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Niche vs. central firms: Pattern of technology choice and cost-price dynamics in a differentiated oligopoly*

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Abstract

We investigate whether and how positions in the characteristics space influences technological adoption and how price levels are affected; furthermore we assess the effects of policy interventions. In an industry where a central firm competes with two peripheral/niche ones, two technologies are available: one with low marginal and high fixed costs and one with opposite pattern. The central firm is in direct competition with the all the rivals. We show that this firm has higher incentives to adopt the technology efficient at large production scale; consequently if fixed cost decreases, the diffusion of this technology in the industry starts from the center and then spreads over to the niche firms. Changes in fixed and marginal costs affect long-run prices in non-obvious way. On the normative side, subsidies affect the technology pattern and deliver relevant effects: lump-sum subsidies increase consumer surplus, but can reduce profits. A price-cap that forestalls a technological change improves welfare. Our analysis is well-suited to analyze the digitalization process that has taken place in the last years.

Keywords: Technology adoption, technological change, digitalization, price competition, product differentiation, subsidies, price caps.

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1 Introduction

An important instance of choice between alternative technologies is the switch to digitalized and robotized processes implying low marginal costs and high fixed costs. Firms face this choice in many sectors, especially in the wake of new and cheaper ways of transforming their business by making use of increased digitalization and IT. Think of examples as "Fintech" in finance (Gomber et al., 2017), information technology applications in retailing (Hagberg et al., 2016), in telecommunications, in film and music production and distribution (Waldfogel, 2017), in the news industry, the data collection industry—and even in old fashioned sectors, like agriculture, the digitalization of processes makes its dramatic inroads (Walter et al., 2017).

In this technology adoption decision, however it needs not be that the choice involves a "new" versus an "old" technology: it may simply be between a more labor intensive and one which is more capital intensive.\(^1\) With this interpretation, the more capital intensive technology is the "digital" one, whereas the more labor intensive is the "analog" one. Of course in the real world, this is often debated as a race to the new frontier of the technology, but as it is well known, the adoption may come long after the discovery of a new technology. The digitalization of operations in retailing (like the use of bar codes) is an often quoted example. Asymmetric costs resulting from the adoption of different technologies are widely discussed in the theory of oligopoly and have been empirically documented as early as Griliches (1957); Dunne (1994); Doms et al. (1995). With respect to the existing analyses, however, the present paper adopts a distinctive approach: it uses an address model where price competition is localized, and it introduces the definition of "niche" versus "central" firms.

A branch of the theory analyzes why firms adopt different technologies among those available, without considering the firms as the agents involved in the R&D leading to the discovering of new technologies. To be more precise, technology asymmetries of two main types have been analyzed: the first framework admits a ranking of technologies, according to the associated total costs where a dominant technology achieves lower costs at all output levels (e.g. Bester and Petrakis, 1996, and more recently Amir et al., 2011); the second is with technologies that allow one technology to dominate over the other at a large production scale, namely for output levels higher than a threshold because of its lower variable cost and higher fixed costs.\(^2\)

\(^1\)Another branch of the literature analyzes the different adoption times of technologies; in a sense one can argue that if a new technology is adopted at different times it becomes an "existing" technology for the late adopters. As Mahathi et al. (2016) remark: "Adoption of productivity-enhancing ‘computer-aided design technology’ and ‘computer numerically controlled machine tools’ by firms in metalworking industries and machining-intensive industries, respectively, had also been delayed and sequential", see also Åstebro (2002); Milliou and Petrakis (2011).

\(^2\)Hence although it is not necessary to think of the low variable cost technology as of an innovation in the "canonical" sense, like, e.g. in Cohen and Klepper (1996), it is legitimate to do so. Of course, as our examples suggest the distinction is somewhat blurred in the applications as we shall further discuss in due course. Nevertheless, increasing the production efficiency at larger scale is desirable as it will lead to lower prices for consumers.
second type of asymmetry is the object of our analysis, similarly to various related works (Mills and Smith, 1996; Wauthy and Zenou, 2000; Elberfeld, 2003; Götz, 2005; Hansen and Nielsen, 2010). In our approach, a technology improves when the associated production cost decreases, leading the firms to possibly review their current technology choice. For instance the cost in the digital technology can decrease due to an improved "state of the art" coming from a wide range of scientific and technological advances, a better exploitation of current knowledge, better utilization of the associated information systems, better quality of the associated capital goods—or a combination of all these elements. Then, as we show, the nature of price competition and the position vis-à-vis the rivals enter into the determinants of a firm’s decision whether or not to change technology. The technology choice set for each firm in our model is composed of two elements: a technology implying lower total costs for small production scale than the alternative, and one implying lower total costs than the alternative at large production scales. Both technologies feature economies of scale, with the first having smaller average cost than the second for low-scale production, and the converse. We will refer to the first technology as of the "Small Fixed Cost-Large Marginal Cost" (SF-lm) technology and to the second one as of the "Large Fixed Cost Small Marginal Cost" (LF-sm) technology, which can represent the "digital" technology, or more generally the "new" one.

