Decisions in Economics and Finance
https://doi.org/10.1007/s10203-025-00506-z

Implied dividend bounds in option prices: anatomy of two markets

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Received: 15 February 2024 / Accepted: 3 January 2025

Abstract

We propose a measure of uncertainty of a dividend paying asset based on the cross-section of bid-ask spreads of options. This is the difference between the cheapest synthetic long position of the asset that it is possible to construct using European options and the most expensive short position that can be constructed at the same time for the same maturity. For index and stock option applications this measure can be compared with other direct measures of uncertainty such as the bid-ask spread of the futures market on the underlying or their dividend. It turns out that for index options the measure is tighter than the bid-ask spread of the futures market for maturities longer than the first two futures contracts. The comparison of the measure for individual stock options with dividend futures gives mixed results. Finally, applying a two-tail distortion (2TD) model we find an asymmetric setting of option ask and bid prices with respect to the reference model. Most of the distortion is loaded on the ask call price and bid put prices. This corresponds to asymmetric uncertainty in dividend yield expectations, which put more weight on low dividends.

Keywords Bid-ask spreads · Put-Call parity · Implied dividends

JEL Classification G12 · G13

1 Introduction

Two-price models of asset prices have always been motivated by the presence of illiquidity and market frictions (transactions costs, limits to short selling, and the like) and uncertainty in strong form, that is uncertainty on the dynamics and the probability measure of the asset prices. Of course, these two elements reinforce themselves in

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Published online: 22 January 2025

a vicious circle: on the one hand, illiquid markets transmit noisy information and increase uncertainty; on the other hand, higher uncertainty keeps investors away from the market and impairs market liquidity. Uncertainty is particularly relevant for the prospects of dividends, and recent literature has been devoted to implied dividend information in option prices (Golez 2014; Bilson et al. 2015; Tunaru 2017; Desmettre et al. 2017).

Yet, the complexity of the issue goes beyond this feedback between illiquidity and uncertainty. First, the analysis is often limited, because of the availability of data, to best bid and ask prices, not taking into account the volume associated to these prices. It may happen that very tight bid and ask spreads be associated with tiny amounts, so that buying or selling even a small amount of the asset may result in a relevant impact on the execution cost of the trade ("slippage"). Second, market microstructure may make a difference: the pricing bounds of an order driven market may differ from those of quote driven markets. In the former all market participants are free to participate on both sides of the market by posting their best prices to buy and sell the asset, while in the latter there is a particular agent, the "market maker", that is in charge of setting the bid and ask prices. A market maker must include a profit mark-up to fund her activity and may also behave strategically: she can manage the two sides of the market setting artificially high ask prices and low bid prices to discourage people for buying or selling the asset.

If we consider frictionless markets both for the underlying asset and its derivatives, a natural no-arbitrage condition is the so called put-call parity (PCP) relationship. Actually, put-call parity is the key condition that has been imposed to extend the fundamental theorem of no-arbitrage pricing to the two-price setting. Cerreia-Vioglio et al. (2015) recast the no-arbitrage pricing theorem in Chateauneuf et al. (1996) including the put-call parity relationship, together with translation invariance, as a condition to ensure the existence of arbitrage-free prices; Bastianello et al. (2022) prove that put-call parity is a sufficient condition to rule out arbitrage opportunities. As for the practical consequences of these theoretical results, Cherubini and Mulinacci (2023) show that in a frictionless market in which bid and ask prices are posted for call and put options with different strikes, put-call parity implies a very stringent link between the bid-ask spreads of the underlying asset and those of the call and put options: the sum of call and put bid-ask spreads should be invariant across strikes. We have:

$$C^a(K) - C^b(K) + P^a(K) - P^b(K) = \pi^a(S) - \pi^b(S)$$

where $C^i(K)$ and $P^i(K)$ represent the call and put bid ($i = b$) and ask ($i = a$) prices and $\pi^i(S)$ are bid and ask pricing functions of the underlying. In practice, the pricing function can be considered as a linear contract, i.e. a forward contract, on S.

This restriction is clearly quite strong for a no-arbitrage market. Even though Cherubini and Mulinacci (2023) find that in general it is approximately verified for a reasonable range of strikes around the at-the-money, it is important to stress that this condition needs not be verified for all the strike prices in quote driven markets. The reason is that investors and arbitrageurs cannot set the prices at both sides of the market. They can only accept to buy at the ask price they find in the market or to sell at the bid. If ask are set too high or bid too low there is nothing that arbitrageurs can do about it. The only mechanism that may restore arbitrage is that some other market maker could intervene by setting more favorable prices.

Cherubini and Mulinacci (2023) report evidence that in general the sum of call and put bid-ask spreads are set higher for strikes in the two tails, the reason being that in-the-money call and put bid-ask spreads are set at a constant high level outside a range of moneyness. This generates a typical piece-wise constant shape of the sum of call and put bid-ask spreads across the strikes. The shape is a flat valley around the at-the-money and two cliffs generated by in-the-money call and put options setting the schedule flat at a higher value.

This evidence raises two main questions. The first: would it be possible to make arbitrage profits setting trades at different strikes? The second: would it be possible to make arbitrage profits by trading options against the underlying? Here we show that the answer is negative to the first question, not always to the second. The answer to the first question allows us to introduce the concept of "*Implied Bid-Ask Price*" (IBAP) bounds. This is the difference between the best long and short synthetic forward positions chosen across all the strike prices traded in the market. We show that this is positive. As for the second question we find that this lower bound is in general different from the typical bid-ask spread that we find in the futures market. While one would expect that a linear contract would be the best way to synthetically buy and sell the underlying at a future maturity date, we show that this is not always the case. We document that option based bounds are tighter than futures spread for the longer maturities (those beyond the first two). Evidence is mixed for options on individual stocks.

When applied to dividend paying stocks the pricing bounds that can be obtained from options and other derivatives can be used to extract bounds for the implied derivatives, that account for most of the uncertainty of the stocks. Bounds for implied derivatives can be extracted non parametrically, by simply using the cross section of derivative prices, or by resorting to a parametric model for a more parsimonious analysis. In this paper we use the two-tail-distortion (2TD) model in Cherubini and Mulinacci (2023). This belongs to a family of models that rely on probability distortions (Cherubini 1997, Cherny and Madan (2009), Madan and Cherny (2010), Cherubini and Mulinacci 2021, 2023). The probability distortion model must not be confused with pure jump models that apply measure distortions (Madan et al. (2016, 2017, 2023)). Finally, Cinfrignini et al. (2023), Petturiti and Vantaggi (2023) propose an approach based on Dempster–Shafer theory.

The plan of the paper is as follows. In section 2 we introduce and describe the concept of IBAD in a model free context, showing how to extract dividends bounds. In section 3 we use the analysis for an application to a set of European options on the Euronext market (live.euronext.com). In section 4 we apply several versions of the 2TD model to calibrate the markets and extract dividend bounds. Section 5 concludes.

2 Implied bid-ask pricing bounds

We start discussing arbitrage opportunities in a quote driven market. The task is to keep the analysis model free and as close as possible to the real market rules. The only condition that we impose is PCP, which is the main condition required for no-arbitrage pricing (Cerreia-Vioglio et al. 2015; Bastianello et al. 2022). The concept of put-call parity in a two price setting should be handled with care. Here we refer to the composition of a long and a short position in call and put options with the same strike and maturity to build a synthetic ex-dividend position in the underlying asset. An alternative definition, called CPP (call-put parity) in Bastianello et al. (2022) refers to combining long and short positions of call options and the underlying to synthetically build a put option. The two concepts are different, with equality holding only if the bid-ask spread is zero. The choice to use the PCP definition here is not only motivated on the grounds that is the one used in the no-arbitrage pricing theorems quoted above, but also because it enables to split the no-arbitrage argument in two parts. First, one may investigate whether it is possible to earn arbitrage gains within the option market, by comparing the synthetic underlying assets that can be generated using options with different strikes. Second, one may compare the best synthetic underlying position constructed in the option market with those traded in other markets, such as the futures or forward market. It is obviously more reasonable to expect segmentation between option and futures markets than within the option market itself.

