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Analysis and decomposition in Peirce

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In a long article written in autumn 1903 and titled "Logical Tracts No. 2" (R 492)[1] Peirce says that "every proposition has one predicate and one only. But what that predicate is considered to be depends upon how we choose to analyze it" (R 492, p. 33 = CP 4.438). A similar point is made in "The Basis of Pragmaticism", written circa 1905: "A proposition, according to any one analysis of it, has but a single predicate, but it is equally legitimate to throw any proper name or substitute for a proper name into the predicate or to make it a subject" (R 280, pp. 40–41). Peirce defines a "rhema" (or "rheme") as that which remains in a sentence (which is what he means in this kind of contexts with "proposition"[2]) when a part of it is removed which if replaced with a proper name would again yield a complete sentence. The element removed is called by Peirce the "subject" and the rhema the "predicate" of the sentence so analyzed. What the predicate of a sentence is depends on what we choose to consider as subject. Thus, to each of the possible "subjects" of a sentence (i.e. to anything in that sentence replaceable with a proper name) there correspond distinct ways of analyzing that sentence into subject and predicate.

However, Peirce seems also to maintain a thesis that is incompatible with the thesis about multiple analyses. Again in the "Logical Tracts No. 2" Peirce distinguishes between "rhemata of first intention" and "rhemata of second intention", and suggests that a sentence can be analyzed into parts corresponding to rhemata of first intention, which are put together by rhemata of second intention: "every combination of parts of a proposition involves a rhema of second intention" (R 492, p. 40). One may wonder whether the definition of rhema that Peirce gives in connection with his statement of the doctrine of multiple analyses is consistent with the distinction between rhemata of first and second intention. If it is, i.e. if rhemata of first and second intention are that which remains of a sentence after something replaceable by a proper name has been removed from it, then the first model of sentence analysis is incompatible with the second model, because according to the first model there is only one rhema that may be regarded as the *unique* predicate of that sentence according to each analysis of it, while according to the second model a sentence is analyzable into a certain combination of rhemata (of first intention, this combination being effected by rhemata of second intention), and this seems to exclude that any of these rhemata may be regarded as the *unique* predicate of the sentence. Thus, Peirce's thesis that every sentence can be can be subjected to multiple, equally admissible analyses into subject and predicate appears to be incompatible with the thesis that every sentence is analyzable

[1] The following abbreviations will be used for Peirce's works: CP, followed by volume and paragraph number, stands for Peirce (1932–1958); SS, followed by page number, for Peirce (1977); R, followed by manuscript number and, when available, page number, for Peirce's manuscripts (Charles S. Peirce Papers, 1787–1951, MS Am 1632, Houghton Library, Harvard University) as catalogued in Robin (1967).
[2] See below, §2.

into rhemata of first and of second intention; for the former thesis says something which the latter appears to deny. Is Peirce's doctrine of analysis inconsistent?

In this paper I offer an answer to this question by showing that Peirce was under the influence of a distinction, which he never explicitly makes, between two different conceptions of analysis. Those familiar with Frege's works and with the Fregean literature may have recognized that the problem which I have detected in Peirce's conception of analysis is very much the same as the one that Michael Dummett has detected in Frege, and which he has proposed to solve by means of the distinction between "analysis" and "decomposition". In order to show my cards at the outset I will say that my solution coincides with Dummett's. In other words, in this paper I will show that in order to consistently interpret Peirce's doctrine of logical analysis we must draw a distinction which he does not draw, a distinction which is roughly the same as the one drawn by Dummett in order to consistently interpret Frege's doctrine of logical analysis.[3] It is not my main purpose in this paper to point out or comment upon the differences and the similarities in Peirce's and Frege's doctrines of analysis.[4] My purpose is to elucidate Peirce's doctrine of analysis so as to free it from an apparent inconsistency, and I do this by using a philosophical tool which, though it was created in the Fregean literature to solve a Fregean problem, yet turns out to be of the utmost utility for the solution of an analogous Peircean problem which has so far remained unnoticed in the Peircean literature.

1. Dummettian analysis and decomposition

To grasp the sense of a sentence (the thought that the sentence expresses) is to know the way in which that sense is determined by the senses of its parts, which Dummett calls the "constituents" of the sentence. Analysis is the process by which we break down a sentence into its constituents, and reveals the manner in which the sense of the sentence depends on the senses of its constituents. A sentence expresses a thought, which is its sense, and a constituent of a sentence is a part of the sentence whose sense we need to grasp in order to grasp the sense of the sentence of which it is a constituent. The purpose of analysis is to reveal the structure of a sentence and thus the structure of the thought that that sentence expresses.

Analysis is unique: the constituents of the thought are the senses of the constituents of the sentence, and the sense of the sentence is determined by the senses of its constituents; thus if one and the same sentence were capable of distinct analyses, which involves that it would be analyzable into *distinct* constituents, then, since distinct constituents have distinct senses, the sense of the sentence would *not* be determined by the senses of its constituents.

The analysis of a sentence takes place in stages. Consider the sentence "Whoever loves someone is killed by him", which we may more explicitly express in semi-regimented form as "For any *x*, for some *y*, if *x* loves *y*, *x* is killed by *y*". The first step in the analysis of this sentence is to represent it as formed by attaching the quantifier "For any ξ" to the complex expression "For

[3] It has been objected, most notably by Currie (1985) and Levine (2002), that Dummett's distinction between analysis and decomposition fails as a faithful exegesis of Frege's thought. As I am only interested in the usefulness of Dummett's distinction in the interpretation of Peirce, I will ignore the question whether that distinction provides an adequate interpretation of Frege.

[4] This has been done partially but excellently by Hawkins (1993) and Anellis (2012).

some *y*, if ξ loves *y*, ξ is killed by *y*".[5] The second stage is to represent this complex expression as formed by attaching the quantifier "For some ζ" to the complex predicate "If ξ loves ζ, ξ is killed by ζ". The complex predicate so obtained is in turn represented as formed by conjoining the simple relational expressions "ξ loves ζ" and "ξ is killed by ζ" by means of a conditional operator. The sentence is thus completely analyzed into simple constituents, which are simple predicates or simple relational expressions conjoined by truth-functional operators, to which quantifiers are then attached. The sense of the sentence depends on the sense of its constituents: we understand the sense of "Whoever loves somebody is killed by him" by understanding the senses of "ξ loves ζ" and "ξ is killed by ζ" and the manner in which they are put together by the truth-functional operator and are subjected to quantification. As Dummett stressed (1981b, p. 273), when a sentence of natural language is expressed in a regimented notation, such as Frege's symbolism, its analysis is perspicuous and appears to be unique: in expressing the sentence "Whoever loves somebody is killed by him" in semi-regimented form as "For any *x*, for some *y*, if *x* loves *y*, *x* is killed by *y*" we are *thereby* revealing its analysis, i.e. we are revealing the only way in which it can have been built up from primitive constituents.

