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# Temptation in Consumption and Optimal Taxation\*

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## Abstract

This article aims to integrate temptation preferences into the theory of optimal taxation with heterogeneous agents and asymmetric information. Consumers are tempted to over-consume a commodity which leads to an over-supply of labor. Resisting this temptation implies a utility cost and any policy that reduces this cost is welfare improving. We uncover novel channels for government intervention and the interaction between the welfare improving and redistributive roles of public policy. We also identify a commitment mechanism that works through the endogenous labor choice and affects the design and effectiveness of the optimal tax policy.

**JEL:** D03, H21, H24, H31

**Keywords:** Temptation, self-control, optimal taxation, redistribution, commodity taxation, income taxation.

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# 1. Introduction

Experimental and empirical evidence has shown that individuals may exhibit behavioral anomalies such as preference reversals, biases or self-control problems in intertemporal decision-making.<sup>1</sup> Such evidence has led to the development of a vast theoretical literature on time inconsistent behavior. Starting with Strotz (1956), and Phelps and Pollak (1968), a common feature in these frameworks is modelling the individual as a sequence of different “selves” who play a dynamic game vis-a-vis each other where each “self” values the consumption stream in a unique way.<sup>2</sup> In such models, the inherent dynamic inconsistency may call for commitment mechanisms. Gul and Pesendorfer (2001, 2004, 2005) instead suggest a different cause for a preference for commitment. Focusing on preferences over the choice sets rather than choices from the set, Gul and Pesendorfer (henceforth referred to as GP) argue that agents who suffer from, but resist, temptation will always prefer a smaller choice set as it will be associated with a lower cost of exercising self-control, therefore giving rise to a demand for commitment devices. This observation creates an obvious case for government intervention (in the absence of other commitment mechanisms): policies that restrict the choice set of an agent will reduce the cost of exercising self-control, thereby improving welfare. The purpose of the present paper is to analyze how the appearance of self-control costs modifies the optimal tax structure in a mixed tax framework with a linear commodity tax and nonlinear labor income taxes.

Krusell et al (2010) were the first to study how linear tax-transfer schemes can be used to improve the welfare in a representative consumer economy where agents are tempted towards current consumption, thereby distorting the incentive to save for tomorrow. They showed that a savings subsidy improves welfare by making succumbing to temptation less attractive. Using a multi-period framework with a finite time horizon, they also found that optimal savings subsidies increase over time for a logarithmic utility function. Tran (2018) modified this analysis by allowing for a labor/leisure choice and showed that the inclusion of elastic labor introduces an intra-temporal channel for temptation distortions through a consumption-leisure trade off. In this extended

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<sup>1</sup> Frederick, Loewenstein and O’Donoghue (2002) provide an excellent overview of the experimental and empirical literature on this issue. Ameriks et al. (2007) conduct a survey to measure self-control problems and find that self-control problems are smaller in scale for older than for younger individuals. Fang and Silverman (2001) empirically find the existence of time inconsistency that stems from self-control problems. Huang et al (2007) and Bucciol (2012) study the empirical relevance of self-control preferences using household level data from the Consumer Expenditure Survey and find evidence supporting the presence of temptation.

<sup>2</sup> See the literature review in section 2 for other relevant studies.

framework with both an intra-temporal and an inter-temporal channel for temptation distortions, a mix of linear labor and capital taxes appears to be more effective in improving welfare than solely relying on capital taxation.

The studies mentioned above have contributed greatly to our understanding of how taxes can be used to improve welfare when agents have GP preferences. However, key aspects remain unexplored when it comes to linking GP preferences to traditional optimal tax theory. Representative consumer models typically abstract from the information problem that arises in an economy with heterogeneous consumers. The latter feature is accounted for in the literature on optimal nonlinear taxation.<sup>3</sup> In that context, consumers typically differ in terms of their labor market productivities and as long as the latter is private information, the government faces an information constraint when solving the optimal tax and expenditure problem. This information asymmetry imposes a restriction on the government's ability to redistribute resources between different consumer types and is an important determinant of tax policy. Therefore, a natural extension is to incorporate GP preferences into the optimal nonlinear tax framework and analyze how this will affect and modify the optimal policy rules in comparison to those derived in the conventional framework. Such an extension is important (i) because many real-world tax systems feature nonlinear labor income tax schedules accompanied by linear taxes on other tax bases and (ii) because a government's decision to implement distortionary taxes will in that context be an optimal choice subject to informational constraints: the tax rules do not arise because of arbitrary restrictions on the tax instruments. This means that a model that features nonlinear income taxation provides a suitable framework for analyzing the basic question of how the appearance of GP preferences itself motivates the use of distortionary taxes. In particular, it allows us to study two important issues: to what degree GP preferences themselves motivate the use of distortionary taxes and how do GP related components and redistributive components interact in a mixed tax framework.

In line with the discussion above, we extend the analysis of optimal taxation when agents have GP preferences in two directions. The first is by relating temptation to a consumption good: we will refer to a good for which consumers may experience temptation as a temptation preference

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<sup>3</sup> See the seminal work of Stern (1982), Stiglitz (1982), and Edwards, Keen and Tuomala (1994).

(TP) good.<sup>4</sup> The second is by incorporating the GP framework into the theory of optimal nonlinear labor income taxation with two ability types and asymmetric information between the private sector and the government. As such, this is an extension of the two-type optimal income tax model developed by Stern (1982) and Stiglitz (1982). The policy instruments consist of a linear commodity tax and nonlinear labor income taxes. To the best of our knowledge, this is the first paper which uses a self-selection approach to analyze optimal taxation in the presence of GP preferences. Note, that if a nonlinear commodity tax would be available, then it would be easy for the government to circumvent the self-control problem by implementing a commodity tax structure, which reduces the consumer's choice set to a singleton. However, since nonlinear commodity taxes are not commonly observed in reality (e.g., due to informational limitations), we follow convention and focus on linear commodity taxation. If we compare our analysis with that of Tran (2018), we differ from that study by introducing heterogeneity in the agents' earning abilities and by focusing on optimal nonlinear income taxation. This allows us to characterize optimal tax rules in a general framework and to uncover novel channels for government intervention.

Our results contribute to the literature in a number of ways. To start with, we identify a welfare motive to implement a positive tax on the TP commodity and a positive marginal income tax on labor income in a representative consumer framework. In the context of heterogeneous agents, we show that if the low-ability type tries to replicate the income of the high-ability type when he succumbs to temptation, the labor income tax intended for the high-ability type will affect the welfare also for the low-ability type. This mechanism provides a motive to implement a non-zero marginal income tax rate on the high-ability type (the top-income earner). This is a novel result because a conventional optimal tax model prescribes that the top-income earner should face a zero marginal income tax rate.

We are also the first to discuss how the timing of labor decisions can affect the effectiveness of the policy instruments in a framework with GP preferences. In a pre-commitment scenario, labor

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<sup>4</sup> There are many examples of when an individual makes a consumption plan which he later is tempted to deviate from. Consider, for example, a person who in the morning plans to eat a healthy salad at lunch but when lunchtime actually comes, he may face more tempting alternatives such as fish, burgers, steaks etc. Another example is a person who plans to buy a car with a certain set of attributes and within in a certain price range, but after she has test driven some cars, she may be tempted to buy a fancier and more expensive car than originally planned. Although there is a similar interpretation, our approach differs from models focusing on addictive goods (Gruber and Koszegi 2001, 2004) and sin goods (O'Donoghue and Rabin [2006], Allcott, H., Lockwood, B. B., & Taubinsky, D. (2019)).

supply is agreed upon in a contract before consumption takes place which means that the hours of work are determined without the influence of temptation and are fixed at the time of consumption. This implies that effectively the agent acts as a planner and commits to a budget set recognizing how the labor choice will affect the self-control cost that arises from resisting the tempting alternatives when consumption takes place. Therefore, there is no welfare motive for taxing labor income in this scenario. Instead, there arises a self-selection motive for commodity taxation which reflects that the commodity tax affects the utility cost of exercising self-control both for the high-ability type and for the potential mimicker. This provides the government with an additional channel, compared with the conventional optimal tax model, via which the commodity tax can be used to relax a binding self-selection constraint. With iso-elastic functional forms, we identify when this channel contributes to a higher/lower commodity tax.

The outline of the study is as follows: Section 2 contains a literature review. In Section 3, we characterize the basic model and in Section 4 we consider optimal mixed taxation in a representative consumer framework. In Section 5, we introduce GP preferences into an optimal tax model with two agent types. In Section 6 we address the case of the labor choice as a commitment device. The paper is concluded in Section 7 and proofs are presented in the Appendix.

## 2. Literature Review

Our paper belongs to a long stream of literature dealing with issues of present bias and time inconsistency. Strotz (1956) was the first to suggest a model where an agent's future behavior is inconsistent with her optimal plan which, in turn, gives rise to a demand for pre-commitment devices. Following the same idea, Phelps and Pollak (1968) introduced “hyperbolic discounting”, where the discount rate is different in the short and long run. In their work, they focused on second-best national saving when the present generation lacks the power to commit future generations' decisions, while Laibson (1997) models time-inconsistency within an individual in the presence of an imperfect commitment technology. In the same spirit, O'Donoghue and Rabin (1999, 2001) explore the welfare and behavioral implications of present-biased preferences and procrastination.

An alternative framework and the one we follow here, is the “temptation preferences” approach, first suggested by Gul and Pesendorfer (2001). Following Kreps (1979), GP develop an axiomatic approach of temptation and self-control preferences over menus together with a representation theorem in a two-period model, which can be summarized as follows. In the first period, agents

choose over menus of lotteries while in the second, they choose an alternative from the menu. However, agents are subject to temptation: at the time of actual consumption, they suffer from an urge to deviate from their “commitment” preferences,  $u(x)$ , which prescribe what they “should” do, and instead evaluate alternatives according to their “temptation” preferences,  $h(x)$ , which is what they “want” to do. In this framework, an agent’s welfare from a given set is determined by the maximized value of the sum of the commitment and temptation utilities minus the temptation utility evaluated at the most tempting alternative of the menu. Naturally, this representation suggests the following choice behavior in the second period: given a menu  $A$ , an agent’s actual choice maximizes  $u(x) + h(x)$  while the agent at the same time experiences a cost of exercising self-control which is given by  $\max_{\tilde{x} \in A} h(\tilde{x}) - h(x)$ . Therefore, the agent’s second period choice behavior represents a compromise between the utility that could have been achieved under commitment and the cost associated with exercising self-control.

It becomes evident that a key difference between the two approaches lies in the motives that drive a decision maker to intervene: a present biased agent will choose to remove a temptation from his choice set only if he expects to yield to it while an agent with temptation preferences may value commitment even if he expects to never give in, simply because commitment will reduce or even eliminate the cost of self-control. From a policy perspective, this implies that the welfare benefits of policies that restrict access to tempting alternatives might be much larger than what models with exponential or quasi-hyperbolic discounting would imply as non-consequentialist cost such as self-control cost would not be considered. In fact, Toussaert (2018) found that a quarter to a third of subjects exhibit GP preferences in a lab experiment. Therefore, it is of particular importance to study how self-control preferences can affect conventional public policy.

There is indeed a growing literature that studies questions of public policy with present biased agents. Aronsson and Sjögren (2014, 2016) were the first to integrate quasi hyperbolic discounting into the modern literature on optimal mixed taxation. They employ non-linear income taxation and linear commodity taxation to correct for the welfare effects of quasi-hyperbolic discounting and redistribute from high to low skill individuals. Guo and Krause (2015) follow a similar analysis and show analytically and quantitatively how quasi-hyperbolic discounting can actually raise long run utility and social welfare. Lockwood (2020) focuses on the effect of present bias on labor supply decisions and show that present bias tends to lower marginal income tax rates.

On the other hand, the literature on the design of optimal policies when agents have temptation preferences is rather limited. As mentioned in the introduction, Krusell et al (2010) and Tran (2018) were among the first to analyze how taxes can be used to improve welfare. There are also some other studies which are concerned with optimal taxation when agents have GP preferences. Kumru and Thanopoulos (2015) quantitatively examine the impact of fiscal policies in a stochastic OLG-model where agents can have standard or GP preferences and are subject to idiosyncratic shocks and borrowing constraints. They find that the presence of self-control agents puts a downward pressure on the optimal capital tax. While the size of the tax depends on the share of GP preferences and on the self-control cost, it remains positive for all empirically relevant values. Bethencourt and Kunze (2017) study the optimal taxation of education and labor and show that the size and direction of taxes depend on the strength of temptation, the elasticity of earnings and the sensitivity to taxes. St-Amant, P. A. B. & Garon, J. D. (2015) study optimal redistributive pension schemes when agents are tempted by immediate consumption. Tran (2016) shows that the savings subsidy and social security programs can be properly designed to mitigate the adverse effect of succumbing to temptation and release severity of self-control in a two-period partial equilibrium OLG model. Amador et al (2006) also use nonlinear tax instruments but in a different setting: they study the optimal trade-off between commitment and flexibility in a consumption-savings model where agents have GP preferences but are subject to taste shocks. They find that a minimum savings rule is always part of the optimal solution. Instead, Farhi and Gabaix (2020) develop a theory of optimal taxation in a general framework that allows for a wide range of behavioral biases, such as misperceptions and internalities, as well as externalities and population heterogeneity.

