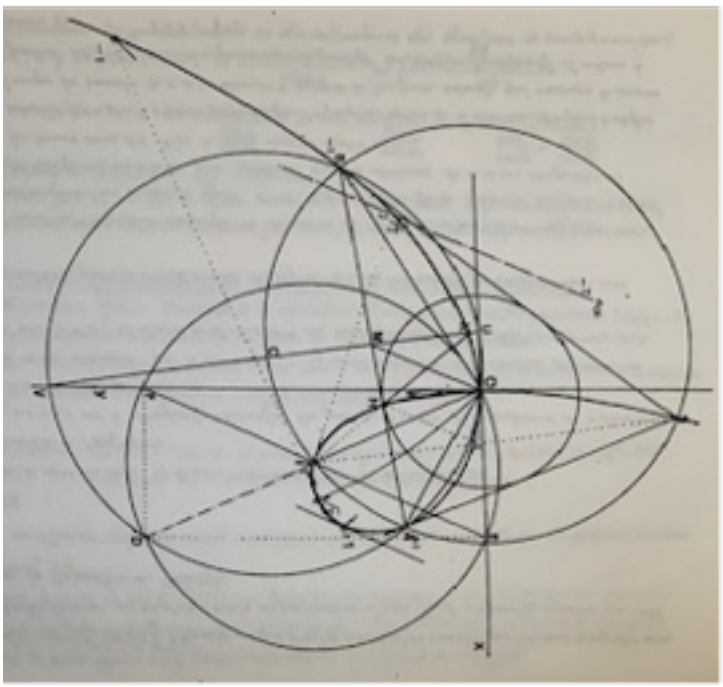
A Personal Tribute to Paul de Faget de Casteljau

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A personal tribute to Paul Faget de Casteljau

By Yvon Gardan - Honorary University Professor - Director of MICADO

The history of CAD has been marked by work on curves and surfaces carried out in France. Among the pioneers in this field, Paul Faget de Casteljau played an important role that was not recognised until long after he had demonstrated the value of his approach.
Paul de Faget de Casteljau (Paul de Casteljau) died in March 2022. His contribution to CAD is undeniable. His subsequent work is also highly original and deserves a closer look.
However, this is not the place to go through his many works, nor to summarise them - others will do that better than me - but to look back on my encounters and my exchanges with this brilliant mathematician. So it is from my meetings with him (and with Pierre Bézier) and the letters he sent me that I have drawn a personal vision of this brilliant mathematician.

I invite those who would like an in-depth presentation of Paul de Casteljau's contributions to refer to the articles by Professor Andreas Mueller (University of Kempten, Germany), the

various books published by Hermes, and the special issue currently being prepared for the journal Computer Aided Geometric Design (Special issue on Paul de Casteljau, a pioneer in CAGD).

So, exceptionally, allow me to write in the first person and give a personal view.