As hinted above, we use an address model à la Hotelling where price competition is localized. By "localized" it is meant that strategic interaction is deeply affected by the positions of the firms' products in the space of characteristics, or product space (in a Hotelling sense).\(^3\) In the demand system, the prices of all competitors enter the demand function of the central firm, while the prices only of the central firm enter the demand of the niche producers. For an instance of such a pattern, one may think of a market for pesticides, where one narrow spectrum product is mostly but not only effective against aphids, another mostly but not only against mites and one is a broad spectrum pesticide in competition with both. In the market for fabrics where one producer is specialized in waterproof cotton another in organic cotton and a third produces standard industrially processed cotton. Also, sports items (as for ski, or tennis) as well as electronic appliances (like cameras or PC’s) are produced in versions targeted to beginners ("dummies" only) or to professionals ("geeks" only) as well as in standard versions.\(^4\)

This feature entails a relationship between technology choices and the position in the

\(^3\)The characteristics space in the literature has long been known also as product space, referring to a product as embedding specific characteristics, e.g. Perloff and Salop (1985); Economides (1989); Lancaster (1990) among others.

\(^4\)A neat example comes from the drill market: there, Black&Decker is mostly specialized in tools for home use, whereas on the other range of the spectrum, Hilti produces tools for use in the construction industry. Bosch, a third company, produces a "blue drill" line, targeted to professional users and one "green drill" line for home users.
product characteristics space.\textsuperscript{5} This also makes for a rich analysis of the long-run implications of technology choices. The Hotelling framework also fits particularly well industries such as broadcasting and retailing, where firms –or platforms– typically sell horizontally differentiated products –or bundles (see, e.g. Gabszewicz et al., 2004; Anderson and Coate, 2005; Liu et al., 2006). In the last decade, these industries have witnessed a fast switch to digital technologies which required huge investments in infrastructure (Galperin, 2004; Peitz and Waldfogel, 2012).

We set our main focus on the following issues: (i) how and if the switch to the low variable cost technology depends upon the position of a firm in the space of characteristics; (ii) what is the impact of exogenous shocks in technology (like a lower fixed cost associated to the digital technology) and in demand parameters on the equilibrium patterns and prices; (iii) how is the transition to the LF-sm technology affected by some widely used policies.

Technologies are chosen at the first stage of the game and at the second stage price competition is resolved. The LF-sm technology implies a higher fixed cost than the SF-lm, and a lower marginal cost. We show that the central firm has a higher incentive to switch to the technology with lowest variable costs – "larger increasing returns to scale"; this is due to its size but also to the centrality of its location, which magnifies the business stealing effect of a decrease in its marginal cost. Accordingly, there is no equilibrium in which the central firm adopts the SF-lm except for the equilibrium where all firms do –obviously when the fixed cost differential between the LF-sm and the SF-lm is too high with respect to the marginal cost differential. A niche firm only adopts the LF-sm technology if the central firm also does so.\textsuperscript{6} In the range of possible asymmetric equilibria there is a parameter region where the two peripheral firms choose the same technology and a region where they choose different technologies.

Changes in the supply conditions affect the equilibrium configurations and hence prices and market shares. An exogenous decrease in the fixed costs differential for a given marginal cost differential (cheaper digitalization), associated with the developments hinted above, may lead to changes in the technology choices, with consequences on the output costs in the form of lower average costs for the firm that switches to the digital technology. Consumers also enjoy lower prices; this never happens in a model where technologies are fixed. For example, a decrease in the fixed cost associated to the LF-sm technology may lead to additional firm(s) adopting it, with a lower price for all firms due to prices being strategic complements. For similar reasons, an increase in the marginal cost differential may lead to a change favoring the LF-sm one and a reduction in prices.

\textsuperscript{5}The 3 × 3 Jacobian matrix of price derivatives in the direct demand system for the differentiated products displays two zeros: the niche firms compete against each other only indirectly. This cannot happen in, say, a Cournot model where all firms simultaneously compete (possibly to different extents) with all the others.

\textsuperscript{6}This confirms the intuition that firms with higher variable costs than those of the competitors are often associated with niche-products, although we do not venture here in giving a cause-effect ordering to the association; this would imply a full analysis of the location choice game.
Consumer loyalty, here represented by the unit transportation cost parameter, increases all prices and profits, since it decreases the intensity of price competition. It also hampers the transition to the LF-sm technology. Conversely, an increase in the competitiveness of the market, represented by a decrease in consumer loyalty, favors the adoption of the digital technology for any cost differential. We then study the welfare maximizing configurations, computed by letting firms choose prices in a non-cooperative way and allowing the Authority to assign technologies to each individual firm. Here it is worth mentioning that consumer surplus in address models is affected by prices as well as by the distance between a consumer preferred specification of the product and the one purchased (a utility loss due to "transportation costs" in the geographic interpretation). Hence a low price by a distant firm may lead to purchase from an inefficient source in terms of these "transportation costs". We show that with respect to the socially optimal configurations the market displays a tendency to over-adopt the SF-lm technology; this is because in oligopoly marginal costs are "passed through prices" on consumers while fixed costs are not.

It is then natural to analyze the effects of some policy actions by the Government. We limit the analysis here to a subsidy to the LF-sm technology and to price regulation because, in the context of the transition from analog to digital technologies, which our setup fits particularly well, those instruments have seen a widespread usage. A subsidy on the fixed input which reduces the fixed cost differential is a relevant policy option, and one often encountered in the real world. We show that the lowest lump-sum subsidy needed to foster adoption of the LF-sm technology decreases with the consumer mass and with the consumer loyalty parameter. The effects of a subsidy to the use of fixed capital may also be paradoxical: since the equilibrium where all firms select the LF-sm technology has the character of a Prisoner Dilemma, a grant to firms adopting a larger quantity of fixed capital may result in lowering the firms profits if it leads to that configuration. We also analyze a production subsidy to the firms running the LF-sm technology to foster the shift to that technology. In this case the subsidy is increasing in consumer loyalty. A price cap, limiting the possibility to adjust the price upward may favor the switch from the SF-lm—the analog—to the LF-sm—digital—technology by one firm, when this would not be done in the absence of a cap. This is an additional limitation to the profit seeking behavior by regulated firms, beyond the simple impediment to raise price for given technologies. A price cap can therefore be used to realign the technology configuration with the socially optimal one and can be welfare improving. When it does not change the technology configuration, a price cap can only reduce total welfare or be ineffective, contrary to what happens in other models where firms have market power. This is due to transportation costs and to an inefficient redistribution of market shares in favor of firms adopting the SF-lm technology.