This two step analysis based on PCP is what we do in this paper, and for this reason we limit our analysis to option contracts with European exercise. As a reference market we have in mind Euronext, which collects most of the operations in derivatives and underlying assets in European securities, and whose rules of operation we will discuss in the next section.

Before addressing quote driven markets, we first introduce a strict no-arbitrage relationship in a frictionless market. Consider a market in which every investor could set her own prices for purchase and sale of the asset. Since PCP holds, one could synthetically construct, at time t, a long position at a future date T in the forward value of the underlying buying a call, selling a put and investing in the risk-free asset:

$$C_t^a(K,T) - P_t^b(K,T) + B^a(t,T)K = \pi_t^a(S,T) \tag{1}$$

where $C_t^a(K,T)$ denotes the ask price of a K-strike call option, $P_t^b(K,T)$ the bid price of the corresponding put option, $B^a(t,T)$ is the risk-free long discount factor (the discount factor for lending operations) and $\pi_t^a(S,T)$ is the pricing operator of a forward long position in S. Likewise, a short position in the forward underlying can be synthetically obtained by setting:

$$P_t^a(K,T) - C_t^b(K,T) - B^b(t,T)K = -\pi_t^b(S,T) \tag{2}$$

where $P_t^a(K,T)$ and $C_t^b(K,T)$ denote the ask put and bid call option prices for strike K, $B^b(t,T)$ represents the discount factor for borrowing positions in the risk free market and $\pi_t^b(S,T)$ is the pricing function of a short forward position in S.

Putting equations (1) and (2) together yields the general relationship

$$C_t^a(K,T) - C_t^b(K,T) + P_t^a(K,T) - P_t^b(K,T) = \pi_t^a(S,T) - \pi_t^b(S,T) - \left(B^a(t,T) - B^b(t,T)\right) K \tag{3}$$

and the sum of call and put bid-ask prices should be non-increasing in the strike. If we further assume perfect markets for the risk-free asset ($B^a = B^b$), PCP implies that the sum of bid-ask spreads of call and put prices should be invariant across strikes:

$$C_t^a(K,T) - C_t^b(K,T) + P_t^a(K,T) - P_t^b(K,T) = \pi_t^a(S,T) - \pi_t^b(S,T). \tag{4}$$

So, if all the participants in the market are free to set their own orders for buying and selling call and put options and their underlying asset, we should observe that the uncertainty on the underlying asset should be distributed between the bid-ask spreads of call and put options. If this is not the case, it is evident that in a purely frictionless market the violation of condition (4) would allow arbitrage opportunities, that will be closed by taking the most advantageous position in one market or the other.

Consider now the two-step no-arbitrage conditions stated above. The first states that it must not be possible to exploit arbitrage oppotunities trading in the option market only. In other words, it must not be possible to synthetically construct *long* positions of the underlying with values lower than synthetic *short* positions. The second condition refers to the comparison of synthetic and actual bid-ask spreads of the underlying asset. This implies that investors would switch between the options market and the underlying asset to exploit differences in the spreads.

We now introduce the assumption that investors are not allowed to freely set bid and ask prices for their orders because the market is quote driven: there is a set of market makers that are responsible for setting bid and ask prices at which the investors may sell or purchase the contracts. In this setting market makers may coordinate, intentionally or not, to set the ask (bid) prices of some options higher (lower) than they should be with respect to those of the underlying, with the goal of discouraging orders for those options. For the market makers, though, setting the ask (bid) price at a higher (lower) level with respect to what it should be is simply a way to curb the orders. Moreover, they can do that for some strike prices and not for others. As for the reason why they want to do it, a large set of the market microstructure literature has proposed several answers. Mostly, the reasons may be due to the cost of holding inventories of the assets or caution about possible informational asymmetries in the general public of investors.

Based on these arguments, our two-step no-arbitrage analysis proceeds as follows. First we ensure no-arbitrage among the synthetic long and short positions. We use the following definitions

Definition 2.1 Given a set of prices for call and put options observed at the same time t for the same maturity T for the same underlying asset S with a set of strike prices

$\{K_1, \ldots, K_N\}$, we call *Lower Ask Price* (LAP) the value of the synthetic position

$$\pi_t^*(S,T) = \min_{i=1,\ldots,N} \left[C_t^a(K_i,T) - P_t^b(K_i,T) + B(t,T)K_i \right]$$

where $C_t^a(K_i,T)$ and $P_t^b(K_i,T)$ are call ask and put bid prices respectively, for strike K_i and maturity T, and $B(t,T)$ is the risk-free discount factor observed at time t for maturity T.

In plain words, $\pi_t^*(S,T)$ denotes the minimum amount of cash that is possible to pay to receive S_T at time T. By the same token, we define the best synthetic short position.

Definition 2.2 Given a set of prices for call and put options observed at the same time t for the same maturity T for the same underlying asset S with a set of strike prices $\{K_1, \ldots, K_N\}$, we call *Upper Bid Price* (UBP) the value of the synthetic position

$$\pi_{*t}(S,T) = \max_{i=1,\ldots,N} \left[C_t^b(K_i,T) - P_t^a(K_i,T) + B(t,T)K_i \right]$$

where $C_t^b(K_i,T)$ and $P_t^a(K_i,T)$ are call bid and put ask prices respectively, for strike K_i and maturity T, and $B(t,T)$ is the risk-free discount factor observed at time t for maturity T.

Symmetrically with respect to LAP, $\pi_{*t}(K_i,T)$ denotes the maximum payment that can be received at time t against a payment S_T that has to be made at time T.

Given that LAP represents the best price to receive an amount of money and UBP is the best price to pay the same amount at the future time T, it is natural to define the no-arbitrage condition:

Proposition 2.1 *No arbitrage requires:*

$$\pi_t^*(S,T) \geq \pi_{*t}(S,T).$$

This leads to the definition of the key feature of this paper:

Definition 2.3 Given a set of prices for call and put options observed at the same time t for the same maturity T for the same underlying asset S with a set of strike prices $\{K_1, \ldots, K_N\}$, we call *Implied Bid-Ask Price* (IBAP) the difference

$$IBAP \equiv \pi_t^*(S,T) - \pi_{*t}(S,T). \tag{5}$$

We stress that here IBAP is defined with respect to the options market alone. In the second step of our no-arbitrage analysis this should be compared with measures obtained from other markets. The most obvious reference is the spot stock market in which we observe the bid and ask prices S_t^b and S_t^a. Of course, the spot pricing bounds may account just for a part of IBAP, because IBAP refers to the value of the underlying in the future, and expected accumulated dividends must be included in the picture. Denote expected accumulated dividends as

$$AD(t,T) = \sum_{\tau=t+1}^{T} D_\tau$$

where D_τ denotes the amount of dividends paid between time $\tau - 1$ and τ. The presence of dividends, then, introduces another reason of segmentation between the synthetic long and short positions constructed in the market and the spot market of the underlying. If we assume that there exists a futures market for dividends and denote $DF_t^i(T)$ the bid and ask prices ($i = a, b$) of the futures contracts we have that

$$\pi_t^*(S,T) = S_t^a - DF_t^b(T) \quad \text{and} \quad \pi_{*t}(S,T) = S_t^b - DF_t^a(T).$$

Now, the no-arbitrage condition requires to compare IBAP with

$$S_t^a - S_t^b + DF_t^a(T) - DF_t^b(T).$$

So, if we add the no-arbitrage condition that bid and ask prices must be aligned also across different markets, we would have

$$\pi_t^*(S,T) - \pi_{*t}(S,T) = S_t^a - S_t^b + DF_t^a(T) - DF_t^b(T).$$

If a market for dividend futures does not exist, the comparison of the best synthetic positions in the underlying built in the option market with different maturities provides a measure of uncertainty of future dividends expectations. We call this measure IBAD