The decomposition of a sentence differs from its analysis in that its purpose is not to reveal how the sense of the sentence is dependent on the senses of its constituents, but to construct quantified sentences in which complex predicates occur and to explain the validity of the inferences involving that sentence. Decomposition is achieved by removing from a complete sentence one or more occurrences of each of one or more proper names or other expression, thus leaving an incomplete expression. In Dummett's terminology, the parts revealed by the decomposition of a sentence are not constituents of that sentence, but "components" of it. While analysis in general requires several steps, decomposition occurs in a single step. If from the sentence "If Desdemona loves Cassio, Desdemona is killed by Othello" we remove the two occurrences of "Desdemona", what we obtain is the complex relational expression "If ξ loves Cassio, ξ is killed by Othello". The recognition that this complex relational expression is a component of the sentence is essential in order to form the quantified sentence "For any *x*, if *x* loves Cassio, *x* is killed by Othello", which we do by attaching the quantifier "For any *x*" to the complex predicate obtained by decomposition. Decomposition is also necessary in order to explain the validity of the inferences in which the decomposed sentence occurs. Thus in order to recognize that from "For any *x*, if *x* loves Cassio, *x* is killed by Othello" it follows that "If Desdemona loves Cassio, Desdemona is killed by Othello" we need to recognize the complex relational expression "if ξ loves Cassio, ξ is killed by Othello" as a component of the quantified sentence. While the analysis of a sentence is unique, multiple decompositions of it are possible. Thus, the atomic sentence "Desdemona loves Cassio" may be decomposed into "Desdemona" and "ξ loves Cassio", but also into "Cassio" and "Desdemona loves ξ", as well as into "Desdemona", "Cassio", and "ξ loves ζ". Each of these decompositions of the atomic sentence highlights a certain pattern within that sentence, and hence it highlights distinct inferential relations into which that sentence can enter.

Decomposition aids in the recognition that distinct sentences may share a common pattern. Thus the component "if ξ loves Cassio, ξ is killed by Othello" is a pattern shared by "If

[5] This involves an explanation of how to treat a predicate with any number of places formed by attaching a quantifier to a predicate with one more place. Dummett (1981b, pp. 284–286) explain this by a substitutional account of quantification, which he argues was more or less what Frege had in mind.

Desdemona loves Cassio, Desdemona is killed by Othello", by "For any *x*, if *x* loves Cassio, *x* is killed by Othello", by "For some *x*, if *x* loves Cassio, *x* is killed by Othello", by "For any *x*, if *x* is a fictional character, then if *x* loves Cassio, *x* is killed by Othello", by "For any *x*, for some *y*, then if *y* is the author of a play featuring *x*, then if *x* loves Cassio, *x* is killed by Othello", and so on. But in order to grasp the sense of "If Desdemona loves Cassio, Desdemona is killed by Othello" we do not need to see "if ξ loves Cassio, ξ is killed by Othello" as occurring in it. The sense of this sentence is grasped by grasping the senses of its constituents, independently of whether there is a complex expression that may figure as a component of that sentence according to one decomposition of it which it shares with other sentences. Thus decomposition presupposes the sense of the complete sentence as somewhat given, while analysis assumes that the sense of the complete sense is given by the senses of its constituents plus the way they are put together.

Analysis and decomposition, though distinct procedures, are inextricably related. In order to decompose a sentence by removing one or more occurrences of an incomplete expression, we have to see that incomplete expression as occurring in that sentence, and this involves analysis. The way in which a proper name occurs in a sentence is quite easy to grasp. But how can an incomplete expression occur in a sentence, and thus be a candidate for removal from it? This Dummett explains inductively (1981b, pp. 275–276). In the first place, an incomplete expression occurs in a complete sentence (or other complete term) which is formed by filling its argument-place(s). Thus, "ξ loves ζ" occurs in "Desdemona loves Cassio" because the latter is formed by filling the argument-places of the former. In the second place, an incomplete expression occurs in a complete sentence or term if it occurs in a complete constituent of that sentence or term. Thus "ξ loves ζ" occurs in "If Desdemona loves Cassio, Desdemona is killed by Othello" because, by the first stipulation, it occurs in "Desdemona loves Cassio", which is a constituent of the longer sentence. In the third place, an incomplete expression occurs in a complete sentence or term if it occurs (without that any part of it is removed if not for being substituted with a bound variable) in a constituent of that sentence. Thus, "ξ loves ζ" occurs in the quantified sentence "For any *x*, if *x* loves Cassio, *x* is killed by Othello" because it occurs in the complex predicate "if ξ loves Cassio, ξ is killed by Othello", which is a constituent of the quantified sentence. Thus decomposition presupposes analysis, because in order to decompose a sentence we must remove some part of it, and in order to be removed that part has to occur in that sentence. But in order to recognize something as occurring in a sentence, appeal has to be made to the constituents of that sentence (as is made clear by Dummett's third stipulation). And constituents are revealed by analysis.

Conversely, while in general the analysis of a sentence does not presuppose that we regard that sentence as decomposable in a certain way, the analysis of a *quantified* sentence presupposes decomposition. The reason is that in the analysis of such a sentence it is required that we see a complex predicate as a constituent of it, and a complex predicate is always arrived at by decomposition of a simpler sentence (Dummett 1981b, p. 276). In order to analyze the quantified sentence "For any *x*, if *x* loves Cassio, *x* is killed by Othello" we need, as the first step in the analysis, to regard it as formed by attaching the quantifier to the complex predicate "if ξ loves Cassio, ξ is killed by Othello". But this complex predicate can only be attained by decomposition of some simpler sentence such as "If Desdemona loves Cassio, Desdemona is killed by Othello" by removal of the two occurrences of "Desdemona" from it. The complex predicate is a *constituent* (i.e. an element in the analysis) of the quantified sentence, but is only a *component* (i.e. an element in the decomposition) of the simpler unquantified sentence. (The complex

predicate "If ξ loves Cassio, ξ is killed by Othello" is a component but not a constituent of the sentence "If Desdemona loves Cassio, Desdemona is killed by Othello" because in order to grasp the sense of this sentence we do not need to regard the complex predicate as occurring in it.) Thus its being a constituent of the quantified sentence, which we have to recognize it to be in order to take the first step in the analysis of that sentence, depends on its being regarded as a component of the simpler unquantified sentence. Thus the analysis of a quantified sentence presupposes the decomposition of a simpler sentence.

