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An R Tool for Computing and Evaluating Fuzzy Poverty Indices. The package FuzzyPovertyR

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Abstract
Fuzzy set theory has become increasingly popular for deriving uni- and multi-dimensional poverty estimates. In recent years, various authors have proposed different approaches to defining membership functions, resulting in the development of various fuzzy poverty indices. This paper introduces a new R package called FuzzyPovertyR, designed for estimating fuzzy poverty indices. The package is demonstrated by using it to estimate three fuzzy poverty indices – one multi- and two uni-dimensional – at regional level (NUTS 2) in Italy. The package allows users to select from a range of membership functions and includes tools for estimating the variance of these indices by the ad-hoc Jack-Knife repeated replication procedure or by naive and calibrated non-parametric bootstrap methods.

KEYWORDS
Bootstrap; EU-SILC; Jack-Knife; Multidimensional poverty; Unidimensional Poverty

1. Introduction

Analysing and monitoring increasingly complex socio-economic phenomena, such as social vulnerability, poverty and quality of life, requires sophisticated data structures and advanced statistical methods. Traditional statistical software often lacks the necessary tools to perform these intricate analyses. As a result, there is a growing need for specialized statistical packages that can be integrated into widely used software environments, facilitating the implementation of these advanced methods.

In this paper, we introduce FuzzyPovertyR, a new statistical package developed in the R environment [36]. The package is designed to estimate and evaluate fuzzy indices and their variances, addressing the challenge of measuring complex socio-economic phenomena, including uni- and multi-dimensional poverty [1, 7, 33, 39, 44]. The motivation for this package stems from the increasing number of studies on fuzzy indices in recent decades and ongoing criticism of classical crisp indices. In particular, poverty measurement has traditionally relied on a dichotomous classification that separates individuals into "poor" and "non-poor" based on a predefined poverty threshold. How-

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ever, as noted by [38], this binary approach is inherently imprecise, as it fails to capture the continuous nature of poverty. A more realistic perspective acknowledges a gradual transition from severe poverty to wealth rather than an abrupt division.

To address this limitation, FuzzyPovertyR provides a framework for computing poverty indices that supersedes dichotomization [18, 24, 40]. The package enables researchers to apply fuzzy set theory [49] to poverty measurement, allowing a more nuanced representation of economic hardship. In the following sections (and in Section 1 of the supplementary materials), we present a range of fuzzy indices that enable this gradual transition, along with their estimation formulas and variance computation methods.

Modern research increasingly adopts a multidimensional perspective on poverty, recognizing determinants beyond income and consumption. Socio-economic hardship encompasses aspects such as education, health and living conditions, requiring the aggregation of multiple indicators [4, 27, 29, 32]. This in turn increases the complexity of analysis and calls for advanced methods of constructing multi-dimensional poverty indices. Fuzzy set theory offers a robust approach to this challenge, as it effectively captures the degree of poverty rather than relying on strict thresholds. Several studies have explored the theoretical and applied aspects of this approach [5, 28, 31], comparing it with other multi-dimensional techniques.

In this context, FuzzyPovertyR provides researchers with a user-friendly tool for computing fuzzy indices, including the fuzzy multi-dimensional index proposed by [12] and various uni-dimensional fuzzy indices. The package also supports estimation of fuzzy index variance by non-parametric techniques such as Jack-Knife Repeated Replications (JRR) and bootstrap methods [14, 15, 17, 46]. It also includes a dedicated function for computing equivalised income or consumption variables, allowing users to adjust for household size and composition using four predefined or self-defined equivalence scales.

FuzzyPovertyR also introduces two new S3[1] classes that offer specific plotting and summary methods, enhancing data visualization and interpretation. The package aims to fill gaps in existing statistical tools by complementing and extending the functions of well-known R packages such as Laeken [3], which computes the 17 Laeken poverty and inequality indices, and the ineq family of packages [6, 37, 41, 48, 51], which estimate various inequality indices. Existing multi-dimensional poverty packages, such as MPI, mpindex and mpitbR, primarily use Alkire-Foster methodology [2, 25, 30]. In contrast, FuzzyPovertyR can be a fuzzy counterpart to these tools, enabling computation of several fuzzy uni-dimensional indices, a fuzzy multi-dimensional index, and robust estimation of variance.