There is a closely related literature on optimal commodity taxation when agents suffer from self-control issues: Gruber and Koszegi (2001, 2004) focus on addictive goods and O'Donoghue and Rabin (2006), Allcott, H., Lockwood, B. B., & Taubinsky, D. (2019) study sin goods and the interaction between the corrective and redistributive motives in designing “sin” taxes. A key difference between these setups and our setup is that agents’ GP preferences are time-consistent, which is not the case for sin goods.

### 3. The Basic Model



Consider an economy made up of a large number of identical consumers whose number is normalized to one. Each individual consumes  $c$  units of a numeraire good,  $x$  units of a non-numeraire good which will be referred to as a temptation preference (TP) good, and  $z$  units of leisure. The consumer has Gul-Pesendorfer (GP) preferences which are captured by two functions; a normative utility function  $u(c, x, z) = a(c) + d(x) + f(z)$  which prescribes what an agent should do, and a temptation utility function  $h(c, x, z) = a(c) + \rho d(x) + f(z)$  which shows what she is tempted to do.<sup>5</sup> These functions are twice continuously differentiable, increasing and concave in their respective arguments. The parameter  $\rho$  exceeds one which ensures that the marginal temptation utility of consuming the TP good exceeds the corresponding marginal normative utility for given levels of  $x$ . The decision problem of the consumer is stated as follows

$$\max_{\check{c}, \check{x}, \check{z}} [u(\check{c}, \check{x}, \check{z}) + h(\check{c}, \check{x}, \check{z})] - \max_{\tilde{c}, \tilde{x}, \tilde{z}} h(\tilde{c}, \tilde{x}, \tilde{z}) \quad (1)$$

subject to a budget constraint that we specify below. Here  $\check{c}$ ,  $\check{x}$  and  $\check{z}$  denote the consumer's *actual* choices which maximize the sum of the normative and temptation utilities,  $u(\cdot) + h(\cdot)$ , while  $\tilde{c}$ ,  $\tilde{x}$  and  $\tilde{z}$  are the *temptation* choices that arise if the consumer would only maximize the temptation utility  $h(\cdot)$ . Note that the actual choices are the ones that materialize while the temptation choices affect the consumer's welfare only through the self-control cost that arises from resisting temptation, which we define as  $h(\tilde{c}, \tilde{x}, \tilde{z}) - h(\check{c}, \check{x}, \check{z})$ . Next, we present step by step the maximization problem of the consumer/worker as stated in equation (1). We start with the actual choices and continue with the temptation choices.

The maximization problem associated with the actual choices can be stated as follows

$$\max_{\check{c}, \check{x}, \check{z}} [u(\check{c}, \check{x}, \check{z}) + h(\check{c}, \check{x}, \check{z})] \quad \text{subject to} \quad \tilde{w}\check{l} - \check{T} = \check{c} + q\check{x}, \quad \check{z} = 1 - \check{l} \quad (2)$$

where  $\tilde{w}\check{l} - \check{T} = \check{c} + q\check{x}$  is the budget constraint,  $\tilde{w} = (1 - \check{\tau})w$  is the post-tax wage,  $w$  is the pre-tax wage,  $\check{\tau}$  is the (marginal) labor income tax rate,  $\check{l}$  is the actual hours of work and  $\check{T}$  is a lump-sum tax payment (or a lump-sum subsidy if  $\check{T} < 0$ ). The consumer price of the TP good is given by  $q = p + t$ , where  $p$  is a fixed producer price and  $t$  is a linear commodity tax. Actual leisure is defined as a time endowment normalized to one less the actual hours of work, i.e.  $\check{z} = 1 - \check{l}$ . When

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<sup>5</sup> A more general formulation would be to define  $h(c, x, z) = \lambda(\mu a(c) + \rho d(x) + \nu f(z))$  which allows for temptation related to both goods and leisure while  $\lambda$  is the strength of temptation. We leave this for future research.

solving the optimal tax problem to be defined below, it is convenient to work with indirect utility functions which are conditioned on leisure. To define these functions, let  $\tilde{b} = \tilde{\omega}\tilde{l} - \tilde{T}$  denote post-tax income and let us use  $\tilde{b} = \tilde{c} + q\tilde{x}$  as the budget constraint when choosing  $\tilde{c}$  and  $\tilde{x}$  to maximize the objective function defined in (2) conditional on  $\tilde{z}$ . The first-order condition associated with this maximization problem can be written as  $\overline{MRS}_{x,c} = q$ , where

$$\overline{MRS}_{x,c} = \frac{u_x(\tilde{x}) + h_x(\tilde{x})}{u_c(\tilde{c}) + h_c(\tilde{c})} = \left(\frac{1+\rho}{2}\right) \frac{d_x(\tilde{x})}{a_c(\tilde{c})} \quad (3)$$

Together with  $\tilde{b} = \tilde{c} + q\tilde{x}$ , this first-order condition implicitly defines the demand functions  $\tilde{x}(\tilde{b}, q)$  and  $\tilde{c}(\tilde{b}, q) = \tilde{b} - q\tilde{x}(\tilde{b}, q)$ . These functions can now be used to define the following conditional indirect utility functions associated with the actual consumption choices

$$\tilde{U} = \tilde{U}(\tilde{b}, \tilde{z}, q) = u(\tilde{c}(\tilde{b}, q), \tilde{x}(\tilde{b}, q), \tilde{z}) = a(\tilde{c}(\tilde{b}, q)) + d(\tilde{x}(\tilde{b}, q)) + f(\tilde{z}) \quad (4a)$$

$$\tilde{H} = \tilde{H}(\tilde{b}, \tilde{z}, q) = h(\tilde{c}(\tilde{b}, q), \tilde{x}(\tilde{b}, q), \tilde{z}) = a(\tilde{c}(\tilde{b}, q)) + \rho d(\tilde{x}(\tilde{b}, q)) + f(\tilde{z}) \quad (4b)$$

Substituting  $\tilde{b} = \tilde{\omega}\tilde{l} - \tilde{T}$  and  $\tilde{z} = 1 - \tilde{l}$  into (4a) and (4b) and maximizing  $\tilde{U} + \tilde{H}$  w.r.t  $\tilde{l}$  produces the first-order condition for the hours of work,  $\overline{MRS}_{b,z} = \tilde{\omega}$ , where

$$\overline{MRS}_{b,z} = \frac{\tilde{U}_z(\tilde{z}) + \tilde{H}_z(\tilde{z})}{\tilde{U}_b(\tilde{b}, q) + \tilde{H}_b(\tilde{b}, q)} = \frac{f_z(\tilde{z})}{a_c(\tilde{c}(\tilde{b}, q))} \quad (5)$$

The second maximization problem in (1) defines the temptation choices  $\tilde{c}$ ,  $\tilde{x}$  and  $\tilde{z}$  as the solution to

$$\max_{\tilde{c}, \tilde{x}, \tilde{z}} h(\tilde{c}, \tilde{x}, \tilde{z}) \quad \text{subject to} \quad \tilde{\omega}\tilde{l} - \tilde{T} = \tilde{c} + q\tilde{x}, \quad \tilde{z} = 1 - \tilde{l} \quad (6)$$

where  $\tilde{\omega} = (1 - \tilde{\tau})w$ . The government is able to tax different income levels at different rates. That is why  $\tilde{\tau}$  and  $\tilde{T}$  denote the (marginal) tax rate and the lump-sum tax payment associated with the *temptation* labor income  $w\tilde{l}$  while  $\tilde{\tau}$  and  $\tilde{T}$  above denote the (marginal) tax rate and the lump-sum tax payment associated with the *actual* labor income  $w\tilde{l}$ . We follow the same approach as above and define temptation post-tax income as  $\tilde{b} = \tilde{\omega}\tilde{l} - \tilde{T}$ . Then we use  $\tilde{b} = \tilde{c} + q\tilde{x}$  as the budget constraint when maximizing the objective function in (6) w.r.t.  $\tilde{c}$  and  $\tilde{x}$ , conditional on  $\tilde{z}$ . This produces the first-order condition  $\overline{MRS}_{x,c} = q$ , where

$$\widetilde{MRS}_{x,c} = \frac{h_x(\tilde{x})}{h_c(\tilde{c})} = \rho \frac{d_x(\tilde{x})}{a_c(\tilde{c})} \quad (7)$$

Together with  $\tilde{b} = \tilde{c} + q\tilde{x}$ , this first-order condition implicitly defines the temptation consumption functions  $\tilde{x}(\tilde{b}, q)$  and  $\tilde{c}(\tilde{b}, q) = \tilde{b} - q\tilde{x}(\tilde{b}, q)$ . These functions can now be used to define the conditional indirect maximum temptation utility function (henceforth referred to as the maximum temptation utility) as

$$\tilde{H} = \tilde{H}(\tilde{b}, \tilde{z}, q) = h(\tilde{c}(\tilde{b}, q), \tilde{x}(\tilde{b}, q), \tilde{z}) = a(\tilde{c}(\tilde{b}, q)) + \rho d(\tilde{x}(\tilde{b}, q)) + f(\tilde{z}) \quad (8)$$

Substituting  $\tilde{b} = \tilde{\omega}\tilde{l} - \tilde{T}$  and  $\tilde{z} = 1 - \tilde{l}$  into equation (8) and maximizing w.r.t  $\tilde{l}$  produces the first-order condition for the temptation hours of work as  $\widetilde{MRS}_{b,z} = \tilde{\omega}$ , where

$$\widetilde{MRS}_{b,z} = \frac{\tilde{H}_z(\tilde{z})}{\tilde{H}_b(\tilde{b}, q)} = \frac{f_z(\tilde{z})}{a_c(\tilde{c}(\tilde{b}, q))} \quad (9)$$

By using (4a), (4b) and (8), we can define the conditional overall indirect utility function as

$$V = V(\tilde{b}, \tilde{z}, q, \tilde{b}, \tilde{z}) = \tilde{U}(\tilde{b}, \tilde{z}, q) + \tilde{H}(\tilde{b}, \tilde{z}, q) - \tilde{H}(\tilde{b}, \tilde{z}, q) \quad (10)$$

We end this part by comparing the consumer's actual choices with the corresponding temptation and normative choices<sup>6</sup> in an unregulated market economy ( $t = \tilde{t} = \tilde{\tau} = \tilde{T} = \tilde{T} = 0$ ) where  $q = p$  and  $\tilde{\omega} = \tilde{\omega} = w$ . To make this comparison, consider a point  $(x, c)$ . By using equations (3) and (7), and the first-order condition for the optimal normative consumption choice in footnote 6, we obtain the following chain of inequalities (conditional on  $z$ )

$$\frac{d_x(x)}{a_c(c)} = MRS_{x,c} < \left(\frac{1+\rho}{2}\right) \frac{d_x(x)}{a_c(c)} = \widetilde{MRS}_{x,c} < \rho \frac{d_x(x)}{a_c(c)} = \widetilde{MRS}_{x,c} \quad (11)$$

Since the private first-order conditions imply  $MRS_{x,c} = \widetilde{MRS}_{x,c} = \widetilde{MRS}_{x,c} = p$ , the chain of inequalities in (11) suggests that the temptation level of consumption of the TP good is higher than the actual level which, in turn, is higher than the normative level, i.e.  $\tilde{x} > \check{x} > x$  and  $c < \tilde{c} < \check{c}$ . This comparison confirms that the actual choices reflect a compromise between the normative and temptation utilities.

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<sup>6</sup> If the consumer were to only maximize her normative utility, then the optimal consumption choice would satisfy  $MRS_{x,c} = d_x(x)/a_c(c) = p$ . The optimal choice of the hours of work would satisfy  $MRS_{b,z} = f_z(z)/a_c(c(b, q)) = w$ .

