi+\phi^2}{(\Gamma_H-\phi)(1-\Gamma_O\phi)},\quad \Delta_O(\Gamma_H,\Gamma_O,\phi)\equiv\frac{1-(\Gamma_H+\Gamma_O)\phi+\phi^2}{(\Gamma_O-\phi)(1-\Gamma_H\phi)}$$

as trade and FDI frictions terms. I derive the following lemma.

Lemma 2. *The terms Δ_H and Δ_O are greater than zero, and they are decreasing as the cost of investing abroad goes up and as freeness of trade goes up.*

Since $\sigma>1$, the equilibrium values of $\overline{S}_O^1$ and $\overline{S}_H^2$ are associated in a more than proportional fashion with the world total number of foreign varieties, $n_O^1+n_H^2$, through what I call the *Market share effect* term, MSE hereafter. The more than proportional relation originates from the fact that the two terms

$$\Lambda_H(\Gamma_H,\Gamma_O,\phi)\equiv\frac{1-(\Gamma_O-\Gamma_H)\phi-\phi^2}{1-(\Gamma_H+\Gamma_O)\phi+\phi^2},\quad\text{and}\quad \Lambda_O(\Gamma_H,\Gamma_O,\phi)\equiv\frac{1-(\Gamma_H-\Gamma_O)\phi-\phi^2}{1-(\Gamma_H+\Gamma_O)\phi+\phi^2}$$

are greater than one. I derive the following lemma.

Lemma 3. *The terms Λ_H and Λ_O are greater than one, and they are increasing as the FDI frictions go up and as freeness of trade goes up.*

In the proofs of Lemmas 2 and 3 in the Appendix I show the following.

Assumption 3. The condition $1-(\Gamma_H+\Gamma_O)\phi+\phi^2>0$ is necessarily satisfied when the world stock of FDI is strictly positive, $n_O^1+n_H^2>0$.

The MSE term is boosted or dampened by the magnitude of σ: the higher it is the degree of substitutability between varieties, the stronger it is the connection of this term to the equilibrium world number of foreign varieties. When σ approaches 1 (varieties are almost independent) this term vanishes. This term is nurtured by the interrelatedness among market shares, markups, and profitability to invest in a given market. We know from previous sections that such interrelatedness depends on σ. The economic intuition behind the MSE term is the following. An increment in the foreign market share by MNC 1, $\overline{S}_O^1$, is positively associated with the stock of FDI belonging to MNC 1, n_O^1, through Eq. (34). But as $\overline{S}_O^1$ rises, the equilibrium market share of MNC 2, $\overline{S}_O^2$, goes down. A fall in $\overline{S}_O^2$ pushes MNC 2 to have a larger stock of foreign capital, n_H^2, through the term $2(\sigma-1)\overline{S}_O^2$ in Eq. (36).[12] In turn, a larger n_H^2 will be associated with a higher $\overline{S}_H^2$, a lower $\overline{S}_H^1$, a higher n_O^1, etc. The cumulation of these reciprocal adjustments explains the presence of the MSE term; that is, a term that links in a more than proportional fashion the foreign market share of MNCs to the total number of varieties worldwide.

Lemma 3 implies that there is a *magnification* of the MSE when investment frictions are high and trade costs are low. To understand this, let us consider the impact of a decrease (for whatever reason) in $\overline{S}_H^2$ on $n_O^1+n_H^2$, and how the impact changes at different levels of the investment frictions, Γ_H and Γ_O. If $\overline{S}_H^2$ goes down, then $\overline{S}_H^1$ goes up. I have argued that this triggers further reallocation of investment stocks and market shares in market O, through the MSE term: in country O MNC 1 will decrease the investment stock, n_O^1, since market H has become relatively more attractive, and MNC 2 will increase the investment stock, n_O^2, since market H has become relatively less attractive. The extent of the decrease of FDI by MNC 1 in market O depends in turn on Γ_O: the larger it is Γ_O, the more sizeable it is the negative reaction of n_O^1 to the increase in $\overline{S}_H^1$. The increase of the investment stock by MNC 2 in country O, n_O^2, is associated with a larger market share by MNC 2, $\overline{S}_O^2$ (and a lower $\overline{S}_O^1$), and with a lower stock of investment in country H, n_H^2, with the latter effect being dependent on the magnitude of Γ_H: the larger it is Γ_H, the more sizeable it is the negative reaction of n_H^2 to the increase in $\overline{S}_O^2$. This gives the intuition of why the MSE term is magnified by large investment frictions in both countries. Turning to the magnification induced by low transport costs, let us see what happens with a decrease (for whatever reason) in $\overline{S}_H^2$, and how the impact changes for different levels of ϕ. If $\overline{S}_H^2$ goes down, then $\overline{S}_H^1$ goes up. These changes will determine, through the MSE term, a reallocation of market shares also in country O, with $\overline{S}_O^1$ going down and $\overline{S}_O^2$ going up. The intensity of these reallocations depends on freeness of trade, because the easier it is to trade, the more sensitive

[12] From Proposition 4 it descends that the sign of the derivative of n_H^2 with respect to $\overline{S}_O^2$ is negative if and only if $\overline{S}_O^2<\overline{S}^+$. However, what goes in the MSE term is only $2(\sigma-1)\overline{S}_O^2$, and so this term relates unambiguously a decrease in $\overline{S}_O^2$ to a rise in n_H^2.

MNCs are with respect to the relative profitability of the two countries, since it becomes increasingly more profitable to expand investment (and market share) in one market and reach the other one by export.

Relying on the lemmas above, I can then assess what is the effect of a change in FDI frictions and freeness of trade on the total FDI stock, $n_O^1 + n_H^2$.

Proposition 7 (*Comparative Statics on the Total Stock of FDI*)**.** *The total foreign investment stock, $n_O^1 + n_H^2$, is decreasing as the cost of investing abroad in one of the two countries, Γ_H or Γ_O, goes up, and as the degree of freeness of trade, ϕ, goes up.*

Using the fact that $\overline{S}_H^1 + \overline{S}_H^2 = 1$ and $\overline{S}_O^1 + \overline{S}_O^2 = 1$, the total number of domestic varieties in the economy, $n_H^1 + n_O^2$, can be written in terms of foreign market shares, $\overline{S}_O^1$ and $\overline{S}_H^2$, as:

$$n_H^1 + n_O^2 = \frac{eL_O\overline{S}_O^1(1-\overline{S}_O^1)}{\sigma^2 F}\underbrace{\frac{\Gamma_O - 2\phi + \Gamma_H\phi^2}{(1-\Gamma_H\phi)(\Gamma_O-\phi)}}_{\text{Trade and FDI frictions}}\left\{1 + 2(\sigma-1)\underbrace{\left[\frac{1}{2} + \frac{\Gamma_O - \Gamma_H\phi^2}{\Gamma_O - 2\phi + \Gamma_H\phi^2}\left(\frac{1}{2} - \overline{S}_O^1\right)\right]}_{\text{Market share effect}}\right\}$$
$$+\frac{eL_H\overline{S}_H^2(1-\overline{S}_H^2)}{\sigma^2 F}\underbrace{\frac{\Gamma_H - 2\phi + \Gamma_O\phi^2}{(1-\Gamma_O\phi)(\Gamma_H-\phi)}}_{\text{Trade and FDI frictions}}\left\{1 + 2(\sigma-1)\underbrace{\left[\frac{1}{2} + \frac{\Gamma_H - \Gamma_O\phi^2}{\Gamma_H - 2\phi + \Gamma_O\phi^2}\left(\frac{1}{2} - \overline{S}_H^2\right)\right]}_{\text{Market share effect}}\right\}. \quad (38)$$

Along the lines of the proofs described above, it can be shown that in (38) the *Trade and FDI frictions* terms are greater than zero, but, differently from before, they are increasing in Γ_H, Γ_O and ϕ. When it comes to the terms entering the *Market share effect*, they are greater than one, but, differently from before, they are decreasing in Γ_H, Γ_O and ϕ (we get an *attenuation* of the MSE for domestic investment).