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7We refer to the WP version for a treatment of unit taxes and subsidies on the variable input in an earlier version of this paper.
technology.\textsuperscript{8}

Related literature can be found in different traditions. Asymmetric choices and resulting patterns and costs have been analyzed under an Industrial Organization approach, where strategic choices are determined in a Game Theoretic context, and asymmetric choices across firms arise as a natural by-product of oligopoly Nash equilibria.\textsuperscript{9} Policy implications of asymmetric costs have been first highlighted in Salant and Shaffer (1999), and more recently in Krysiak (2008). In an oligopoly context, cost asymmetries have been explained also as the result of firms strategic manipulation of their cost structures (Van Long and Soubeyran, 2001) or as firms' attempts to "help" less efficient rivals in order to achieve higher profits (Ishida et al., 2011). Bustos (2011) adopts a definition of the technology set that is similar to ours, in a model of international trade with monopolistic competition (see also Yin, 1998) where however strategic interaction is absent. Also, less related, the choice of flexible versus rigid technologies in terms of adaptability to demand has been analyzed in Goyal and Netessine (2007).

The plan of the paper is as follows. In Section 2, we lay down the model. In Section 3 we analyze the second stage Nash equilibria in prices, and the first stage in technology configurations as a function of the cost differentials; then we analyze the effects of changes in supply and demand parameters on the equilibrium outcomes. In Section 4 we determine the welfare maximizing technology configurations as functions of the parameters. In Sections 5 and 6 we analyze the effects of some frequently used policies like subsidies and price regulations. Finally, in the Concluding Section 7 we summarize our findings and add some comments.

## 2 Model

We consider a Hotelling "linear city" with uniform distribution of a mass $M$ of consumers over the interval $[0, 1]$ which represents the space of characteristics. We assume linear transportation costs $t(x - x_i)$ where $x_i$ is the location of firm $i$ in the unit interval and $x$ the location of a consumer. Consumers are uniformly distributed and share the willingness to pay for one unit of the good, which is $V > 0$. There are three firms with exogenous locations $(x_1, x_2, x_3) = (0, 1/2, 1).$\textsuperscript{10} The extreme location firms take the role of "peripheral" or "niche". The three firms play a two-stage game: at the first stage they choose a technology and at stage 2 they choose prices.

Two cost functions are available to the firms. Both cost functions display economies of

\textsuperscript{8}We argue accordingly that this result is not due to the total demand being inelastic.

\textsuperscript{9}On similar grounds, Février and Linnemer (2004) study the effects of \textit{exogenous} asymmetric shifts in marginal costs in Cournot oligopolies.

\textsuperscript{10}Endogenous locations in a 3-firms Hotelling model are not obvious and are studied in De Palma et al. (1987).
scale, yet to a different extent. The first cost function (for the SF-lm technology) is \( C_s(q_i) = c_s q_i + f_s \) and the second (for the LF-sm technology) is \( C_l(q_i) = c_l q_i + f_l \), with \( c_s > c_l \geq 0 \) and \( f_l > f_s \geq 0 \). Let \( \Delta c \equiv c_l - c_s > 0 \) and \( \Delta f \equiv f_l - f_s > 0 \). The two cost functions cross at \( \tilde{q} = \Delta f / \Delta c \), so that \( C_s(\cdot) \) is best suited for a small production scale and \( C_l(\cdot) \) is best suited for large production scale.\(^{11}\)

Let \( Q_i(p_1, p_2, p_3) \) be the demand to firm \( i \) and define \( q_i = Q_i / M \), the normalized demand. The indifferent consumers are \( x_{12} = \frac{1}{4} + \frac{p_2 - p_1}{2t} \) (indifferent between firms 1 and 2) and \( x_{23} = \frac{3}{4} - \frac{p_2 - p_3}{2t} \) (indifferent between firms 2 and 3). Letting \( x_i \) denote the consumer indifferent between buying from \( i \) and not buying (e.g. \( x_1 = (V - p_1)/2 \)), it can be shown that the condition

\[
t < 4V
\]

guarantees that all market shares overlap. Thus, we assume henceforth (1) to hold. The normalized demand system is given by the following equations:

\[
q_1(p_1, p_2) = \frac{1}{4} + \frac{(p_2 - p_1)}{2t}
\]

\[
q_2(p_1, p_2, p_3) = \frac{1}{2} + \frac{(p_1 + p_3 - 2p_2)}{2t}
\]

\[
q_3(p_2, p_3) = \frac{1}{4} + \frac{(p_2 - p_3)}{2t}
\]

It is apparent that the demand to the central firm at symmetric prices, has a higher intercept and a higher own price elasticity than that of those for the peripheral firms. It is important for the following to consider that the Jacobian matrix of price coefficients displays two zeros at the positions referring to \( \partial q_1(\cdot)/\partial p_3 \) and \( \partial q_3(\cdot)/\partial p_1 \), since these two firms do not directly compete but only through the price reaction of firm 2.