In a similar way we can, conditional on  $q$ , compare the choices for the hours of work in an unregulated market economy. For a point  $(z, b)$ , we can use equations (5) and (9), and the first-order condition for the optimal normative choice of the hours of work, to obtain

$$\widetilde{MRS}_{b,z} = \frac{f_z(z)}{a_c(\tilde{c}(b,q))} < \frac{f_z(z)}{a_c(\check{c}(b,q))} = \overline{MRS}_{b,z} < \frac{f_z(z)}{a_c(c(b,q))} = MRS_{b,z} \quad (12)$$

The inequalities in (12) arise because  $\tilde{c}(b,q) < \check{c}(b,q) < c(b,q)$  implies  $a_c(\tilde{c}(b,q)) > a_c(\check{c}(b,q)) > a_c(c(b,q))$  when the actual post-tax income is equal to the temptation and normative post-tax income. The private first order conditions imply  $\widetilde{MRS}_{b,z} = \overline{MRS}_{b,z} = MRS_{b,z} = w$  which means that  $\tilde{b} > \check{b} > b$  and  $\tilde{z} < \check{z} < z$ . This outcome reflects that if the consumer succumbs to temptation, she is prepared to give up more leisure in order to have a higher post-tax income so that she can consume more of the TP good.

The analysis above can be summarized as follows;

**Proposition 1:** *In the absence of taxation, temptation consumption of the TP good exceeds actual consumption and normative consumption ( $\tilde{x} > \check{x} > x$ ) while the temptation hours of work exceed the actual hours of work and the normative hours of work ( $\tilde{l} > \check{l} > l$ ).*

## 4. Taxation and Welfare with a Representative Consumer

We now analyze how income and commodity taxation can be used to improve welfare in a representative consumer framework. This allows us to highlight the key motives for using tax policy to improve the welfare when consumers have temptation preferences. It also provides us with a benchmark which can be used as a point of reference to highlight the novel aspects that arise when consumers are heterogenous in terms of their labor market productivities.

Note first that within the framework outlined above, the first-best outcome can be obtained in a command optimum where the choice set is reduced to a singleton. Let  $(c^{**}, x^{**}, z^{**})$  denote this consumption bundle. Since there are no other bundles to compare with when the choice set is reduced to a singleton, the utility cost of exercising self-control is zero and the first-best level of welfare is given by  $u(c^{**}, x^{**}, z^{**})$ . The first-best levels  $c^{**}$ ,  $x^{**}$  and  $z^{**} = 1 - l^{**}$  are therefore obtained by maximizing the normative utility function subject to the resource constraint. To implement this allocation in the market economy, and thereby achieving the first-best welfare in

the market economy, a planner would need to have access to nonlinear commodity and income tax schedules which feature crushingly high tax payments for all  $x \neq x^{**}$  and  $wl \neq wl^{**}$ . Since a nonlinear commodity tax system would be difficult to implement in practice (because of information asymmetries between the private agents and the government), the analysis below will focus on optimal mixed taxation where the government uses a linear commodity tax together with a nonlinear income tax to improve the welfare. Naturally, this puts us in a *second-best* world and the challenge is to use the tax instruments available to get as close as possible to the *first-best* outcome, i.e., to reach the constrained efficient allocation. The second-best nature of the optimal tax problem is exacerbated in Section 5 where we introduce asymmetric information between the private agents and the government.

We conduct the analysis in this section in two steps. First, we use a graphical analysis to explain why and how linear commodity taxation and labor income taxation can be used to improve the consumer's welfare under temptation. In the second step, we formally derive these results.

#### 4.1 Commodity Taxation

Let us first consider a simplified framework where the consumer does not make any labor supply decision. Instead, the consumer is endowed with a fixed income  $m$  which means that the consumer's budget constraint can be written as  $c = m - T - (p + t)x$ . If we initially are in an unregulated market economy with no taxes, this budget constraint reduces to  $c = m - px$  which is the solid budget line depicted in Figure 1. In the unregulated market economy, the consumer's actual (optimal) choices  $\check{c}_0$  and  $\check{x}_0$  are at point  $A_0$  while the temptation (optimal) choices  $\tilde{c}_0$  and  $\tilde{x}_0$  are at point  $B_0$ . In this situation, the consumer's overall utility is given by  $v_0 = \check{u}_0 + \check{h}_0 - \tilde{h}_0$ .

**Figure 1.** Consumption taxation.

Consider now the effect on overall welfare of a tax reform where the commodity tax increases from the initial level of zero and the consumer is compensated via a lump-sum transfer  $T < 0$  so that  $u + h$  is unchanged at the level  $\check{u}_0 + \check{h}_0$ . This corresponds to the move from  $A_0$  to  $A_1$  in Figure 1 where the dotted line is the new budget line. The new temptation choice is at point  $B_1$ , where  $\tilde{h}_1 < \tilde{h}_0$ . Since this tax reform, all else equal, reduces the utility cost of exercising self-control,

overall welfare increases, i.e.  $v_1 = \tilde{u}_0 + \tilde{h}_0 - \tilde{h}_1 > v_0$ .<sup>7</sup> This positive welfare effect arises because the amount paid in tax if the consumer succumbs to the temptation exceeds the actual tax payment (and the amount that is transferred back, i.e.  $t\tilde{x} > t\tilde{x}$ ) which means that succumbing to temptation is less attractive than before. We will refer to this as the *GP welfare motive for taxing the TP good* and this result is summarized as follows:

**Proposition 2:** *When the representative consumer has temptation preferences for a consumption good, the welfare can be improved by implementing a positive commodity tax on the TP good.*

Note that there is no corrective motive for implementing this tax. Instead, the distortionary tax reflects an opportunity to improve the consumer's welfare by reducing the utility of succumbing to the temptation. This result is analogous to a result derived by Krusell et al (2010) where they showed that it is optimal to implement a subsidy on saving in a framework where the temptation reflects impatience between consuming today and tomorrow.

## 4.2 Labor Income Taxation

Let us now turn to the motive for taxing labor income when the consumer has temptation preferences. To do that, consider Figure 2 where point *A* is the consumer's actual leisure-income choice and point *E* is the temptation leisure-income choice<sup>8</sup> on the budget line  $b = w(1 - z)$  in an unregulated market economy. In this situation, the consumer's overall utility is given by  $V_{AE} = \tilde{U}_A + \tilde{H}_A - \tilde{H}_E$ , where  $\tilde{U}_A = \tilde{U}(\tilde{b}_A, \tilde{z}_A, p)$ ,  $\tilde{H}_A = \tilde{H}(\tilde{b}_A, \tilde{z}_A, p)$  and  $\tilde{H}_E = \tilde{H}(\tilde{b}_E, \tilde{z}_E, p)$ .

**Figure 2.** The GP welfare motive for taxing temptation income.

Let us now consider how labor income taxation can improve the welfare. First, consider implementing a piecewise linear income tax schedule which involves taxing labor income above

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<sup>7</sup> More generally, we consider the following policy reform. Consider a small increase in  $t$  by  $\Delta t$  from the initial level  $t = 0$ . The tax revenue  $x\Delta t$  is redistributed back via a lump-sum transfer to the consumer so that  $b$  increases by (approximately)  $\Delta b = \Delta tx$ . This policy has zero marginal effect on  $\tilde{U} + \tilde{H}$ . To evaluate how this policy mix affects the maximum temptation utility, we differentiate  $\tilde{H}(\tilde{b}, \tilde{z}, q)$  w.r.t.  $b$  and  $q$ , conditional on  $z$ . This produces (approximately)  $\Delta \tilde{H} = \tilde{H}_b \Delta b + \tilde{H}_q \Delta q$ . By using  $\Delta q = \Delta t$ ,  $\Delta b = \Delta tx$  and  $\tilde{H}_q = -\tilde{x}\tilde{H}_b$  (Roy's Identity), we can rewrite the expression for  $\Delta \tilde{H}$  to read  $\Delta \tilde{H} = (x - \tilde{x})\tilde{H}_b \Delta t < 0$ .

<sup>8</sup> The first inequality in (12) implies that the slope of the indifference curve for  $H(b, z, p)$  is smaller in absolute value than the slope of the indifference curve for  $U(b, z, p) + H(b, z, p)$  at a given point in  $(z, b)$  space.

the income level  $w\check{l}_A$  at a very high rate  $\tau^c \simeq 1$  while taxing labor income at or below the income level  $w\check{l}_A$  at a zero rate, where  $\check{l}_A = 1 - \check{z}_A$ . The tax function and the resulting consumer budget function are therefore defined as follows

$$T(wl) = \begin{cases} \tau^c w(l - \check{l}_A) & \text{if } l > \check{l}_A \\ 0 & \text{if } l \leq \check{l}_A \end{cases}, \quad b = \begin{cases} w\check{l}_A + (1 - \tau^c)w(l - \check{l}_A) & \text{if } l > \check{l}_A \\ wl & \text{if } l \leq \check{l}_A \end{cases} \quad (13a)$$

After this piecewise linear tax schedule has been implemented, the consumer budget function is represented by the solid line segment to the right of point  $A$  and the dotted line segment to the left of point  $A$ , as illustrated in Figure 2. Since the new budget line has a kink at point  $A$ , we see in Figure 2 that if  $\tau^c$  is sufficiently large, then the highest temptation utility the consumer can obtain if she succumbs to temptation is  $\tilde{H}_A = \tilde{H}(\check{b}_A, \check{z}_A, p)$ , i.e. the utility level associated with the indifference curve for the temptation utility which passes through point  $A$ . At point  $A$ , temptation leisure equals actual leisure and temptation labor income equals actual labor income. The new overall utility is  $V_{AA} = \check{U}_A + \check{H}_A - \tilde{H}_A$  and since  $\tilde{H}_A < \tilde{H}_E$ , this tax reform increases the welfare, i.e.  $V_{AA} > V_{AE}$ . We will refer to this as the *GP welfare motive for taxing temptation income*.

The welfare can be improved further. To illustrate this, assume that the consumer initially faces the piecewise linear budget line defined in (13a) so that she is situated at point  $A$  in Figure 3 (which corresponds to point  $A$  in Figure 2) where the overall utility is given by  $V_{AA} = \check{U}_A + \check{H}_A - \tilde{H}_A$ . Now, consider a tax reform which involves taxing labor income below the kink point at a positive rate  $\check{\tau} > 0$  while the consumer is compensated by a lump-sum subsidy ( $\check{T} < 0$ ) so that she remains on the original indifference curve associated with the utility level  $\check{U}_A + \check{H}_A$ . If the consumer faces a crushingly high rate  $\tau^c$  above the new kink point, this tax reform shifts the consumer's budget function to the dotted piecewise linear budget line depicted in Figure 3. The tax reform is calibrated so that the consumer's new optimal choice is at point  $B$ . Therefore, the new tax function and the corresponding new piece-wise linear budget line depicted in Figure 3 are defined as follows

$$T(wl) = \begin{cases} \check{T} + \check{\tau}w\check{l}_B + \tau^c w(l - \check{l}_B) & \text{if } l > \check{l}_B \\ \check{T} + \check{\tau}wl & \text{if } l \leq \check{l}_B \end{cases}$$

$$b = \begin{cases} (1 - \check{\tau})w\check{l}_B + (1 - \tau^c)w(l - \check{l}_B) - \check{T} & \text{if } l > \check{l}_B \\ (1 - \check{\tau})wl - \check{T} & \text{if } l \leq \check{l}_B \end{cases} \quad (13b)$$

where  $\check{l}_B = 1 - \check{z}_B$ .