When establishing the sign of the derivative of the total domestic investment stock, $n_H^1 + n_O^2$, with respect to the investment and trade policy parameters Γ_H, Γ_O, and ϕ one has to take into account that the terms $\overline{S}_O^1(1-\overline{S}_O^1)$ and $\overline{S}_H^2(1-\overline{S}_H^2)$ are decreasing when the investment and trade policy parameters go up. The following proposition is proved in the Appendix and provides a characterization of what I call the strategic proximity-concentration trade-off.

Proposition 8 (*Strategic Proximity-Concentration Trade-Off*)**.** *Increasing the cost of investing abroad in one of the two countries, Γ_H or Γ_O, or increasing the degree of freeness of trade, ϕ, brings a relative concentration of investment in the domestic markets in place of the proximity of investment to the foreign markets:*

$$\frac{\partial(n_H^1 + n_O^2)}{\partial\psi} > \frac{\partial(n_H^2 + n_O^1)}{\partial\psi},$$

where ψ is any of the investment and trade policy parameters, $\psi = \{\Gamma_H, \Gamma_O, \phi\}$.

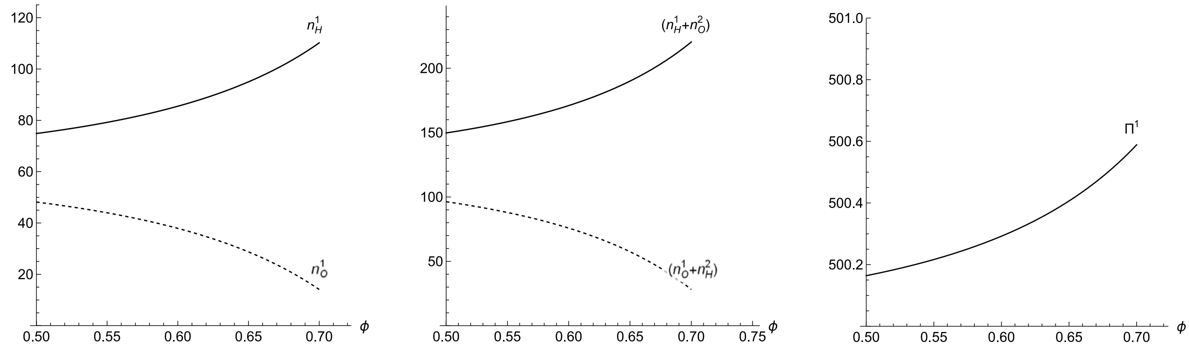
This proposition is quite general, since it holds for any value of the model's parameters and market shares, and shows when, in relative terms, there is a concentration of investment in the domestic markets in place of the proximity to foreign markets. The economic mechanism is clear. Consider for example an increase in freeness of trade that brings a reduction in foreign market shares, but also a reduction in the total stock of FDI. The reason why lower transport costs are conducive to a lower stock of FDI and lower foreign market shares is the concentration of production by MNCs in the markets where they enjoy a stronger market power. Hence, lowering barriers to trade makes for them more attractive to concentrate production in the domestic market, something that is facilitated by the possibility of reaching the foreign one through exports that are now cheaper, thanks to the lower impediments to trade.

Increasing in relative terms investment at home while reducing it abroad is reminiscent of the literature on the proximity-concentration trade-off (see e.g. Brainard, 1993), although in my framework the incentives to concentrate production domestically and do relatively less FDI following a change in exogenous parameters hinge upon strategic considerations related to market power that are absent in that literature.

6. Numerical analysis

In what follows I present a numerical analysis that aims at providing some examples related to the behaviour of the number of varieties and profits with respect to the investment and trade policy parameters. I assume that $F = 1$, without losing generality in the sense explained above. Moreover I set as baseline a symmetric configuration of the model, such that $\Gamma_H = 1.04$, $\Gamma_O = 1.04$, $\phi = 0.70$, $\sigma = 4$, $eL_H = 1000$, $eL_O = 1000$. This implies, approximately, $\tau \approx 1.13$. These values satisfy simultaneously the parameters' restrictions

$$\Gamma_H < \frac{1}{\phi}, \quad \Gamma_O < \frac{1}{\phi}, \quad k\phi < \frac{L_O}{L_H} < \frac{k}{\phi}, \quad z\phi < \frac{L_O}{L_H} < \frac{z}{\phi},$$



(a) ϕ and domestic and foreign investment of MNC 1.
(b) ϕ and strategic proximity-concentration.
(c) ϕ and total profits of MNC 1.

Fig. 1. Impact of trade costs, $0.5 \leq \phi \leq 0.7$.

so that the number of varieties $n_H^1, n_O^1, n_O^2, n_H^2$ are all strictly positive at equilibrium,

$$n_H^1 = n_O^2 = 110.2, \quad n_O^1 = n_H^2 = 14.1,$$

and market shares are approximately equal to

$$S_H^1 = S_O^2 \approx 0.52, \quad S_H^2 = S_O^1 \approx 0.48.$$

Total profits are $\Pi^1 = \Pi^2 \approx 500.6$. In order to understand the comparative statics results, it is also important to compute the value of the threshold $\overline{S}^+$ from Proposition 4, whose exact expression is provided in the Appendix. I get that when $\sigma = 4$, then $\overline{S}^+ \approx 0.64$.

In what follows I will change only one parameter per time, leaving all other parameters at their baseline value.

6.1. Impact of trade costs

I consider how different values of ϕ affect MNC 1's domestic and foreign varieties, the total stock of domestic and foreign investment (the sum of the two MNCs investment) and MNC 1's profits. I assume that $0.5 \leq \phi \leq 0.7$.

Proposition 6 tells that, when domestic market shares are below $\overline{S}^+$ (something that is always verified in my numerical analysis), a larger freeness of trade affects positively domestic investment and negatively foreign investment, while Proposition 8 provides the characterization of the strategic proximity-concentration trade-off. Figs. 1(a) and 1(b) confirm the theoretical results. In the third panel, I plot total profits of MNC 1, Π^1, and I verify that they are increasing in freeness of trade (although the increase is quite small in absolute value), which means that MNCs are more profitable when transport costs are lower.

6.2. Impact of MNC 1's own foreign investment costs, Γ_O

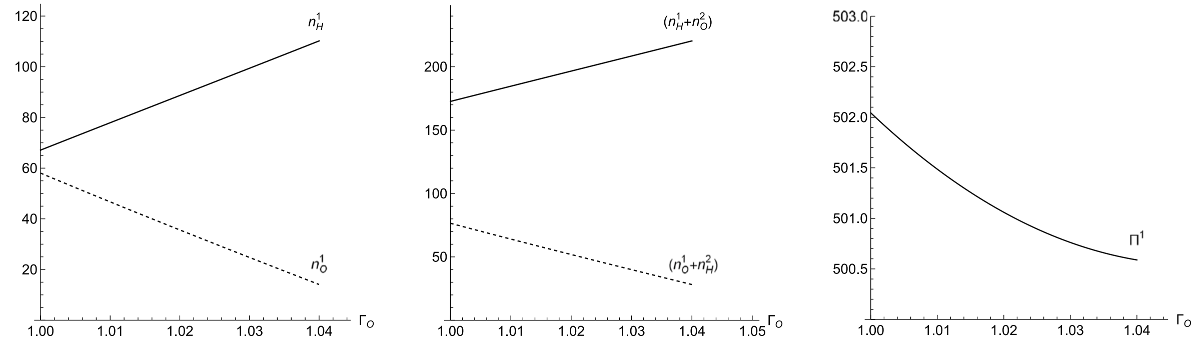
I now consider how a change in MNC 1's own foreign investment friction parameter, Γ_O, impacts on the number of varieties and total profits of MNC 1, and on the total stock of domestic and foreign investment by the two MNCs. I assume that $1 \leq \Gamma_O \leq 1.04$.