If firms have different marginal costs, following the adoption of different technologies, their best replies write as

\[
p_i = \frac{(t + 2c_i)}{4} + \frac{p_2}{2}, \text{ for } i \in \{1, 3\},
\]

\[
p_2 = \frac{(t + 2c_2)}{4} + \frac{(p_1 + p_3)}{4}.
\]

The equilibrium price triplet, with \( c_i \in \{c_s, c_l\} \) is:

\[
(p_1^e, p_2^e, p_3^e) = \left( \frac{t}{2} + \frac{7c_1 + 4c_2 + c_3}{12}, \frac{t}{2} + \frac{c_1 + 4c_2 + c_3}{6}, \frac{t}{2} + \frac{c_1 + 4c_2 + 7c_3}{12} \right).
\]

\(^{11}\)It is immediate to observe that the average cost of the LF-sm technology is smaller than that of the SF-lm one when \( q > \tilde{q} \), and the opposite, which entails that the scale economies are larger for the LF-sm one for large production scales.
3 Equilibrium analysis

Firms play a two-stage game with observable actions, at the first stage they simultaneously choose which technology to adopt, at the second they simultaneously set their prices. Let the triplet 123 represent the technology choices of firms 1, 2, 3 respectively, so that, for instance, sls means that firms 1 and 3 select the SF-lm technology and firm 2 selects the LF-sm one. There are nine possible technology configurations, but, clearly, configurations lls and sll are equivalent to each other up to a permutation in the labels of the peripheral firms, and so are ssl and lss. Hence we have in total only six possible non-equivalent configurations that are to be analyzed, namely lll, lls, lsl, lss, sls and sss.

3.1 Price stage

The three firms set their prices simultaneously; hereafter we report the equilibrium outcomes given the technology configurations. The prices for the configurations ssl and lls are omitted for brevity as they are the mirror images of lss and sll. Superscripts refer to the technology configuration chosen at stage 1.

<table>
<thead>
<tr>
<th>Tech. conf.</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sss</td>
<td>( c_s + t/2 )</td>
<td>( c_s + t/2 )</td>
<td>( c_s + t/2 )</td>
</tr>
<tr>
<td>sls</td>
<td>( (7c_s + 5c_l + 6t)/12 )</td>
<td>( (c_s + 5c_l + 3t)/6 )</td>
<td>( (c_s + 11c_l + 6t)/12 )</td>
</tr>
<tr>
<td>lss</td>
<td>( (4c_s + 2c_l + 3t)/6 )</td>
<td>( (4c_s + 2c_l + 3t)/6 )</td>
<td>( (4c_s + 2c_l + 3t)/6 )</td>
</tr>
<tr>
<td>lls</td>
<td>( (5c_s + 7c_l + 6t)/12 )</td>
<td>( (5c_s + c_l + 3t)/6 )</td>
<td>( (11c_s + c_l + 6t)/12 )</td>
</tr>
<tr>
<td>lsl</td>
<td>( (2c_s + 4c_l + 3t)/6 )</td>
<td>( (2c_s + 4c_l + 3t)/6 )</td>
<td>( (2c_s + 4c_l + 3t)/6 )</td>
</tr>
<tr>
<td>lll</td>
<td>( c_l + t/2 )</td>
<td>( c_l + t/2 )</td>
<td>( c_l + t/2 )</td>
</tr>
</tbody>
</table>

3.2 Technology stage

Firms simultaneously and costlessly adopt their technologies at stage 1 of the Game. In the following, we shall assume

**Assumption 1 (A.1).** \( \Delta c < (6/5)t \) and \( \Delta f < (1/8)Mt \).

A.1 guarantees that under all the possible technological candidate-equilibrium configurations the quantities and profits of the firms are non-negative. In Appendix B we prove the following.

**Proposition 1.** Under A.1, let \( F^I = \frac{5M\Delta c}{24t} \left( t - \frac{5\Delta c}{12} \right) \), \( F^{II} = \frac{5M\Delta c}{24t} \left( t - \frac{\Delta c}{4} \right) \) and \( F^{III} = \frac{M\Delta c}{36} \left( t + \frac{\Delta c}{3} \right) \), where the ranking \( F^I < F^{II} < F^{III} \) holds; neither lss (and ssl) nor lls can be equilibrium configurations, and the unique equilibrium is

\[ \text{We provide a fuller characterization in the Appendix B.} \]
Figure 1: Equilibrium technology choice.

(i) lll, for $0 \leq \Delta f < F_I$, all firms adopt the LF-sm technology;

(ii) sll (or lls) for $F_I < \Delta f < F_{II}$, all but one peripheral firms adopt the LF-sm technology;

(iii) sls for $F_{II} < \Delta f < F_{III}$, only the central firm adopts the LF-sm technology;

(iv) sss for $F_{III} < \Delta f$, no firm adopts the LF-sm technology.

Figure 1 graphically represents the regions in Proposition 1, in particular Panel 1a plots these regions as function of the marginal (and average) cost difference $\Delta c$, for given $t$ and Panel 1b as a function of $t$ for given $\Delta c$.\(^\text{13}\)

It is useful to observe that under A.1 neither lss (and ssl) nor lsl can be equilibrium configurations. In fact, in the first case, either firm 1 wants to deviate (when $\Delta f$ is "large" relative to $\Delta c$) or firm 2 wants to deviate (when $\Delta f$ is "small" relative to $\Delta c$). In the second case a similar reasoning applies, but in this case, both peripheral firms want to deviate to the LF-sm technology.