Closely allied to Dummett's distinction between analysis and decomposition is the distinction between simple and complex predicates. A simple predicate is a constituent of a sentence in the sense that it is obtained by the analysis of it. It is necessary to recognize a simple predicate as occurring in a sentence in order to grasp the sense of that sentence, i.e. in order to recognize the contribution that the sense of that predicate makes to the sense of the sentence in which it occurs. By contrast, a complex predicate is a component of a sentence in the sense that it is obtained by the decomposition of the sentence; in general, a complex predicate is not regarded as a constituent of, but only as a pattern within, the sentence in which it occurs, because it is not necessary to recognize that predicate as occurring in the sentence in order to grasp the sense of that sentence.

Dummett (1981a, pp. 30–31) allows, as a limiting case, that in which a simple predicate coincides with a predicate obtained by decomposition. Consider the predicate "ξ walks". It can be seen as having being obtained by decomposition from the atomic sentence "Dion walks" by omission of the proper name "Dion". It is thus a component of that sentence. But it is also a constituent of that sentence, because it is one of the elements revealed by the analysis of it. When considered as a component rather than as a constituent, a simple predicate is, like any other complex predicate, to be regarded as having been extracted from a complete sentence by decomposition. The limiting case of a simple predicate that is considered as a result of decomposition, and thus as a complex predicate, sufficiently evidences that complex predicates are not a kind of entity distinct from simple predicates, but are ways in which sentences can be seen for certain inferential purposes.[6] Dummett suggests (1981a, p. 63) that it is only for the sake of economy that Frege silently assimilates complex and simple predicates, but in fact these notions are theoretically distinct in that they serve quite distinct purposes.

2. Peircean decomposition

Peirce defines a "rhema" as that which results if from a complete sentence we remove a part which, if substituted with a proper name, would again result in a complete sentence (CP 2.95, 3.363, 4.354, 4.438). Thus, if from the sentence "God gives some good to every man" we remove "every man" what we obtain is "God gives some good to ξ". This is a rhema, because if we were to replace "ξ" with a proper name, say "Hamlet", we would get "God gives some good to Hamlet", which is a complete sentence. The argument-place that is produced in a sentence by the removal of a part of it, and which following Dummett I represent with a Greek letter, is called by Peirce a "blank". He specifies that "[t]here may be any integer non-negative number of blanks, so that the term rheme is extended even to a full proposition when it is looked upon as having a number of blanks which happens to be gone" (R 280, pp. 19–20, 1905). A rhema with one blank, like "God gives some good to ξ", is called a "monad"; a rhema with two blanks, like "God gives

[6] Cf. Sullivan (2010, pp. 107–108).

ζ to ξ" is called a "dyad"; a rhema with three blanks, like "η gives ζ to ξ", is called a "triad"; and so on. When nothing is removed from the sentence, the complete sentence itself is considered as a rhema, and is called by Peirce a "medad".

In fact, the notion of the medad is important for a purpose quite distinct from that of decomposition, namely for the purpose of distinguishing the expression of a propositional content from the assertion of it. Thus Peirce says that "a medad can only be a mental proposition, which one may call a judgment, or *Urtheil*, provided it will be well understood that in so far as it is a medad, it need not be asserted, or mentally assented to: it need only be understood" (R 280, p. 20). Apart from the otherwise important distinction between the locutionary and the illocutionary dimensions of analysis, however, medads play no special role in Peirce's doctrine of analysis, and accordingly I will ignore them in what follows.[7]

Rhemata are obtained by Dummettian decomposition from complete sentences. One and the same sentence may be subjected to distinct decompositions. Here is the whole passage from the "Logical Tracts no. 2" from which the quotation at the beginning of this paper was taken:

Every proposition has one predicate and one only. But what that predicate is considered to be depends upon how we choose to analyze it. Thus, the proposition

God gives some good to every man

may be considered as having for its predicate either of the following rhemata:
— gives —to —
— gives some good to —
— gives — to every man
God gives — to —
God gives some good to —
God gives — to every man
— gives some good to every man
God gives some good to every man.

In the last case the entire proposition is considered as predicate. (R 492, p. 33 = CP 4.438)

The same point is made in the already-mentioned "The Basis of Pragmaticism" (c. 1905):

A proposition, according to any one analysis of it, has but a single predicate, but it is equally legitimate to throw any proper name or substitute for a proper name into the predicate or to make it a subject. It is a mere question of convenience. Thus, take the following proposition:

The serpent beguiled Eve to give an apple to Adam so that he should disobey God.

This may be regarded as having a pentad predicate

— beguiled — to give — to — so that he should disobey —

or as having a predicate of any lesser valency. If the predicate be taken a monad, the one subject will be any one we choose of the five: "The serpent", "Eve", "some apple", "Adam", "God". (R 280, p. 41)

[7] On Peirce's speech-act theoretical analysis of assertion see Bellucci (2017, pp. 295–321).

Peirce's use of the term "proposition" in these passages requires some qualification. Peirce's "standard" definition of "proposition" is framed in the terms of his theory of signs: a proposition is a sign that separately represents an object.[8] This definition is designed precisely to distinguish a proposition from a rhema (in the sense of Peirce's definition). For a rhema is a sign that does *not* separately represent an object, i.e. is that which remains of a proposition after something that may represent an object (and is thus replaceable by a proper name) is removed from it.

However, Peirce's use of "proposition" is not wholly consistent. Sometimes he uses "proposition" for that which is expressed by a sentence, and which is the propositional content that may be the object of different speech acts.[9] At other times with "proposition" he means an assertive sentence, as opposed to interrogative and imperative sentences (R 515, 1904). To add to the confusion, in R 544 (1896) a "syntactical arrangement of words" is said to be a proposition, a position later echoed in "The Rationale of Reasoning" (1910), where a proposition is characterized as a "product of language" (R 664, p. 7); in "The Logic of Relatives" of 1897 (CP 3.461) "proposition" and "sentence" are used as synonymous.