**Figure 3.** The GP welfare motive for taxing actual income.

The overall utility at point  $B$  is given by  $V_{AB} = \tilde{U}_A + \tilde{H}_A - \tilde{H}_B$ , where  $\tilde{H}_B = \tilde{H}(\tilde{b}_B, \tilde{z}_B, p)$ . Since  $\tilde{H}_B < \tilde{H}_A$ , the consumer's overall utility is higher at point  $B$  than at point  $A$ . i.e.  $V_{AB} > V_{AA}$ . The positive welfare effect that arises from this tax reform will be referred to as the *GP welfare motive for taxing actual income* at the margin, and can be summarized as follows:

**Proposition 3:** *When the representative consumer has temptation preferences for a consumption good, the welfare can be improved by implementing a positive (marginal) tax rate on actual labor income.*

### 4.3 Optimal Tax Rules

Let us now derive the optimal tax rules for the commodity tax  $t$  and the labor income tax rate  $\tilde{\tau}$ , associated with the graphical analyses conducted in Figures 1 - 3. To do that, note first that the government can use the (marginal) tax rate  $\tilde{\tau}$  and the lump-sum component  $\tilde{T}$  to induce the consumer to choose whatever bundle  $(\tilde{b}, \tilde{z})$  the government finds optimal. We can therefore either use  $(\tilde{\tau}, \tilde{T}, t)$  or  $(\tilde{b}, \tilde{z}, t)$  as decision variables when solving the government's optimal tax problem. Below we will use the latter approach. Recall also from the graphical analysis that the temptation post-tax income and the temptation hours of work will be equal to their actual counterparts, i.e.,  $\tilde{b} = \tilde{b} = b$ ,  $\tilde{z} = \tilde{z} = z$  and  $\tilde{l} = \tilde{l} = l$  (in the following we omit the symbol “ $\sim$ ” for the actual choices). This implies that the conditional indirect maximum temptation utility function is defined by  $\tilde{H} = \tilde{H}(b, z, q) = h(\tilde{c}(b, q), \tilde{x}(b, q), z)$ . By using this definition together with equations (4a) and (4b), the consumer's overall indirect utility function, which is the welfare function that the government wants to maximize, is given by

$$V(b, z, t) = U(b, z, t) + H(b, z, t) - \tilde{H}(b, z, q) \quad (14)$$

The government redistributes all tax revenue back to the consumer. This means that the public budget constraint is given by  $\tau wl + T + tx(b, q) = 0$ . By using the private budget constraint  $b = (1 - \tau)wl - T$ , we can rewrite the public budget constraint to read  $tx(b, q) + wl - b = 0$ . The optimal tax problem is to choose  $b$ ,  $z$  and  $t$  to maximize the welfare function  $V(b, z, t)$  subject to  $wl - b + tx(b, q) = 0$ . Solving this problem allows us to characterize the tax rule for the optimal



marginal labor income tax rate associated with point  $B$  in Figure 3, as well as the corresponding tax rule for the optimal linear commodity tax associated with the dotted budget line in Figure 1. The Lagrange function associated with this maximization problem is stated as follows

$$L = U(b, z, t) + H(b, z, t) - \tilde{H}(b, z, q) + \gamma[wl - b + tx(b, q)] \quad (15)$$

where  $\gamma$  is the Lagrange multiplier associated with the government's budget constraint. The first-order conditions are presented in the Appendix where we derive the following results:

**Proposition 4:** *When the representative consumer has temptation preferences for a consumption good, the optimal linear commodity tax and the optimal (marginal) tax rate on actual labor income can be written as follows*

$$t = \frac{(x - \tilde{x})}{\left(\frac{\partial x}{\partial q} + x \frac{\partial x}{\partial b}\right)} \frac{\tilde{H}_b}{\gamma} > 0 \quad (16a)$$

$$\tau = \left(\overline{MRS}_{b,z} - \widetilde{MRS}_{b,z}\right) \frac{\tilde{H}_b}{\gamma w} - t \frac{\overline{MRS}_{b,z}}{w} \frac{\partial x}{\partial b} \quad (16b)$$

Since  $x < \tilde{x}$ , and since the compensated price effect on the temptation preference good is negative ( $\partial x / \partial q + x \partial x / \partial b < 0$ ), the optimal commodity tax will be positive. This reflects the GP welfare motive for taxing the TP good stated in Proposition 2. Turning to the formula for the optimal (marginal) labor income tax rate in equation (16b), we recall that the first inequality in equation (12) implies  $\overline{MRS}_{b,z} > \widetilde{MRS}_{b,z}$ . Therefore, the first term on the right-hand side (RHS) in equation (16b) is positive. It reflects the GP welfare motive for taxing actual labor income stated in Proposition 3. Next, we turn to the second term on the RHS in equation (16b) which is proportional to the commodity tax. This term is negative (including the minus sign) as  $t > 0$  and  $\partial x / \partial b > 0$ . To explain the role of this term, recall that if a nonlinear commodity tax schedule would be available, the government would be able to reduce the consumer's consumption set to a singleton by implementing a crushingly high commodity tax for any  $x \neq x^{**}$  and a zero-commodity tax for  $x = x^{**}$ . The fact that the commodity tax would be zero at  $x = x^{**}$  reflects that there is no direct motive (such as an externality correction) to influence the consumption of the TP good. This implies that when the government is restricted to use a linear commodity tax for the TP good, and since  $t > 0$  in the second-best optimum,  $t$  is interpretable as being too high (relatively to the zero marginal commodity tax rate of the first-best optimum). From this perspective, it is (on the margin)

welfare improving to stimulate the consumption of the TP good. Since  $\partial x / \partial b > 0$ , this can be achieved by implementing a lower marginal tax on labor income which induces the consumer to supply more hours of work, thereby increasing her post-tax income  $b$ . It is this compensatory motive which is captured by the term which is proportional to the commodity tax (and which vanishes in the absence of commodity taxation).

## 5. Optimal Taxation with Heterogeneous Agents

In the previous section, we showed that temptation preferences for a consumption good has implications for the taxation of labor income when the consumers are identical. In this section, we extend the analysis into a framework where the consumers differ in terms of their respective labor market productivities, i.e. in terms of their pre-tax wages. In this context, we pose the following question: does labor market heterogeneity have implications for the *GP welfare motive for taxing actual labor income*? To address this question, we use the two-type version of the Mirrleesian optimal tax model.<sup>9</sup> The two-type model allows us to show how the presence of GP preferences modifies the conventional second-best optimal tax rules in an analytically tractable framework, and it is straightforward to extend this analysis into a more general setting with more than two agent types. In the two-type model, we distinguish between two consumer types who differ in terms of their innate earnings-abilities (i.e. the agent types differ in terms of their respective productivity levels on the labor market); there is a low-ability type ( $i = 1$ ) who faces a lower pre-tax hourly wage than a high-ability type ( $i = 2$ ). Each consumer is atomistic and for notational convenience we normalize the number of consumers of each ability type to one. The output of the numeraire good is produced by a linear technology that employs both labor types and given competitive markets, the pre-tax hourly wage rate  $w^i$  facing ability-type  $i$  equals the corresponding marginal productivity, where  $w^1 < w^2$ . In addition, it is assumed that one unit of the TP good can be attained by using up  $p$  units of the numeraire good. Since the price of the numeraire good is one, the producer price of the TP good is  $p$ . As in Section 4, we first use a graphical analysis to provide intuition for the results derived thereafter.

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<sup>9</sup> The two-type version of the Mirrleesian optimal income tax model originates from Stern (1982) and Stiglitz (1982), and was later extended to a model of optimal mixed taxation by Edwards, Keen and Tuomala (1994).

## 5.1 Second-Best Taxation without Temptation Preferences

To illustrate how the presence of TP preferences modifies the government's optimal tax problem when agents are heterogenous, let us begin by briefly recapitulating the key results in the conventional optimal tax model where TP preferences are absent (see e.g. Stiglitz [1982]). To do that, we omit the temptation part of consumer  $i$ 's preferences, which means that the conditional indirect utility function reduces to  $U(b^i, z^i, q)$ . In a first-best setting where the government maximizes a utilitarian welfare function, and where consumption is additively separable along the lines presented above, it is straightforward to show that the first-best policy features  $b^1 = b^2$  and  $z^2 < z^1$  where the latter result reflects that the high-ability type is more productive on the labor market than the low-ability type. As a consequence,  $U(b^2, z^2, q) < U(b^1, z^1, q)$  holds in the first-best, which implies that if individual productivity and hours of work are not observable by the government, the high-ability type would prefer to mimic the income of the low-ability type. To deter mimicking, one of the key results in the literature on second-best optimal taxation is that the government implements a nonlinear income tax schedule where the low-ability type faces a positive marginal tax on labor income while the high-ability type (the top-income earner) faces a zero marginal tax rate. To illustrate this outcome, let  $y^i = w^i l^i$  denote pre-tax income, which implies that post-tax income can be written as  $b^i = y^i - T(y^i)$  where  $T(y^i)$  is a nonlinear income tax schedule. Let us now use  $z^i = 1 - y^i/w^i$  to rewrite the consumer's utility function in terms of the observable variables  $b^i$  and  $y^i$ , such that  $U(b^i, 1 - y^i/w^i, q)$ . The slope of the indifference curve for  $U(b^i, 1 - y^i/w^i, q)$  in  $(y, b)$  space is given by  $U_z^i / (w^i U_b^i) > 0$  and it can be shown that the high-ability type's indifference curve is flatter than that of the low-ability type. The solution to the government's second-best optimal tax problem under asymmetric information (i.e. when  $y^i$  is observable but not  $w^i$  and  $l^i$ ) is illustrated in Figure 4. Here, the government implements a nonlinear income tax schedule where only two points are feasible; the low-ability type chooses point A1 where she faces a positive marginal income tax rate to deter mimicking while the high-ability type chooses point A2 where the marginal tax rate is zero. Since the government implements a tax policy where the high-ability type is indifferent between mimicking and not mimicking at the optimum, agent type 2's indifference curve passes through point A1 in Figure 4.

**Figure 4.** Optimal taxation in the conventional model.

## 5.2 Second-Best Taxation with Temptation Preferences

Let us now introduce TP preferences into the second-best tax framework. To do that, we first use  $z^i = 1 - y^i/w^i$  to write equations (4a), (4b) and (8) as follows

$$\bar{U}^i = \bar{U}\left(\bar{b}^i, 1 - \frac{\bar{y}^i}{w^i}, q\right) = u\left(\bar{c}(\bar{b}^i, q), \bar{x}(\bar{b}^i, q), 1 - \frac{\bar{y}^i}{w^i}\right) \quad (17a)$$

$$\bar{H}^i = \bar{H}\left(\bar{b}^i, 1 - \frac{\bar{y}^i}{w^i}, q\right) = h\left(\bar{c}(\bar{b}^i, q), \bar{x}(\bar{b}^i, q), 1 - \frac{\bar{y}^i}{w^i}\right) \quad (17b)$$

$$\tilde{H}^i = \tilde{H}\left(\tilde{b}^i, 1 - \frac{\tilde{y}^i}{w^i}, q\right) = h\left(\tilde{c}(\tilde{b}^i, q), \tilde{x}(\tilde{b}^i, q), 1 - \frac{\tilde{y}^i}{w^i}\right) \quad (17c)$$

As above, we will in the following omit the symbol “ $\bar{\cdot}$ ” for the actual choices. Next, recall from the analysis conducted in the representative consumer framework that the government has a GP welfare motive to tax actual labor income at a positive rate when the temptation choices of  $b$  and  $z$  (and hence  $y$ ) coincide with the actual choices. This motive was captured by the first term on the RHS in equation (16b).

With heterogenous consumers, the analogous term associated with ability type  $i$  would be written as  $(\overline{MRS}_{b,z}^i - \widetilde{MRS}_{b,z}^i)\bar{H}_b^i/(w^i\gamma) > 0$ . This term will appear in the formula for the marginal income tax rate facing ability type  $i$  if ability type  $i$ 's temptation choices of  $b$  and  $y$  coincide with her actual choices. However, this term will not appear in the marginal income tax formula for ability type  $i$  if the temptation and actual choices do not coincide. To address whether ability type  $i$ 's temptation choices coincide with her actual choices, we proceed as follows. First, we graphically illustrate the outcome when the government implements a policy conditional on that the temptation choices coincide with the actual choices. Second, we ask if this outcome is feasible and illustrate that it is feasible for the high-ability type (i.e. the temptation choices coincide with the actual choices for the high-ability type) but this outcome may not be feasible for the low-ability type (i.e. the temptation choices may not coincide with the actual choices for the low-ability type). Third, we illustrate graphically the implications for tax policy when the low-ability type's temptation choices do not coincide with her actual choices. In Section 5.3, we formally derive the corresponding optimal tax rules.

Let us begin by illustrating the outcome when the government implements a policy conditional on that the temptation choices coincide with the actual choices for both ability types. This policy involves adding the term  $(\overline{MRS}_{b,z}^i - \widetilde{MRS}_{b,z}^i)\bar{H}_b^i/(w^i\gamma) > 0$  to the marginal income tax rule

facing each agent type in the conventional optimal tax model. As such, the high-ability type now faces a positive marginal tax rate while the low-ability type faces a marginal tax rate which exceeds that which is implemented to deter mimicking. These outcomes are depicted in Figure 5 where the low-ability type's actual choices are at point B1 while the high-ability type's actual choices are at B2. To avoid clutter, we do not depict the post-tax income function in Figure 5.