Definition 2.4 Given a set of prices for call and put options observed at the same time t for a set of maturity dates $\{t+1,\ldots,T\}$ for the same underlying asset S with a set of strike prices $\{K_1,\ldots,K_N\}$, we call *Implied Bid-Ask Dividends* (IBAD) for dividends expected in time $j = t+1,\ldots,T$ the difference

$$IBAD_t(j) \equiv \pi_t^*(S,t+j) - \pi_{*t}(S,t+j) - \pi_t^*(S,t+j-1) + \pi_{*t}(S,t+j-1).$$

An alternative measure of dividend expectations uncertainty is expressed in terms of average growth rate expressing the difference in terms of dividend yields defined as

$$\pi_t^*(S,T) = S_t^a e^{-\delta^*(t,T)\cdot(T-t)}$$

and

$$\pi_{*t}(S,T) = S_t^b e^{-\delta_*(t,T)\cdot(T-t)}.$$

The bounds $\delta^*(t, T)$ and $\delta_*(t, T)$ for the dividend yield from t to T are obtained computing

$$\delta^*(t, T) = \frac{1}{T-t} \log \left(\frac{S_t^a}{\pi_t^*(S, T)} \right) \quad \text{and} \quad \delta_*(t, T) = \frac{1}{T-t} \log \left(\frac{S_t^b}{\pi_{*t}(S, T)} \right), \tag{6}$$

as in Tunaru (2017) and Kragt (2018).

Computing and comparing these implied price and dividend bounds is the main goal of this paper.

3 Evidence from the Euronext market

Since all the analysis in section 2 was carried out in a model free setting, here we report market measures computed with raw data collected from a market. We use the Euronext market which collects most of the trading activity on securities and derivative contracts in the European Union. A rich set of options contracts with European exercise are traded in this market and futures markets are also traded for both stock indexes and a selection of individual stocks. For some stocks, dividend futures are also available.

For each maturity, all derivatives are settled the third Friday of the corresponding month, that in the Anglosaxon world is called the “triple witching hour”. On that day, stock index options and futures are regulated by cash settlement. The settlement price is determined as the average of all index values calculated and disseminated between 15.40 and 16.00 pm of the settlement day. For single stock options, the same rules apply except that settlement occurs by physical delivery, typically for an amount of 100 pieces of the underlying stock. Dividend futures are instead referred to an amount of 10000 underlying stocks. In all cases, exercise is automatic for options that are in-the-money, so that every issue of irrational exercise is excluded, unless deliberately intended by the option buyer. In what follows, we report IBAP and IBAD data for a selected sample of stock indexes, for which we report both a time series and a term structure analysis, and a representative sample of individual stocks.

3.1 Options on stock indexes: dynamics

We begin analyzing the dynamic behavior of IBAP, the difference between the minimum cost of replicating a long and the maximum cost of replicating a short position in the underlying stock index. We compare this cost with the average difference, which corresponds to a strategy in which long and short positions are replicated selecting the strike price randomly. This difference should be zero in a market in which the strategy to construct the underlying should have the same cost, no matter which strike prices one selects. The presence of a difference represents an additional source of cost, accruing to the profit of the market maker. Here we address the question how this source of profit evolves during the lifetime of the contract.

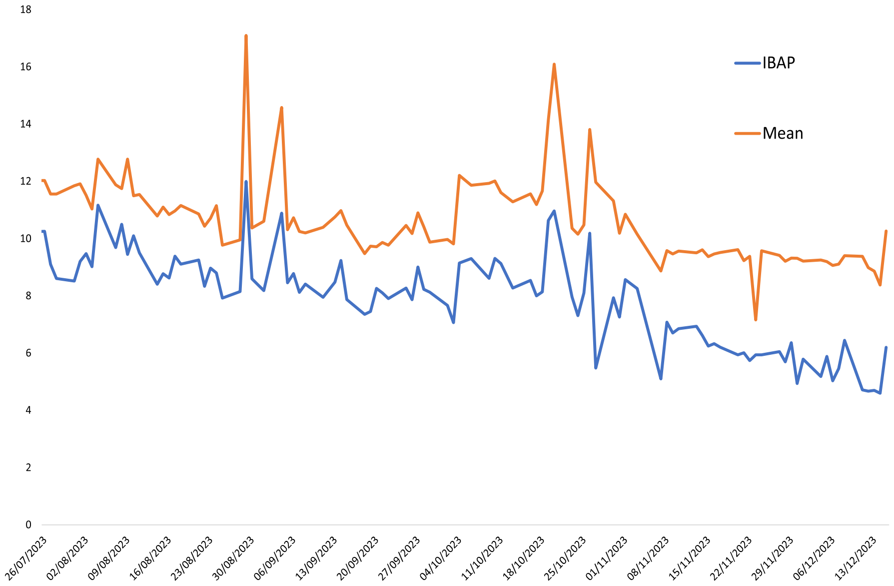

Fig. 1 Dynamics of IBAP and average sum of call and put bid-ask spreads: CAC40 index, December 2023, July 26th-December 15th 2023

For this purpose we use the data collected in Cherubini and Mulinacci (2023). Bid and ask put and call option prices were collected daily, when available, for the CAC40 index, referred to the French market, and the AEX index referred to the Dutch one, for the contract expiring on December 15th 2023, starting July 26th. In Figs 1 and 2 we report the daily dynamics of the IBAP figure and the mean sum of call and put bid-ask spreads. We see that in all cases the replicating costs are decreasing, as we approach maturity. We notice some spikes, that are common to both the IBAP and the average measure. The two markets behave differently concerning the difference between the average and the minimum measures. For the CAC40 index this difference increases from an average value of 2.76 index points in the whole sample to 3.60 in the last trading month. The evidence is opposite for the AEX index, for which the IBAP figure decreases more slowly than the mean cost and the difference decreases from 0.67 index points in the whole sample to 0.47 index points in the last month.

3.2 Options on stock indexes: term structure

We now move to the analysis of the minimum replicating cost of long and short positions across different strike prices and maturities. The dataset was collected by downloading synchronous bid and ask quotes displayed for call and put options with different strikes and maturities. For reference, we also downloaded bid and ask quotes on the corresponding futures contracts. In a first step, data were collected on a single shot on January 29th 2024. These data will be used to illustrate our analysis. For robustness check, several months after a new dataset was extracted for the same indexes

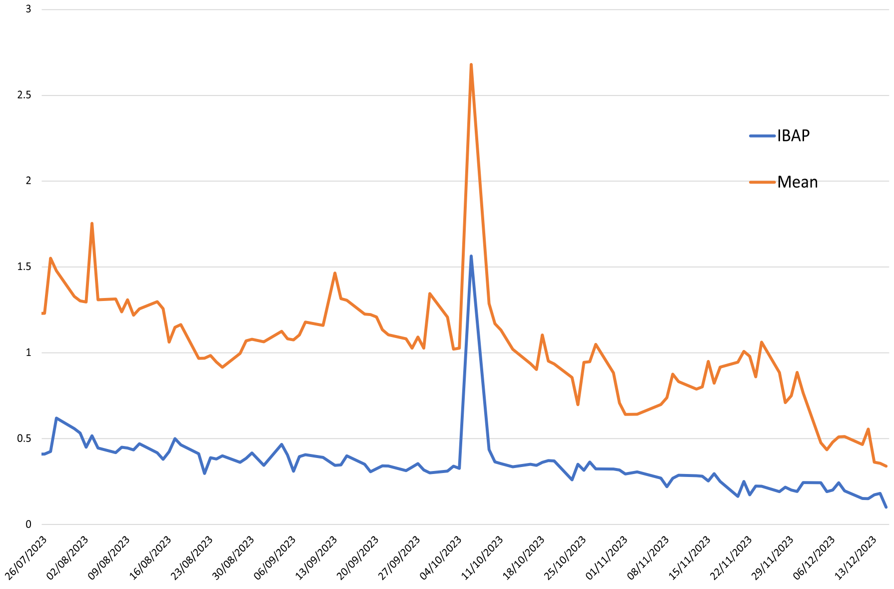

Fig. 2 Dynamics of IBAP and average sum of call and put bid-ask spreads: AEX index, December 2023, July 26th-December 15th 2023

and an entire week, spanning from September 9th to September 20th 2024, a "triple witching hour".