In order to understand Proposition 1, start with a high level of $\Delta f$ relative to $\Delta c$, case (iv), then all firms select the SF-lm technology, which allows to save on variable costs. For lower values of $\Delta f$, however, the LF-sm technology becomes more attractive and, eventually, only the central firm switches to this technology (case (iii)). If $\Delta f$ decreases further either firm 1 or 3 joins in choosing the LF-sm technology. It is interesting to observe that in spite

\(^{13}\)At the boundaries, where $\Delta f \in \{F_I, F_{II}, F_{III}\}$, two equilibria coexist.
of their symmetry with respect to the central location, the niche firms make different choices in this region. When \( \Delta f \) is low enough all firms adopt the **LF-sm** technology.

Furthermore, if \( \Delta f \) increases or if \( \Delta c \) decreases, a firm that switches to the **SF-lm** technology increases its marginal cost and therefore its optimal price. Because of the strategic interaction the prices of all the rivals increase too. The difference between the address approach and non-localized competition models is highlighted observing that not only size, but also location affect the technology choice.

Indeed, one could suspect that the pattern of technological adoption reported in Proposition 1 is an artifact of the model, due to the assumption that firm 2 has "naturally" a larger market share than do firms 1 and 3, and consequently higher profits, which, ultimately, favor the adoption of the **LF-sm** technology by the central firm—consistent with findings in Bengtsson et al. (2007). Certainly, size matters since the unit cost after the adoption of the technology is lower for the firm with the largest demand. We argue however that size is not the only driver in the decision concerning technological adoption, but location is a crucial determinant too. Two streams of reasoning support our claim, which are summarized in the ensuing Remarks.

**Remark 1.** The central firm benefits from a larger business stealing effect than the niche ones if it is the first to adopt the **LF-sm** technology.

**Proof.** Consider the general prices as defined in (3), and let \( q_i^e(c_1, c_2, c_3) \equiv q_i(p_i^e, p_2^e) \) be the quantity of firm \( i = 1, 3 \) and \( q_2^s(c_1, c_2, c_3) \equiv q_2(p_1^s, p_2^s, p_3^s) \) that of firm 2 at these prices. Start at the symmetric \( sss \) configuration and let \( \Delta q_i^e \) denote the business volume stolen by firm \( i \) if it unilaterally switches to the LSE technology; as an example, if firm 1 is switching, then

\[
\Delta q_1^e = q_1(p_1^{lss}, p_2^{lss}) - q_1(p_1^{sss}, p_2^{sss}).
\]

Because \( q_i^e \) is linear in own marginal cost, we consider the linear approximation \( \Delta q_i^e = \Delta c_i \frac{\partial q_i^e}{\partial c_i} \). For firms 1 and 3 we have \( \frac{\partial q_1^e}{\partial c_1} = \frac{\partial q_3^e}{\partial c_3} = -\frac{5M}{24t} \) and, for firm 2, \( \frac{\partial q_2^e}{\partial c_2} = \frac{M}{2t} \). It is instructive to observe here that the derivatives are independent of the size of the demand of the firm. Let \( p_2^n = p_2^{sls} = p_1^n = p_1^{sss} = p_3^n = p_3^{sls} \) stand for the equilibrium price of firm \( i \) after its unilateral drop in marginal cost from \( c_s \) to \( c_l \) due to the adoption of the LSE technology. Further, denote the value of the stolen business as \( \Delta V_{1^n} = p_1^n \left. \frac{\partial q_1^e}{\partial c_1} \right| \Delta c_i \), then

\[
\Delta V_{2^n} = \left( \frac{2A_c+3A_cM}{6} \right) \left( \frac{1}{3t} \right) \Delta c = \frac{2A_c+3A_cM}{18t} \Delta c 
\]

and \( \Delta V_{1^n} = \Delta V_{3^n} = \left( \frac{5A_c+6A_cM}{12} \right) \left( \frac{5}{24t} \right) \Delta c = \frac{25c+30cM}{288t} \Delta c \). It is immediate to ascertain that the value of the business stealing effect is larger for the centrally located firm as \( \Delta V_{2^n} > \Delta V_{1^n} \).

It can also be shown that the comparison of the business stealing effects depends only on location and not on size. This can be done by letting the three firms be located at \( (x_1, x_2, x_3) = (a, 1/2, 1-a) \); under the resulting demand system, if \( a \geq 1/6 \) at symmetric prices firm 2 has the lowest market share. Under this change of location, however, the business stealing effect

is still larger for firm 2 than for firm 1 for any value of \( a < 1/3 \).\(^{14}\) We complete our argument with the following.

**Remark 2.** Central location favors the adoption of the LF-sm technology even with more than three firms.

This Remark expands on the ideas expressed in Remark 1. Its proof follows from extending the model to include 5 firms, with location vector \((0, 1/4, 1/2, 3/4, 1)\). In any equilibrium with symmetric technology adoption by all firms, the firms 2,3,4 have an identical market share. However, the equilibria where only one firm adopts the LF-sm technology display the central firm as the unique firm (except for a region of the parameters where there are multiple equilibria, with either one of the three central firms adopting the LF-sm technology).\(^{15}\) This supports our claim that the central location fosters the adoption of the LF-sm technology.

We conclude this Section by observing a further feature of the equilibrium described in Proposition 1, which will turn out to be useful in the sequel.

**Remark 3.** The equilibrium III has the character of a prisoner dilemma (all firms prefer sss to lll). Also, in region \( F^{II} < \Delta f < F^{III} \) the gain to firm 2 from adopting the LF-sm technology is smaller than the total loss inflicted to 1 and 3.