**Figure 5.** The possible temptation choices with two consumer types.

Since only two points are feasible on the income tax schedule, the low-ability type can either use  $\tilde{b}^1 = b_{B1}^1$  and  $\tilde{y}^1 = y_{B1}^1$  or  $\tilde{b}^1 = b_{B2}^2$  and  $\tilde{y}^1 = y_{B2}^2$  as her temptation choices. If she uses  $\tilde{b}^1 = b_{B1}^1$  and  $\tilde{y}^1 = y_{B1}^1$  as her temptation choices, the temptation utility will be given by  $\tilde{H}_{B1}^1 = \tilde{H}(b_{B1}^1, 1 - y_{B1}^1/w^1, q)$ . The indifference curve for  $\tilde{H}_{B1}^1$  passes through point B1, where the first inequality in equation (12) implies that this indifference curve has a flatter slope than the indifference curve for  $U_{B1}^1 + H_{B1}^1$  in  $(y, b)$  space. If the low-ability type instead would use point B2 as the basis for her temptation choices, then  $\tilde{b}^1 = b_{B2}^2$  and  $\tilde{y}^1 = y_{B2}^2$  in which case the temptation utility would be given by  $\tilde{H}_{B2}^1 = \tilde{H}(b_{B2}^2, 1 - y_{B2}^2/w^1, q)$ . The indifference curve for  $\tilde{H}_{B2}^1$  is the dotted curve passing through point B2 and from Figure 5, it follows that  $\tilde{H}_{B1}^1 < \tilde{H}_{B2}^1$ . Therefore, the low-ability type will choose  $\tilde{H}_{B2}^1$  to be her maximum temptation utility, implying that her temptation choices do not coincide with her actual choices. Instead, the low-ability type *frames* her maximum temptation utility on replicating the labor income of the high-ability type.<sup>10</sup> Her overall utility will therefore be given by

$$V_{12}^1 = \underbrace{U\left(b_{B1}^1, 1 - \frac{y_{B1}^1}{w^1}, q\right)}_{U_{B1}^1} + \underbrace{H\left(b_{B1}^1, 1 - \frac{y_{B1}^1}{w^1}, q\right)}_{H_{B1}^1} - \underbrace{\tilde{H}\left(b_{B2}^2, 1 - \frac{y_{B2}^2}{w^1}, q\right)}_{\tilde{H}_{B2}^1} \quad (18a)$$

Turning to the high-ability type, we note that if she uses  $\tilde{b}^2 = b_{B2}^2$  and  $\tilde{y}^2 = y_{B2}^2$  as temptation choices, then the temptation utility will be given by  $\tilde{H}_{B2}^2 = \tilde{H}(b_{B2}^2, 1 - y_{B2}^2/w^2, q)$  but if she instead would use  $\tilde{b}^2 = b_{B1}^1$  and  $\tilde{y}^2 = y_{B1}^1$  as temptation choices, then the temptation utility would be given by  $\tilde{H}_{B1}^2 = \tilde{H}(b_{B1}^1, 1 - y_{B1}^1/w^2, q)$ . From Figure 5, we see that  $\tilde{H}_{B1}^2 < \tilde{H}_{B2}^2$  which implies that the high-ability type will choose  $\tilde{H}_{B2}^2$  to be her maximum temptation utility. Therefore, the

<sup>10</sup> If the indifference curve for  $\tilde{H}_{B2}^1$  would be steeper than the indifference curve for  $U_{B2}^2 + H_{B2}^2$  at each point in  $(y, b)$  space, then framing would not occur. If there are circumstances when this could happen is an empirical question.

high-ability type's temptation choices coincide with her actual choices and the overall utility is given by

$$V_{22}^2 = \underbrace{U\left(b_{B2}^2, 1 - \frac{y_{B2}^2}{w^2}, q\right)}_{U_{B2}^2} + \underbrace{H\left(b_{B2}^2, 1 - \frac{y_{B2}^2}{w^2}, q\right)}_{H_{B2}^2} - \underbrace{\tilde{H}\left(b_{B2}^2, 1 - \frac{y_{B2}^2}{w^2}, q\right)}_{\tilde{H}_{B2}^2} \quad (18b)$$

Under *framing*, the labor income tax intended for the high-ability type will influence the welfare of the low-ability type via the maximum temptation utility  $\tilde{H}_{B2}^1 = \tilde{H}(b_{B2}^2, 1 - y_{B2}^2/w^1, q)$ . This provides a novel channel via which the income tax intended for the high-ability type affects the welfare of the low-ability type. To illustrate how this particular mechanism affects the marginal taxation of the high-income earner, consider a tax reform where the high-ability type's marginal income tax rate increases from the level at point B2 while the high-ability type is simultaneously compensated via a lump-sum transfer so that she remains on the indifference curve associated with the utility level  $U_{B2}^2 + H_{B2}^2$ . This compensated tax reform is illustrated in Figure 6 and corresponds to the move from point B2 (which corresponds to point B2 in Figure 5) to the new point B3. In Figure 6, we only depict the indifference curves for the utility levels  $U_{B1}^1 + H_{B1}^1$  and  $U_{B2}^2 + H_{B2}^2$ , as well as for the maximum temptation utility for the low-ability type under *framing*. The move from B2 to B3 changes the low-ability type's maximum temptation utility from  $\tilde{H}_{B2}^1 = \tilde{H}(b_{B2}^2, 1 - y_{B2}^2/w^1, q)$  to  $\tilde{H}_{B3}^1 = \tilde{H}(b_{B3}^2, 1 - y_{B3}^2/w^1, q)$ . Since  $\tilde{H}_{B3}^1 < \tilde{H}_{B2}^1$ , the low-ability type's overall utility increases from  $V_{12}^1 = U_{B1}^1 + H_{B1}^1 - \tilde{H}_{B2}^1$  to  $V_{13}^1 = U_{B1}^1 + H_{B1}^1 - \tilde{H}_{B3}^1$ .

**Figure 6.** The cross GP welfare motive for taxing the high-ability type under framing.

The positive welfare effect of this tax reform provides a motive for implementing a positive marginal income tax rate on the high-ability type which only appears when the economy is made up of heterogenous agents. We will refer to this as the *cross GP welfare motive* for taxing the high-income earner's actual income. This result is summarized as follows;

**Proposition 5:** *When the low-ability type frames her maximum temptation utility on replicating the labor income of the high-ability type, the welfare for the low-ability type can, ceteris paribus, be improved by implementing a positive (marginal) tax rate on the high-ability type's income.*

Finally, it is worth reiterating that under *framing*, there is no GP welfare motive for taxing the low-income earner's actual income because her maximum temptation utility is unaffected by the taxation of  $y^1$ .

### 5.3 Optimal Tax Rules under Framing

In this part, we will focus on solving the second-best optimal tax problem when the low-ability type frames her maximum temptation utility on the labor income earned by the high-ability type. This means that we will use<sup>11</sup>  $\tilde{H}^{1,2} = \tilde{H}^1(b^2, 1 - w^2 l^2 / w^1, q)$  and  $\tilde{H}^{2,2} = \tilde{H}^2(b^2, 1 - l^2, q)$  as maximum temptation utilities for the two consumer types. The government maximizes a utilitarian welfare function  $W = V^1 + V^2$  where the overall indirect utility functions  $V^1$  and  $V^2$  are given by

$$V^1 = U(b^1, 1 - l^1, q) + H(b^1, 1 - l^1, q) + \tilde{H}^1\left(b^2, 1 - \frac{w^2 l^2}{w^1}, q\right) \quad (19a)$$

$$V^2 = U(b^2, 1 - l^2, q) + H(b^2, 1 - l^2, q) + \tilde{H}^2(b^2, 1 - l^2, q) \quad (19b)$$

We assume (in line with the convention in the optimal tax literature) that the innate earnings ability (as measured by the before-tax wage) is private information. This implies that the government observes the post-tax income ( $w^i l^i$ ) of each consumer but the individual consumer's productivity level ( $w^i$ ) and hours of work ( $l^i$ ) are unobserved. Therefore, the government cannot differentiate taxes by ability. Instead, the government must base its redistribution policy on observable income where the tax policy needs to satisfy a self-selection constraint which ensures that the high-ability type does not prefer to mimic the before-tax income of the low-ability type<sup>12</sup>

$$V^2 = U^2 + H^2 - \tilde{H}^{2,2} \geq \hat{U}^2 + \hat{H}^2 - \tilde{H}^{2,2} = \hat{V}^2 \quad (20)$$

where “ $\wedge$ ” denotes the mimicker. The left-hand side in equation (20) defines the utility of the high-ability type when he/she does not mimic the before-tax income of the low-ability type while the right-hand side defines the utility of the high-ability type when she does mimic the before-tax income of the low-ability type. If the high-ability type mimics the before-tax income of the low-ability type then the mimicker's actual labor supply is  $\hat{l}^2 = w^1 l^1 / w^2$  while the corresponding level

<sup>11</sup> In the superscripts for  $\tilde{H}^{1,2}$  and  $\tilde{H}^{2,2}$ , the first number refers to ability type while the second indicates that ability type  $i = 1, 2$  uses the actual choices of the high-ability type as temptation choices.

<sup>12</sup> The other possible self-selection constraint, which serves to prevent the low-ability individual from mimicking the high-ability type, is assumed not to be binding. This is a common assumption in the optimal tax literature.

of leisure is  $\hat{z}^2 = 1 - \hat{l}^2$ . Hence  $\hat{z}^2 > z^1$ . Substituting  $b^1$  and  $\hat{z}^2$  into equations (4a) and (4b) allows us to define  $\hat{U}^2 = U(b^1, \hat{z}^2, q)$  and  $\hat{H}^2 = H(b^1, \hat{z}^2, q)$ . Finally, we note that the temptation hours of work which provides the high-ability type with maximum temptation utility is independent of whether the high-ability type actually acts as a mimicker or not. Therefore,  $\tilde{H}^{2,2} = \tilde{H}^2(b^2, 1 - l^2, q)$  is the maximum temptation utility on both sides of the inequality sign in (20). As a consequence, the self-selection constraint effectively reduces to  $U^2 + H^2 \geq \hat{U}^2 + \hat{H}^2$ .

Since the number of consumers of each ability-type is normalized to one, and by using the private budget constraints, the government's budget constraint can be written as  $\sum_i [w^i l^i + t x^i - b^i] = 0$ . The Lagrange function associated with the government's optimization problems is specified as follows

$$\begin{aligned} L = & U(b^1, 1 - l^1, q) + H(b^1, 1 - l^1, q) - \tilde{H}^1\left(b^2, 1 - \frac{w^2 l^2}{w^1}, q\right) \\ & + U(b^2, 1 - l^2, q) + H(b^2, 1 - l^2, q) - \tilde{H}^2(b^2, 1 - l^2, q) \\ & + \lambda \left[ U(b^2, z^2, q) + H(b^2, z^2, q) - U\left(b^1, 1 - \frac{w^1 l^1}{w^2}, q\right) - H\left(b^1, 1 - \frac{w^1 l^1}{w^2}, q\right) \right] \\ & + \gamma [t x^1(b^1, q) + t x^2(b^2, q) + w^1 l^1 + w^2 l^2 - b^1 - b^2] \end{aligned} \quad (21)$$

where  $\lambda$  is the Lagrange multiplier associated with the self-selection constraint. The first-order conditions are presented in the Appendix where we derive all results to be presented below.

Let us introduce the following short notations

$$\begin{aligned} \theta^i &= -\frac{1}{\Omega} \left( \frac{\partial x^i}{\partial q} + x^i \frac{\partial x^i}{\partial b^i} \right), \quad t^{1,2} = \frac{(x^2 - \tilde{x}^{1,2})}{\frac{\partial x^1}{\partial q} + x^1 \frac{\partial x^1}{\partial b^1}} \frac{\tilde{H}_b^{1,2}}{\gamma}, \quad t^2 = \frac{(x^2 - \tilde{x}^2)}{\frac{\partial x^2}{\partial q} + x^2 \frac{\partial x^2}{\partial b^2}} \frac{\tilde{H}_b^{2,2}}{\gamma} \\ \Omega &= -\sum_i \left( \frac{\partial x^i}{\partial q} + x^i \frac{\partial x^i}{\partial b^i} \right), \quad \overline{MRS}_{b,z}^i = \frac{U_z^i + H_z^i}{U_b^i + H_b^i}, \quad \overline{MRS}_{b,z}^2 = \frac{\hat{U}_z^2 + \hat{H}_z^2}{\hat{U}_b^2 + \hat{H}_b^2}, \quad \overline{MRS}_{b,z}^{1,2} = \frac{\tilde{H}_z^{1,2}}{\tilde{H}_b^{1,2}} \end{aligned}$$

where  $\Omega > 0$ , and where  $\theta^i > 0$  reflects the relative size of agent type  $i$ 's compensated price sensitivity in relation to the compensated price sensitivity summed over both agent types. These definitions imply that  $\theta^1 + \theta^2 = 1$ . The term  $t^{1,2}$  is the tax rule for the optimal linear commodity tax that the government would implement for consumer type 1 under framing if type-specific linear commodity taxes would be available, while  $\tilde{x}^{1,2} = \tilde{x}^1(b^2, q)$  is consumer type 1's temptation demand function for the TP good under framing. As for  $t^2$ , it is the tax rule for the optimal linear commodity tax that the government would implement for consumer type 2 if type-specific linear



commodity taxes would be available. The latter tax rule is equivalent to the commodity tax formula presented in equation (16a).