Finally, like the Laeken package, FuzzyPovertyR allows researchers to estimate indices at aggregate and disaggregated levels using a single function. This flexibility enables analysis across different population groups or regions without requiring multiple functions, making it a valuable addition to the toolkit of researchers studying poverty and inequality.

In summary, FuzzyPovertyR, allows uni- and multi-dimensional poverty indices to be estimated on the basis of different datasets with different data structure. For instance, all the data used in the following studies can be used in FuzzyPovertyR. In the European context, European Statistics on Income and Living Conditions (EU-SILC) is a source of data for poverty analysis [45]. In Asia, [35] developed fuzzy set analy-

[1] S3 classes represent a type of object-oriented system in R, providing a simple way to define specific methods (e.g., plot, summary) for objects, without requiring any additional operation by the users. For more details, see [47]

sis based on household data on housing, education, health and living standards. For Africa, [19] used consumption data. FuzzyPovertyR also enabled researchers to reproduce the results of [16] by creating a fuzzy index to measure violence against women with data from the survey on violence against women conducted by the Fundamental Rights Agency. It can also be applied to quality-of-life assessments, such as those derived from the Eurofound survey [13, 42], or in the health domain, as demonstrated by [26]. In conclusion, FuzzyPovertyR can even be used with non-probabilistic data to compute specific indices, such as the one developed by [34], a fuzzy index for bank geo-marketing.

This article is organized as follows. Readers already familiar with the fuzzy approach to poverty measurement may wish to proceed directly to Section 3, where the well-known Head Count Ratio (HCR[2]), two fuzzy poverty indices, and the fuzzy multi-dimensional index are estimated and compared using Italian EU-SILC data. Sections 2 and 3 introduce the concept of membership functions to explain the basics of fuzzy sets applied to poverty measurement, as well as estimation of variance through Bootstrap or JRR of these poverty measures. Specifically, the simplified ad-hoc procedure for fuzzy poverty measures is discussed in detail [14]. Section 4 concludes the paper. Readers interested in further details on the package, its structure, S3 classes, the specifics of the functions implemented, and the code for reproducing results based on a simulated dataset are referred to the supplementary materials or the package vignettes available on CRAN, or may simply run vignette("FuzzyPovertyR") in R.

2. Methodology

The package introduces new functions for computing uni- and multi-dimensional fuzzy indices and their variances. The functions are: fm_construct, fs_construct, fs_construct_all, fs_equate, fs_order, fs_transform, fs_weight, fm_var and fs_var. The first function computes eight different uni-dimensional fuzzy indices, while the functions starting with "fs" are useful for computing the multi-dimensional index step by step. The package also includes the functions eq_predicate, HCR, and the dataset eusilc. In the following section, we briefly review fuzzy set theory and some of the indices that can be computed using the package. For a detailed description of all the options and arguments of the functions, see supplementary materials.

2.1. Poverty analysis using fuzzy sets theory

Fuzzy sets represent the initial endeavour to surpass the Aristotelian two-valued logic that underlies crisp sets. In fuzzy logic, the statement a is a member of A, is not necessarily true or false. It can be partly true, and its veracity is given by a certain degree. The way of establishing such degree is one of the main issues of fuzzy sets as it represents the extent of how much a belongs to A, and it is a number in the interval $[0, 1]$.

The function

$$\mu : A \to [0, 1] \subseteq R,$$

that assigns to a its degree of membership to A is known as the membership function.

[2]HCR is also known as "At Risk of Poverty Rate" and it is one the Laeken indices [3].

When the membership degree is equal to zero, the object does not belong to the set at all, whereas when it is equal to 1 the object belongs fully to the set. Since full membership and full non-membership to fuzzy sets are allowed, the concept of a crisp set can be considered a special case of the more general fuzzy set. The traditional approach to poverty measurement based on the definition of a poverty line can therefore be seen as a crisp set where the population is divided into two subsets (1 poor, 0 not poor). The poorest individual or household will have value of 1, while the least poor will have value of 0, and all the others will have values in the interval $[0, 1]$ depending on their situation.