In the present framework the prisoner dilemma property holds—even if mark-ups are independent of marginal costs. A prisoner dilemma of the same kind arises under Cournot competition, where, with two technologies, the equilibrium with both firms choosing the LF-sm technology can display the prisoner dilemma character, though only under some specific parameter configurations.\(^{16}\)

### 3.3 Comparative Statics

#### 3.3.1 Changes in Technology

First we note that a change in the technology configuration originated by an increasingly cheaper digital technology, here represented by a decrease in the parameter \( \Delta f \), leads to a

\(^{14}\)Computing the demand system and the price equilibria with locations \( a \) and \( 1 - a \) for the niche firms and \( 1/2 \) for the central one gives \( q_i^* = q_i + a/2 \) for \( i = 1, 3 \) and \( q_2^* = q_2 - a \). The equilibrium prices are \( p_i = p_i^c + \frac{a}{2} \), for \( i \in \{1, 3\} \) and \( p_2 = p_2^c - \frac{a}{4} \) where \( p_i^c \) is as defined in (3) for \( i = 1, 2, 3 \).

\(^{15}\)The computations are available upon request. A short summary is provided in Appendix E.

\(^{16}\)To see this point consider a duopoly with inverse market demand equal to \( P = 1 - Q \), with \( Q = q_1 + q_2 \). In our notation, one has a \( 2 \times 2 \) payoff matrix at the technology choice stage that can be constructed form the following values for profits: \( \pi_1^{ss} = (1 - c_s)^2 / 9 - f_s \), \( \pi_1^{sl} = (1 - 2c_s + c_l)^2 / 9 - f_s \), for the firm adopting the SSE technology and \( \pi_1^{ls} = (1 - c_l)^2 / 9 - f_l \) and \( \pi_1^{ls} = (1 - 2c_l + c_s)^2 / 9 - f_l \) for that adopting the LSE one. Simple computations show that \( ll \) is the unique Nash equilibrium for \( \Delta f < \frac{4\Delta c}{3} (1 - c_s) \equiv F_o \), while \( \pi_1^{ss} > \pi_1^{ls} \) is true for \( \Delta f > \frac{1}{3} [4\Delta c + c_s (1 - c_s) - c_l (1 - c_l)] \equiv F_d \). The interval \([F_d, F_o]\) is not empty for \( \Delta c < \frac{c_s (1 - c_s) - c_l (1 - c_l)}{4(1 - c_l)^2}\). The same outcome obtains in a model with three firms, at the cost of dealing with slightly more cumbersome conditions.
decrease in average costs of the firm that embraces the LF-sm technology. In fact it can be shown that the equilibrium quantity \(q_2\) of the central firm involves a lower average cost in sls than in sss. The average cost of a peripheral firm—say 3— is lower under configuration sll than under sls. Also, the average cost of firm 1 is lower under lll than under sll. In this sense the switch to the LF-sm technology is associated with a more efficient utilization of input for the switching firm. This is summarized in the following result.

**Proposition 2.** As \(\Delta f\) decreases from a level such that \(\Delta f > F^{III}\) continuously, then at each change in configuration there corresponds a lowering of the average production cost of the firm that switches to the LF-sm technology.

*Proof.* See Appendix C.

Furthermore, when a firm switches from the SF-lm to the LF-sm technology, for given technologies of the rivals, it will lower its own price and trigger a downward shift in the best reply of the rivals.

As an example of the price effect of changes in fixed costs suppose that the cost parameters initially lead to the configuration sss. In this situation the price is given by \(p^{sss}\) for all firms, namely \(c_s + \frac{t}{2}\). Suppose now that a decrease in \(\Delta f\) is large enough to lead to the configuration sls (because \(\Delta f\) decreases from \(F^{III} < \Delta f < F^{II} < \Delta f < F^{III}\)). Then, the highest price under sls is the common price of the niche firms as given in (4), which is lower than the common price prevailing before the change of technology by firm 2. More generally, holding \(\Delta c\) constant, the price of any firm is a function \(p_i(\Delta f; \Delta c)\) of \(\Delta f\), with the graph of a step function displaying positive upward "jumps" at the critical points \(F^I, F^{II}, F^{III}\). The relationship between firm \(i\)'s price level and the difference in marginal costs between the SF-lm and LF-sm technology for a given \(\Delta f\), \(p_i(c_s; c_l, \Delta f)\), is a saw-like function with vertical downward drops at critical points where firm \(i\) or a rival of it change technology and adopt the LF-sm one. Figure 2 depicts the behavior of \(p_2\) as a function of \(\Delta f\), for a constant \(\Delta c\) (panel 2a) and as a function of \(c_s\), for given \(c_l\) and \(\Delta f\) (panel 2b).\(^{17}\) One can state the following results.

**Proposition 3.** (a) As \(\Delta f\) decreases from a level such that \(\Delta f > F^{III}\) continuously, then at each change in configuration there corresponds a lowering of the average price paid by the consumers. (b) for \(i = 1, 2, 3\) the relations \(p_i(c_s; c_l, \Delta f)\) linking the marginal cost, \(c_s\), in the SF-lm technology and the equilibrium price of firm \(i\), for a given value of \(c_l\) and \(\Delta f\), are discontinuous and non-monotonic; furthermore, at the discontinuities, an increase in \(\Delta c\) leads to a generalized drop in the prices of all firms.