Proposition 5 tells that, when domestic market shares are below $\overline{S}^+$, larger FDI costs affect positively domestic investment and negatively foreign investment, while Proposition 8 provides the characterization of the strategic proximity-concentration trade-off. Figs. 2(a) and 2(b) confirm the theoretical results. Lastly, I plot total profits of MNC 1, Π^1, and I find that they are decreasing in the cost of investing abroad, which means that MNCs are less profitable when they have to pay higher fixed costs for their foreign varieties.

6.3. Impact of MNC 2's foreign investment costs, Γ_H, on MNC 1

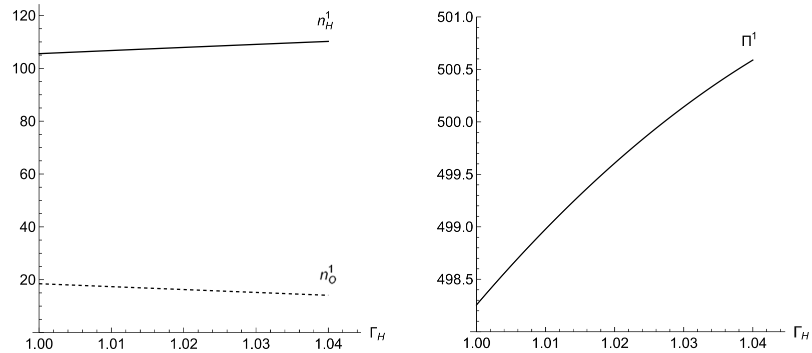
I then consider how a change in the foreign investment friction parameter for MNC 2 in MNC 1's domestic market, Γ_H, impacts on the number of varieties and profits by MNC 1 (the impact of Γ_H on total domestic and foreign investment stocks is symmetric to the one of Γ_O presented in Fig. 2(b) and I omit it here). Even if MNC 1 does not experience any direct change in investment costs when Γ_H raises, the strategic interaction mechanisms described above raises MNC 1's domestic market share and lowers MNC 1's foreign market share. I assume that $1 \leq \Gamma_H \leq 1.04$.

Proposition 5 tells that, when domestic market shares are below $\overline{S}^+$, larger FDI costs affect positively domestic investment and negatively foreign investment for both firms. This holds true both for a rise in own foreign investment costs (Γ_O in the case of MNC 1) and for a rise in the competitor's foreign investment costs (Γ_H, which are the costs of foreign investment for MNC 2 in country



(a) Γ_O and domestic and foreign investment of MNC 1. (b) Γ_O and strategic proximity-concentration. (c) Γ_O and MNC 1's total profits.

Fig. 2. Impact of investment costs Γ_O, $1 \leq \Gamma_O \leq 1.04$.



(a) Γ_H and domestic and foreign investment of MNC 1. (b) Γ_H and MNC 1's total profits.

Fig. 3. Impact of investment costs Γ_H, $1 \leq \Gamma_H \leq 1.04$.

H). Fig. 3(a) confirms that the increase in Γ_H indirectly increases the domestic number of varieties, n_H^1, and decreases the foreign number of varieties, n_O^1. The figure shows that the derivative of MNC 1's investment with respect to Γ_H (an indirect effect going through strategic interaction) is smaller in absolute value than in the case of Γ_O represented in Fig. 2(a) (a direct effect). The last panel shows that total profits of MNC 1, Π^1, are increasing in the cost of investing abroad of MNC 2, which means that MNC 1 takes advantage from the higher FDI frictions that MNC 2 has to pay.

7. Discussion and conclusions

I propose a model with multiproduct multinational firms and horizontal product differentiation and I retrieve gravity equations for the number of domestic and foreign varieties located in each country. The model delivers simple testable implications related to international investment and trade policy. Policies attracting FDI in a host country by lowering taxes or subsidizing foreign investment end up decreasing market shares and markups of multinationals in their domestic market. Moreover, protectionist trade policies that increase the cost of exporting between countries end up decreasing the domestic market shares and markups of MNCs. The reason is that both these policies push towards a reallocation of investment in the host markets, with a subsequent increase of foreign market shares and markups. At the aggregate level, the total stock of domestic investment (a measure of concentration of economic activity in the domestic markets) decreases relative to the total stock of foreign investment (a measure of proximity to foreign markets) whenever there is a decline in source market shares with respect to host market shares. This is what I call the strategic proximity-concentration trade-off.

A promising direction for future research is to deepen the empirical investigations. The alternative strategic mechanism regulating the proximity-concentration trade-off that I propose has stark implications in terms of testable predictions. The predictions based on models such as Helpman et al. (2004) imply an elasticity between domestic and foreign sales by MNCs that is positive and equal to one. This is not what is found empirically by Yeaple (2009), who estimates (cross-sectionally) an elasticity which is still positive but half in magnitude with respect to the theory. He attributes the weakening of the coefficient precisely to the presence of market power and variable markups by US MNCs. On the contrary, my model predicts the substitutability among domestic and foreign sales of MNCs, or, put differently, a negative elasticity (in the model the substitutability of the level of sales is isomorphic to the substitutability of market shares). While introducing variable markups is definitely important in accounting for a more realistic estimation of the proximity-concentration trade-off, more empirical work is warranted in order to exactly identify the sign and magnitude of these coefficients. Another empirical exercise could involve the quantification of the model, based on the estimation of the gravity equations for domestic and foreign investment. On the theoretical side, a possible extension could be the solution of a more general oligopolistic market structure, with more than two firms in the economy.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

A.1. Positive employment in the homogeneous and differentiated sectors

Total consumption and employment for the homogeneous sector in country H is equal to

$$L_H A = (1-e)L_H + \frac{L_H}{L_H + L_O}(\Pi^1 + \Pi^2)$$

that is positive if $e < 1$, $\Pi^1 > 0$, and $\Pi^2 > 0$, something which holds true in our model. Total employment in the differentiated sector in country H is, consequently,

$$L_H(1-A) = eL_H - \frac{L_H}{L_H + L_O}(\Pi^1 + \Pi^2). \tag{A.1}$$

The right-hand side of (A.1) is greater than zero if $e(L_H + L_O) > \Pi^1 + \Pi^2$; that is, total expenditure on the differentiated good in the two countries is greater than the sum of MNCs total profits. This is obviously the case, because, by definition, total profits are equal to expenditure on the differentiated good minus fixed and variable costs. So, in each country there is positive employment in both the homogeneous and the differentiated sector. I also derive that, necessarily, $A < 1$.