In Table 1 we report the IBAP computed for the AEX and CAC40 stock indexes on the synchronous set of exercise maturities observed on January 29th 2024. The values are reported in percentage of the underlying index, and are expressed in basis points. For the AEX data we observed quotes for a richer set of maturities extending to December 2028. For the CAC40 market the quotes extend only up to the June 2025 maturity. For comparison, we also report the bid-ask spreads of the futures contracts quoted at the same time on the same stock indexes. As is well known, most of the activity of futures contract is concentrated on the closest maturities. In fact, we observed quotes for the March and June 2024 delivery only. Moreover, also for these maturities, the trading activity was quite low, as most of the action was still concentrated on February, the nearest maturity. In fact, for the AEX market we observed an open interest of 406 contracts on the March maturity and only 9 on the June maturity. In the CAC40 market we have open interest of 705 contracts on the March contract and no contracts at all open for the June maturity, so it is no surprise that the corresponding bid-ask spread is completely out-of range: almost 82 basis points as a percentage of the value of the index.

So, it seems that the stock option market, and the IBAP, is the only source of information available if one wants to draw a term structure of the pricing kernel bounds of the underlying. For the AEX contract, it is encouraging that the bounds computed from the option measures are tighter than the bid-ask spreads on the futures contract for both the March and the June maturity. For the CAC40 contract, the bounds for the futures market are much tighter than those measured on the option market for

Table 1 IBAP and Futures Term Structure on January 29th, 2024

Exercise	AEX IBAP	AEX Futures	CAC40 IBAP	CAC40 Futures
March 2024	0.22	0.35	7.18	2.51
June 2024	0.44	0.46	7.80	62.44
September 2024	1.15	–	9.04	–
December 2024	0.95	–	2.76	–
March 2025	–	–	10.43	–
June 2025	3.30	–	6.44	–
December 2025	3.78	–	–	–
December 2026	2.69	–	–	–
December 2027	5.19	–	–	–
December 2028	12.56	–	–	–

the March maturity, while as observed above the bid-ask futures value is completely out of range.

Figure 3 describes the relationships of the bid-ask spreads used in our analysis. Call spreads are decreasing with strike while put spreads are increasing. The two do not balance out and the schedule of their sum is not flat as it should be in a theoretical no-arbitrage model. Instead, as in section 3 above, the sum of the two bid-ask schedule takes on a U-shaped form, with lower values around the at-the-money strike. The distance between the sum of the bid-ask spreads of call and put options and the flat line representing IBAP represents the profit accruing to market makers over and above any possible arbitrage deal. Moreover, we find evidence of segmentation between the option market and the futures market in favor of the former. The straight line representing the futures contract bid-ask spread crosses the schedule of the sum of call and put bid-ask spreads in two points. The region between these two points is the set of possible arbitrage opportunities that are left unexploited.

Tables 2 and 3 report the same analysis for the control dataset. It is notable that only in one case we find evidence of an arbitrage opportunity: on September 17th the AEX IBAP figure is negative for the June 2025 maturity, meaning that in principle it could have been possible to construct a synthetic bid position on the index for a value higher than the best synthetic ask. However, a closer inspection of the case reveals that the result is explained by a single very far out-of-the money position, so that exploiting this arbitrage opportunity would most likely have run into liquidity problems. As for the futures market, we have four maturities that may be compared with the IBAP figures computed for the same maturities on the option market. Here we find that the bid-ask spread of the futures contract is always stricter than the IBAP measure for the closest maturity, September (even for the AEX contract). In the AEX contract the two figures are comparable for the second maturity, October, with two observations (9/13 and 9/19) in which the IBAP falls short of the futures bid-ask spread, and two days (9/12 and 9/16) in which the two are equal. The difference is instead large, in favour of the futures market, for the CAC40 contracts. For the remaining longer maturities, November and December, however, the futures bid-ask spread is by far larger than the IBAP measures for both contracts.

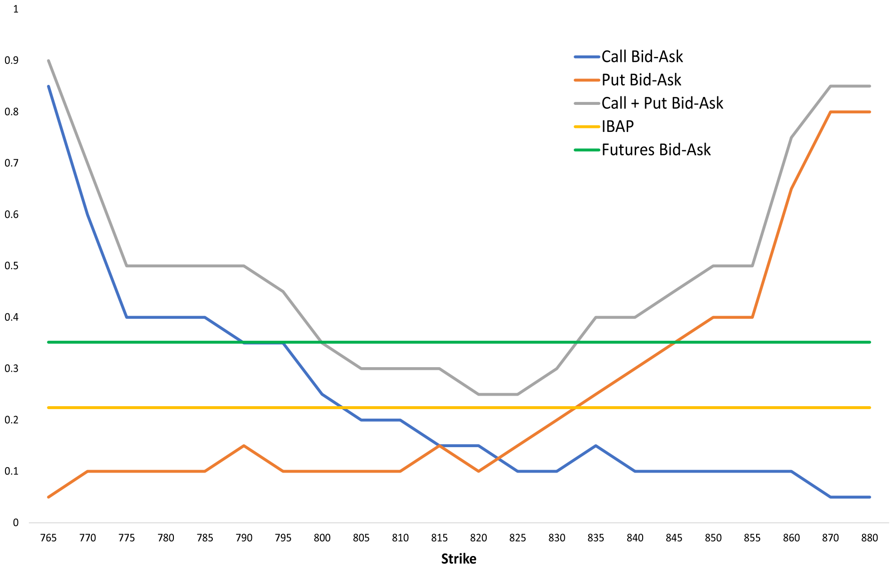

Fig. 3 Bid-ask spreads of call, put, their sum, IBAP bound and futures bid-ask spread: AEX index, March 2024 contracts, January 29th 2024

Table 2 AEX: FUTURES and IBAP Term Structure

FUTURES								
Exercise	09/11	09/12	09/13	09/16	09/17	09/18	09/19	09/20
Sept-24	0.1	0.1	0.15	0.1	0.15	0.1	0.1	0.35
Oct-24	0.25	0.3	0.3	0.2	0.15	0.15	0.15	0.1
Nov-24	6.15	6.2	5.1	6.05	6.15	5.6	5.6	6.55
Dec-24	5.15	4.3	4.25	3.2	4.95	5.2	5.2	3
IBAP								
Exercise	09/11	09/12	09/13	09/16	09/17	09/18	09/19	09/20
Sept-24	0.27	0.29	0.25	0.28	0.28	0.26	0.27	0.22
Oct-24	0.35	0.30	0.28	0.20	0.21	0.26	0.11	0.21
Nov-24	0.55	0.75	0.58	0.35	0.30	0.37	0.47	0.30
Dec-24	0.45	0.40	0.37	0.44	0.38	0.41	0.71	0.36
Mar-25	1.20	1.04	0.98	0.93	1.03	0.84	0.86	0.60
Jun-25	1.65	1.83	1.56	1.35	– 0.43	1.34	1.55	1.32
Dec-25	3.34	3.19	3.35	2.62	2.76	2.92	4.05	3.59
Dec-26	7.60	6.68	6.87	7.30	5.36	3.40	7.15	7.65
Dec-27	13.26	12.67	10.80	7.34	12.13	12.93	27.04	27.58
Dec-28	22.36	22.43	23.23	9.00	17.40	16.46	35.82	39.32

Table 3 CAC40: FUTURES and IBAP Term Structure

FUTURES								
Exercise	09/11	09/12	09/13	09/16	09/17	09/18	09/19	09/20
Sept-24	0.5	1	0.5	1	1	0.5	0.5	4
Oct-24	2	2.5	1.5	2	1	1	1	0.5
Nov-24	52	49.5	49.5	50	49.5	49.5	49.5	53.5
Dec-24	43.5	51.5	42	51.5	31.5	49.5	49.5	37.5
IBAP								
Exercise	09/11	09/12	09/13	09/16	09/17	09/18	09/19	09/20
Sept-24	8.79	6.54	6.10	5.90	5.92	6.20	6.10	5.90
Oct-24	10.60	8.85	9.50	10.39	9.75	10.70	10.60	11.00
Nov-24	13.67	12.13	11.13	13.69	11.64	13.86	15.51	12.66
Dec-24	13.64	10.80	12.46	13.22	12.33	11.41	14.30	14.59
Mar-25	14.92	14.56	14.62	14.13	14.18	14.38	15.72	13.88
Jun-25	17.75	15.43	17.86	15.88	15.17	16.01	16.71	15.13
Dec-25	28.27	28.33	28.33	28.34	28.42	28.63	43.75	28.87

3.3 Options on stocks: implied dividends

For stock index options we have evidence that the IBAP measure is the only one available for longer maturities and for shorter maturities the bound may be even tighter than the bid-ask spread of the corresponding futures contract. An interesting question is whether the same holds for options on single stocks in which idiosyncratic risk may make the difference. To answer this question we constructed a sample of European stock options and compared the IBAP value with the sum of the spread of the underlying on the spot market and the bid-ask spread of the dividend future contract observed at the same time. The data refer to January 29th 2024 and September 11th 2024.