A comparison with Frege is helpful here. Frege says that corresponding sentences in the active and passive voice may express the same thought: "the only essential thing for us is that a different thought does not correspond to every difference in the words used […] The distinction between the active and the passive voice belongs here too. The sentences 'M gave document A to N', 'Document A was given to N by M', […] express exactly the same thought" (Frege 1979, p. 141). Peirce makes the very same point in "The Basis of Pragmaticism" in terms of the contrast between sentence and proposition: "A single form of proposition expresses indifferently every set of sentences such as the following: Abel gives a Bolometer to Cain; Abel enriches Cain with the gift of a Bolometer; Cain receives a Bolometer as a gift from Abel; Cain is enriched by Abel with the gift of a Bolometer; A Bolometer is given by Abel to Cain; A Bolometer is added to Cain's possession as a gift from Abel" (R 280, p. 34). Likewise, in "The Rationale of Reasoning" he says: "Take the proposition 'Cain killed Abel'. This is identically the same Proposition as 'Abel was killed by Cain'; it is only the grammatical dress that is different" (R 664, pp. 10–11, 1910).[10] More generally, the contrast is between the abstract type and its realization in natural language: "it is necessary to distinguish between a proposition and a sentence, i.e. this or that expression of it, in writing, speech, thought, etc. A *sentence*, in the sense here used, is a single object. Every time it is copied or pronounced, a new sentence is made. But a proposition is not a single thing and cannot properly be said to have any *existence*. […] It is the same proposition every time it is thought, spoken, or written, whether in English, German, Spanish, Tagálog, or how" (R 599, p. 6, c. 1902).

Now, the problem with Frege's idea that structurally distinct sentences (like corresponding sentences in the active and the passive voice) may have the same sense is that such sentences may then be capable not only of multiple decompositions, but also of multiple analyses. For distinct sentences would be analyzable into distinct constituents; but then, since distinct constituents have

[8] CP 1.559, 1867; R 484, p. 7, 1898; CP 2.357, 1901; RL 75, p. 21, 1902; CP 5.139, 1903; R 491, p. 9, 1903; R 7, p. 16, c. 1903. Hilpinen (1992) was the first to see that this is the standard definition of the proposition in Peirce; see also Stjernfelt (2015) and Bellucci (2017, pp. 95–96, 197–198, 287–288, 294).
[9] Cf. e.g. R 599 (c. 1902); CP 2.252 (1903); R 517 (1904); CP 5.424n (1905); R 280, pp. 25–26 (1905).
[10] Cf. also R 641, p. 24 (1909); R 659, p. 19 (1910).

distinct senses, structurally distinct sentences would *not* have the same sense. To remedy this, some commentators have proposed to restrict the principle of the uniqueness of analysis to the sentences of a regimented language. In a natural language, it is not always the case that the manner in which a sentence is structured into parts can be taken as an indication of the manner in which the thought expressed by that sentence is structured into parts; but in a regimented language like Frege's Begriffsschrift it is possible to take the structure of a sentence as an indication of the structure of the thought expressed by that sentence. Therefore, while Frege's principle of the uniqueness of analysis does not hold generally for any language, it does hold for a regimented language as the Begriffschrift. In such a language, sentences that in natural language would be distinctly structured, as "*m* gave document *a* to *n*" and "Document *a* was given to *n* by *m*" are, would be symbolized by one and the same formula: "G*anm*".[11]

There is reason to think that a parallel remedy would work for Peirce. In the *Short Logic* of 1895 he says: "the sentence 'every man loves a woman' is equivalent to 'whatever is a man loves something that is a woman'" (R 595, p. 15). To say that the latter is "equivalent" to the former amounts to saying that the latter expresses the former in a regimented or semi-regimented notation: Peirce is here saying that the sentence "every man loves a woman" can be expressed in a semi-regimented notation as "For every *x*, for some *y*, if *x* is a man then *x* loves *y*, and *y* is a woman". He continues thus: "In the eye of logic, two propositions expressing the same fact are *equivalent*, or virtually (at least) identical. Accordingly, the practice of logicians has always been to adopts certain canonical forms in which they require that judgments should be expressed" (pp. 20–21). The same is true of any pair of corresponding sentences in the active and passive voice, for they are "identically the same proposition". He is most clear in the following passage from "The Basis of Pragmaticism":

> a proposition is nothing existent, but is a general model, type, or law according to which existents are shaped. Here, for instance, are half a dozen existent writings:
>
> *Solomon built him an house* [sic]
> *Σολομὼν ᾠκοδόμησεν αὐτῷ οἶκον*
> *Solomon built him a house.*
> *Salomon aedificavit illi domum.*
> *Solomon built a house for himself.*
> *Solomon a adeiladodd dŷ iddo ef.*
>
> But they are all existing singular *instances* of one self-same proposition. Such a form of expression of any proposition as is adapted to use in this system, and which not only represents a determinate proposition but represents it in a determinate way, while indeterminate as to characters which are not significant, will be termed a *graph*, for short. (R 280, pp. 27–28)

Though Peirce is here presenting his system of Existential Graphs – a notation for sentential and quantificational logic which he invented in 1896 – the point that he makes is independent of the particular notation in question: a proposition is that which can be expressed by, and thus can for the purposes of logic be identified with, a type sentence of a regimented or semi-regimented language ("form of expression") that corresponds to an equivalence class of sentences of natural

[11] Dummett (1991, p. 291) and Textor (2018) are in favour of such emendation of Frege's principle of the uniqueness of analysis.

language. It is in this sense, I think, that "proposition" has to be taken not only in "The Basis of Pragmaticism", but also, at least, in those contexts in which Peirce expounds his doctrine of the logical analysis *of propositions* in terms of an analysis *of sentences*. When he says that a proposition can be "analyzed" (i.e., as we shall see, "decomposed" in Dummett's sense) in multiple ways into subject and predicate, he means that the sentence type of a regimented language that expresses that proposition can be so analyzed. Accordingly, in what follows I will use "sentence" in the sense just specified to refer to what in these contexts Peirce calls a proposition.

With this qualification in mind, let us now return to the problem of the multiple decompositions of a sentence. According to each of its decompositions, a sentence has one predicate, which is the rhema obtained by that decomposition. What that predicate is simply depends on what we choose to consider as a subject, i.e. on what elements are removed from the complete sentence. Peirce usually uses lines or "em dashes" to indicate the "blanks", i.e. the argument-places of the rhema which I indicate by Greek letters. Thus, from the sentence "Desdemona loves Cassio" we may remove "Desdemona", thus obtaining the monadic rhema "ξ loves Cassio", or we may remove both "Desdemona" and "Cassio" from it, thus obtaining the dyadic rhema "ζ loves ξ". In the first case, "Desdemona" is regarded as the subject, while in the second case, the ordered pair <Desdemona, Cassio> is so regarded.[12] The first decomposition displays a pattern within the sentence which it shares with, e.g., "Bianca loves Cassio"; the second decomposition displays a pattern within the same sentence which it shares with, e.g., "Juliet loves Romeo". The first decomposition shows why "Desdemona loves Cassio" follows from "Anyone loves Cassio", the second shows why it follows from "Anyone loves everyone".