A.2. Price and markup under maximization with respect to quantity

Thanks to the definition of the elasticity of the price of a certain variety with respect to the quantity of variety i I can write

$$p^1_{HH}(i) - 1 + \frac{\partial \ln p^1_{HH}(i)}{\partial \ln c^1_{HH}(i)} p^1_{HH}(i) + \sum_{j\in\{\Omega^1_H \setminus i\}} c^1_{HH}(j) \frac{\partial \ln p^1_{HH}(j)}{\partial \ln c^1_{HH}(i)} \frac{p^1_{HH}(j)}{c^1_{HH}(i)} +$$
$$\sum_{j\in\Omega^1_O} c^1_{OH}(j) \frac{\partial \ln p^1_{OH}(j)}{\partial \ln c^1_{HH}(i)} \frac{p^1_{OH}(j)}{c^1_{HH}(i)} = 0. \tag{A.2}$$

I now use the following results. The inverse demand function for a generic variety j in log-linear form is

$$\ln p^1_{HH}(j) = -\frac{1}{\sigma}\ln c^1_{HH}(j) - \frac{\sigma-1}{\sigma}\ln M_H + \ln\left(eL_H\right),$$

where M_H is the CES quantity index in market H:

$$M_H = \left[\sum_{j\in\Omega^1_H} c^1_{HH}(j)^{(\sigma-1)/\sigma} + \sum_{j\in\Omega^1_O} c^1_{OH}(j)^{(\sigma-1)/\sigma} + \sum_{j\in\Omega^2_H} c^2_{HH}(j)^{(\sigma-1)/\sigma} + \sum_{j\in\Omega^2_O} c^2_{OH}(j)^{(\sigma-1)/\sigma}\right]^{\frac{\sigma}{\sigma-1}}.$$

The price elasticity of variety i with respect to its own quantity is

$$\frac{\partial \ln p^1_{HH}(i)}{\partial \ln c^1_{HH}(i)} = -\frac{1}{\sigma} - \frac{\sigma-1}{\sigma}\frac{\partial \ln M_H}{\partial \ln c^1_{HH}(i)}. \tag{A.3}$$

The price elasticity of variety j with respect to the quantity of variety i, with $j \neq i$, is

$$\frac{\partial \ln p^1_{HH}(j)}{\partial \ln c^1_{HH}(i)} = -\frac{\sigma-1}{\sigma}\frac{\partial \ln M_H}{\partial \ln c^1_{HH}(i)}. \tag{A.4}$$

Furthermore, it can be checked that the elasticity of M_H with respect to the quantity of variety i is positive and equal to

$$\frac{\partial \ln M_H}{\partial \ln c^1_{HH}(i)} = \frac{\partial M_H}{\partial c^1_{HH}(i)}\frac{c^1_{HH}(i)}{M_H} = \left[\frac{c^1_{HH}(i)}{M_H}\right]^{(\sigma-1)/\sigma} > 0.$$

Moreover, it is equal to the market share in value of variety i:

$$s^1_{HH}(i) = \frac{p^1_{HH}(i)c^1_{HH}(i)}{eL_H} = \frac{c^1_{HH}(i)^{-1/\sigma} M_H^{-(\sigma-1)/\sigma} eL_H c^1_{HH}(i)}{eL_H} = \left[\frac{c^1_{HH}(i)}{M_H}\right]^{(\sigma-1)/\sigma} = \frac{\partial \ln M_H}{\partial \ln c^1_{HH}(i)}.$$

When σ approaches 1 the own and cross price elasticities (A.3) and (A.4) tend to -1 and 0, respectively. I can also write

$$\frac{\partial \ln p^1_{HH}(i)}{\partial \ln c^1_{HH}(i)} = -\frac{1}{\sigma} - \frac{\sigma-1}{\sigma} s^1_{HH}(i),$$
$$\frac{\partial \ln p^1_{HH}(j)}{\partial \ln c^1_{HH}(i)} = -\frac{\sigma-1}{\sigma} s^1_{HH}(i).$$

If I substitute into (A.2) I get

$$p^1_{HH}(i) - 1 - \frac{1}{\sigma}p^1_{HH}(i) - \frac{\sigma-1}{\sigma}\frac{\partial \ln M_H}{\partial \ln c^1_{HH}(i)} p^1_{HH}(i) + \sum_{j\in\{\Omega^1_H \setminus i\}} p^1_{HH}(j)\frac{c^1_{HH}(j)}{c^1_{HH}(i)}\left[-\frac{\sigma-1}{\sigma}\frac{\partial \ln M_H}{\partial \ln c^1_{HH}(i)}\right] +$$
$$\sum_{j\in\Omega^1_O} p^1_{OH}(j)\frac{c^1_{OH}(j)}{c^1_{HH}(i)}\left[-\frac{\sigma-1}{\sigma}\frac{\partial \ln M_H}{\partial \ln c^1_{HH}(i)}\right] = 0$$

and then

$$p^1_{HH}(i) - 1 - \frac{1}{\sigma}p^1_{HH}(i) = \frac{\sigma-1}{\sigma}\sum_{j\in\Omega^1_H} p^1_{HH}(j)\frac{c^1_{HH}(j)}{c^1_{HH}(i)}\left[\frac{c^1_{HH}(i)}{M_H}\right]^{(\sigma-1)/\sigma} +$$
$$\frac{\sigma-1}{\sigma}\sum_{j\in\Omega^1_O} p^1_{OH}(j)\frac{c^1_{OH}(j)}{c^1_{HH}(i)}\left[\frac{c^1_{HH}(i)}{M_H}\right]^{(\sigma-1)/\sigma}. \tag{A.5}$$

I can rewrite Eq. (A.5) as

$$\left[\frac{c^1_{HH}(i)}{M_H}\right]^{-(\sigma-1)/\sigma} c^1_{HH}(i)\left[p^1_{HH}(i) - 1 - \frac{1}{\sigma}p^1_{HH}(i)\right] = \frac{\sigma-1}{\sigma}\left[\sum_{j\in\Omega^1_H} p^1_{HH}(j)c^1_{HH}(j) + \sum_{j\in\Omega^1_O} p^1_{OH}(j)c^1_{OH}(j)\right]. \tag{A.6}$$

Now I notice that the demand function $c^1_{HH}(i)$ is equal to

$$c^1_{HH}(i) = p^1_{HH}(i)^{-\sigma}\left(eL_H\right)^{\sigma} M_H^{-(\sigma-1)}.$$

Substituting this expression into (A.6) we end up with an equation that implicitly defines the price $p^1_{HH}(i)$, and it holds for every variety i located in market H and sold to the same market. Then there is a unique price p^1_{HH} that is the same for every $i \in \Omega^1_H$. I suppress the index related to the specific variety, and indicate the price charged by MNC 1 simply as p^1_{HH}. I can also suppress the index related to quantities, c^1_{HH}, because if the price is the same also the quantity demanded will be the same. A similar relationship can be derived for a generic variety located in market O and shipped to market H, whose price is indicated with p^1_{OH} and quantity with c^1_{OH}. Eq. (A.5) can also be written as

$$p^1_{HH} - 1 - \frac{1}{\sigma}p^1_{HH} = \frac{\sigma-1}{\sigma}n^1_H p^1_{HH} s^1_{HH} + \frac{\sigma-1}{\sigma}n^1_O p^1_{OH}\frac{c^1_{OH}}{c^1_{HH}} s^1_{HH} \tag{A.7}$$

where n^1_H and n^1_O is the number of varieties belonging to MNC 1 that are located in markets H and O, respectively. I employ now the following result:

$$p^1_{OH}\frac{c^1_{OH}}{c^1_{HH}} s^1_{HH} = p^1_{HH}\frac{p^1_{OH}c^1_{OH}}{eL_H} = p^1_{HH}s^1_{OH}. \tag{A.8}$$

Consequently, I can write Eq. (A.7) as

$$p^1_{HH} - 1 = \frac{1}{\sigma}p^1_{HH} + \frac{\sigma-1}{\sigma}p^1_{HH}\left(n^1_H s^1_{HH} + n^1_O s^1_{OH}\right)$$

and then

$$\frac{p^1_{HH} - 1}{p^1_{HH}} = \frac{1}{\sigma} + \frac{\sigma-1}{\sigma}S^1_H, \tag{A.9}$$

where $S^1_H = n^1_H s^1_{HH} + n^1_O s^1_{OH}$. After some simple algebra I get that

$$p^1_{HH} = \frac{\sigma}{(\sigma-1)(1-S^1_H)}. \tag{A.10}$$

When I maximize π_H^1 with respect to $c_{OH}^1(i)$, following steps similar to the ones described above, I get the condition

$$\frac{p_{OH}^1 - \tau}{p_{OH}^1} = \frac{p_{HH}^1 - 1}{p_{HH}^1} = \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} S_H^1. \tag{A.11}$$