The sample on the Euronext market was selected among the stocks for which a European option contract is traded on the market and there are dividend futures quotes for the December 2024 and 2025 maturities. This way we selected 6 stocks.

Table 4 reports the results obtained in our sample. We find that evidence is quite mixed. While the option implied measures are generally tighter for the December 2025 maturity, the evidence for the December 2024 maturity leads to different conclusions for the January 29th and September 11th maturities. The difference is mainly driven by the bid-ask spreads on the futures market, which are very small in the January sample and quite larger in the September one.

Evidence is mixed also if we compare the implied dividend bound (IBAD) for year 2025 with the uncertainty on the December 2024 maturity. In both the maturities, dividend uncertainty is found to increase in three cases out of six.

Table 4 Bid-Ask Price and Dividend Bounds

January 29, 2024						
Stock	Spot	Div. 2024	IBAP 2024	Div. 2025	IBAP 2025	IBAD
AXA	0.01	0.0723	0.2670	0.5184	0.5904	0.3234
BNP	0.01	0.1136	0.2470	1.8620	0.4360	0.1891
CREDAGR	0.002	0.69180	0.15152	0.6454	0.33	0.1785
DANONE	0.01	0.0723	0.8124	0.7512	1.5721	0.7596
SOCGEN	0.01	0.0619	0.46	0.0846	0.99	0.53
TOTAL	0.01	0.0929	0.9897	0.1375	1.7862	0.7965
September 11, 2024						
Stock	Spot	Div. 2024	IBAP 2024	Div. 2025	IBAP 2025	IBAD
AXA	0.01	0.353099	0.2100	0.55938	0.573404	0.363404
BNP	0.18	0.6	0.539201	1.64	0.840212	0.301011
CREDAGR	0.005	0.706198	0.165106	0.714774	0.29	0.124894
DANONE	0.02	0.403542	0.45	0.735492	1.176808	0.726808
SOCGEN	0.01	0.302656	0.398777	0.331489	0.889319	0.490453
TOTAL	0.02	0.161417	0.53	1.129135	1.026808	0.496808

4 Parametric analysis: the 2TD model

Here we provide a parametric approach to represent the pricing bounds implied in options bid and ask quotes. We apply the Cherubini and Mulinacci (2023) model with the task of investigating any asymmetric behavior in the tails. The model is well suited to accomplish this task because it is based on different distortions applied to each tail, under suitable consistency requirements.

The 2TD model is based on three ingredients:

- a reference probability cumulative distribution function H which represents the fundamental model for the valuation of the asset, that is the pricing kernel that would generate the price in a liquid frictionless market;
- a strictly increasing bijection function $\phi : [0, 1] \to [0, 1]$, with $\psi(0) = 0$ and $\psi(1) = 1$, such that

$$\psi(u) + \psi(1 - u) \leq 1 \tag{7}$$

  which represents uncertainty and generates pricing bounds;
- two strictly increasing bijections $\psi : [0, 1] \to [0, 1]$ and $\gamma : [0, 1] \to [0, 1]$ such that

$$\psi(\phi(u)) + \psi(\gamma(1 - u)) = 1 \tag{8}$$

  which assign different distortions to the lower and upper tail.

Table 5 Option pricing formulas in the 2TD model

Type	Call	Put
Bid	$C^b = \int_K^{+\infty} (1 - \phi(H(s))\, ds$	$P^b = \int_0^K (1 - \gamma(1 - H(s)))\, ds$
Ask	$C^a = \int_K^{+\infty} \gamma\, (1 - H(s))\, ds$	$P^a = \int_0^K \phi(H(s))ds$

More precisely, in Cherubini and Mulinacci (2023) it is shown that it is possible to construct a fuzzy measure μ on $\mathbb{R}$ with left and right tails given by $\mu\left((-\infty, x]\right) = \phi\left(H(x)\right)$ and $\mu\left((x, +\infty)\right) = \gamma\left(1 - H(x)\right)$, respectively, for every $x \in \mathbb{R}$.
Prices are obtained by applying the Choquet integral. The pricing formulas, assuming zero interest rate, are reported in Table 5. It is immediate to see that:

- the LAP pricing functionals $\pi_t^*(S, T)$ depend only on the distortion function γ;
- the UBP pricing functionals $\pi_{*t}(S, T)$ depend only on the distortion function ϕ.

The two distortions have the role of raising LAP or decreasing UBP around the fundamental model represented by the reference probability H. Following (Cherubini and Mulinacci 2023) the distortion ϕ is maximal when $\gamma(x) = x$ and vice versa: the maximum distortion γ is obtained when $\phi(x) = x$. These are the one tail distortion models that are typical of fuzzy measure theory (Wang and Klir 1992). So, estimates of the tail distortions allow us to measure whether the IBAP pricing bounds are driven by LAB only, by UBP only, or both.

In the analysis we use a basic parametric form based on Sugeno distortion and two standard models for the underlying.

4.1 Model specification

The distortion function is obtained from the class of λ-fuzzy measures μ introduced by Sugeno (see, among the others, Wang and Klir (1992)). This class of measures is characterized by

$$\mu\left(A \cup B\right) = \mu\left(A\right) + \mu\left(B\right) + \theta\mu\left(A\right)\mu\left(B\right)$$

for all disjoint measurable sets A and B and with $\theta \in (-1, +\infty)$. As shown in Cherubini and Mulinacci (2021) this relation can be rewritten in terms of a ψ-sum as

$$\mu\left(A \cup B\right) = \mu\left(A\right) \oplus_\psi \mu\left(B\right) = \psi^{-1}\left(\psi\left(\mu(A)\right) + (\psi(\mu(B))\right)$$

where

$$\psi(x) = \frac{\log(1 + \theta x)}{\log(1 + \theta)} \quad \theta \in (-1, 0)$$

satisfies (7) being super-additive.