That the idea of the multiple decompositions of a sentence was not a late product of Peirce's logical investigations is attested by the following passage from the sixth chapter of *How to Reason*, the unpublished multi-volume work on logic that Peirce had completed in 1894:

> In the logical analysis of the sentence, we disregard the forms and consider the sense. Isolating the indices as well as we can, of which there will generally be a number, we term them the *logical subjects*, though more or less of the symbolic element will adhere to them unless we make our analysis more recondite than it is commonly worth while to do; while the purely symbolic parts, or the parts whose indicative character needs no particular notice, will be called the *logical predicate*. As the analysis may be more or less perfect, – and perfect analyses are very complicated, – different lines of demarcation will be possible between the two logical members. In the sentence John marries the mother of Thomas, John and Thomas are the logical subjects, marries-the-mother-of is the logical predicate. (R 409, p. 97 = CP 4.58)

In the analysis of the sentence "John marries the mother of Thomas", that which Peirce here terms the "logical predicate", namely the composite expression "marries-the-mother-of" (or more perspicuously, "ζ marries the mother of ξ"), perfectly satisfies the definition of rhema, because it is what remains of the complete sentence after something replaceable by proper names (in this case, the proper names "John" and "Thomas") has been removed from it. A different analysis (i.e. decomposition) of the sentence is obtained, however, by drawing a "different line of demarcation" between the two "logical members", i.e. the removed subjects and the remaining predicate: if we remove "the mother of Thomas" from the sentence we get "John marries ξ",

[12] That Peirce regarded the subject of a relative proposition as an ordered set is evident, for example, from CP 2.230, 2.316, 4.453, 8.177.

which, evidently, is also a rhema, because if we replace the "ξ" in it with a proper name, say "Mary", we obtain again a complete sentence ("John marries Mary").

Further evidence that Peirce conceived the decomposition of a sentence in Dummettian terms is the following. In a draft of the entry "predicate" which Peirce wrote for Baldwin's *Dictionary of Philosophy and Psychology*, we read: "If in any proposition, or sign which must be true or false, such a part is (or such parts are) struck out that what remains is not a proposition but will become a proposition as soon as the blank is (or blanks are) filled up each with a proper name, or index of a known individual, such a residue is a predicate, and is *the* predicate of the original proposition in reference to the particular mode of mutilation used" (R 1147, pp. 269–270, c. 1901). In the Harvard Lectures of 1903 the expression used is "mode of analysis": "How much shall be embraced in the predicate and how many subjects shall be recognized depends, for the ordinary analyses of logic, upon what mode of analysis will answer the purpose in hand" (R 308). It is clear that, despite the variation in terminology, with "mode of mutilation" and "mode of analysis" Peirce means Dummettian decomposition, not Dummettian analysis.

Later[13] Peirce also makes the point that the fact that multiple decompositions of one and the same sentence are possible by no means implies that the distinction between subject and predicate remains undetermined in each particular decomposition of that sentence. Peirce expresses this idea in sign-theoretical terms in a draft of a letter to Samuel Barnett in 1909:

> The determination by a Sign of its Interpreting Mind, – i.e. the idea that mind gets, or the feeling it sets up, or the action it stimulates, I call its 'Interpretant'; and there is all the difference in the world between the *Object* of a sign, of which the Interpreter must have some *collateral* experience, immediate or mediate, or he won't know at all what it is that the Sign represents [,] and whoever questions that point simply fails to understand what I mean by the Object, and confounds it with the Interpretant. The latter is *all* that the sign *conveys*. The Object is the otherwise known something concerning which what it conveys relates. The distinction is a *real* distinction and yet it is *purely relative*, in the sense that the line of demarcation between the two can just as well be drawn in one place as another […] The point is that the artificiality of a line of demarcation does not prove that the *twoness* of the parts that line of demarcation may be regarded as separating does not correspond to any twoness *in re.* (RL 36)

The subject of a proposition is, in sign-theoretical terms, its "object", i.e., that which must be given through collateral observation and which the sign cannot by itself express, while the interpretant of a proposition is everything which the proposition says of the object, i.e., that which remains after the object has been removed. It follows from the possibility of multiple decompositions of a proposition that the line of demarcation between the object of a sign and its interpretant can shift. This does not mean that no line of demarcation between object and interpretant (in the case of propositions, between subject and predicate) can be determined in every special case, however: from the fact that different decompositions into components are possible it does not follow that in each case there is no distinction between the components obtained by decomposition.[14]

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[13] There are passages that indicate that by 1908 he still admitted the possibility of multiple decompositions of one and the same sentence: "A proposition can be separated into a predicate and subjects in more ways than one" (R 278, 1908); "more or fewer objects may be regarded as subjects while the remainder of the assertion is the predicate" (R 339, p. 332, 1908).

[14] For a discussion of the "interpretability" of the subject/predicate distinction (in sign-theoretical terms, the object/interpretant distinction) see Stjernfelt (2015).

3. Peircean analysis

A rhema is obtained by Dummettian decomposition. Is a rhema a "part" of the sentence from which it has been obtained by decomposition? In "Logical Tracts No. 2" Peirce says that "every combination of parts of a proposition involves a rhema of second intention. If two propositions agree exactly in respect to their rhemata of second intention, differing consequently only in respect to their simple rhemata of first intention, they are said to have the same *logical form*" (R 492). A rhema of first intention is a rhema that expresses "differences of real fact", while a rhema of second intention is one that expresses "differences between symbols" (R 492, p. 38).[15] The same distinction had been made in "The Regenerated Logic" of 1896, where Peirce had proposed to distinguish within a sentence a subject, which is an *index* of the object or objects the sentence is about, and a predicate, which is an *icon* of the object of the index. Then, he had distinguished between two different sorts of icons:

> But instead of a single *icon*, or sign by resemblance of a familiar image or "dream," evocable at will, there may be a complexus of such icons, forming a composite image of which the whole is not familiar. But though the whole is not familiar, yet not only are the parts familiar images, but there will also be a familiar image of its mode of composition. In fact, two types of complication will be sufficient. For example, one may be conjunctive and the other disjunctive combination. Conjunctive combination is when two images are both to be used at once; and disjunctive when one or other is to be used. […] The sort of idea which an icon embodies, if it be such that it can convey any positive information, being applicable to some things but not to others, is called a *first intention*. The idea embodied by an icon which cannot of itself convey any information, being applicable to everything or to nothing, but which may, nevertheless, be useful in modifying other icons, is called a *second intention* (CP 3.433).