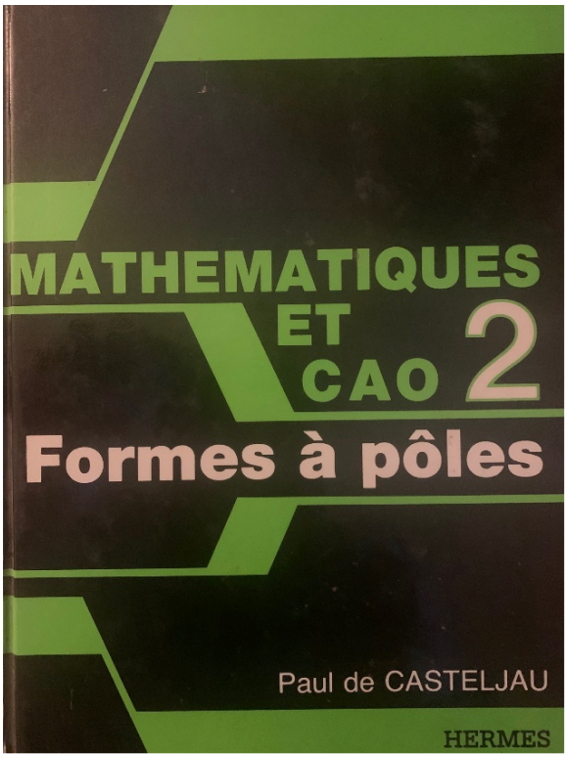
First encounter

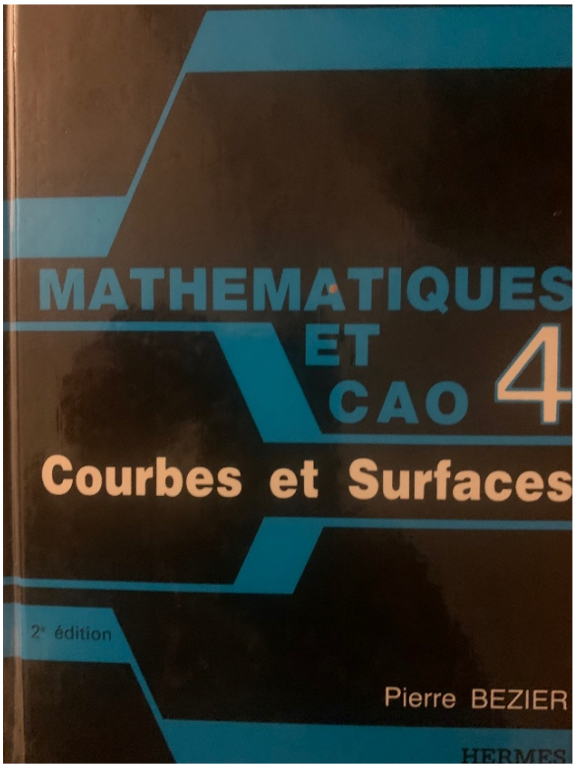
In the early 1980s, I was in charge of a collection of books entitled "Mathématiques et CAO" (Mathematics and CAD) published by Hermes. This collection was very successful, thanks in particular to the high quality of the authors of each of the books in it. For obvious reasons of chronology, we wanted Casteljau's book to be published before Pierre Bézier's book. After reading the manuscript, I was convinced that the content would make a significant contribution to understanding curves and surfaces in CAD, but I felt it was impossible not to ask for some formal changes. In fact, the document began with an introduction that included a few sentences no doubt reflecting a certain bitterness that his work on pole forms had not been recognised as seminal. It was understandable, but it was a little too direct, and its excess somewhat spoiled the quality of the work.

It had to be said, however, that others than him had attracted the spotlight, even though his work was undeniably the basis for all the developments carried out in the 1960s on curves and surfaces in CAD. That was why we wanted to publish his work as soon as possible in the collection.

Paul de Faget de Casteljau worked at Citroën and his superiors had to agree to the publication of his book. So we agreed to meet at their offices to discuss the few changes we wanted to make. It would have been easier to do this directly with Paul de Casteljau, but we had to pay tribute to how the company works internally. The meeting was held in a huge basement room, with no windows or at least little natural light. The tables were arranged in a U-shape. Paul de Casteljau was at one end of the U and I was in the middle of the other leg, with Sami Ménascé, the head of Hermes Publishing, whose contribution to the promotion of new technologies, particularly CAD, must be underlined. The Citroën managers concerned stood at the head of the U. I remember feeling very uncomfortable because of the way the room was organised, but especially when it came to discussing the few words I wanted to take out of the introduction, which I felt added nothing to the book and rather gave a bad impression of the author.

One of the responsibles addressed Paul de Casteljau, calling on him to take into account my request with a very curt "you have heard it ...". I can still feel the embarrassment of the young university professor that I was at having embarrassed one of the builders of CAD. I also had the feeling, perhaps wrongly, that Paul de Casteljau was regarded as a researcher whose work, brilliant though it was, received little recognition even within his own company. Each company has its own internal rules, and the desire to keep confidential any work that might be of interest to competitors certainly played a part in the limited dissemination of Paul Faget de Casteljau's results.