Then I can write

$$p_{OH}^1 = \tau p_{HH}^1 = \tau \frac{\sigma}{(\sigma - 1)(1 - S_H^1)}. \tag{A.12}$$

A.3. Derivation of market shares in terms of the number of varieties

I provide the derivation for S_H^1. I start from the fact that:

$$S_H^1 = n_H^1 s_{HH}^1 + n_O^1 s_{OH}^1.$$

Then, I realize that

$$s_{HH}^1 = \left[\frac{c_{HH}^1}{M_H}\right]^{(\sigma-1)/\sigma} = M_H^{-(\sigma-1)/\sigma} \left[{p_{HH}^1}^{-\sigma} \left(eL_H\right)^{\sigma} M_H^{-(\sigma-1)} \right]^{(\sigma-1)/\sigma} = M_H^{-(\sigma-1)} \left(eL_H\right)^{\sigma-1} {p_{HH}^1}^{-(\sigma-1)} = \left[\frac{p_{HH}^1}{P_H}\right]^{-(\sigma-1)},$$

$$s_{OH}^1 = \left[\frac{p_{OH}^1}{P_H}\right]^{-(\sigma-1)}$$

where

$$P_H = \left[n_H^1 {p_{HH}^1}^{-(\sigma-1)} + n_O^1 {p_{OH}^1}^{-(\sigma-1)} + n_H^2 {p_{HH}^2}^{-(\sigma-1)} + n_O^2 {p_{OH}^2}^{-(\sigma-1)} \right]^{-1/(\sigma-1)}.$$

It follows that

$$S_H^1 = \frac{{p_{HH}^1}^{-(\sigma-1)} (n_H^1 + \phi n_O^1)}{P_H^{-(\sigma-1)}},$$

where I use the fact that ${p_{OH}^1}^{-(\sigma-1)} = \phi {p_{HH}^1}^{-(\sigma-1)}$, with $\phi \equiv \tau^{-(\sigma-1)}$. Employing (A.10) I get

$$S_H^1 = \frac{\sigma^{-(\sigma-1)}}{(\sigma-1)^{-(\sigma-1)} (1 - S_H^1)^{-(\sigma-1)}} \cdot \frac{n_H^1 + \phi n_O^1}{n_H^1 {p_{HH}^1}^{-(\sigma-1)} + n_O^1 \phi {p_{HH}^1}^{-(\sigma-1)} + n_H^2 {p_{HH}^2}^{-(\sigma-1)} + n_O^2 \phi {p_{HH}^2}^{-(\sigma-1)}}.$$

Since

$$p_{HH}^2 = \frac{\sigma}{(\sigma-1)(1 - S_H^2)} = \frac{\sigma}{(\sigma-1) S_H^1},$$

after some rearrangement and simplification I get

$$S_H^1 = \frac{(n_H^1 + \phi n_O^1)(1 - S_H^1)^{\sigma-1}}{(n_H^1 + \phi n_O^1)(1 - S_H^1)^{\sigma-1} + (n_H^2 + \phi n_O^2)(S_H^1)^{\sigma-1}}. \tag{A.13}$$

I explicit with respect to $1 - S_H^1$ and I get

$$1 - S_H^1 = \left(\frac{n_H^2 + \phi n_O^2}{n_H^1 + \phi n_O^1}\right)^{1/\sigma} S_H^1.$$

Substituting this expression in (A.13) and simplifying I finally get:

$$S_H^1 = \frac{(n_H^1 + \phi n_O^1)^{1/\sigma}}{(n_H^1 + \phi n_O^1)^{1/\sigma} + (n_H^2 + \phi n_O^2)^{1/\sigma}}.$$

A.4. Derivation of total profits

Total profits of MNC 1 can be written as

$$\Pi^1 = \pi_H^1 + \pi_O^1 - (n_H^1 + n_O^1 \Gamma_O) F = (n_H^1 \pi_{HH}^1 + n_O^1 \pi_{OH}^1) + (n_H^1 \pi_{HO}^1 + n_O^1 \pi_{OO}^1) - (n_H^1 + n_O^1 \Gamma_O) F,$$

where π_{HH}^1 are the operating profits in country H from sales of a variety located in country H by MNC 1, π_{OH}^1 are the operating profits in country H from sales of a variety located in country O by MNC 1, etc. It is also true that

$$\pi_H^1 = n_H^1 \pi_{HH}^1 + n_O^1 \pi_{OH}^1 = n_H^1 (p_{HH}^1 - 1) c_{HH}^1 + n_O^1 (p_{OH}^1 - \tau) c_{OH}^1 = S_H^1 e L_H \left(\frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} S_H^1\right),$$

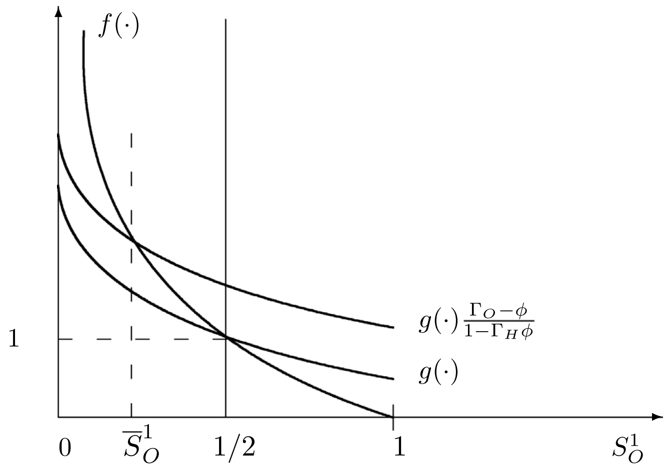


Fig. A.1. Graphical representation of the solution $\overline{S}^1_O$.

where I use (A.9) and (A.11) to infer that

$$p^1_{HH} - 1 = p^1_{HH}\left(\frac{1}{\sigma} + \frac{\sigma-1}{\sigma}S^1_H\right) \quad (A.14)$$

$$p^1_{OH} - \tau = p^1_{OH}\left(\frac{1}{\sigma} + \frac{\sigma-1}{\sigma}S^1_H\right) \quad (A.15)$$

and I use the fact that $n^1_H p^1_{HH} c^1_{HH} + n^1_O p^1_{OH} c^1_{OH} = S^1_H e L_H$. Working out similar expressions for the operating profits in country O by MNC 1, π^1_O, it is then possible to write total profits by MNC 1 as

$$\Pi^1 = \frac{e}{\sigma}\left\{S^1_H L_H[1+(\sigma-1)S^1_H] + S^1_O L_O[1+(\sigma-1)S^1_O]\right\} - (n^1_H + n^1_O \Gamma_O)F.$$

A.5. Proof of Lemma 1

Let us start from the equilibrium condition

$$\left(\frac{1-S^1_O}{S^1_O}\right)^{\sigma} = \frac{1+2(\sigma-1)(1-S^1_O)}{1+2(\sigma-1)S^1_O}\,\frac{\Gamma_O-\phi}{1-\Gamma_H\phi} \quad (A.16)$$

which implicitly defines the equilibrium market share $\overline{S}^1_O$. The domain for S^1_O is $(0,1)$, and $\sigma > 1$. The equilibrium condition can be written as

$$f(S^1_O;\sigma) = g(S^1_O;\sigma)\frac{\Gamma_O-\phi}{1-\Gamma_H\phi}, \quad (A.17)$$

where

$$f(S^1_O;\sigma) = \left(\frac{1-S^1_O}{S^1_O}\right)^{\sigma}, \quad g(S^1_O;\sigma) = \frac{1+2(\sigma-1)(1-S^1_O)}{1+2(\sigma-1)S^1_O}.$$