2.2. Membership functions in a uni-dimensional setting

Let Y be a poverty predicate (e.g. income or expenditure) and y_i be the level of Y taken by the i-th statistical unit with $i = 1, \ldots, n$. In the history of the fuzzy approach to poverty measurement, the first proposal of a membership function for the set of poor individuals (or households) was that of [20] who suggested specifying the following membership function, μ_i for the i-th statistical unit:

$$\mu_i = \begin{cases} 1, \quad 0 < y_i < z_1 \\ \frac{z_2 - y_i}{z_2 - z_1}, \quad z_1 \leq y_i < z_2. \\ 0 \quad y_i \geq z_2 \end{cases} \tag{1}$$

This membership function has a simple trapezoidal form and leads to a fuzzy index in which the values z_1 and z_2 have to be chosen by the researcher, and a linear function is used for all intermediate values.

[22] proposed the Totally Fuzzy and Relative (TFR) approach, where the membership function is defined as

$$\mu_i = (1 - F_Y(y_i))^{\alpha - 1} = \left(\frac{\sum_{j=i+1}^{n} w_j \mid y_j > y_i}{\sum_{j \geq 2}^{n} w_j > y_1} \right)^{\alpha - 1}, \tag{2}$$

where F_Y is the empirical cumulative distribution function of Y calculated for the i-th individual and w_i is the sampling weight of statistical unit i. The parameter $\alpha \geq 2$ is chosen so that the average over the function equals the head count ratio (i.e. the proportion of units whose equivalised disposable income falls below the poverty line) of the whole population. Interestingly, the head count ratio may be seen also as the average of a membership function defined as $\mu_i = 1$ if $y_i < z, \mu_i = 0$ if $y_i \geq z$, where y_i is the equivalised disposable income of individual i, and z is the chosen poverty line.

[10] suggested to change the cumulative distribution function of Y in the preceding Equation (2), with the Lorenz curve L, obtaining a new membership function defined

$$\mu_i = (1 - L_Y(y_i))^{\alpha - 1} = \left(\frac{\sum_{j=i+1}^{n} w_j y_j \mid y_j > y_i}{\sum_{j \geq 2}^{n} w_j y_j \mid y_j > y_1} \right)^{\alpha - 1}, \tag{3}$$

where again, the parameter $\alpha \geq 2$ is chosen as above.

Subsequently, [11] considered he proportion of individuals less poor than the person concerned, $1 - F_Y(y_i)$, and the share of the total equivalised income received by all individuals less poor than the person concerned, in order to obtain a new membership function

$$\mu_i = \left(1 - F_Y(y_i)\right)^{\alpha-1} \left(1 - L_Y(y_i)\right), \tag{4}$$

where α is computed as in Equation (2) and Equation (3). The other indices (see supplementary material) that can be obtained with the package are: [21], [8], [9], and [50].

2.3. Multi-dimensional approach

Besides in terms of monetary income (or expenditure), the standard of living of households and persons can also be described in terms of a set of items that measure different aspects of household deprivation (i.e. non-monetary aspects of poverty), such as housing conditions, possession of durable goods, expectations, and perception of general financial situation and hardship. Exploitation of these various items (henceforth deprivation items) marks a shift in the way we approach the study of poverty: a shift from a uni- to a multi-dimensional perspective.

Since deprivation items are usually measured on an ordinal scale, the combination of multi-dimensionality and the non-quantitative nature of the variables greatly complicates the fuzzy approach to poverty analysis. Indeed, it raises the problem of quantifying and then aggregating the items in an attempt to synthesize them into a composite index that represents supplementary information with respect to that provided by an economic variable alone [43].

In particular, the quantification and aggregation of these items to construct composite indices requires decisions about: i) assigning numerical values to ordered categories; ii) the weighting system, which is often chosen on the basis of the statistical characteristics of the data; iii) the method of aggregation.

In endeavouring to integrate the fuzzy measurement of poverty based solely on a monetary predicate with a supplementary poverty index combining other dimensions of poverty, [11] proposed the Fuzzy Supplementary (FS) approach. This step-by-step method can be summarized as follows (see supplementary materials for a detailed description of each step, including all the functions);