The sentences in question have been changed, and all that remains on the first page of Volume 2 of the collection is a sentence that reflects the author's state of mind: "In this respect, the pioneer is now the only one who knows in depth all the difficulties he had to iron out." [1] In fact, Paul de Casteljau would afterwards often display his humour by referring to the "poles" as points of "Béziers" (the town, far away from the town of Casteljau).

There is no doubt that Paul Faget de Casteljau was a pioneer in the field of curves and surfaces for CAD, and we can understand a certain bitterness on his part at not having been recognised as such, at least not as much or as soon as he deserved.

This brief meeting was followed some time later by a lunch, the atmosphere of which remains in my mind as if it were yesterday. The word historic may be overused, but this meal, which has remained confidential, is in a way part of the history of CAD.

A rather special lunch...

It is the mid-1980s, in a small restaurant in Paris. Paul Faget de Casteljau's book was published as Volume 2 of the Mathématiques et CAO collection, and Pierre Bézier's as Volume 4.

[1] The English book Shape Mathematics and CAD (1986) translated the original sentence to *"We had to learn from our own experience and build up our own system on solid foundations."*

The table itself is small and rectangular. I'm sitting opposite Pierre Bézier, and Sami Ménascé, President of Hermes Editions, the company behind the lunch, is next to me, opposite Paul de Casteljau. The first moments were marked by palpable tension and I still have vivid memories of the first meeting between these two CAD pioneers. By their own admission, it is the first time they have the opportunity to talk to each other, barely had they said 'hello' to each other from afar at some conference or industrial exchange. Sami and I wish this to happen as pleasant as possible. At the start of the meal, I get the feeling that de Casteljau is looking for some well-deserved recognition and that Pierre Bézier is a bit on the defensive. Swiftly, Pierre Bézier will strike up the dialogue, acknowledging that certain transfers from Citroën to Renault had unarguably facilitated his work, without having the theoretical elements of all that at his disposal. His work was therefore inspired by that of de Casteljau in that he had had the opportunity to see how curves were constructed at Citroën, but it was also original in that he did not have access to the underlying theoretical elements. He readily admitted that de Casteljau's work predated his own. We soon realised that Paul de Casteljau was, in a way, "relieved". The rest of the meal was very pleasant, with fascinating exchanges. Their discussions also reflected their more mathematical education ("Normale sup") for one, and more mechanical education ("Arts et Métiers") for the other. The two men were also very different 'physically', both in height and in their ease to make conversation.

Don't misinterpret what I said. I felt it was important for de Casteljau's work to be recognised. However, that in no way neglects the quality of Pierre Bézier's work. I had a very pleasant relationship with Pierre Bézier. Sami Ménascé had submitted his book, subject of Volume 4 of the "Mathématiques et CAO" collection, for the Roberval prize for the "best technology book for higher education". Without telling me, he had also nominated the book I had written ("La CFAO"). So it was to my great surprise that I won the prize. Perhaps that is what gave me a certain legitimacy towards Pierre Bézier. Every time we met, especially for thesis committees, I appreciated his conversation and his culture. In fact, I was more than happy to let him take over the direction of the “Mathématiques et CAO” collection from Volume 5 onwards. His contacts and knowledge of researchers in this field were a great asset to the collection.
We never reported this lunch. Social networks certainly didn't exist, but even if they had, we certainly wouldn't have published anything. That moment belonged to both of them, we were mere witnesses, hardly actors, but it was a very special moment.

Fascinating exchanges

In 1990, Sami Ménascé and I organised a day on curves and surfaces to celebrate Pierre Bézier's eightieth birthday. It was a time when the interest of Nurbs (Non Uniform Rational B-Splines) over so-called 'Bézier' models was being passionately discussed in the small world of CAD. We have, of course, invited Paul de Casteljau.