The "predicate" of a sentence may be a "complexus" of icons of first intention and icons of second intention. An icon of first intention conveys positive information about an object or set of objects; in the terminology of the "Logical Tracts", it expresses "differences of real fact". An icon of second intention, by contrast, does not convey any positive information, but only conveys information about icons of first intention; in the terminology of the "Logical Tracts", it only expresses "differences between symbols". In the same vein, in "On Logical Graphs" (1896) he had written:

> When we first begin the study of any department of experiential science, we form classes and systems of objects of experience and these give the ordinary verbs, or verbs of first intention. But when we come to observe the signs themselves which we have so created, we make classes and systems based on the characters of the signs, and these are verbs of second intention. (R 480)

It is scarcely dubitable that what Peirce calls "icons" of first and second intention in "The Regenerated Logic", and "verbs" of first and second intention in "On Logica Graphs",

[15] Avicenna, in a passage frequently quoted at the end of the thirteenth century, had affirmed that second intentions are the subject matter of logic: "Subiectum vero logicae, sicut scisti, sunt intentiones intellectae secundo, quae apponuntur intentionibus intellectis primo" (*Liber de philosophia prima*, I, 2). In his questions on Porphyry, Scotus attributes this thesis to Boethius: "Aliter ponitur, quod [subiectum logicae] est de secundis intentionibus applicatis primis, sicut dicit Boethius, quia illae sunt communes omnibus in logica determinatis" (qu. III). It is probably to this passage that Peirce refers in the "New List of Categories" when he says that "Logic is said to treat of second intentions as applied to first" (CP 1.559). Peirce explicitly refers to Avicenna's claim in the "Logical Tracts No. 2" in the context of his explanation of the difference between rhemata of first and of second intention (R 492, pp. 79–80).

correspond to the rhemata of first and second intention of the "Logical Tracts".[16]

In order to understand how a sentence is analyzed into rhemata of first and second intention, let us consider the sentence "For every *x*, if *x* is a man, then *x* is mortal". Just like Dummettian analysis, Peircean analysis takes place in stages. The first stage in the analysis of this sentence consists in regarding it as having being obtained by applying the universal quantifier to the complex predicate "if ξ is a man, then ξ is mortal". As Dummett explains, we get the notion of a complex predicate by *decomposition*, i.e. by removing one or more occurrences of a proper name from a complete sentence. Our complex predicate may thus be regarded as having been obtained from the sentence "If Socrates is a man, then Socrates is mortal" by removal of the two occurrences of the proper name "Socrates". Decomposition, we saw, is necessary in order to construct a quantified sentence, because a quantified sentence is formed by attaching a quantifier to an incomplete expression, and an incomplete expression or "rhema" according to Peirce's definition of it, is something that we obtain by decomposition of a complete sentence. Frege regarded quantifiers as second-order functions applied to first-level functions, while Peirce regarded quantifiers as rules specifying the manner and the order in which the objects to which the predicate of a sentence indefinitely refers are to be selected. But this difference is inessential for our purposes.

The second stage of the analysis consists in regarding our complex predicate as having being obtained by combining the icons of first intention "ξ is a man" and "ξ is mortal" by the icon of second intention "if Ξ, then Φ". The analysis of a sentence, therefore, is obtained through distinct steps, and is successful if it yields the rhemata of first and second intention that are genuine constituents of the sentence. That rhemata of second intention are not obtained by decomposition is evident from the fact that Peirce's definition of "rhema" does not apply to them: we cannot regard the icon of second intention "if Ξ, then Φ" as a rhema in Peirce's definition of it, i.e. as having been obtained by decomposition of, e.g., the sentence "If it rains, then it thunders" by omission of "It rains" and "It thunders", because if we replace both with proper names, e.g., "Desdemona" and "Cassio", what we obtain is not a complete sentence, but a ill-formed expression ("If Desdemona, then Cassio").

4. Peircean analysis *vs* Peircean decomposition

Peirce never draws the distinction between analysis and decomposition. In point of fact, "analysis" and "decomposition" (and their cognates) are often used interchangeably in Peirce's writings.[17] However, if analysis and decomposition are not distinguished, the model of analysis

[16] Peirce speaks of "rhemata of second intention" also in "On Logical Graphs" (R 482) and in the fourth section of the *Syllabus* of 1903 (R 508, 478 = CP 4.394–417). In "Schroeder's Logic of Relations" (an earlier draft of "The Logic of Relatives" of 1897) the distinction made is between indices (i.e. algebraical symbols) that denote first intentions and those that denote second intentions (R 521, p. 2).

[17] For example, in R S-36 Peirce explains that a "spot" (about which more below) is an "undecomposed", i.e. "unanalyzed", element of a logical graph (p. 12). In a draft of a letter to his former student Christine Ladd-Franklin, probably written in 1901, he subsumes the question "what does the logical decomposition of a reasoning consist in" under the heading of "logical analysis" (RL 237, p. 6). In a 1909 letter to James we read: "there are concepts which, however we may attempt to *analyze* them, will always be found to enter intact into one or the other or both of the *components* into which we may fancy that we have analyzed them" (RL 224, *emphases added*). In discussing the inferential rules ("rules of transformation") of the Existential Graphs, he says that "an *indecomposable* transformation is either an omission or an insertion, since any other may be *analyzed* into an omission followed by an insertion" (R 515, p. 1, 1904, *emphases added*); cf. also CP 4.564.

of a sentence into "parts", some of which are rhemata of first intention and others rhemata of second intention, turns out to be inconsistent with the idea that one and the same sentence can be analyzed in multiple ways into subject and predicate. For if a sentence has a number of "rhemata of first intention" as parts, none of these can be considered as the "predicate" of the sentence. Nor can any of the rhemata of second intention that connect the rhemata of first intention be so considered, not even the "main" rhema of second intention of a sentence (i.e. the truth-functional operator that has the greater scope in the sentence). For according to the model of analysis into subject and predicate, that which is removed from a sentence in order to extract its unique predicate must be replaceable by a proper name, and this is hardly the case if from the sentence "For every *x*, if *x* is a man, then *x* is mortal" we extract its main rhema of second intention "If Ξ, then Φ".