- Step 1: Identification of k items ($j = 1, \ldots, k$) of deprivation to be included in the analysis;
- Step 2: Transformation of the items into the unit interval by using:

  $$s_{ij} = 1 - \frac{1 - F(c_{ij})}{1 - F(1)}, \quad i = 1, \ldots, n \quad \text{and} \quad j = 1, \ldots, k, \tag{5}$$

  where c_{ij} is the value of the category of the j-th item for the i-th household and $F(c_{ij})$ is the value of the j-th item cumulative function for the i-th household;
- Step 3: Exploratory and confirmatory factor analysis to identify h dimensions of deprivation ($h = 1, \ldots, H$);
- Step 4: Calculation of weights of individual items of deprivation within each

dimension. The weight of item j belonging to dimension h is taken as:

$$\omega_{hj} = \omega^a_{hj} \times \omega^b_{hj} \quad h = 1, \ldots, H \quad \text{and} \quad j = 1, \ldots, k_h. \tag{6}$$

The first factor ω^a_{hj} is taken as proportional to the coefficient of variation of the complement to one of the positive score for the variable concerned

$$\omega^a_{hj} \propto \frac{\sigma_{s_{hj}}}{1 - \bar{s}_{hj}}, \tag{7}$$

where $\sigma_{s_{hj}}$ is the standard deviation of the deprivation score s for item j in dimension h and $\bar{s}_{hj}$ its sample mean. The second factor, ω^b_{hj}, is defined in terms of the Kendall's correlation coefficient between deprivation indicators corresponding to items j and j^* in the h-dimension and it is detailed in the supplementary materials;

- Step 5: Calculation of scores for each dimension;
- Step 6: Calculation of an overall score. This step applies the membership function seen in Equation (4) to the scores as follows

$$\mu_i = (1 - F_S(s_i))^{\alpha - 1} (1 - L_S(s_i)). \tag{8}$$

- Step 7: Construction of the fuzzy deprivation measure in each dimension. The parameter α is computed as in Equation (3). The FS index for the h-th deprivation dimension for the i-th individual is defined as combination of the $(1 - F_{S,hi})$ indicator and the $(1 - L_{S,hi})$ indicator.

$$\mu_{hi} = (1 - F_{S_h}(s_{hi}))^{\alpha - 1} (1 - L_{S_h}(s_{hi})). \tag{9}$$

The term $(1 - F_{S_h}(s_{hi}))$ is the proportion of individuals who are less deprived than i in the h-th dimension.

2.4. Variance Estimation

The paper, through two simple functions, fm_var and fs_var, allows the computation of the the Mean Squar Error (MSE). Several options, summarized in the supplementary material, enable users to choose not only the estimator — either "jackknife," "bootstrap_naive," or "bootstrap_calibrated" — but also whether to recompute the poverty line and, consequently, the membership function at each replicate of the estimator. The three different methods provide the possibility to use various estimators, each of which can be preferred in specific situations. The Jack-Knife method implemented here is the one described by [14]. It is ad hoc for fuzzy indices but can only be performed when design information (i.e., strata and primary selection units) is available. The two bootstrap methods can be adopted without that information, but the calibrated method requires knowledge of population totals and can outperform the naive method for complex designs. The bootstrap methods align with those of the Laeken package [3].

3. Application

The database we use to illustrate the package FuzzyPovertyR is the EU-SILC survey. More specifically, we use Italian data for the year 2017 (now on IT-SILC). This complex survey is widely used to measure and compare the living conditions of households and individuals in Europe. There are numerous contributions in the literature that use this data for poverty analysis, employing both traditional and multi-dimensional measures. This data source is also suitable for poverty studies based on the fuzzy methodology illustrated in the previous paragraphs. The data from this survey can be accessed through Eurostat by submitting a research project, and for privacy reasons, cannot be shared. In the next section, we describe the results obtained with the package using Italian data from 2017. However, in the package as described in the supplementary materials, we provide a simulated dataset that replicates the same structure and can be easily used to understand the functionalities of the proposed functions.

3.1. What can we learn by using fuzzy indices?

In this application, we now demonstrate how the use of fuzzy indices (both uni- and multi-dimensional) can aid our understanding of poverty. We analyse living conditions at regional level in Italy. Italy shows sharp geographical differences, especially between north and south. Fuzzy methods can help identify more nuanced differences that are less evident between regions in the same macro-areas of the country.

To highlight this remarkable potential of the fuzzy approach, we estimate: i) the HCR, considering it the target index among those based on dichotomization of the population with a poverty line set at 60% of the equivalised median income; ii) the index proposed by [20], regarded as the seminal fuzzy index, with thresholds of 1500 and 15000; (iii) the FM index, which is likely the most widely used; and finally, (iv) the multi-dimensional FS index. The uni-dimensional indices are based on equivalised income, while the multi-dimensional index is computed using the variables shown in the supplementary materials. The equivalised income, fuzzy indices and their variability are computed using the functions eq_predicate, fm_construct, fs_construct, fm_var, and fs_var, all within the FuzzyPovertyR package. The HCR index and its variance are computed using functions from the Laeken package [3].