\(^{17}\) Panel 2b, indirectly reports the relationship between the marginal cost difference \(\Delta c\) and the price level of firm \(i\) by depicting the behavior of \(p_2\) as \(c_s\) increases for given \(c_l\), which clearly implies an increase in \(\Delta c\). In that panel, for \(\Delta c \in [0, \hat{c}_2]\) the technological configuration is sss, for \(\Delta c \in [\hat{c}_2, \check{c}_2]\) it is sls, for \(\Delta c \in [\check{c}_2, C_2]\) it is sll/lls and finally, when \(\Delta c > C_2\) it is lll.
The proposition underlines that fixed costs variations can generate changes in equilibrium prices even in the absence of entry or exit of firms. This is in contrast with oligopoly models where technology is fixed.\footnote{Indeed, under standard Cournot competition, a reduction in fixed costs allows for the entry of new firms, thereby increasing competition which, in turn, is the actual cause of the decline in prices. In the present framework, the degree of competition, as proxied by the number of competitors, is constant.}

### 3.3.2 Changes in Demand parameters

Our last comparative statics results explores how the demand parameter $t$ affects the equilibrium profits. The parameter $t$ represents consumer loyalty to a brand, a higher $t$ decreases substitutability between goods and hence the degree of competition in a Bertrand framework. We summarize the relevant effects of an increase in consumer loyalty as follows.

**Corollary 1.** (a) The profits of all firms increase in consumer loyalty, the positive effect is strongest on the central firm in all equilibrium configurations; (b) the impact of an increase in $t$ on prices is positive. (c) under the asymmetric configurations the equilibrium market share of a niche firm increases with consumer loyalty.

The implication of this corollary is that the central firm has the highest benefit in terms of profits from any change in consumer tastes that leads to higher consumer loyalty, such as those induced, e.g., by advertising. If investments by firms in specific advertising—which increases brand loyalty to their own brand— are possible, they would lead to eventually asymmetric values of $t$ for each firm, then the central firm would be the one investing the largest amount.
Here the causality linking advertising and size of a firm would run from the latter to the former.

Also, the niche firms’ market shares increase with consumer loyalty when they use the SF-lm technology and the central firm the LF-sm one, so that it is apparent that consumer loyalty is a condition favoring the entry and survival of such products. By contrast a more competitive market, with high substitutability (low \( t \)) reduces the scope for niche firms under the asymmetric configurations.

**Corollary 2.** An increase in consumer loyalty disfavors the initial switch to the LF-sm technology and reduces the overall measure of the parameter space where this technology is operated. It also delays adoption of this technology when it becomes cheap enough to minimize the costs of producing the current output by the central firm.

Starting in configuration \( sss \), only a sizeable decrease in consumer loyalty will lead to equilibrium \( sls \), where a technology change by the central firm ignites the switch to the LF-sm technology. More generally, an increase in \( t \) reduces \( F^{III} \), therefore shrinking the measure of the overall parameter space where the LF-sm technology is operated. Furthermore, the equilibrium output of firm 2 under \( sss \) is \( M/2 \); it is easy to see that the cost of producing this quantity is minimized by using the LF-sm technology if \( \Delta f < M\Delta c/2 \). However, if \( t \) is high (precisely, if \( 3t > 10c \)) then the central firm sticks to the choice of the SF-lm technology even if, costwise, it would be more efficient to switch to the LF-sm one.

An interesting alternative interpretation consists in considering \( t \) as inversely related to the switching costs borne by consumers. According to the digital transformation interpretation of our model, the Corollary suggests that higher switching costs for consumers slow down the pace of adoption of digital technologies. This is in accordance with the findings of Forman (2005), who shows that a higher competitive pressure –here represented by lower unit transportation costs– spurs firms to adopt digital technologies.\(^{19}\)

To complete this Section, let us briefly hint at the implications of Assumption A.1. Relaxing this Assumption implies that, at the prices identified by the best replies (3), quantities and profits may turn negative. Nonetheless, an equilibrium analysis can be carried out, which features two main differences w.r.t. the present one. First, in some parameter constellations one or more firms set a non-negative price though having a zero output and nil profits. Second, in a subset of the region where \( sls \) is part of a SPNE, a second equilibrium may emerge, where the technological configuration is \( lsl \).\(^{20}\)

\(^{19}\)Beggs (1989) finds that consumer switching costs may lead to the adoption of inferior technologies.

\(^{20}\)The complete analysis of the unrestricted parameter space is available upon request, in Appendix D we briefly comment on and diagrammatically represent its outcomes.
4 Welfare maximizing configurations

A comparison of the market outcome with the socially optimal allocations of technologies can be performed by letting a social planner assign technologies to firms while allowing them to freely choose their prices.\(^{21}\) Compared with traditional Cournot analysis, the main difference here is that the allocation of consumers to firms matters; this is due to the disutility cost borne by the consumer (the "transportation cost") in address models, which has no counterpart in Cournot analyses or in non spatial models. We define total welfare in the standard way as the sum of producer and consumer surplus. Because the market is assumed to be covered, total welfare is, in this case, the aggregate gross consumer surplus

\[ W^k = M \left[ V - \left( \frac{t}{2} \right) \sum_{i \in \{1,3\}} (q^k_i)^2 - t \int_{x^k_{12}}^{x^k_{13}} |1/2 - x| \, dx \right] - \sum_{i=1}^{3} C_t(q^k_i), \]

where \( C_t(q^k_i) \) is the production cost borne by firm \( i \) with the selected technology \( t \) in configuration \( k \), while \( x^k_{12} = 1/4 + (p^k_2 - p^k_1)/(2t) \) and \( x^k_{23} = 3/4 - (p^k_2 - p^k_3)/(2t) \) are the indifferent consumers. Let \( W^I \equiv M \left( \frac{\Delta c}{4} - \frac{47\Delta c^2}{288t} \right), W^{II} \equiv M \left( \frac{\Delta c}{4} - \frac{11\Delta c^2}{96t} \right) \) and \( W^{III} \equiv M \left( \frac{3\Delta c^2}{18t} + \frac{\Delta c}{2} \right), \) with \( 0 < W^I < W^{II} < W^{III} \), it is then possible to prove the following result.