Once analysis and decomposition are distinguished, however, there is no conflict in Peirce's holding that one and the same sentence can be decomposed into a subject (anything removable which could be replaced by a proper name) and a predicate (the rhema extracted from a sentence by removing one or more occurrences of a subject), and at the same time – indeed, in the very same paper, the "Logical Tracts", in which he has exposed his doctrine of decomposition – that every sentence can be analysed into a combination of rhemata of first intention and rhemata of second intention. The apparent conflict only derives from the assumption that Peirce is talking of the same kind of analysis in the two cases.

The two models are distinguishable because serve quite distinct purposes. The model of decomposition has the purpose of highlighting a pattern within a sentence, i.e. the predicate that a sentence may have in common with other sentences; thus, the sentence "If Desdemona loves Cassio, Desdemona is killed by Othello" is decomposed in one way into the subject "Desdemona" and the rhema "If ξ loves Cassio, ξ is killed by Othello", and this rhema is a pattern that the sentence in question shares with, e.g., "If anyone loves Cassio, she is killed by Othello"; but the same sentence can be decomposed in another way into the subject "Cassio" and the rhema "If Desdemona loves ξ, Desdemona is killed by Othello", and this rhema is a pattern that the sentence in question shares with, e.g., "If Desdemona loves anyone, Desdemona is killed by Othello". The model of analysis, by contrast, has the purpose of displaying the constituent parts of which a sentence is composed: by the first step in the analysis, "If Desdemona loves Cassio, Desdemona is killed by Othello" is analyzed into its three subjects "Desdemona", "Cassio", and "Othello" and the complex predicate "If η loves ζ, η is killed by ξ". This complex predicate is then analyzed into the rhemata of first intention "η loves ζ" and "η is killed by ξ" and the rhema of second intention "If Ξ, then Φ".

In order to more clearly see that there is no conflict between analysis and decomposition, we must explain in what sense the rhemata of first and second intention into which a sentence can be analysed are not, in general, rhemata obtained by decomposing that sentence. In the context of his works on Existential Graphs Peirce terms a rhema whose analysis is not exhibited a "spot": "A rhema which is made a part of a graph and is not itself compounded according to the rules of existential graphs is called a *spot*" (R 450, 1903).[18] Though "spot" is a technical term of the theory of logical graphs, I shall retain this terminology without entering the subjects of the

[18] Cf. also CP 4.441 (1903); CP 4.403 (1903); R 295 (1906). Roberts, who was fully aware of Peirce's doctrine of multiple analyses, did not differentiate between spots and rhemata (1973, p. 115); nor does Pietarinen, who thinks that a spot is simply the "iconic version" of a rhema (2006, p. 115; cf. also pp. 123–125).

graphs. Thus, more generally, for us a spot is a rhema which cannot be regarded as having been compounded according to logical relations. This means that a spot is a simple rhema, i.e. either a rhema representing a logical relation or a rhema that cannot be regarded as the result of the combination of rhemata by logical relations. For example, the rhema "If η loves ζ, η is killed by ξ" can be regarded as having been obtained by combining the rhemata "η loves ζ" and "η is killed by ξ" by means of the sentential operator "If Ξ, then Φ"; but neither "η loves ζ" and "η is killed by ξ" can be so regarded. Therefore, "η loves ζ" and "η is killed by ξ" qualify as spots, while "If η loves ζ, η is killed by ξ" does not so qualify. It is evident that a spot is either a rhema of first intention or a rhema of second intention.

At one point, Peirce suggests that while in general spots of second intention represent logical relations, not all of them represent sentential operators. We have seen that in "Logical Tracts No. 2" Peirce says that two sentences constituted by the same rhemata (i.e. spots) of second intention, whether differing or not in respect to their rhemata (i.e. spots) of first intention, have the same "logical form". In a 1908 letter to Lady Welby he makes a similar point: "When we have analyzed a proposition so as to throw into the subject everything that can be removed from the predicate, all that it remains for the predicate to represent is the form of connection between the different subjects as expressed in the propositional *form*" (SS, p. 71). As he proceeds to explain, the operation of "throwing into the subject everything that can be removed from the predicate" is the operation that turns "Cain kills Abel" into "Cain is to Abel in the relation of killing". This is not, obviously enough, to be considered as a decomposition of the sentence, but as a translation of it that highlights how it is constituted by spots of first intention and spots of second intention, i.e. as a translation of the sentence that makes its analysis perspicuous: the sentence is formed by three spots of first intention ("Cain", "Abel", "killing") put together by a spot of second intention ("ζ is to ξ in the relation of η"). This spot of second intention is not, as in previous examples, a sentential operator, but only the logical notion of relation (one might say, the *pure* idea of a relation). In this context, Peirce calls such spot of second intention a "continuous predicate", and concludes: "when we have carried *analysis* so far as to leave only a continuous predicate, we have carried it to its *ultimate elements*" (SS, p. 71, *emphases added*). This only means: there is only one analysis of a sentence into ultimate elements, and these are the proper names and spots of first intention that can be "thrown into the subject" and the spot or spots of second intention that connect them. There is no sense in which the decomposition of a sentence can yield its "ultimate elements": "analysis" here means Dummettian analysis, not Dummettian decomposition.[19]

A spot of first intention is a Dummettian simple predicate. We need to recognize a sentence as being built up from spots of first intention in order to account for its sense: the sense of a spot of first intention is the contribution that that spot makes to the sense of the sentence. In a letter to William James of February 1909 Peirce wrote:

> In the sentence ["Napoleon was a lethargic creature"] Napoleon is not the only Object. Another Partial Object is Lethargy; and the sentence cannot convey its meaning unless collateral experience has taught its Interpreter what Lethargy is, or what that is that 'lethargy' means in this sentence. (CP 8.178)

In the same vein, he had written to Lady Welby in the 1908 letter quoted above:

[19] On Peirce's notion of "continuous predicate" see Bellucci (2013) and Bellucci (2017, pp. 331–334).

Thus the statement, "Cain killed Abel" cannot be fully understood by a person who has no further acquaintance with Cain and Abel than that which the proposition itself gives. Of course, Abel is as much a subject as Cain. But further, the statement cannot be understood by a person who has no collateral acquaintance with killing. Therefore, Cain, Abel, and the relation of killing are the subjects of this proposition (SS, p. 70).