Cherubini and Mulinacci (2023) consider the class of distortions of the left tail of type

$$\phi_\alpha(u) = \frac{u}{1+\alpha\theta(1-u)} \quad \alpha \in [0,1].$$

When $\alpha = 1$ this gives the one tail distortion:

$$\phi_{\max}(u) \equiv \frac{u}{1+\theta(1-u)}$$

used in Cherubini (1997) and Cherubini and Mulinacci (2021). When $\alpha = 0$ the left tail is not distorted. The corresponding distortion for the right tail is obtained from (8) once that ψ and ϕ_α are given:

$$\gamma_\alpha(u) = \frac{u(1+\theta\alpha)}{1+\theta-\theta u(1-\alpha)}.$$

Notice that for $\alpha = 0$ the maximum distortion is loaded on γ:

$$\gamma_{\max}(u) \equiv \frac{u}{1+\theta(1-u)}.$$

As for the reference models, represented by the cumulative distribution function H of a risk-neutral probability, we propose:

- The Black and Scholes (1973) model: the parameter of the model is the volatility of the log-normal distribution, that is assumed to be constant across the strikes. It is well known that this provides a poor fit of market option prices, particularly for short maturities. Nevertheless, here we are interested to check whether this standard model can still be useful at least for the estimation of the bid-ask spreads.
- The Carr and Torricelli (2021) model: this model is well suited for our analysis because it builds on the put-call parity relationship. More to the point, it is based on the idea of pricing a product giving the payoff: $m_T \equiv \max(S_T, K)$ and then recovering the prices of options from put-call parity. The price of this product at time t is obtained as

  $$m_t = B(t,T)E_Q(\max(S_T, K)) = B(t,T)\left(F_t(T)^{1/b_\tau} + K^{1/b_\tau}\right)^{b_\tau}$$

  where $B(t,T)$ is the risk-free discount factor for maturity T, $F_t(T)$ is the forward price for delivery at time T, $\tau \equiv T - t$ is the time to maturity and b_τ is the parameter of the model, assumed to be in the unit interval, and such that $\lim_{\tau\to 0} b_\tau = 0$. The corresponding cumulative distribution function implied by this model is

  $$H(y) = \left(1 + \left(\frac{y}{F_t(T)}\right)^{-1/b_\tau}\right)^{b_\tau - 1}$$

  which is known as a specification of the Dagum distribution.

Denoting $\hat{C}^i(S, K)$ and $\hat{P}^i(S, K), i = a, b$ the theoretical ask and bid prices of call and put options, and $C^i(S, K)$ and $P^i(S, K)$ the corresponding observed prices, the calibration was carried out by minimizing

$$\min_{\{\mathbb{H},\Theta\}} \sum_{i=a,b} \sum_{j=1}^{n} \left[(\hat{C}^i(K_j; \mathbb{H}, \Theta) - C^i(K_j))^2 + (\hat{P}^i(K_j; \mathbb{H}, \Theta) - P^i(K_j))^2 \right] \quad (9)$$

where $\mathbb{H}$ denotes the set of parameters of our reference models and Θ contains the parameters of the distortion functions (in our case $\Theta \equiv \{\alpha, \theta\}$). Forward prices for the reference conditional distributions were computed as the mid of the cross-section averages of implied bid and ask forward prices (equations (1) and (2)) with respect to strikes.

4.2 Implied dividend bounds: AEX index

The model described above is applied to the calibration of a term structure of the uncertainty bounds. Since for the AEX index options we have data up to the 2028 maturity, we limit our analysis to this market. Even in this case, we focus our discussion on the January 29th case and use the data from September 9th to 20th as a robustness check. For the January 29th data we report both the reference models. The results of the calibration using the Black and Scholes and the Carr and Torricelli reference models are shown in Table 6. We can observe that:

- the parameter θ shows a decreasing trend with the maturities;
- up to the December 2026 maturity, α is zero.

The first fact tells us that the uncertainty trend increases with maturities. The second one implies that up to the December 2026 maturity, $\phi(x) = x$ while $\gamma(x) = \frac{1}{1+\theta(1-x)}$, that is we are dealing with a one-tail distortion model and the distortion is concentrated on the right tail. This implies that the UBP price (that depends on the function ϕ)

Table 6 AEX Parameters' term structure on January 29th, 2024

	Black and Scholes			Carr and Torricelli		
Exercise	α	θ	σ	α	θ	b
March 2024	0.0	−0.061	0.1172	0.0	−0.0582	0.0234
June 2024	0.0	−0.0572	0.1257	0.0	−0.0569	0.0439
September 2024	0.0	−0.074	0.1336	0.0	−0.0719	0.0594
December 2024	0.0	−0.0642	0.1392	0.0	−0.0637	0.0725
June 2025	0.0	−0.1032	0.1411	0.0	−0.0828	0.1077
December 2025	0.0	−0.0826	0.144	0.0	−0.0828	0.1077
December 2026	0.0	−0.0982	0.146	0.0	−0.0994	0.1336
December 2027	0.0075	−0.1159	0.15	0.0219	−0.1159	0.1581
December 2028	0.0866	−0.1399	0.1565	0.1136	−0.1393	0.1838

coincides with the reference one and all the uncertainty is concentrated on the LAP figure, that is on the ask position. When the time to maturity becomes larger then some uncertainty is also affecting the UBP figure, that is the bid position. However, the values of the α parameters remain very small, and the effect on the bounds quite mild.

The September dataset confirms most of the results. The estimation results of the distortions parameters θ and α assuming as reference model the Black and Scholes one, are shown in Tables 7 and 8. The results from the Carr and Torricelli models are similar and are omitted in order to save space. The tables show that the α parameters are generally equal to zero up to the December 2025 maturity and reach material values only for maturities from December 2026 to 2028. The same pattern emerges for parameter θ, which shows an increase of uncertainty on the 2026 maturity and beyond. The additional result that can be found in the September dataset is that both these patterns become more evident in the day before the "triple witching hour".

Overall, we may say that the uncertainty is increasing, even though not uniformly and at a fast rate. As we saw in the non parametric analysis most of the size of these bounds are due to uncertainty about accumulated dividends, since the bid-ask spreads of the underlying stocks are very tiny. It is then important to cast the pricing bounds into dividend yields

$$\delta^*(0, T) = \frac{1}{T} \log\left(\frac{S_0}{\pi_0^*(S, T)}\right) \quad \text{and} \quad \delta_*(t, T) = \frac{1}{T} \log\left(\frac{S_0}{\pi_{*0}(S, T)}\right).$$

Figure 4 reports the plot of the dividend bounds, together with the reference bound (that is mostly coincident and in any case very close to the upper bound) for the Black and Scholes model (the results of the other model are quite similar). The picture shows that the effect on the distance between the bounds is not evident in the long run. Even though the parameters show an increase in uncertainty, this is not enough to outweigh the averaging effect of maturity.

4.3 Implied dividend bounds: stocks

Tables 9 and 10 report the calibrations results of the 2TD models for the 6 stocks for which we have option data for the December 2024 and 2025 maturities in both the January 29th and the September 11 2024 samples. While in the model free analysis the evidence was quite mixed, the parametric analysis shows some clear trends:

- parameter θ decreases with maturity and the parameter α increases in the first sample, while the evidence is mixed in the second;
- lower parameters θ are associated to higher parameters α;
- the α parameters are closer to zero than to one.

These results are in line with those obtained for the AEX index. There is an increase of uncertainty with maturities, even though the results are less clear-cut when the maturity gets closer. Associated to this, the distortion of the UBP (that depends on the function ϕ) becomes more relevant: in fact, ϕ is a decreasing function of $\alpha \cdot \theta$, and, in all cases, it can be verified that $\alpha \cdot \theta$ decreases with the maturity. Taken together,

Table 7 θ term structure of AEX index

Exercise	09/11	09/12	09/13	09/16	09/17	09/18	09/19	09/20
Sept-24	−0.0787	−0.0978	−0.0443	−0.0411	−0.0458	−0.0426	−0.074	–
Oct-24	−0.073	−0.0697	−0.0529	−0.0498	−0.0594	−0.0509	−0.0668	−0.0642
Nov-24	−0.0658	−0.068	−0.0635	−0.0622	−0.0549	−0.0592	−0.0647	−0.06255
Dec-24	−0.0658	−0.0577	−0.0545	−0.0515	−0.0506	−0.053	−0.0709	−0.0503
Mar-25	−0.0691	−0.0705	−0.0671	−0.0675	−0.0686	−0.0709	−0.0710	−0.0503
Jun-25	−0.0752	−0.0756	−0.0747	−0.0753	−0.0699	−0.0762	−0.1217	−0.122
Dec-25	−0.0793	−0.0793	−0.081	−0.0801	−0.0771	−0.0782	−0.1781	−0.1742
Dec-26	−0.1165	−0.1115	−0.112	−0.1119	−0.1076	−0.1056	−0.2193	−0.2186
Dec-27	−0.1352	−0.1276	−0.1273	−0.1211	−0.1236	−0.1245	−0.2427	−0.2444
Dec-28	−0.1751	−0.162	−0.1697	−0.1644	−0.1601	−0.16300	−0.2805	−0.2857