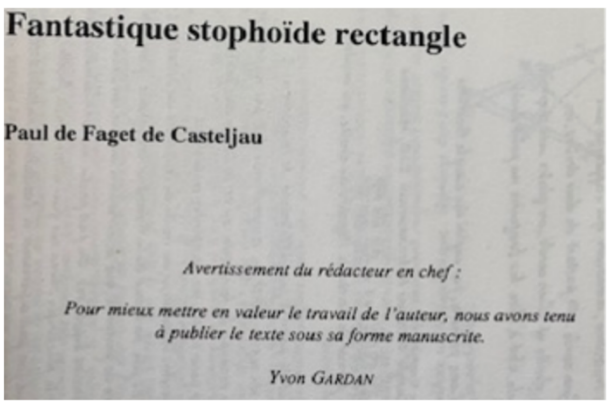
For the anecdote, he had told my assistant on the phone that it was 'also' his sixtieth birthday, in a mixture of humour and 'grievance' that was peculiar to him.

From my point of view, the day was magnificent, thanks to the quality of the speakers and the fact that the lecture hall was full and lively. There were several aspects of the day that stood out for me, but Paul de Casteljau's talk in particular. Very much at ease, he had presented his work by navigating with ease through the sea of indexes, without looking at the figure reproduced in the magazine, which many of us tried to follow.

It was a brilliant presentation, worthy of its author. It was a beautiful moment, not only in terms of content, but also in terms of form, which I would almost qualify as 'lyrical'.

I then saw Paul de Casteljau almost every year at successive editions of the MICAD congress-exhibition. He was kind enough to come and see me and we would take the time to chat for a few minutes. Talking to Paul de Casteljau meant above all trying to understand his latest work, and, to be honest, that wasn't always easy. On these occasions, I found him to be a very pleasant and convivial person. Long after his retirement, he continued to work on various points, in particular quaternions. He was prolific and presented some of his results at MICAD editions. His presentations were not always well attended, perhaps because we didn't know how to 'sell' them. However, his 'historical' presentation of the birth of his work was very interesting. You can always learn from the history of technology. He had come to the 25th anniversary of MICADO (founded in 1974) at a restaurant in the Eiffel Tower. It was an opportunity for his table-mates to see that his favourite subject was always revolving around mathematics, and that his mind was sharp and his passion alive.

From my point of view, Paul Faget de Casteljau was highly original, in the most interesting sense of the word, not only in the content of his work, but also in his behaviour. When I was in contact with him for publications, either articles or books, he only wanted to publish in French and refused to make use of mathematical formula editors. It's true that formula editors were a bit stammering at the time. This sometimes led us to publish hand-written articles in the *Revue de CFAO et d'informatique graphique*.



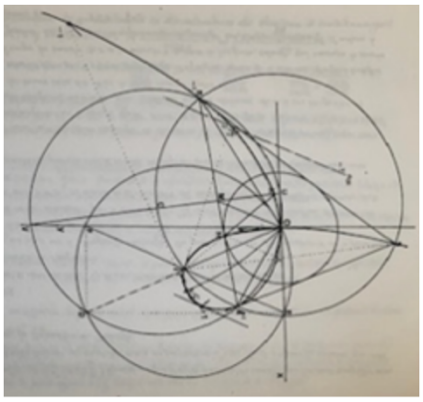
Fantastique stophoïde rectangle

Paul de Faget de Casteljau

Avertissement du rédacteur en chef :

Pour mieux mettre en valeur le travail de l'auteur, nous avons tenu à publier le texte sous sa forme manuscrite.

Yvon GARDAN



$$\frac{2XY - 2Xy_0 - 2Yx_0 + 2x_0y_0}{X^2 - Y^2 - 2Xx_0 + 2Yy_0 + x_0^2 - y_0^2} = \frac{2XY - X(y_1+y_2) - Y(x_1+x_2) + x_1y_2 + x_2y_1}{X^2 - Y^2 - X(x_1+x_2) + Y(y_1+y_2) + x_1x_2 - y_1y_2} = \frac{X(2y_0 - y_1 - y_2) + Y(2x_0 - x_1 - x_2) + x_1y_2 + x_2y_1 - 2x_0y_0}{X(2x_0 - x_1 - x_2) - Y(2y_0 - y_1 - y_2) + x_1x_2 - y_1y_2 - y_0^2}$$