With these definitions at hand, and by letting  $\tau^i$  denote the marginal tax rate facing ability type  $i = 1, 2$ , we can derive the following results;

**Proposition 6:** *When the maximum temptation utility of the low-ability type is framed on replicating the income of the high-ability type, the second-best linear commodity tax and the marginal income tax rates can be written as follows*

$$t = \theta^1 t^{1,2} + \theta^2 t^2 \quad (22a)$$

$$\tau^1 = -t \frac{\overline{MRS}_{b,z}^1}{w^1} \frac{\partial x^1}{\partial b^1} + \frac{\lambda(\hat{U}_b^2 + \hat{H}_b^2)}{w^1 \gamma} \left( \overline{MRS}_{b,z}^1 - \frac{w^1}{w^2} \overline{MRS}_{b,z}^2 \right) \quad (22b)$$

$$\tau^2 = -t \frac{\overline{MRS}_{b,z}^2}{w^2} \frac{\partial x^2}{\partial b^2} + \frac{\tilde{H}_b^2}{w^2 \gamma} \left( \overline{MRS}_{b,z}^2 - \overline{MRS}_{b,z}^2 \right) + \frac{\tilde{H}_b^{1,2}}{w^2 \gamma} \left( \overline{MRS}_{b,z}^2 - \frac{w^2}{w^1} \overline{MRS}_{b,z}^{1,2} \right) \quad (22c)$$

We begin with the commodity tax in equation (22a). The expression on the RHS in equation (22a) is a weighted average of  $t^{1,2}$  and  $t^2$ , where the consumer type who is most price sensitive is attached the highest weight in the calculation of this weighted average. Note that the definition of  $t^{1,2}$  contains the term  $(x^2 - \tilde{x}^{1,2})\tilde{H}_b^{1,2}$ . To explain why this is so, let us conduct the following policy experiment. Assume that the commodity tax is initially zero. Consider now an (infinitesimally) small increase in  $t$  by  $\Delta t$  where the additional tax revenue  $(x^1 + x^2)\Delta t$  is redistributed back via lump-sum transfers to the two ability types according to  $\Delta b^i = x^i \Delta t$  for  $i = 1, 2$ . This policy mix has zero marginal effects on  $U^1 + H^1$  and  $U^2 + H^2$  but will affect the maximum temptation utility levels for both ability types. To evaluate the effects on  $\tilde{H}^{1,2} = \tilde{H}^1(b^2, 1 - w^2 l^2 / w^1, q)$  and  $\tilde{H}^{2,2} = \tilde{H}^2(b^2, 1 - l^2, q)$ , we differentiate these expressions w.r.t.  $b^2$  and  $t$ . This produces

$$\Delta \tilde{H}^{1,2} = \tilde{H}_b^{1,2} \Delta b^2 + \tilde{H}_q^{1,2} \Delta t = (x^2 - \tilde{x}^{1,2})\tilde{H}_b^{1,2} \Delta t < 0 \quad (23a)$$

$$\Delta \tilde{H}^2 = \tilde{H}_b^2 \Delta b^2 + \tilde{H}_q^2 \Delta t = (x^2 - \tilde{x}^2)\tilde{H}_b^2 \Delta t < 0 \quad (23b)$$

where we have used  $\Delta b^2 = x^2 \Delta t$ ,  $\tilde{H}_q^{1,2} = -\tilde{x}^{1,2} \tilde{H}_b^{1,2}$  and  $\tilde{H}_q^2 = -\tilde{x}^2 \tilde{H}_b^2$ .<sup>13</sup> Equations (23a) and (23b) show that an increase in the commodity tax from zero has a negative impact on both  $\tilde{H}^{1,2}$

<sup>13</sup> The latter two equations follow from Roy's Identity.

and  $\tilde{H}^2$ , which implies that the welfare is improved for both ability types. In particular, it follows from equation (23a) that the effect on  $\Delta\tilde{H}^{1,2}$  is proportional to  $(x^2 - \tilde{x}^{1,2})\tilde{H}_b^{1,2}$ , which explains why the definition of  $t^{1,2}$  is a function of  $(x^2 - \tilde{x}^{1,2})\tilde{H}_b^{1,2}$ .

Let us now turn to the tax rules for the marginal income tax rates. To interpret these formulas let us, as a benchmark, first consider what they look like in a conventional model where agents do not have GP preferences (i.e. in the framework described in Section 5.1). In this case, the temptation utility part vanishes and  $V^i = U^i$ . In this case equations (22b) and (22c) reduce to

$$\tau^1 = -t \frac{MRS_{b,z}^1}{w^1} \frac{\partial x^1}{\partial b^1} + \frac{\lambda \hat{U}_b^2}{\gamma w^1} \left( MRS_{b,z}^1 - \frac{w^1}{w^2} \widehat{MRS}_{b,z}^2 \right), \quad \tau^2 = -t \frac{MRS_{b,z}^2}{w^2} \frac{\partial x^2}{\partial b^2} \quad (24)$$

where  $MRS_{b,z}^i = U_z^i/U_b^i$  and  $\widehat{MRS}_{b,z}^2 = \hat{U}_z^2/\hat{U}_b^2$ . In the absence of a commodity tax ( $t = 0$ ), the marginal income tax formulas in (24) coincide with those derived by Stiglitz (1982), who showed that the government implements a positive marginal labor income tax for the low-ability type. The intuition is that the government can relax a binding self-selection constraint by taxing the mimicked agent at the margin (which makes mimicking less attractive). This is so because a potential mimicker attaches a lower marginal value to leisure (and is thus hurt more by being forced to spend more time on leisure instead of earning income) than the low-ability type. This is captured by the second term on the RHS in the equation for  $\tau^1$  which therefore can be labelled the *conventional self-selection motive* for taxing the labor income of the low ability-type. As for the second marginal income tax formula in (24), it reflects the classic Mirrlees result that the marginal income tax for the top-income earner should be zero (as long as  $t = 0$ ).

Let us now return to the marginal income tax formulas in Proposition 6 and we begin with the marginal income tax formula for the low-ability type. Here we note that equation (22b) is analogous to the marginal income tax formula in the conventional optimal tax model. Note also that if we compare equation (22b) with the corresponding marginal income tax formula in the representative consumer framework (equation [16b] in Proposition 5), we see that the *GP welfare motive* for taxing labor income is absent in equation (22b). This reflects that when the low-ability type's maximum temptation utility is framed on replicating the labor income of the high-ability type, then the marginal income tax rate, and the income tax payment facing the low-ability type, cannot be used to influence the low-ability type's maximum temptation utility.

Turning to the marginal income tax formula for the high-ability type, we see that equation (22c) contains two additional terms in comparison with the marginal income tax formula in the conventional optimal tax model. The first additional term (i.e. the second term on the RHS in [22c]) reflects the *GP welfare motive* for implementing a positive marginal tax which was stated in Proposition 3 while the second additional term (i.e. the third term on the RHS in [22c]) reflects the *cross GP welfare motive* for implementing a positive marginal tax on the high-ability type's labor income in the presence of framing, which was stated in Proposition 5. Since the latter motive only arises when the indifference curve for  $U^2 + H^2$  has a steeper slope than the indifference curve for  $\tilde{H}^{1,2}$  (see Figures 5 and 6), the term inside brackets is positive  $\left(\overline{MRS}_{b,z}^2 > \frac{w^2}{w^1} \overline{MRS}_{b,z}^{1,2}\right)$ .

## 6. Using a Labor Market Contract as a Commitment Device

In the analysis so far, we have assumed that the consumption and the labor supply decisions are taken simultaneously. This implies that both choices are made under the influence of temptation which we have demonstrated in Proposition 1: actual consumption of the TP good and actual labor supply are both higher than in the case without temptation preferences (i.e., the case where only the normative utility is maximized). We then showed that implementing a positive commodity tax, as well as a positive (marginal) labor income tax, improves welfare by making succumbing to temptation less attractive.

Alternatively, we could think of a setup where the hours of work (and the wage) are specified in a contract between the consumer/worker and the firm at a time *before* consumption takes place and therefore before temptation has kicked in. This means that although the consumer/worker is free to commit herself to any hours of work in the contract, once the contract is signed, the consumer/worker treats the hours of work that is specified in the contract as fixed during the contract period (in contrast to the case where the worker may be employed on an hourly basis or even decide on the spot e.g. in food delivery services) and at the time of consumption. Hence, even if the consumer/worker succumbs to temptation during the contract period, it is not possible to deviate from the hours of work specified in the contract, i.e., the temptation hours of work is fixed at the contracted level. Therefore, the labor market contract can be used by the consumer as a commitment device if the contracted hours of work are determined before temptation kicks. We will refer to this as *pre-commitment* of the hours of work. In a standard model without GP

preferences, the distinction between having pre-commitment or not is of no consequence for consumer behavior and optimal policy. However, when agents have GP preferences, pre-commitment does matter, and in this section, we will analyze the implications for optimal tax policy.<sup>14</sup>

## 6.1 Consumer Behavior under Pre-Commitment

Let us first look at how pre-commitment affects the individual consumer's choices. Here, the key difference is that since the consumer treats the hours of work as fixed once the labor contract has been signed, temptation hours of work is not a decision variable in the temptation choices problem. Instead, the temptation consumption choices are made conditional on  $z = 1 - l$  and  $b = wl - T(wl)$ . This means that the maximum temptation utility function defined in equation (8) is defined conditional on the pre-committed levels of  $b$  and  $z$ , i.e.  $\tilde{H} = \tilde{H}(b, z, q)$ . This implies that the conditional overall indirect utility function in (10) becomes

$$V = U(b, z, q) + H(b, z, q) - \tilde{H}(b, z, q) \quad (25)$$

Substituting  $b = wl - T(wl)$  and  $z = 1 - l$  into (25) and maximizing w.r.t.  $l$  produces the first-order condition  $\overline{\overline{MRS}}_{b,z} = \omega$  for the hours of work that the worker commits herself to in the labor contract, where

$$\overline{\overline{MRS}}_{b,z} = \frac{U_z + H_z - \tilde{H}_z}{U_b + H_b - \tilde{H}_b} \quad (26)$$

Note the difference between the definition of  $\overline{\overline{MRS}}_{b,z}$  in (26) and the definition of  $\overline{MRS}_{b,z}$  in equation (5): under pre-commitment, the consumer recognizes that the actual choices of  $b$  and  $z$  directly affect the maximum temptation utility  $\tilde{H}(b, z, q)$  while this is not the case in the basic model presented in Section 3. Furthermore, since  $a_c(c(b, q)) < a_c(\tilde{c}(b, q))$ , it follows that for a point  $(z, b)$ , the following chain of inequalities hold

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<sup>14</sup> To reiterate, in the model specified in the previous sections, the consumer/worker treats the hours of work as a variable which is determined *simultaneously* with consumption during the contract period. Therefore, more than the timing, what distinguishes the two cases is whether the labour supply decision is made under the influence of temptation or not: in the *pre-commitment* case, labour is chosen before temptation kicks in while in the other case, labour is chosen after temptation kicks in.

$$\widetilde{MRS}_{b,z} = \frac{f_z(z)}{a_c(\tilde{c}(b,q))} < \frac{f_z(z)}{a_c(c(b,q))} = \overline{MRS}_{b,z} < \frac{f_z(z)}{2a_c(c(b,q)) - a_c(\tilde{c}(b,q))} = \overline{\overline{MRS}}_{b,z}$$

As a result, an individual consumer's hours of work under pre-commitment will be smaller than his temptation hours of work and the actual hours of work he would supply under non-commitment (the case outlined before Section 6) in an unregulated market economy. This verifies that the timing for the choice of labor indeed matters in a framework with temptation preferences. When labor is chosen before consumption takes place and temptation has kicked in, which is the case under pre-commitment, labor supply acts a *commitment mechanism* by restricting the budget set and reducing the cost of self-control associated with the temptation utility. This implies, as we show below, that there is no *GP welfare motive* for income taxation. Instead the marginal labor income taxes will be analogous to those that appear in the conventional optimal tax model.

## 6.2 Optimal Taxation with Heterogenous Agents Revisited

The optimal tax problem under pre-commitment is similar to that outlined in Section 5.3. Following the discussion in Section 6.1, the maximum temptation utility for ability type  $i$  is  $\tilde{H}^i = \tilde{H}(b^i, z^i, q)$ . This has implications for the self-selection constraint because if the high-ability type chooses to mimic the income of the low-ability, then the mimicker commits to supplying  $\hat{l}^2 = w^1 l^1 / w^2$  hours of work in the contract in return for the post-tax income  $b^1$ . Therefore, the maximum temptation utility under mimicking is given by  $\hat{\tilde{H}}^2 = \tilde{H}(b^1, \hat{z}^2, q)$ . Since the high-ability type's maximum temptation utility under non-mimicking is given by  $\tilde{H}^2 = \tilde{H}(b^2, z^2, q)$ , these two functions no longer cancel out in the self-selection constraint (recall that this occurred in the analysis conducted in Section 5.3). This is a key difference in comparison with the analysis conducted in Section 5.3 which will have implications for the optimal tax policy to be defined below.