Figure 1 shows the estimates of the four indices across the regions of Italy, while Table 1 shows the estimates, their variability and the ratios of the two fuzzy measures and the HCR for each region (respectively FM/HCR and FS/HCR). These ratios facilitate comparison of the fuzzy measures and HCR, since their values are not directly comparable. In fact, HCR is expressed as a percentage, whereas the fuzzy measures range from 0 to 1 and indicate the degree of membership to the set of the poor. Furthermore, these ratios demonstrate how fuzzy measures can highlight or smooth the results obtained with the traditional poverty measure based on simple dichotomization, i.e. poor versus non-poor.

The maps clearly show a geographical trend of the HCR index with a sharp increase in poverty rates from the northern to the southern regions of Italy. This north-south divide in poverty levels, where the south tends to be poorer, is well-documented in poverty research.

However, when we focus on the ratios calculated in Table 1, interesting conclusions can be drawn. Let us consider, for example, Piedmont and Sicily, two regions that traditionally have low and high poverty rates, respectively. The numbers on the maps

Table 1. Regional Estimates of HCR and fuzzy indices and the ratios FM/HCR and FS/HCR. In brackets the square root of the MSE multiplied by 100.

	Region	HCR	Cerioli-Zani	FM	FS	FM/HCR	FS/HCR
1	Piedmont	0.13	0.12	0.15	0.23	110.03	169.05
		(0.70)	*(0.48)*	*(0.52)*	*(0.56)*		
2	Aosta Valley	0.14	0.12	0.14	0.26	103.23	190.24
		(1.54)	*(1.10)*	*(1.07)*	*(1.26)*		
3	Lombardy	0.13	0.11	0.13	0.14	98.20	108.14
		(0.64)	*(0.36)*	*(0.38)*	*(0.34)*		
4	Trentino-Alto Adige	0.09	0.08	0.10	0.15	113.24	161.50
		(1.04)	*(0.65)*	*(0.76)*	*(1.00)*		
5	Veneto	0.10	0.09	0.12	0.16	117.76	161.53
		(0.56)	*(0.34)*	*(0.41)*	*(0.38)*		
6	Friuli-Venezia Giulia	0.09	0.09	0.11	0.20	116.61	216.67
		(0.69)	*(0.45)*	*(0.45)*	*(0.64)*		
7	Liguria	0.13	0.12	0.14	0.20	105.90	148.57
		(0.75)	*(0.60)*	*(0.57)*	*(0.71)*		
8	Emilia-Romagna	0.11	0.09	0.11	0.18	104.87	164.90
		(0.60)	*(0.39)*	*(0.34)*	*(0.49)*		
9	Tuscany	0.13	0.11	0.14	0.19	107.85	154.33
		(0.64)	*(0.36)*	*(0.44)*	*(0.48)*		
10	Umbria	0.11	0.11	0.14	0.19	121.90	167.54
		(1.07)	*(0.60)*	*(0.61)*	*(0.67)*		
11	Marche	0.16	0.13	0.16	0.19	97.94	121.36
		(0.84)	*(0.66)*	*(0.58)*	*(0.62)*		
12	Lazio	0.18	0.15	0.18	0.15	96.25	81.86
		(0.80)	*(0.44)*	*(0.39)*	*(0.38)*		
13	Abruzzo	0.19	0.17	0.20	0.20	106.14	106.45
		(1.53)	*(0.96)*	*(0.97)*	*(0.82)*		
14	Molise	0.33	0.23	0.27	0.25	80.88	75.25
		(1.75)	*(1.21)*	*(1.16)*	*(1.12)*		
15	Campania	0.34	0.28	0.32	0.23	92.92	67.78
		(1.24)	*(0.64)*	*(0.61)*	*(0.61)*		
16	Apulia	0.26	0.20	0.24	0.17	93.26	66.51
		(1.10)	*(0.61)*	*(0.61)*	*(0.58)*		
17	Basilicata	0.28	0.22	0.26	0.25	93.66	90.41
		(1.61)	*(1.03)*	*(0.94)*	*(1.26)*		
18	Calabria	0.36	0.29	0.33	0.26	89.78	71.42
		(1.43)	*(1.08)*	*(0.88)*	*(1.03)*		
19	Sicily	0.41	0.31	0.35	0.27	85.65	66.20
		(1.18)	*(0.91)*	*(0.75)*	*(0.64)*		
20	Sardinia	0.28	0.21	0.24	0.18	86.40	65.48
		(1.67)	*(0.98)*	*(1.02)*	*(0.85)*		

match those in Table 1 to facilitate easier visualization of the regions.