First of all notice that, for every $0 \le \phi < 1$,

$$\frac{\Gamma_O-\phi}{1-\Gamma_H\phi} \ge 1$$

with the strict inequality holding when: Γ_O is greater than one; or $\Gamma_O = 1$, $\Gamma_H > 1$, and $0 < \phi < 1$. The two functions f and g are monotonically decreasing and convex on the domain. Moreover, when S^1_O approaches zero f goes to infinity, while g is positive and finite. When S^1_O goes to one, f is zero and g is positive and finite. Evaluating f and g for $S^1_O = 1/2$, I get that $f(1/2;\sigma) = 1$, and $g(1/2;\sigma) = 1$. Then, there exists a unique value for S^1_O so that (A.17) is verified, and it must also be that such a unique value is $0 < \overline{S}^1_O \le 1/2$. The same kind of reasoning applies to $\overline{S}^2_H$. In Fig. A.1 I provide a graphical representation of the solution. From the figure it is also apparent that in order to have $\overline{S}^1_O = 1/2$ it is needed that both Γ_H and Γ_O be equal to one; or $\Gamma_O = 1$ and $\phi = 0$. The solution is well-defined provided that $0 \le \phi < 1$.

A.6. Proof of Proposition 2

I perform comparative statics analysis on the parameters Γ_H, Γ_O, and ϕ. These parameters enter (A.17) through the last multiplicative term only (a non-negative constant). Notice that

$$\partial\left(\frac{\Gamma_O-\phi}{1-\Gamma_H\phi}\right)\Big/\partial\Gamma_k>0,\quad\text{with}\quad k=\{H,O\}$$
$$\partial\left(\frac{\Gamma_O-\phi}{1-\Gamma_H\phi}\right)\Big/\partial\phi>0,$$

so that rising FDI costs and rising freeness of trade increase the right-hand side in (A.17). Given the shapes of f and g portrayed in Fig. A.1, an increase in the right-hand side of (A.17) induces a decrease in the value of S_O^1 that solves (A.17).

A.7. Proof of Proposition 3

After totally differentiating with respect to S_O^1 and σ in (A.17), the derivative of S_O^1 with respect to σ can be expressed as

$$\frac{\mathrm{d}S_O^1}{\mathrm{d}\sigma}=-\frac{\frac{\partial f}{\partial\sigma}-\frac{\partial g}{\partial\sigma}\frac{\Gamma_H-\phi}{1-\Gamma_O\phi}}{\frac{\partial f}{\partial S_O^1}-\frac{\partial g}{\partial S_O^1}\frac{\Gamma_H-\phi}{1-\Gamma_O\phi}}. \tag{A.18}$$

The denominator in (A.18) can be written as $\partial\left(f-g\frac{\Gamma_H-\phi}{1-\Gamma_O\phi}\right)\Big/\partial S_O^1$ and is negative given the shape of the two functions. Then, I get that S_O^1 is monotonically increasing in σ if and only if the numerator in (A.18) is positive:

$$\left(\frac{1-S_O^1}{S_O^1}\right)^{\sigma}\log\left(\frac{1-S_O^1}{S_O^1}\right)>\frac{2-4S_O^1}{[1+2S_O^1(\sigma-1)]^2}\frac{\Gamma_H-\phi}{1-\Gamma_O\phi}. \tag{A.19}$$

I am interested in evaluating the expressions in (A.19) when (A.16) is verified; that is, when $S_O^1=\overline{S}_O^1$. Substituting (A.16) in (A.19) and rearranging I get a simplified expression:

$$\log\left(\frac{1-S_O^1}{S_O^1}\right)>\frac{2(1-2S_O^1)}{1+2S_O^1(\sigma-1)}\cdot\frac{1}{1+2(1-S_O^1)(\sigma-1)}. \tag{A.20}$$

First of all, it is easy to check that the right-hand side of (A.20) is smaller than $2(1-2S_O^1)$:

$$2(1-2S_O^1)>\frac{2(1-2S_O^1)}{1+2S_O^1(\sigma-1)}\cdot\frac{1}{1+2(1-S_O^1)(\sigma-1)}.$$

Then, the final step is to check that the following inequality holds:

$$\log\left(\frac{1-S_O^1}{S_O^1}\right)>2(1-2S_O^1).$$

Through numerical methods, it can be checked that this inequality is always verified if $S_O^1\in(0,1/2)$.

A.8. Proof of Proposition 4

Taking the partial derivative of

$$n_O^1=\frac{eL_O[1+2(\sigma-1)\overline{S}_O^1]\overline{S}_O^1(1-\overline{S}_O^1)}{\sigma^2F(\Gamma_O-\phi)}-\frac{\phi}{F(1-\Gamma_O\phi)}\frac{eL_H[1+2(\sigma-1)\overline{S}_H^1]\overline{S}_H^1(1-\overline{S}_H^1)}{\sigma^2}$$

with respect to $\overline{S}_H^1$ we get that

$$\frac{\partial n_O^1}{\partial\overline{S}_H^1}=-\frac{\phi}{F(1-\Gamma_O\phi)}\frac{eL_H}{\sigma^2}\left[-6(\overline{S}_H^1)^2(\sigma-1)+4\overline{S}_H^1\left(\sigma-\frac{3}{2}\right)+1\right]. \tag{A.21}$$

The function in square brackets is a parabola. The parabola opens downward. In order to identify the intercepts with the horizontal axis of the parabola, when the parabola has a positive value, and hence the sign of (A.21), I need to solve the following second order equation:

$$-6(\overline{S}_H^1)^2(\sigma-1)+4\overline{S}_H^1\left(\sigma-\frac{3}{2}\right)+1=0.$$

I get that the two solutions are

$$\overline{S}_H^1=\frac{2\sigma-3\mp\sqrt{4\sigma^2-6\sigma+3}}{6(\sigma-1)}. \tag{A.22}$$

The intercept with the vertical axis is always equal to 1, so this means that one solution in (A.22) is negative and the other is positive. I call the positive solution $\overline{S}^+$, and it equals

$$\overline{S}^+ = \frac{2\sigma - 3 + \sqrt{4\sigma^2 - 6\sigma + 3}}{6(\sigma - 1)}.$$

$\overline{S}^+$ is characterized by the following limits:

$$\lim_{\sigma\to 1} \overline{S}^+ = \frac{1}{2},$$
$$\lim_{\sigma\to\infty} \overline{S}^+ = \frac{2}{3}.$$

Moreover, it can be easily checked that

$$\frac{\partial \overline{S}^+}{\partial \sigma} = \frac{1 - \frac{\sigma}{\sqrt{4\sigma^2-6\sigma+3}}}{6(\sigma - 1)^2} > 0.$$

So, given any value of σ, it is always true that $1/2 < \overline{S}^+ < 2/3$. Whenever the parabola has a positive value, the derivative (A.21) is negative, and this happens for every $\overline{S}^1_H$ such that $1/2 < \overline{S}^1_H < \overline{S}^+$. The length of the interval $(1/2, \overline{S}^+)$ is increasing in σ. The length shrinks to zero when σ tends to one and reaches the maximum value of $2/3 - 1/2 = 1/6$ when σ tends to infinity. Then, a necessary condition for the occurrence of $\partial n^1_O/\partial\overline{S}^1_H < 0$ is that $1/2 < \overline{S}^1_H < 2/3$.

Turning to the partial derivative of n^1_H with respect to $\overline{S}^1_H$, it is equal to

$$\frac{\partial n^1_H}{\partial \overline{S}^1_H} = \frac{1}{F(1-\Gamma_O\phi)}\frac{eL_H}{\sigma^2}\left[-6(\overline{S}^1_H)^2(\sigma-1) + 4\overline{S}^1_H\left(\sigma - \frac{3}{2}\right) + 1\right].$$

Following the same reasoning as above, it is straightforward to check that it is greater than zero when $1/2 < \overline{S}^1_H < \overline{S}^+$, and less than zero when it is above the threshold.