Table 8 α term structure of AEX index

Exercise	09/11	09/12	09/13	09/16	09/17	09/18	09/19	09/20
Sept-24	0.077	0.1886	0	0	0.0718	0.02976	0.3299	–
Oct-24	0	0	0	0	0	0	0	0
Nov-24	0	0	0	0	0	0	0	0
Dec-24	0	0	0	0	0	0	0.0365	0
Mar-25	0	0	0	0	0	0	0	0
Jun-25	0	0	0	0	0	0	0.1065	0.112
Dec-25	0.0298	0.0407	0.046	0.0516	0.0306	0.0384	0.3274	0.326
Dec-26	0.1105	0.0863	0.08	0.0904	0.0598	0.0719	0.3222	0.328
Dec-27	0.1496	0.1145	0.1095	0.0934	0.0916	0.1048	0.3358	0.3505
Dec-28	0.2294	0.1816	0.2019	0.1958	0.18132	0.1936	0.3642	0.3789

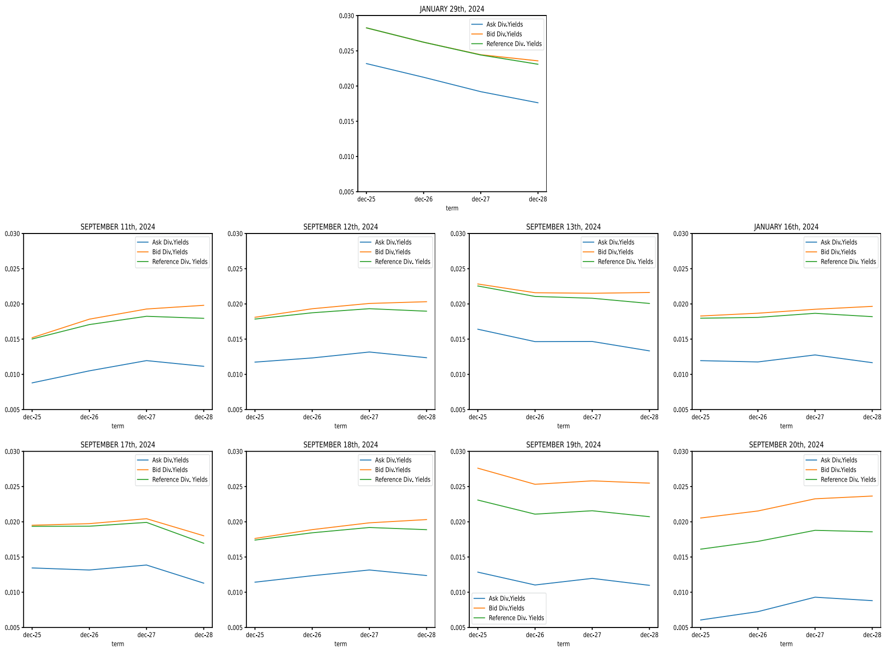

Fig. 4 AEX Index Dividend Yields Term Structure

they confirm, also for the stocks, an increase of the uncertainty with the investment horizon and an increase in the uncertainty of bid positions (at which the market maker sells the option). As done for the AEX index, we report in Fig. 5, in the Black and Scholes model, the bounds for the dividend yields for all the stocks. The green line, representing the dividend yield dynamics under the Black and Scholes model, is closer to the upper bound given by δ_* for all stocks, showing that most of the uncertainty is

Table 9 Stock Parameters' term structure on January 29th 2024

Black and Scholes model

	2024			2025		
Stock	α	θ	σ	α	θ	σ
AXA	0.1122	−0.1292	0.2094	0.3131	−0.1663	0.2042
BNP	0.1751	−0.0798	0.2417	0.2834	−0.1333	0.2431
CREDAGR	0.2852	−0.1855	0.2296	0.343	−0.225	0.2357
DANONE	0.3313	−0.1644	0.1791	0.3928	−0.2054	0.1765
SOCGEN	0.3995	−0.2144	0.2737	0.4214	−0.2453	0.2706
TOTAL	0.3922	−0.1413	0.2263	0.4178	−0.1861	0.2323

Carr and Torricelli model

	2024			2025		
Stock	α	θ	b	α	θ	b
AXA	0.1367	−0.1294	0.108	0.2881	−0.1658	0.1508
BNP	0.15923	−0.0792	0.1248	0.2533	−0.1324	0.178
CREDAGR	0.2935	−0.1867	0.1176	0.3364	−0.2263	0.1716
DANONE	0.3333	−0.1651	0.0928	0.3845	−0.2057	0.131
SOCGEN	0.397	−0.2149	0.1395	0.4311	−0.2436	0.197
TOTAL	0.3772	−0.1419	0.1163	0.3996	−0.1853	0.1704

Table 10 Stock Parameters' term structure on September 11th 2024

Black and Scholes model

	2024			2025		
Stock	α	θ	σ	α	θ	σ
AXA	0.017	−0.1248	0.2071	0.1107	−0.1372	0.2166
BNP	0.182	−0.113	0.2693	0.1651	−0.1108	0.2577
CREDAGR	0.1993	−0.1726	0.2418	0.179	−0.164	0.2322
DANONE	0.3754	−0.1868	0.1684	0.3657	−0.1685	0.1751
SOCGEN	0.3246	−0.2071	0.3041	0.4315	−0.2375	0.2879
TOTAL	0.3484	−0.1406	0.2311	0.4024	−0.1318	0.2271

Carr and Torricelli model

	2024			2025		
Stock	α	θ	b	α	θ	b
AXA	0.0702	−0.1251	0.06039	0.1176	−0.1384	0.1313
BNP	0.1878	−0.1142	0.0773	0.1416	−0.1115	0.1552
CREDAGR	0.2154	−0.1741	0.0698	0.1793	−0.1661	0.14011
DANONE	0.3725	−0.1877	0.0491	0.3712	−0.1701	0.107
SOCGEN	0.3354	−0.2085	0.0873	0.4203	−0.2381	0.1724
TOTAL	0.3333	−0.1416	0.0669	0.3436	−0.1323	0.138

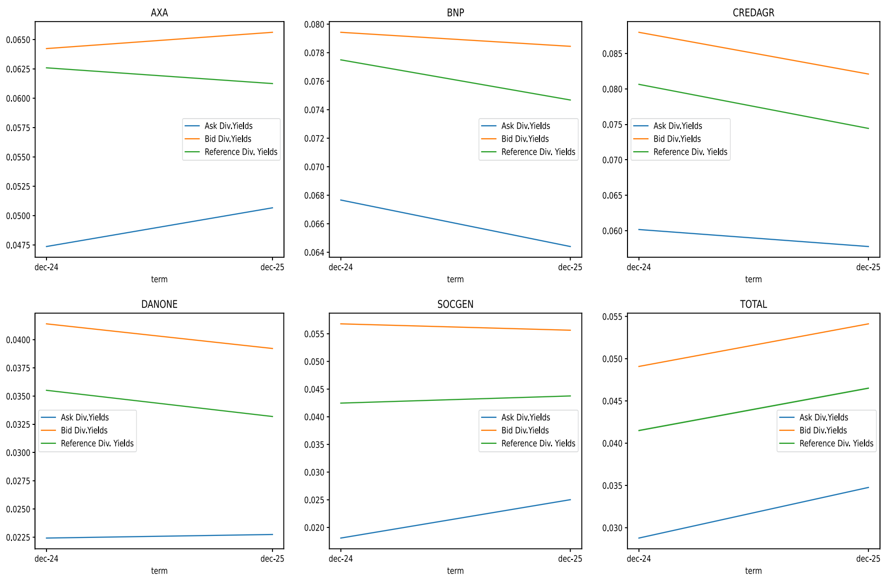

Fig. 5 Stock Dividend Yields Term Structure: January 29th 2024

in the ask positions, but the distance between the two decreases with the time horizon, confirming the above comments.