The FM/HCR ratio for Piedmont is 110. This suggests that the fuzzy measure estimates a poverty level that is 10% higher than the traditional measure. It indicates that some individuals are very close to the poverty threshold but are classified as non-poor by the traditional method. Conversely, the FM/HCR ratio for Sicily is 86, indicating a 24% decrease with respect to the traditional measure. This suggests that many households are just below the poverty threshold and are classified as poor by the traditional method.

Thus FM indices depict a more nuanced and less pronounced disparity between the northern and southern regions. The differences captured by these indices are significantly below the HCR. This divergence is partly due to the inherent limitations of the HCR. The HCR often overestimates poverty when a large portion of the population hovers just below the poverty line. It treats individuals who are just below the poverty threshold as just as poor as those who are much further below, despite the substantial difference in their actual living conditions. Fuzzy indices offer a more refined approach. They account for the fact that people closer to the poverty line are relatively better off than those with extremely low incomes. While still considered poor, those near the poverty threshold are treated as less impoverished than individuals at the very bottom of the income distribution.

These differences illustrate that although aligned with the HCR at national level, the FM index provides a more varied and arguably more accurate picture of poverty across regions.

Focusing now on the Cerioli-Zani index, some conclusions derived from the FM index are already evident. However, one of the key issues of the Cerioli-Zani index is the selection of the two thresholds, which is largely at the researcher's discretion, leading to results that are in a sense subjective. Depending on how these thresholds are set, the outcomes of the analysis can vary significantly. The two parameters of the Cerioli-Zani index work like the inequality aversion parameter of the Atkinson index. Indeed, the higher the two parameters, the greater the aversion to inequality; in other words, the Cerioli-Zani index increases as the two parameters increase. The same argument regarding definition of the thresholds applies to the other fuzzy indices presented in the supplementary materials, and also holds for some crisp sets. For an in-depth analysis of the impact of the choice of subjective parameters on the estimation and variability of the indices, see [23].

The FS index shows a somewhat different pattern. This index incorporates three distinct dimensions of poverty and is computed using categorical variables, setting it apart from the other indices. Like the FM index, the FS index is designed to have the same national mean value as the HCR (see Equation (8)). However, when we analyse the regional distribution of poverty, the FS index partially corroborates the patterns observed with the other fuzzy indices, albeit with some minor deviations. Notably, the FS index diverges more significantly from the HCR, as we see from the FM/HCR ratios. The north-south poverty divide is less pronounced when measured by the FS index. Let us again take Piedmont and Sicily as examples. In terms of FM, the values are 0.15 for Piedmont and 0.35 for Sicily, while in terms of FS, they are 0.23 and 0.27, respectively. The difference in monetary poverty levels between the two regions is therefore quite pronounced, but becomes much less evident when considering non-monetary dimensions of poverty or measures alternative to income. This suggests that the FS index is capturing dimensions of poverty not reflected in a univariate monetary dimension, and is highlighting the complex, multi-dimensional nature of poverty in these regions.

Finally, Figure 2, we also provide maps showing the estimated Coefficient of Variation (CV). The CVs were calculated as the square root of the variance obtained by the Jack-Knife method, divided by the estimated value of the index.