It easy to check that the threshold for $\overline{S}^2_O$, such that $\partial n^2_H/\partial\overline{S}^2_O < 0$ and $\partial n^2_O/\partial\overline{S}^2_O > 0$, is also equal to $\overline{S}^+$.

A.9. Proof of Proposition 5

The starting point for the proof is Proposition 2. That proposition predicts that lowering, say, Γ_H leads to a rise in $\overline{S}^1_O$ and $\overline{S}^2_H$, and to a decline in $\overline{S}^1_H$ and $\overline{S}^2_O$. With these comparative statics results in mind it is possible to analyze the behaviour of the number of varieties in each market. Let us focus on the equation characterizing n^2_H,

$$n^2_H = \frac{eL_H[1 + 2(\sigma-1)\overline{S}^2_H]\overline{S}^2_H(1-\overline{S}^2_H)}{\sigma^2 F(\Gamma_H - \phi)} - \frac{\phi}{F(1-\Gamma_H\phi)}\frac{eL_O[1+2(\sigma-1)\overline{S}^2_O]\overline{S}^2_O(1-\overline{S}^2_O)}{\sigma^2}.$$

Decreasing Γ_H will increase the first right-hand side term. It will also decrease the second right-hand side term, provided that $\overline{S}^2_O$ is less than $\overline{S}^+$. Since the second right-hand side term is preceded by a minus, the overall impact will be positive again. A similar reasoning leads to the following behaviour for the other stocks of varieties after a decrease in Γ_H: n^1_H goes down, n^1_O goes up, and n^2_O goes down.

A.10. Proof of Lemmas 2 and 3

Let us begin with Lemma 2 by showing that $\Delta_H > 0$ and $\Delta_O > 0$. The two terms are greater than zero under the following assumptions. The first assumption is that the numerator be greater than zero, i.e. $1 - (\Gamma_H + \Gamma_O)\phi + \phi^2 > 0$. I will prove in a while that this condition is necessarily met when $n^1_O + n^2_H$ is greater than zero. Turning to the denominator, it is greater than zero under the parameters' restrictions of the model. For example, the denominator of Δ_H is greater than zero when the inequalities $(\Gamma_H - \phi) > (1 - \Gamma_O\phi) > 0$ are verified, something which is true (see Fig. A.2).

Moreover, it can be easily proved that Δ_H and Δ_O are decreasing in ϕ. Additionally, they are also decreasing in the cost of investing abroad, $\partial\Delta_i(\Gamma_H, \Gamma_O, \phi)/\partial\Gamma_j < 0$, for $i = \{H, O\}$ and $j = \{H, O\}$.

Let us now turn to Lemma 3. In order to verify that Λ_H and Λ_O are greater than 1 let us check the following. First, one needs to check that both the numerator and the denominator are greater than zero. In the case of Λ_H, for example, it is easily checked that the numerator is greater than zero since $(1-\Gamma_O\phi)+\phi(\Gamma_H-\phi) > 0$. The denominator, $1-(\Gamma_H+\Gamma_O)\phi+\phi^2$, is equal to the numerator of Δ_H and Δ_O and has to be greater than zero as well. This condition has to be satisfied if we want the total number of foreign varieties $n^1_O + n^2_H$ to be greater than zero. To understand this, consider Eq. (37) and assume that $1 - (\Gamma_H + \Gamma_O)\phi + \phi^2 < 0$. If this is the case, the terms Δ_H, Δ_O, Λ_H and Λ_O are all negative. Since the terms in curly brackets in (37) are positive in this case, both addends on the right-hand side of (37) are negative and so we get that $n^1_2 + n^2_1$ is negative as well. But such a case is not meaningful from an economic point of view, and so, without loss of generality, we can assume that $1-(\Gamma_H+\Gamma_O)\phi+\phi^2 > 0$. Finally, it is easily checked that the numerator in Λ_H and Λ_O is greater than the denominator, and so both Λ_H and Λ_O are greater than one.

It can also be easily checked that the derivative of the MSE terms with respect to Γ_H and Γ_O are greater than zero. Imposing that the derivative of Λ_H with respect to ϕ is positive boils down to $\Gamma_H-\phi-\phi(1-\Gamma_O\phi) > 0$, which is true provided that $(\Gamma_H-\phi) > (1-\Gamma_O\phi)$.

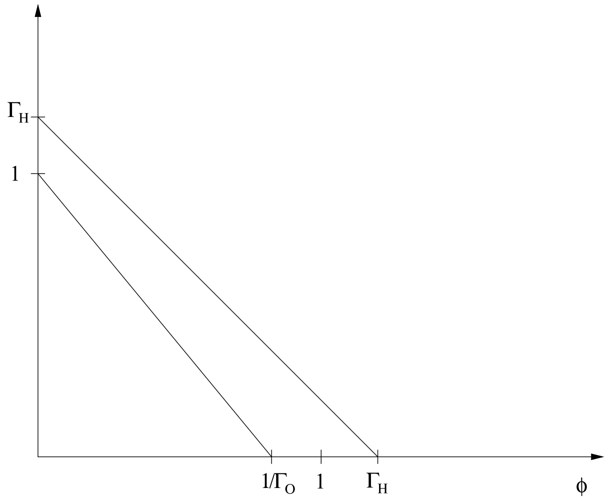


Fig. A.2. Graphical proof that, under the parameters' restrictions of the model, $\Gamma_H - \phi > 1 - \Gamma_O \phi$.

A.11. Proof of Proposition 7

The proof follows from Proposition 2, Lemmas 2, and 3. Proposition 2 shows that foreign market shares go down when FDI frictions and freeness of trade go up. Hence also the two terms $\overline{S}^1_O(1-\overline{S}^1_O)$ and $\overline{S}^2_H(1-\overline{S}^2_H)$ are decreasing. The terms Δ_H and Δ_O are also decreasing, according to Lemma 2. Finally, Lemma 3 states that Λ_H and Λ_O are increasing in FDI frictions and freeness of trade. This implies that the whole MSE term is decreasing (remember that $\overline{S}^1_O$ and $\overline{S}^2_H$ are decreasing). The combination of these effects unambiguously points towards a decrease of the world FDI stock $n^1_O + n^2_H$.

A.12. Proof of Proposition 8

Let us rewrite the total domestic and foreign investment stocks. For the total domestic investment stock I have

$$
\begin{aligned}
n^1_H + n^2_O = {} & \frac{eL_O\overline{S}^1_O(1-\overline{S}^1_O)}{\sigma^2 F}\left\{\Theta_O + 2(\sigma-1)\Theta_O\left[\frac{1}{2} + \Xi_O\left(\frac{1}{2} - \overline{S}^1_O\right)\right]\right\} \\
& + \frac{eL_H\overline{S}^2_H(1-\overline{S}^2_H)}{\sigma^2 F}\left\{\Theta_H + 2(\sigma-1)\Theta_H\left[\frac{1}{2} + \Xi_H\left(\frac{1}{2} - \overline{S}^2_H\right)\right]\right\},
\end{aligned}
\tag{A.23}
$$

where

$$
\begin{aligned}
\Theta_O &\equiv \frac{\Gamma_O - 2\phi + \Gamma_H\phi^2}{(1-\Gamma_H\phi)(\Gamma_O-\phi)}, \\
\Xi_O &\equiv \frac{\Gamma_O - \Gamma_H\phi^2}{\Gamma_O - 2\phi + \Gamma_H\phi^2}, \\
\Theta_H &\equiv \frac{\Gamma_H - 2\phi + \Gamma_O\phi^2}{(1-\Gamma_O\phi)(\Gamma_H-\phi)}, \\
\Xi_H &\equiv \frac{\Gamma_H - \Gamma_O\phi^2}{\Gamma_H - 2\phi + \Gamma_O\phi^2}.
\end{aligned}
$$