In sign-theoretical terms, anything of which "collateral knowledge" is necessary in order to understand the sign is an "object" of it. This knowledge is collateral in that no mere inspection of the sign itself would reveal the identity of the object or objects named; knowledge of the object is presupposed by the understanding of the sign, and is given in terms of the sense (i.e. the collateral knowledge) we associate with the name of that object. In the case of sentences, anything is an "object" of it collateral knowledge of which must be presupposed in order to grasp the sense of the sentence.[20] Thus, the possibility of grasping the sense of the sentence "Cain kills Abel" is not only determined by the sense (i.e. the collateral knowledge of the object) of the proper names "Cain" and "Abel", but also by the sense (i.e. the collateral knowledge of the object) of the simple predicate or spot of first intention "ζ kills ξ". In like manner, the possibility of grasping the sense of the sentence "Napoleon is lethargic" is not only determined by the sense (i.e. the collateral knowledge of the object) of the proper name "Napoleon" but also by the sense (i.e. the collateral knowledge of the object) of the simple predicate or spot of first intention "ξ is lethargic". The sense of a sentence is determined by the senses of its constituent parts, which parts are either proper names or spots of first intention (plus, if the sentence is not atomic, the sense of the spots of second intention that connect the spots of first intention).[21] Since the

[20] The question whether Peirce could be said to embrace the Fregean distinction between *Sinn* and *Bedeutung* is not easy to answer. With respect to general terms, which for Peirce are *symbols*, it is safe to say that they have both *Sinn* and *Bedeutung*: a symbol is a sign that both *connotes* and *denotes*, and denotes whatever satisfies the characters it connotes (CP 1.599, 2.344, 4.544). With respect to proper names, which for Peirce are *indices*, the question is more complex: on the one hand, he clearly says that proper names denote without connoting, i.e. have no signification (R 280); he thus seems to embrace a sort of Kripkean doctrine of proper names, and so he is usually taken to do by the commentators (see e.g. Hilpinen 1995). On the other hand, however, he clearly explains that the object of a proper name can only be given by "collateral observation" (also "collateral experience", "collateral acquaintance"). The question is thus whether such collateral knowledge can be identified with the Fregean *Sinn* of the proper name, i.e. with a definite description that is associated to the name. Since I have no space to argue for it here, I limit myself to say that such identification is legitimate only if we restrict it to the *referential* use of definite descriptions (as opposed to their *attributive* use; see Donnellan 1966). In saying that the object of a proper name is given by collateral knowledge, Peirce is saying that its use is regulated by a description of that object that referentially determines the denotation, whether the object satisfy the description or not. It is in this restricted sense that I say above that in order to grasp the sense of a sentence we need to grasp the sense of the proper names it contains. In any case, Peirce's notion of collateral knowledge may perhaps more easily be considered a variant of Russell's "principle of acquaintance": for Russell, anything, be it an object or a relation, can be a constituent of a proposition, provided we have acquaintance with it; cf. Russell (1905, p. 492); like for Russell, for Peirce not only subjects, but also predicates must be previously or collaterally known if a proposition is to function as such.

[21] It must be observed that at some places Peirce regards proper names as spots (of first intention); cf. CP 3.469, 1897); R 491 (1903); R S51, p. 6. Evidence of such a treatment of proper names can also be found in Peirce's papers on the logical graphs; cf. CP 3.471–477 (1897); CP 4.445 (1903); R 669, p. 11 (1911); R 670, p. 8 (1911). In Peirce's 1903 taxonomy of signs, proper names are classified as indexical rhemata, and a rhema that thus serves as the subject of a proposition is called an "onome" (R 478, p. 89). But in this context "rhema" cannot be taken in the strict sense of the definition. For since a rhema is defined as that which remains of a sentence after something replaceable by a proper name is removed from it, and since a proper name is of course replaceable by a proper name, then in a sentence a rhema should be replaceable by a proper name, which is untrue: if in "Cain kills Abel" I replace "ξ kills

constituent parts of a sentence are obtained by the analysis of it (as opposed to decomposition), Peircean analysis, just like Dummettian analysis, reveals the manner in which the sense of a sentence is determined by the senses of its constituent parts (i.e. by the collateral knowledge of the objects of those parts). This does not apply to the components of a sentence obtained by decomposition: the sense of "ξ kills Abel", which is a component of the complete sentence "Cain kills Abel" obtained by decomposition, is not (in sign-theoretical terms) one of the "objects" of the sign, i.e. is *not* one of things collateral knowledge of which is necessary in order to grasp the sense of the sentence.

5. Conclusion

Peirce seems to maintain two incompatible theses: that a sentence is multiply analyzable into subject and predicate, and that a sentence is uniquely analyzable as a combination of rhemata of first intention and rhemata of second intention. The incompatibility disappears as soon as we distinguish, following Dummett, two distinct notions of analysis: "analysis" proper, whose purpose is to display the manner in which the sense of a sentence is determined by the senses of its constituent parts, which Peirce calls rhemata or spots of first intention (Dummett's simple predicates) and rhemata or spots of second intention (the sentential operators and other purely logical relations); and "decomposition", which is the process of dividing a sentence into a predicate, which Peirce calls rhema and which is that which remains of a sentence when something replaceable by a proper name is removed from it, and a subject, and whose purpose is both to explain how quantified sentences are constructed and to evidence a pattern within a sentence which it shares with other sentences. Peirce – like Dummett's Frege – was at no great pains to distinguish the two processes, and this is why we may perceive the inconsistency between what he says when he has analysis in mind – as when, like in "The Regenerated Logic" (1896), "On Logical Graphs" (1896), and the "Logical Tracts no. 2" (1903) he talks of rhemata of first and second intention as the "parts" of which sentences are composed, or as when, like in the letters to James and Welby of 1908–1909, he talks of the sense of an atomic sentence as being given by the senses (i.e. the collateral knowledge of the object) of the proper names and of the simple predicates contained in it – and what he says when he has decomposition in mind – as when, in *How to Reason* (1894), in Baldwin's *Dictionary* (1901), in the Harvard Lectures (1903), in the "Logical Tracts no. 2" (1903) and at other places, he says that a sentence is multiply decomposable into subject and predicate. Though Peirce never explicitly distinguishes the two processes from one another, yet I believe I have shown that the distinction is not only implicit in what he says, but constitutes the only interpretation that will make clear sense of his doctrine of logical analysis.

Abel" with "Abel", I obtain "Cain Abel", which is *not* a sentence. This means that though it is harmless to regard proper names as spots of first intention ("The spots are of two kinds, rhemata and onomata, although the former are superfluities of which I make little use", R 491), it is impossible to regard them as rhemata in the strict sense of the definition. Thus, the fact that proper names can be regarded as spots of first intention not only confirms that the "analysis" (in Dummett's sense) of a sentence is into spots of first intention (whether "predicative" or "subjectal") and spots of second intention (plus the quantifiers), but also, and more importantly, it supports the distinction between spots and rhemata.

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