4.4 A comparison with the Dempster–Shafer Model

We conclude our analysis comparing the probability distortion approach with a close, but different model. Cinfrignini et al. (2024) apply the Dempster–Shafer approach to the pricing of option, and make their data publicly available. The comparison is useful because, even though technically the Dempster–Shafer approach is not based on the distortion of a probability measure, it exploits the perturbation of a reference model. Since the Dempster–Shafer model is applied to a discrete sample space, the natural choice is a recombining tree. We perform our comparison on call and put options data written on the META stock on 2023-09-29 with maturity 2023-10-27, corresponding to a term of T = 20 trading days. We use the same parameters reported in the paper, including the discount factor and the META stock market bid-ask prices $S_b = \$300.11$ and $S_a = \$300.30$. We work on the same restricted dataset made of 42 call and 33 put options. We perform the calibration minimizing the same squared error functional given by

$$\min_{\{\mathbb{H},\Theta\}} \sum_{i=a,b} \left\{ \sum_{K \in \mathcal{K}_C} (\hat{C}^i(K; \mathbb{H}, \Theta) - C^i(K))^2 + \sum_{K \in \mathcal{K}_P} (\hat{P}^i(K_j; \mathbb{H}, \Theta) - P^i(K_j))^2 \right\}$$

where $\mathcal{K}_C$ and $\mathcal{K}_P$ denote the set of strikes in the considered dataset of call and put options, respectively, while the rest of the notation is the same as in our equation (9).

Table 11 Calibration of the Meta data set

Parameters	Black and Scholes	Carr and Torricelli
θ	−0.046062432	−0.046152504
α	0.787042467	0.830881593
σ	0.460194811	0.071818857
Squared Error	18.77163057	20.02325984

Applying the 2TD model with the specification described in subsection 4.1, we get the results shown in Table 11. The mean squared error is slightly lower than that in the comparable model presented in Cinfrignini et al. (2024). The comparable model is the one in which the gross rate growth of the stock is u in upward movements and $1/u$ in downward movements, and that in the continuous time limit converges to the Black and Scholes model. The difference between our MSE of 18.77 and the 21.95 figure reported by Cinfrignini et al. (2024) can be at least partially explained by the discretization bias of the binomial model. In the model in which the downward movement is an additional calibration parameter their MSE decreases to 15.44, pointing out to the relevance of including skewness in the specification of the reference binomial tree. The relevance of skewness is also confirmed by the result that the Carr and Torricelli model, in our estimation, delivers a worse MSE result. It is well known, in fact, that this model is characterized by a symmetric smile.

Figure 6 shows the theoretical bid and ask options prices compared to the corresponding market quotes, in line with Figure 12 in Cinfrignini et al. (2024).

As for the other parameters, we obtain a volatility figure of 46% which is about 10% higher than the value consistent with the u parameter calibrated in the Dempster–Shafer application. The two parameters characterizing our 2TD model show an evident difference with respect to the results that we find for European stocks. On the one hand the θ parameter is much higher (and comparable with what we find for index options in the European market). On the other hand, the α parameter is much higher and closer to one than to zero, the opposite that we find in the European sample, even for single stocks. This means that bid prices are more distorted than ask prices, opposite to what we find in the European sample.

5 Conclusion

In this paper we discuss and measure the uncertainty implied in the options market. Using a cross-section of bid and ask prices of options, we investigate the implied pricing bounds of a forward position on the underlying. An analysis of stock options shows that most of the size of these pricing bounds are due to dividend bounds, since the bid-ask spreads on the spot markets are very small. So, using options data for several maturities we can extract bounds of implied dividends.

We applied the 2TD model of Cherubini and Mulinacci (2023) to calibrate the bounds and to assess the relative relevance of distortion of the two tails. In all cases, it turns out that most of the distortion is on the ask side of the call options, and the bid

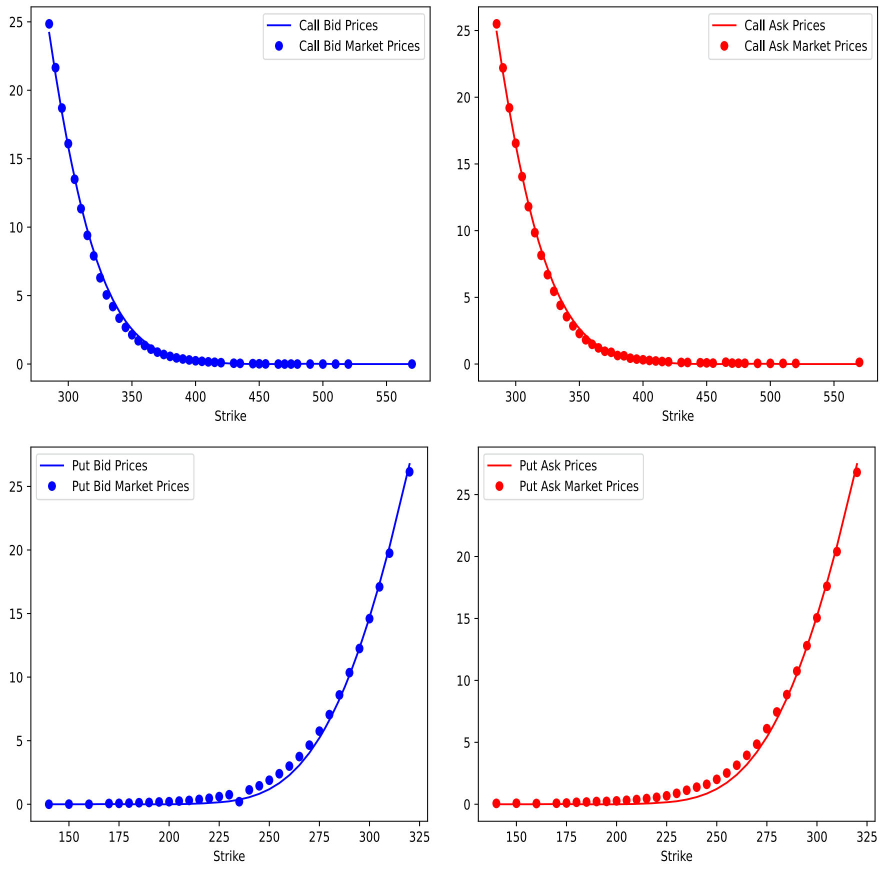

Fig. 6 Theoretical and Market prices of Meta options

side of the put options. This is particularly evident for index options, when the right tail, that is relevant for the ask quotes of call options and bid prices of put options, is the only one that is distorted. In other words, our evidence is consistent with a behavior of market makers setting the bid quotes of call options (and ask of put options) at a reference model and then distorting the reference model to obtain the ask (for call options) and the bid (for put options). This asymmetry is evident in all our data, but is less marked in stock options and for longer maturities. For individual stocks, buying options is also an uncertain business, the more so the longer the maturity.

Putting together this uncertainty asymmetry argument with the observation of the preponderance of dividend uncertainty in setting the pricing bounds, it turns out that ask quotes imply a conservative evaluation of implied dividends. Given a reference model for dividends, uncertainty suggests to reduce this value to get the ask price of a call option and the bid of a put option. Curiously, performing the same analysis for options on a US stock (META) we find opposite evidence. The analysis of this stock was carried out only to compare our approach with the Dempster–Shafer model (applied in Cinfrignini et al. (2024)) so that this finding is left as an interesting research question for future research.

The term structure model shows an increase in the uncertainty parameter of the 2TD model for longer maturities, together with a decrease of the asymmetry, even though the effect in terms of dividend yield bounds is less evident in the index than in the individual stocks.

Finally, the paper also offers a preliminary analysis of the relative size and the relationship between pricing bounds on different markets. To our surprise, we find cases in which option markets can offer pricing bounds which are tighter than those in the futures market. This is the case of index options, except the first two maturities of the futures market. We also compare the option based measure for individual stocks with the corresponding dividend future, finding mixed evidence. This leaves the study of the no arbitrage relationships across different markets and their segmentation as a very relevant topic of this line of research.

Funding Open access funding provided by Alma Mater Studiorum - Università di Bologna within the CRUI-CARE Agreement.

Declarations

Conflict of interest The authors declare no Conflict of interest.

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