Considering the CV across the traditional HCR and fuzzy measures, we observe that the HCR consistently shows higher CV values than the fuzzy indices. This suggests

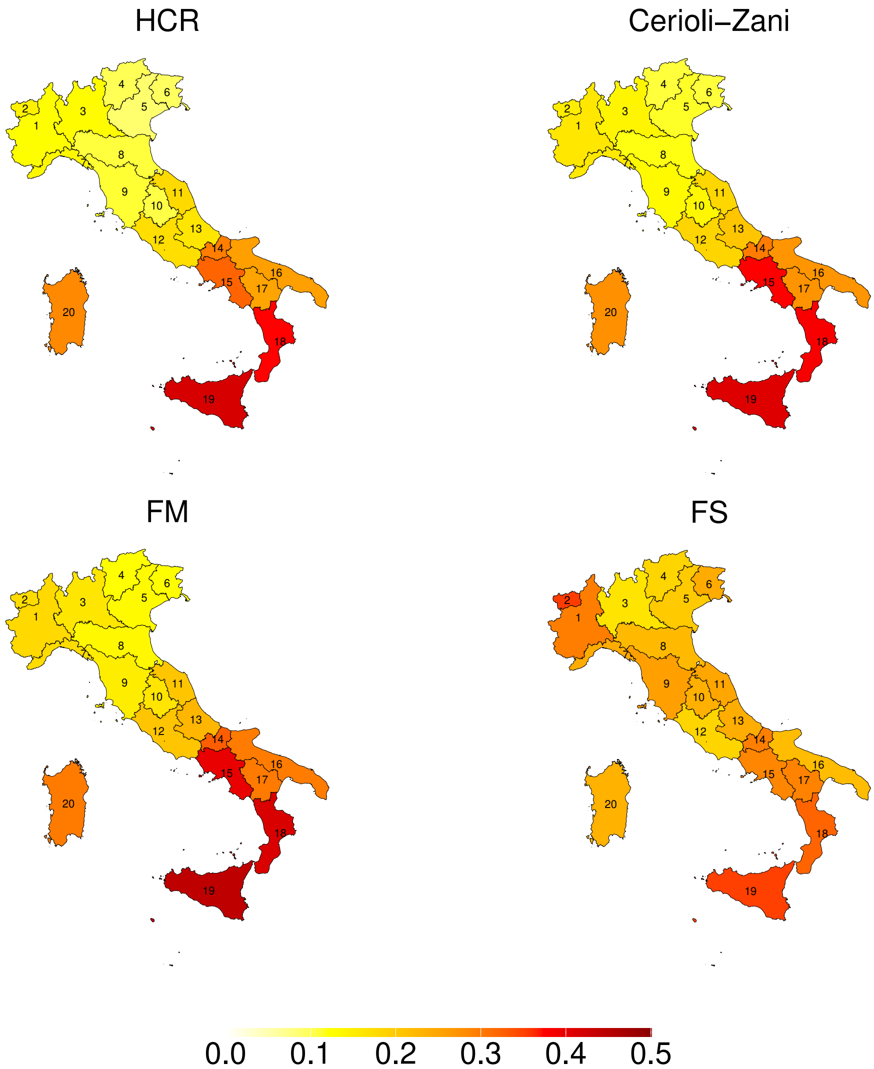

Figure 1. Choroplet map of regional estimates of HCR and fuzzy indices. The numbers over the map correspond to those in Table 1.

that the estimates provided by the HCR are less reliable due to greater variability. On the other hand, estimates of the fuzzy indices show lower variability, indicating more stable and reliable measurements of poverty across regions. This characteristic of fuzzy indices, particularly their reduced variability, has already been validated in simulation studies, as shown in [23].

3.2. Comparing Indices Beyond the Head Count Ratio

The comparison between fuzzy indices and traditional non-fuzzy indices can be further explored by considering two additional uni-dimensional indices: i) the Watts index and ii) the Sen index (for further comparison see [14, 23]). The former is similar to the HCR, with the difference that it considers the gap between the poverty line and the predicted value. The latter, like the FM index, links the HCR and the concentration index [14].