The Lagrange function associated with the government's maximization problem is specified as follows

$$\begin{aligned} L = & U(b^1, 1 - l^1, q) + H(b^1, 1 - l^1, q) - \tilde{H}(b^1, 1 - l^1, q) \\ & + U(b^2, 1 - l^2, q) + H(b^2, 1 - l^2, q) - \tilde{H}(b^2, 1 - l^2, q) \\ & + \lambda [U(b^2, z^2, q) + H(b^2, z^2, q) - \tilde{H}(b^2, 1 - l^2, q)] \\ & - \lambda \left[ U\left(b^1, 1 - \frac{w^1}{w^2} l^1, q\right) + H\left(b^1, 1 - \frac{w^1}{w^2} l^1, q\right) - \tilde{H}\left(b^1, 1 - \frac{w^1}{w^2} l^1, q\right) \right] \end{aligned}$$

$$+\gamma[tx^1(b^1, q) + tx^2(b^2, q) + w^1l^1 + w^2l^2 - b^1 - b^2] \quad (27)$$

By using the definitions  $\overline{MRS}^i = V_z^i/V_b^i$  and  $\widehat{MRS}^2 = \hat{V}_z^2/\hat{V}_b^2$ , and by following the same procedures as in Section 5.3, we can derive the following results;<sup>15</sup>

**Proposition 7:** *Under pre-commitment, the second-best linear commodity tax and the marginal income tax rates can be written as follows*

$$t = \sum_i \theta^i t^i + \lambda \Psi \quad (28a)$$

$$\tau^1 = -t \frac{\overline{MRS}_{b,z}^1}{w^1} \frac{\partial x^1}{\partial b^1} + \frac{\lambda \hat{V}_b^2}{w^1 \gamma} \left( \overline{MRS}_{b,z}^1 - \frac{w^1}{w^2} \widehat{MRS}_{b,z}^2 \right) \quad (28b)$$

$$\tau^2 = -t \frac{\overline{MRS}_{b,z}^2}{w^2} \frac{\partial x^2}{\partial b^2} \quad (28c)$$

where

$$\Psi = \underbrace{\frac{\tilde{H}_b^2}{\gamma \Omega} (\tilde{x}^2 - x^2)}_{\text{SC1} > 0} - \underbrace{\frac{\hat{H}_b^2}{\gamma \Omega} (\hat{x}^2 - \hat{x}^2)}_{\text{SC2} < 0} \quad (29)$$

Beginning with the optimal tax rule for the commodity tax, we note that  $\sum_i \theta^i t^i$  is a weighted average of the type-specific GP commodity tax rules for the two agent types, similar to the one presented in Section 5.3. The term  $\lambda \Psi$  is new and to interpret it, we first note that in a conventional framework without GP preferences, the second-best commodity tax formula would be given by  $t = \lambda(\hat{x}^2 - x^1) \hat{U}_b^2 / (\gamma \Omega)$ . Since the term on the RHS in this expression is proportional to the shadow price associated with the self-selection constraint,  $\lambda$ , the motive underlying this term is related to how the commodity tax affects the self-selection constraint. Therefore, the term  $\lambda(\hat{x}^2 - x^1) \hat{U}_b^2 / (\gamma \Omega)$  can be labelled the *conventional self-selection motive* for taxing a commodity. Note that this motive for taxing a commodity vanishes when consumption and leisure are uncorrelated (because  $\hat{x}^2 = x^1$ ). These results are well known in the optimal tax literature (see e.g. Edwards et al [1994], and Pirttilä and Tuomala [2001]).

Returning to equation (28a), we see that the *conventional self-selection motive* is absent in our model as leisure is additively separable from leisure, implying that  $\hat{x}^2 = x^1$ . Instead, a novel term,  $\lambda \Psi$ , appears on the RHS in equation (28a). Since this term is proportional to  $\lambda$ , it captures a self-

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<sup>15</sup> The derivations of these results are analogous to the derivations underlying the optimal tax rules presented in Proposition 6, and are therefore omitted.

selection motive for implementing a non-zero commodity tax which is directly related to the presence of GP preferences. Let us refer to this term as the *GP self-selection motive* for taxing the TP good. The *GP self-selection motive* reflects that the commodity tax affects the utility cost of exercising self-control both for the high-ability type and for the potential mimicker. This provides the government with two additional channels, compared with the conventional optimal tax model, via which the commodity tax can be used to relax a binding self-selection constraint. These channels are reflected in the definition of  $\Psi$ . The first term on the RHS in the definition of  $\Psi$  reflects a channel that works via the utility cost of exercising self-control for the high-ability type when she does not mimic the low-ability type while the second term reflects a channel that works via the mimicker's utility cost of exercising self-control. We will refer to these as self-control cost 1 (SC1) and self-control cost 2 (SC2).

To interpret SC1, recall that the high-ability type's temptation demand for the TP good exceeds her actual demand, i.e.  $\tilde{x}^2 > x^2$ . This implies that a marginally higher tax on the TP good will have a larger negative impact on  $\tilde{H}^2$  than on  $H^2$  which, in turn, reduces the high-ability type's utility cost of exercising self-control;  $\tilde{H}^2 - H^2$ . The reduction in  $\tilde{H}^2 - H^2$ , in turn, has a positive impact on  $V^2 = U^2 - (\tilde{H}^2 - H^2)$  which contributes to relaxing the self-selection constraint if it is initially binding. As such, this mechanism provides the government with an incentive to implement a higher commodity tax than otherwise and explains why SC1 is positive.

As for SC2, it is related to the potential mimicker's utility cost of exercising self-control. The interpretation of SC2 is analogous to that of SC1; a marginally higher tax on the TP good has a positive impact on the mimicker's utility,  $\hat{V}^2 = \hat{U}^2 - (\hat{\tilde{H}}^2 - \hat{H}^2)$ , and since an increase in  $\hat{V}^2$  tightens the self-selection constraint, this mechanism provides the government with a motive to set the commodity tax lower than otherwise. This explains why SC2 is negative.

Since SC1 and SC2 go in opposite directions, the net sign of  $\Psi$  in the commodity tax formula is indeterminable without making functional form assumptions. Let us therefore consider the following iso-elastic functional forms for the consumption parts of the normative and temptation utility functions (we do not need to specify a functional form for the leisure part in these functions)

$$u(c, x, z) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} + \beta \frac{x^{1-\sigma}}{1-\sigma} + f(z) & \sigma > 0, \sigma \neq 1 \\ \ln(c) + \beta \ln(x) + f(z) & \sigma = 1 \end{cases} \quad (30a)$$

$$h(c, x, z) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} + \rho\beta \frac{x^{1-\sigma}}{1-\sigma} + f(z) & \sigma > 0, \sigma \neq 1 \\ \ln(c) + \rho\beta \ln(x) + f(z) & \sigma = 1 \end{cases} \quad (30b)$$

where  $\beta$  is a positive parameter and where we recall that  $\rho > 1$ . We can now derive the following results (see the Appendix);

**Corollary 1:** *With the iso-elastic functional forms in (30), the GP self-selection motive provides the government with*

- (i) *a motive to implement a higher commodity tax if  $\sigma < 1$  (since  $\Psi > 0$ ),*
- (ii) *a motive to implement a lower commodity tax if  $\sigma > 1$  (since  $\Psi < 0$ ),*
- (iii) *no motive to influence the commodity tax if  $\sigma = 1$  (since  $\Psi = 0$ ).*

Recall that when consumption and leisure are uncorrelated, a key result in the conventional optimal tax model is that the commodity tax cannot be used to relax the self-selection constraint. Corollary 1, illustrates that this result need not hold when agents have GP preferences as there are cases when the commodity tax affects the high-ability agent's utility cost of exercising self-control by a different magnitude compared with how the tax affects the potential mimicker's utility cost of exercising self-control.

Finally, turning to the marginal income tax formulas in (28b) and (28c), they are analogous to those that appear in a conventional optimal tax model (see the marginal income tax formulas in [24] above). We therefore conclude that the presence of GP preferences does not affect the basic motives underlying marginal income taxation under pre-commitment. The reason is that when the consumer makes her labor supply choice under pre-commitment, she recognizes how this choice affects the maximum temptation utility. As such, there is no *GP welfare motive* remaining for the government to improve the welfare via the marginal labor income taxes. In a way, the consumer acts as *planner* when choosing her labor supply, making the choice before temptation has kicked in and taking into account the effect on the maximum temptation utility.

## 7. Conclusions

To the best of our knowledge, this article is the first to consider optimal redistributive taxation in a second-best economy with asymmetric information where people have temptation and self-



control problems related to a consumption good. Our analysis has focused on channels through which a government can affect welfare and has uncovered novel aspects in the interaction of the welfare improving and redistributive roles of public policy.

When agents are tempted in consumption, welfare can be improved by taxing the temptation good, and the welfare may be improved further by using income taxes. To identify these channels, we start with a model of homogenous agents, and we identify a *GP welfare motive* for taxing labor on the margin: a linear commodity tax complemented by a positive marginal income tax rate improves welfare by reducing the cost of exercising self-control.

Next, we introduce GP preferences into a two-type version of the Mirrleesian optimal tax model. This allows us to study if and how GP preferences affect the redistributive role of public policy when there is asymmetric information between the private agents and the government. Here, we show that if the low-ability type's maximum temptation utility is framed on replicating the labor income of the high-ability type, then this will provide a motive for implementing a non-zero marginal income tax rate for the high-ability type.

Finally, we point out that the timing of the labor decision is crucial for the optimal income policy. We describe a pre-commitment scenario, where labor supply is determined before consumption takes place. The key difference between this scenario and the standard case is that the agent in the pre-commitment scenario recognizes the commitment power of the labor choice via its effect on the budget set and on the cost of self-control, while this is not the case in the standard case. As a result, there is no *GP welfare* motive to use the marginal income tax to influence the consumers' labor supply decisions. Instead, we identify a novel *GP self-selection motive for commodity taxation*. The latter motive reflects that the commodity tax affects the utility cost of self-control both for the high-ability type and the potential mimicker. In particular, we find that one of the key results highlighted in the conventional optimal tax literature, namely that there is no self-selection motive for implementing a non-zero commodity tax when there is no correlation between consumption and leisure, need not hold in under pre-commitment.

Although this article generalizes the literature on optimal taxation by incorporating Gul-Pesendorfer preferences into the analysis, there are still many important aspects left to explore. Examples include the role of optimal nonlinear labor income taxation when agents are tempted to under-save and combining Gul-Pesendorfer preferences with positional preferences in an optimal tax framework.

## Appendix

### Optimal Tax Rules in the Representative Consumer Framework

Differentiating the Lagrange function defined in equation (15) w.r.t.  $b$ ,  $z$  and  $t$  produces

$$\frac{\partial L}{\partial b} = (U_b + H_b - \tilde{H}_b) + \gamma \left( t \frac{\partial x}{\partial b} - 1 \right) = 0 \quad (A1)$$

$$\frac{\partial L}{\partial z} = (U_z + H_z - \tilde{H}_z) - \gamma w = 0 \quad (A2)$$

$$\frac{\partial L}{\partial t} = -[x(U_b + H_b) - \tilde{x}\tilde{H}_b] + \gamma \left( t \frac{\partial x}{\partial q} + x \right) = 0 \quad (A3)$$

where we have used Roy's Identity in (A3). To derive the commodity tax formula in Proposition 4, multiply (A1) with  $x$  and add the resulting expression to (A3). This produces  $0 = (\tilde{x} - x)\tilde{H}_b + \gamma t(\partial x / \partial q + x \partial x / \partial b)$ . Rearranging this expression produces equation (16a) in the text. To derive the income tax formula in Proposition 4, rearrange (A1) and (A2) to read

$$U_b + H_b = \tilde{H}_b + \gamma \left( 1 - t \frac{\partial x}{\partial b} \right) \quad (A4)$$

$$U_z + H_z = \tilde{H}_z + \gamma w \quad (A5)$$

Divide (A5) by (A4) and use the definition of  $\overline{MRS}_{b,z}$

$$\overline{MRS}_{b,z} = \frac{\tilde{H}_z + \gamma w}{\tilde{H}_b + \gamma \left( 1 - t \frac{\partial x}{\partial b} \right)} \quad (A6)$$

Multiply up the denominator on the RHS, divide by  $\gamma$ , rearrange and use the definition of  $\overline{MRS}_{b,z}$

$$w - \overline{MRS}_{b,z} = \frac{\tilde{H}_b}{\gamma} (\overline{MRS}_{b,z} - \overline{MRS}_{b,z}) - t \overline{MRS}_{b,z} \frac{\partial x}{\partial b} \quad (A7)$$

Use that the private first-order condition for the actual hours of work can be rearranged to read  $\tau w = w - \overline{MRS}_{b,z}$ . Substituting this expression into (A7) and rearranging produces equation (16b) in Proposition 4.