For the total FDI stock I have

$$
\begin{aligned}
n^1_O + n^2_H = {} & \frac{eL_O\overline{S}^1_O(1-\overline{S}^1_O)}{\sigma^2 F}\left\{\Delta_O + 2(\sigma-1)\Delta_O\left[\frac{1}{2} - \Lambda_O\left(\frac{1}{2} - \overline{S}^1_O\right)\right]\right\} \\
& + \frac{eL_H\overline{S}^2_H(1-\overline{S}^2_H)}{\sigma^2 F}\left\{\Delta_H + 2(\sigma-1)\Delta_H\left[\frac{1}{2} - \Lambda_H\left(\frac{1}{2} - \overline{S}^2_H\right)\right]\right\}.
\end{aligned}
\tag{A.24}
$$

Proposition 7 tells that $n^1_O + n^2_H$ is always decreasing in any parameter $\psi = \{\Gamma_H, \Gamma_O, \phi\}$. Moreover, each of the two terms in curly brackets in (A.24) is decreasing in ψ. Expressions (A.23) and (A.24) share the terms $eL_O\overline{S}^1_O(1-\overline{S}^1_O)/\sigma^2 F$ and $eL_H\overline{S}^2_H(1-\overline{S}^2_H)/\sigma^2 F$, which are both decreasing in any ψ. When each of the two terms in curly brackets in (A.23) has a positive derivative with respect to any ψ, then necessarily

$$
\frac{\partial(n^1_H + n^2_O)}{\partial\psi} > \frac{\partial(n^2_H + n^1_O)}{\partial\psi},
$$

and this is what we are going to check below. Let us write

$$\Theta_O + 2(\sigma-1)\Theta_O\left[\frac{1}{2} + \Xi_O\left(\frac{1}{2} - \overline{S}^1_O\right)\right] = \Theta_O + 2(\sigma-1)\left[\frac{1}{2}\Theta_O + \Xi_O\Theta_O\left(\frac{1}{2} - \overline{S}^1_O\right)\right], \tag{A.25}$$

where

$$\Xi_O\Theta_O = \frac{\Gamma_O - \Gamma_H\phi^2}{(1-\Gamma_H\phi)(\Gamma_O - \phi)}.$$

While inspecting the righthand side of (A.25), it is straightforward to check that $\partial\Theta_O/\partial\Gamma_i > 0$, with $i = \{H, F\}$. The condition $\partial\Theta_O/\partial\phi > 0$ boils down to checking that

$$\frac{\Gamma_H}{(\Gamma_H\phi - 1)^2} > \frac{\Gamma_O}{(\Gamma_O - \phi)^2},$$

which is verified given that $\phi\Gamma_H < 1 < \Gamma_O/\phi$. The term $\left(\frac{1}{2} - \overline{S}^1_O\right)$ is also increasing in each ψ. The derivative of $\Xi_O\Theta_O$ with respect to Γ_H and ϕ can be easily checked to be positive. But if we consider in isolation the derivative $\partial\Xi_O\Theta_O/\partial\Gamma_O$, it is negative. Then, for the derivative with respect to Γ_O, in order to assess that the partial derivative is positive we differentiate the entire term

$$\frac{\partial\left[\frac{1}{2}\Theta_O + \Xi_O\Theta_O\left(\frac{1}{2} - \overline{S}^1_O\right)\right]}{\partial\Gamma_O} = \frac{1}{2}\frac{\partial\Theta_O}{\partial\Gamma_O} + \frac{\partial\Xi_O\Theta_O}{\partial\Gamma_O}\left(\frac{1}{2} - \overline{S}^1_O\right) + \Xi_O\Theta_O\frac{\partial\left(\frac{1}{2} - \overline{S}^1_O\right)}{\partial\Gamma_O} =$$
$$\overline{S}^1_O\frac{\phi}{(\Gamma_O - \phi)^2} + \Xi_O\Theta_O\frac{\partial\left(\frac{1}{2} - \overline{S}^1_O\right)}{\partial\Gamma_O}.$$

Provided that

$$\Xi_O\Theta_O\frac{\partial\left(\frac{1}{2} - \overline{S}^1_O\right)}{\partial\Gamma_O} > 0$$

we can say that

$$\frac{\partial\left[\frac{1}{2}\Theta_O + \Xi_O\Theta_O\left(\frac{1}{2} - \overline{S}^1_O\right)\right]}{\partial\Gamma_O} > 0,$$

and we can conclude that the term in curly brackets in (A.23)

$$\Theta_O + 2(\sigma-1)\Theta_O\left[\frac{1}{2} + \Xi_O\left(\frac{1}{2} - \overline{S}^1_O\right)\right]$$

is increasing in every investment and trade policy parameter ψ. Given symmetry, the proof that also the other term

$$\Theta_H + 2(\sigma-1)\Theta_H\left[\frac{1}{2} + \Xi_H\left(\frac{1}{2} - \overline{S}^2_H\right)\right]$$

is increasing in every parameter ψ descends in a similar manner.

References

Atkeson, A., Burstein, A., 2008. Pricing-to-market, trade costs, and international relative prices. Amer. Econ. Rev. 98, 1998–2031.

Baldwin, R., Forslid, R., Martin, P., Ottaviano, G., Robert-Nicoud, F., 2003. Economic Geography and Public Policy. Princeton University Press, Princeton.

Baldwin, R., Ottaviano, G.I.P., 1998. Multiproduct Multinationals and Reciprocal FDI Dumping. NBER Working Papers 6483, National Bureau of Economic Research.

Baldwin, R.E., Ottaviano, G.I.P., 2001. Multiproduct multinationals and reciprocal FDI dumping. J. Int. Econ. 54, 429–448.

Brainard, S.L., 1993. A Simple Theory of Multinational Corporations and Trade with a Trade-Off Between Proximity and Concentration. NBER Working Papers 4269, National Bureau of Economic Research.

Brander, J., Krugman, P., 1983. A 'reciprocal dumping' model of international trade. J. Int. Econ. 15, 313–321.

d'Aspremont, C., Dos Santos Ferreira, R., Gérard Varet, L.-A., 1996. On the dixit-stiglitz model of monopolistic competition. Amer. Econ. Rev. 86, 623–629.

Eckel, C., Neary, J.P., 2010. Multi-product firms and flexible manufacturing in the global economy. Rev. Econom. Stud. 77, 188–217.

Feenstra, R., Ma, H., 2008. Optimal choice of product scope for multiproduct firms under monopolistic competition. In: Helpman, E., Marin, D., Verdier, T. (Eds.), The Organization of Firms in a Global Economy. Harvard University Press, Cambridge, MA.

Head, K., Spencer, B.J., 2017. Oligopoly in international trade: Rise, fall and resurgence. Can. J. Econ. 50 (5), 1414–1444.

Helpman, E., Melitz, M.J., Yeaple, S.R., 2004. Export versus FDI with heterogenous firms. Amer. Econ. Rev. 94, 300–316.

Minniti, A., Turino, F., 2013. Multiproduct firms and business cycle dynamics. Eur. Econ. Rev. 57, 75–97.

Neary, J.P., 2003. Globalization and market structure. J. Eur. Econom. Assoc. 1, 245–271.

Tintelnot, F., 2017. Global production with export platforms. Q. J. Econ. 132, 157–209.

Yang, X., Heijdra, J., 1993. Monopolistic competition and optimum product diversity: Comment. Amer. Econ. Rev. 83, 295–301.

Yeaple, S.R., 2009. Firm heterogeneity and the structure of U.S. multinational activity. J. Int. Econ. 78, 206–215.