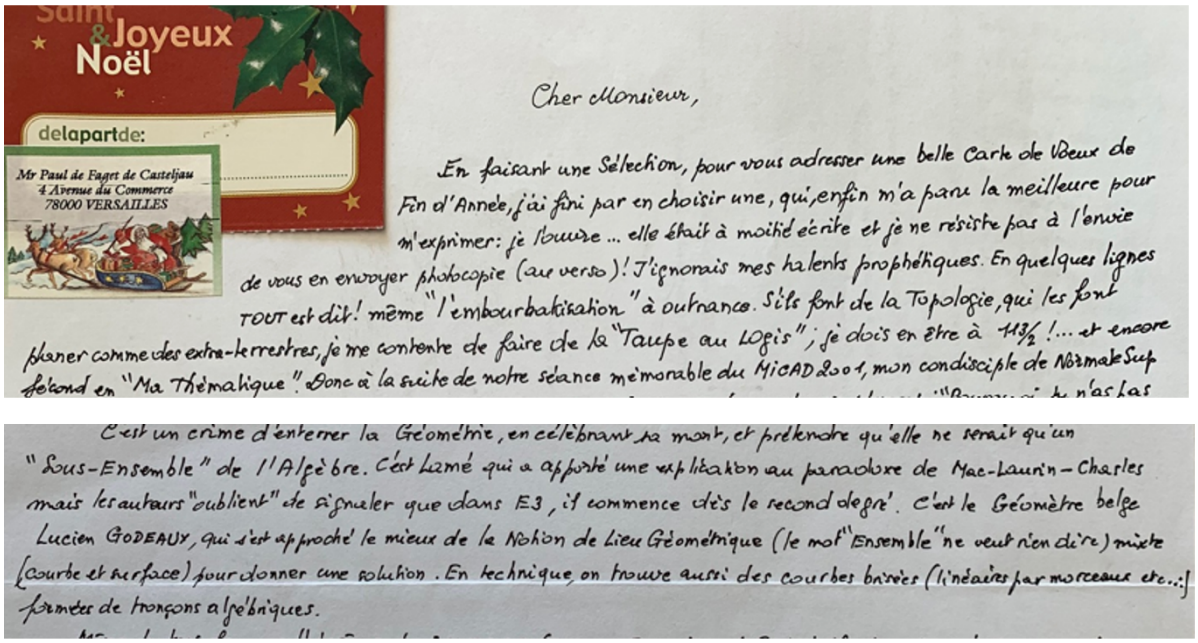
A good illustration of what this could lead to (Revue de CFAO et d'informatique graphique, volume 16- N°3/2001, Hermes editions).

Earlier this summer, I had an e-mail exchange with Professor Andreas Müller (Kempten University of Applied Sciences, Germany), who takes a very close interest in Paul de Casteljau's work. I shared with him my admiration for de Casteljau's handwriting (among other things). He replied that one of his colleagues, who didn't speak French, compared the design of these letters to that of Leonardo Da Vinci.

I had the pleasure of receiving several letters from the hand of Paul Faget de Casteljau. I have kept some of them in my archives. Long, written ('drawn'), practically without erasures, with mixtures of colours, they are a kind of artwork. I leave it to the reader to make up his or her own mind about the graphic quality of these letters with the extracts below. There is a certain amount of childish and mathematical humour in these letters ("I'm content to be the 'mole in the house'"[2], "I must there be at 113/2"[3], "I'm making fun of Gauss"[4], etc.), which enlivens the dense prose that sometimes reveals his desire for legitimate recognition.

He wrote, for example, about a presentation he wanted to give at MICAD: "the organisers have the absolute right to check that I haven't set up an 'enormous student hoax'[5] for them, just like the one at Citroën, when I proposed to 'Mathémastiquer' bodywork shapes, which rhymes with 'rosserie', i.e. to propose a process for the numerical definition of shapes". This kind of sentence could coexist with considerations on Ptolemy's theorem or the ARCHANGE.

To be read and reread with a certain amount of relish.