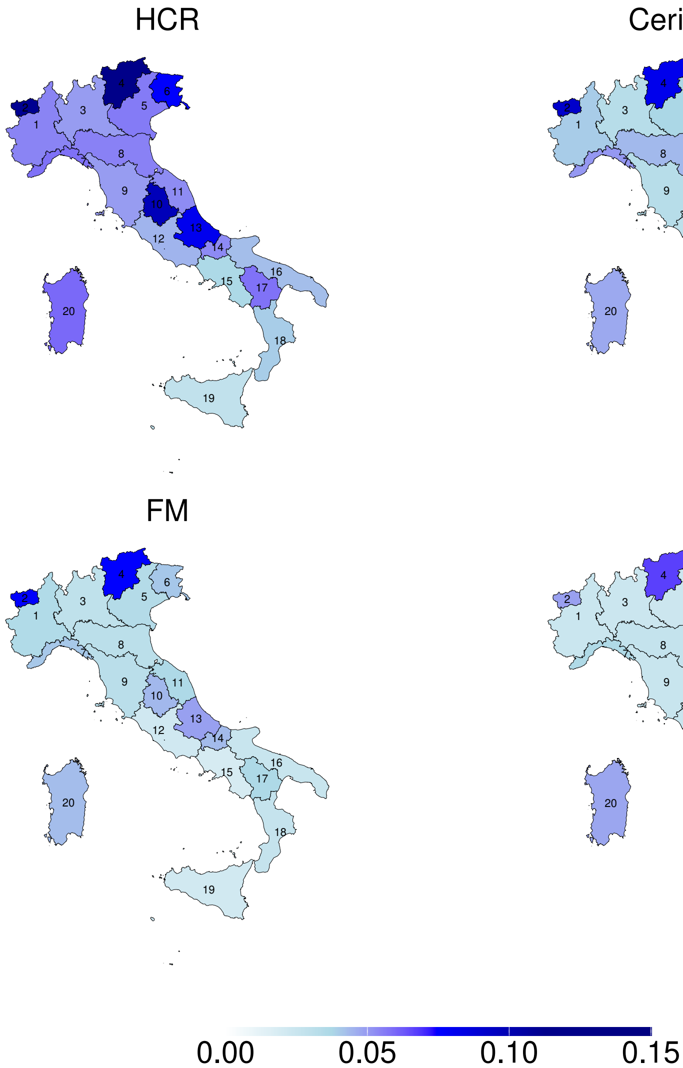

Figure 2. Choroplet map of coefficient of variation of regional estimates of HCR and fuzzy indices. The numbers over the map correspond to those in Table 1.

We computed the uni-dimensional indices for the regions of Italy. We ranked the estimates and computed the correlation between the indices. Table 2 summarizes the results. As expected, the fuzzy indices are strongly correlated and their correlation with the Watts and Sen indices is only slightly weaker.

In terms of variability, the two fuzzy indices outperform the Sen and Watts indices. The MSE for non fuzzy indices is higher than 1. For the fuzzy indices, this value is 0.673 for FM and 0.692 for the Cerioli-Zani index.

Table 2. Correlation of ranks for five uni-dimensional indies.

	HCR	Cerioli-Zani	FM	Sen	Watts
HCR	1.00	–	–	–	–
Cerioli-Zani	0.99	1.00	–	–	–
FM	0.97	0.99	1.00	–	–
Sen	0.96	0.95	0.93	1.00	–
Watts	0.93	0.94	0.91	0.99	1.00

4. Conclusions and further developments

The present note introduces the first R package specifically designed to calculate both uni- and multi-dimensional poverty indices based on fuzzy set theory. The package offers the possibility of estimating eight different uni-dimensional indices, and one supplementary multi-dimensional index.

One of the key features of this package is the ability to estimate variance for each index using three distinct estimators. The first is the Jack-Knife estimator developed by [14], which is known for its robustness in handling complex data. The second option is the more straightforward non-parametric naive bootstrap estimator, which is often preferred for its simplicity and ease of implementation. The third one is a calibrated bootstrap. Notably, this Jack-Knife procedure, which is defined ad-hoc for fuzzy indices, has been integrated into this package for the first time, representing a significant advancement in the tools available for poverty analysis.

To illustrate the practical applications of the package, this note presents an application example based on the IT-SILC data, demonstrating how different patterns of poverty can be identified when using fuzzy indices in comparison to the traditional HCR method. This example highlights the nuanced insights that can be gained from fuzzy set theory in the context of poverty measurement.

Looking ahead, there are plans for future contributions to this package. Fuzzy indices have certain limitations that will be addressed by advances in fuzzy set theory supported by easily implementable code. The first limitation is their 'unofficial nature', since all but a few of them are based on values not used in official statistics. Secondly, more precise studies are needed to refine the definitions of subjective parameters and assess their impact on estimates. Lastly, and perhaps most importantly, obtaining closed-form variance estimates, i.e. that do not rely on bootstrap and/or Jack-Knife methods, would be advisable to facilitate analysis of their analytical properties. Other possible extensions include incorporating additional membership functions.

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