### Optimal Tax Rules Under Framing

Differentiating the Lagrange function defined in equation (21) w.r.t.  $b^1$ ,  $l^1$ ,  $b^2$ ,  $l^2$  and  $t$  produces

$$\frac{\partial L}{\partial b^1} = (U_b^1 + H_b^1) - \lambda(\hat{U}_b^2 + \hat{H}_b^2) + \gamma \left( t \frac{\partial x^1}{\partial b^1} - 1 \right) = 0 \quad (B1)$$

$$\frac{\partial L}{\partial l^1} = -(U_z^1 + H_z^1) + \lambda \frac{w^1}{w^2} (\hat{U}_z^2 + \hat{H}_z^2) + \gamma w^1 = 0 \quad (B2)$$

$$\frac{\partial L}{\partial b^2} = (1 + \lambda)(U_b^2 + H_b^2) - \tilde{H}_b^2 - \tilde{H}_b^{1,2} + \gamma \left( t \frac{\partial x^2}{\partial b^2} - 1 \right) = 0 \quad (B3)$$

$$\frac{\partial L}{\partial l^2} = -(1 + \lambda)(U_z^2 + H_z^2) + \tilde{H}_z^2 + \frac{w^2}{w^1} \tilde{H}_z^{1,2} + \gamma w^2 = 0 \quad (B4)$$

$$\begin{aligned} \frac{\partial L}{\partial t} = & -x^1(U_b^1 + H_b^1) - (1 + \lambda)[x^2(U_b^2 + H_b^2)] + \tilde{x}^2 \tilde{H}_b^2 \\ & + \tilde{x}^{1,2} \tilde{H}_b^{1,2} + \lambda \hat{x}^2 (\hat{U}_b^2 + \hat{H}_b^2) + \gamma \left[ x^1 + x^2 + t \left( \frac{\partial x^1}{\partial q} + \frac{\partial x^2}{\partial q} \right) \right] = 0 \end{aligned} \quad (B5)$$

where we have used Roy's Identity to rewrite (B5). To derive the commodity tax formula in Proposition 6, multiply (B1) by  $x^1$  and (B3) by  $x^2$ . Adding the resulting expressions to (B5) gives

$$\gamma t \Omega = \tilde{H}_b^2 (\tilde{x}^2 - x^2) + \lambda (\hat{U}_b^2 + \hat{H}_b^2) (\hat{x}^2 - x^1) + \tilde{H}_b^{1,2} (\tilde{x}^{1,2} - x^2) \quad (B6)$$

Dividing by  $\gamma \Omega$  produces

$$t = \frac{\tilde{H}_b^{1,2}}{\gamma \Omega} (\tilde{x}^{1,2} - x^2) + \frac{\tilde{H}_b^2}{\gamma \Omega} (\tilde{x}^2 - x^2) \quad (B7)$$

where we have used that  $\hat{x}^2 = x^1$ . Multiply and divide the first term on the RHS by  $(\partial x^1/\partial q + x^1 \partial x^1/\partial b^1)$ , then multiply and divide the second term on the RHS by  $(\partial x^2/\partial q + x^2 \partial x^2/\partial b^2)$ . Then use the definitions of  $\theta^1, \theta^2, t^{1,2}$  and  $t^2$ . This produces  $t = \theta^1 t^{1,2} + \theta^2 t^2$ .

To derive the marginal income tax formula for the *low-ability type*, we rewrite (B1) and (B2) as follows

$$U_b^1 + H_b^1 = \lambda(\hat{U}_b^2 + \hat{H}_b^2) + \gamma \left(1 - t \frac{\partial x^1}{\partial b^1}\right) \quad (\text{B8})$$

$$U_z^1 + H_z^1 = \lambda \frac{w^1}{w^2} (\hat{U}_z^2 + \hat{H}_z^2) + \gamma w^1 \quad (\text{B9})$$

Divide (B9) by (B8) and use the definition of  $\overline{MRS}_{b,z}^1$

$$\overline{MRS}_{b,z}^1 = \frac{\lambda \frac{w^1}{w^2} (\hat{U}_z^2 + \hat{H}_z^2) + \gamma w^1}{\lambda (\hat{U}_b^2 + \hat{H}_b^2) + \gamma \left(1 - t \frac{\partial x^1}{\partial b^1}\right)} \quad (\text{B10})$$

Multiply up the denominator and rearrange

$$\gamma(w^1 - \overline{MRS}_{b,z}^1) = \lambda(\hat{U}_b^2 + \hat{H}_b^2) \left( \overline{MRS}_{b,z}^1 - \frac{w^1 \hat{U}_z^2 + \hat{H}_z^2}{w^2 \hat{U}_b^2 + \hat{H}_b^2} \right) - \gamma t \overline{MRS}_{b,z}^1 \frac{\partial x^1}{\partial b^1} \quad (\text{B11})$$

Using  $\tau^1 w^1 = w^1 - \overline{MRS}_{b,z}^1$  and the definition of  $\overline{MRS}_{b,z}^2$ , and then dividing by  $\gamma w^1$  produces equation (22b) in Proposition 6.

To derive the marginal income tax formula for the *high-ability type*, we rewrite (B3) and (B4) as follows

$$(1 + \lambda)(U_b^2 + H_b^2) = \tilde{H}_b^2 + \tilde{H}_b^{1,2} + \gamma \left(1 - t \frac{\partial x^2}{\partial b^2}\right) \quad (\text{B12})$$

$$(1 + \lambda)(U_z^2 + H_z^2) = \tilde{H}_z^2 + \frac{w^2}{w^1} \tilde{H}_z^{1,2} + \gamma w^2 \quad (\text{B13})$$

Divide (B13) by (B12) and use the definition of  $\overline{MRS}_{b,z}^2$

$$\overline{MRS}_{b,z}^2 = \frac{\tilde{H}_z^2 + \frac{w^2}{w^1} \tilde{H}_z^{1,2} + \gamma w^2}{\tilde{H}_b^2 + \tilde{H}_b^{1,2} + \gamma \left(1 - t \frac{\partial x^2}{\partial b^2}\right)} \quad (\text{B14})$$

Multiply up the denominator and rearrange

$$\gamma(w^2 - \overline{MRS}_{b,z}^2) = \tilde{H}_b^2 \left( \overline{MRS}_{b,z}^2 - \frac{\tilde{H}_z^2}{\tilde{H}_b^2} \right) + \tilde{H}_b^{1,2} \left( \overline{MRS}_{b,z}^2 - \frac{w^2 \tilde{H}_z^{1,2}}{w^1 \tilde{H}_b^{1,2}} \right) - \gamma t \overline{MRS}_{b,z}^2 \frac{\partial x^2}{\partial b^2} \quad (\text{B15})$$

Use  $\tau^2 w^2 = w^2 - \overline{MRS}_{b,z}^2$ , and the definitions of  $\overline{MRS}_{b,z}^{1,2}$  and  $\overline{MRS}_{b,z}^2$ . Dividing by  $\gamma w^2$  produces equation (22c).

### **Corollary 1**

With the iso-elastic functional forms defined in (30a) and (30b), the GP utility function defined in equation (1) can be written as

$$u(c, x, z) + h(c, x, z) - h(\tilde{c}, \tilde{x}, \tilde{z}) = 2 \frac{c^{1-\sigma}}{1-\sigma} + 2B \frac{x^{1-\sigma}}{1-\sigma} + 2f(z) - \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - \rho\beta \frac{\tilde{x}^{1-\sigma}}{1-\sigma} - f(z) \quad (\text{C1})$$

where  $B = (1 + \rho)\beta/2$ . To derive the actual demand functions, we substitute  $c = b - qx$  into (C1) and maximize w.r.t.  $x$ . Solving this problem produces the following demand functions

$$c(b, q) = \alpha(q) \frac{b}{\alpha(q) + q}, \quad x(b, q) = \frac{b}{\alpha(q) + q} \quad (\text{C2})$$

where  $\alpha(q) = (B/q)^{-\frac{1}{\sigma}}$ . To derive the temptation demand functions, we substitute  $\tilde{c} = b - q\tilde{x}$  into  $h(\tilde{c}, \tilde{x}, \tilde{z})$  and maximize w.r.t.  $\tilde{x}$ . The solution to this problem produces the following temptation demand functions

$$\tilde{c}(b, q) = \tilde{\alpha}(q) \frac{b}{\tilde{\alpha}(q) + q}, \quad \tilde{x}(b, q) = \frac{b}{\tilde{\alpha}(q) + q} \quad (\text{C3})$$

where  $\tilde{\alpha}(q) = (\rho\beta/q)^{-\frac{1}{\sigma}}$ . Substituting these expressions back into the temptation utility function gives

$$\tilde{H}(b, z, q) = \frac{1}{1-\sigma} \left[ \left( \frac{\tilde{\alpha}(q)b}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} + \rho\beta \left( \frac{b}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} \right] + f(z) \quad (C4)$$

Differentiating this function w.r.t.  $b$  produces

$$\tilde{H}_b(b, q) = \left[ \left( \frac{\tilde{\alpha}(q)}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} + \rho\beta \left( \frac{1}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} \right] b^{-\sigma} \quad (C5)$$

Conditional on these functional forms, we want to evaluate the sign of  $\Psi$ . Substituting

$$\begin{aligned} \tilde{H}_b^2(b^2, q) &= \left[ \left( \frac{\tilde{\alpha}(q)}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} + \rho\beta \left( \frac{1}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} \right] (b^2)^{-\sigma}, \\ \hat{H}_b^2(b^1, q) &= \left[ \left( \frac{\tilde{\alpha}(q)}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} + \rho\beta \left( \frac{1}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} \right] (b^1)^{-\sigma} \\ x^2 &= x(b^2, q) = \frac{b^2}{\alpha(q)+q}, & \tilde{x}^2 &= \tilde{x}(b^2, q) = \frac{b^2}{\tilde{\alpha}(q)+q} \\ \hat{x}^2 &= x(b^1, q) = \frac{b^1}{\alpha(q)+q}, & \hat{\tilde{x}}^2 &= \tilde{x}(b^1, q) = \frac{b^1}{\tilde{\alpha}(q)+q} \end{aligned} \quad (C6)$$

into the definition of  $\Psi$  in (29) and simplifying produces

$$\Psi = \frac{1}{\gamma\Omega} \left[ \left( \frac{\tilde{\alpha}(q)}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} + \rho\beta \left( \frac{1}{\tilde{\alpha}(q)+q} \right)^{1-\sigma} \right] \left[ \frac{1}{\tilde{\alpha}(q)+q} - \frac{1}{\alpha(q)+q} \right] [(b^2)^{1-\sigma} - (b^1)^{1-\sigma}] \quad (C7)$$

Since  $B < \rho\beta$ , it follows that  $\tilde{\alpha}(q) = (\rho\beta/q)^{-\frac{1}{\sigma}} < \alpha(q) = (B/q)^{-\frac{1}{\sigma}}$ . Hence, the expression inside the second pair of square brackets is positive. Since  $\Omega > 0$ , it follows that the sign of  $\Psi$  depends on the sign of  $(b^2)^{1-\sigma} - (b^1)^{1-\sigma}$ . Since  $b^2 > b^1$ , it follows that  $\Psi > 0$  if  $\sigma < 1$  and  $\Psi < 0$  if  $\sigma > 1$ . This verifies parts (i) and (ii) in Corollary 1.

If the agent instead has logarithmic preferences, the functions in (C6) are modified to read

$$\begin{aligned} \tilde{H}_b^2 &= \frac{\partial H(\tilde{b}^2, q, \tilde{z}^2)}{\partial \tilde{b}^2} = \frac{\partial H(b^2, q, z^2)}{\partial b^2} = \frac{1+\rho\beta}{b^2}, & \hat{H}_b^2 &= \frac{\partial H(\hat{b}^2, q, \hat{z}^2)}{\partial \hat{b}^2} = \frac{\partial H(b^1, q, \hat{z}^2)}{\partial b^1} = \frac{1+\rho\beta}{b^1} \\ x^2 &= x(b^2, q) = \frac{B}{1+B} \frac{b^2}{q}, & \tilde{x}^2 &= \tilde{x}(b^2, q) = \frac{\rho\beta}{1+\rho\beta} \frac{b^2}{q} \\ \hat{x}^2 &= x(b^1, q) = \frac{B}{1+B} \frac{b^1}{q}, & \hat{\tilde{x}}^2 &= \tilde{x}(b^1, q) = \frac{\rho\beta}{1+\rho\beta} \frac{b^1}{q} \end{aligned} \quad (C8)$$

Substituting these expressions into (29) and simplifying produces  $\Psi = 0$ . Hence  $\Psi = 0$  if  $\sigma = 1$ . This verifies part (iii) in Corollary 1.

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