[2] Wordplay ‚topologie' (topology) vs. ‚taupe au logis' (mole at home), while 'taupe' refers to 1st year students in classes which prepare for the Grandes Ecoles.
[3] The preparatory classes count ½ for the first year, and 2/2 for the consecutive ones, so this refers to a time 56 years after having passed the first year preparation
[4] Wordplay ‚Je me gausse de Gauss' (I make fun of Gauss), gausse and Gauss are homologues
[5] Students at ENS (Normale Sup) are known for their student hoaxes

A few short extracts to help you appreciate the form and content...[6]

A word about the poetry of Casteljau's algorithm

I've had the opportunity to introduce several generations of students to the Casteljau algorithm (approximation of polynomials written in the Bernstein basis, which can be used to calculate a point or draw a curve or a so-called 'Bézier' surface).

This algorithm is interesting not only in the area of CAD, but also as an example of recurrence. From my point of view, it's a very elegant algorithm with a very meaningful geometric interpretation. In general, I used to start telling students that this algorithm was as beautiful as a poem by Rimbaud.

Many of them were probably not convinced, perhaps because all they had learned of Rimbaud was the inevitably tedious Dormeur du Val by heart, whereas you need to immerse yourself in the words and their silent music to appreciate their poetry. You also need to immerse yourself in the algorithm to appreciate its power and beauty.

In conclusion

Paul de Casteljau has, in my opinion, received in 2012 a recognition that, while belated, the committee of the SMA (Solid Modeling Association) unanimously awarded him its 2012

[6] "Cher Monsieur, En faisant une Sélection, pour vous adresser une belle Carte de Vœux de Fin d'Année, j'ai fini par en choisir une, qui, enfin m'a paru la meilleure pour m'exprimer: je l'ouvre … elle était à moitié écrite et je ne resiste pas à l'envie de vous en envoyer photocopie (au verso) ! J'ignorais mes talents prophétiques. En quelques lignes TOUT dit ! même "l'embourbakisation" à outrance. S'ils font de la Topologie, qui les font planer comme des extra-terrestres, je me contente de faire de la „Taupe au Logis"; je dois en être à 113/2 ! … et encore fécond en "Ma Thématique". Donc à la suite de notre séance memorable du MICAD 2001, mon condisciple de Normale Sup […]" (= Dear Sir, by making a selection to send you a beautiful End of Year greeting card, I ended up in choosing one that finally appeared the best to explain myself: I open it … it is half written and I do not resist in snding you a copy (see over)! I ignored my prophetic talents. In some lined ALL said! Even the 'in-Bourbaki-sation'extravagently. When the do topology, which makes them float like extra-terrestrians, I confine myself to do the 'mole in the house'; I must be 113/2 on this! And still fertile in 'Ma Thématique' (=my topic / maths). So at the end of our memorable session of MICAD 2001, my classmate from Normale Sup […]) "C'est un crime d'enterrer la Géométrie, en célébrant sa mort, et prétendre qu'elle ne serait qu'un "Sous-Ensemble" de l'Algèbre. C'est Lamé qui a apporté une explication au paradoxe de Mac-Laurin-Chasles mais les auteurs "oublient" de signaler que dans E3, il commence dès le second degré. C'est le Géomètre belge Lucien GODEAUX, qui s'est approché le mieux de la Notion de Lieux Géométrique (le mot "Ensemble" ne veut rien dire) mixte (courbe et surface) pour donner une solution. En technique, on trouve aussi des courbes brisées (linéaires par morceaux etc…) formées de tronçons algébraiques. (= It is a crime to bury Geometry, by celebrating its death, and to pretend that it is but a 'sub-set' of Algebra. It was Lamé who brought an explanation of the MacLaurin-Chasles paradox, but the authors 'forgot' to remind that in E3, it begins with the second degree. It is the Belgian geometer Lucien Godeaux who approached best to the notion of geometric locus (the word 'set' does not say anything), mixed (curves and surfaces) to offer a solution. Technically, one also finds broken curves (piecewise linear etc…) formed of algebraic segments.)

Bézier Prize), is richly deserved. Others than myself will be better placed to present his scientific contribution.

As far as I'm concerned, I felt, if I may say so, a certain tenderness for this great man, because of his talent, with a particular sense of humour, but characteristic of a great mathematician. This modest article is therefore also a testimony to the indisputable human qualities of this CAD pioneer.