**Proposition 4.** Under A.1, welfare maximization is obtained at (i) configuration lll for \( 0 < \Delta f < W^I \); (ii) configuration sll (or lls) for \( W^I < \Delta f < W^{II} \); (iii) configuration sls for \( W^{II} < \Delta f < W^{III} \); (iv) configuration sss for \( W^{III} < \Delta f \).

Total welfare is maximized by the technology configuration that minimizes the sum of the industry cost of production and of the total transportation cost, so for relatively low fixed costs it is welfare-maximizing that all the firms adopt the **LF-sm** technology. As \( \Delta f \) increases relative to \( \Delta c \), welfare maximization requires that the **LF-sm** technology is gradually phased out in favor of the **SF-lm** one. It is interesting to remark that, like in the case where technologies are chosen by the firms, technological configurations lss (or ssl) and lsl are never welfare-maximizing as both are dominated (in the relevant parameter region) by another configuration where the central firm uses the **LF-sm** technology. This is so because, when the central firm adopts the **LF-sm** technology, its price decreases, which increases the number of consumers that purchase from it, resulting in lower aggregate transport costs, whence higher welfare. Remember, in fact, that this firm imposes, for a given demand size,
lower transport costs to consumers than the marginal ones. Thus, socially, it is optimal that "many" consumers patronize this firm (as far as $q_2 < 2/3$). This implies that, when it is socially optimal that at least one firm adopts the LF-sm technology, the central firm must be among the ones doing so. It is also instructive that, letting $\Delta f$ increase, the sequence of socially optimal welfare configurations replicates the equilibrium one, though the threshold values are clearly different. Furthermore, the relative position of the cutoffs depends on $\Delta c$ and $t$. Indeed, direct comparison allows us to state the following.

**Corollary 3.** With respect to the socially optimal choices, the central firm tends to overadopt the SF-lm technology, the niche firms tend to overadopt the SF-lm technology for small values of $c$ (if $\Delta c < 6t/11$) and to underadopt it for large values of $c$.

Proof: (i) if $\Delta c < \frac{4t}{3}$ then $F_{II} < W_{II} < F_{III} < W_{III}$, (ii) if $\frac{3t}{8} < \Delta c < \frac{6t}{11}$, then $F_{I} < W_{I} < F_{II} < W_{II} < F_{III} < W_{III}$, (iii) if $\frac{6t}{11} < \Delta c < \frac{2t}{3}$, then $W_{I} < F_{I} < F_{II} < W_{II} < F_{III} < W_{III}$, (iv) if $\frac{2t}{3} < \Delta c$, then $W_{I} < F_{I} < W_{II} < F_{II} < F_{III} < W_{III}$.

A first observation is that, in all possible instances, $F_{III} < W_{III}$: therefore if $F_{III} < \Delta f < W_{III}$ the central firm switches from sls to sss (adopts the SF-lm technology) when it would be socially optimal lll, namely that it still operated the LF-sm one. A symmetric reasoning holds, for very low levels of $\Delta f$, for the niche firms. Indeed, in case (i) at $\Delta f < F_{II}$ both niche firms choose the SF-lm when welfare is maximized by lll. In case (ii) for $W_{I} < f < F_{II}$ one niche adopts the SF-lm when lll is the social optimal configuration. This is intuitive: in cases (i) and (ii) $\Delta c$ is "relatively low" , so the niche firms have a incentive to adopt that technology to save on their (individual) production costs. The opposite reasoning applies to cases (iii) and (iv), where $\Delta c$ is relatively high: the niche firms "delay" the adoption of the SF-lm technology although social optimality would require to operate it.

5 Policy

5.1 Subsidies

Government taxes and subsides are quite widespread; we shall briefly analyze here the case of subsidies targeted to spurriing the adoption of the LF-sm technology, which corresponds, in our interpretation, to speeding up the digitalization process. Subsidies are, indeed, widely used for funding firms operating in the ICT sector, (see e.g. Colombo and Grilli, 2007; Zhang and Liang, 2012 and the references therein).\footnote{Quoting Colombo and Grilli (2007, p.573), "Even if ICTs are defined as general-purpose technologies, there is growing evidence that the emergence and consolidation of a strong national ICT sector are a fundamental prerequisite for rapid digitalization of a country [...] and have a positive impact on the performance of the whole national economy [...]".} In the context here analyzed a Government, in an effort to encourage technological change and efficiency can introduce such incentives for
the adoption of the **LF-sm** technology in the form of subsidies on the cost of the fixed input, or of a subsidy to production if the firm operates the **LF-sm** technology. Tax exemptions that discriminate some types of firms can also be encountered and they are like discriminatory subsidies. Governments often also provide subsidies in the form of loans at special interest rates, independent of production. Taxes on variable input produce effects that can be analyzed in a way that parallels the analysis in this section, but we do not pursue the issue here for the sake of brevity.\(^{23}\)

**Subsidy on fixed input.** A subsidy on the fixed input for the **LF-sm** technology leads to a reduction in the net cost of adopting the **LF-sm** technology itself and thus can be used by the government to determine the degree of diffusion of that technology in the industry. Suppose indeed that, initially, \(\Delta f > F^{III}\) so that the initial situation is sss. Let \(\sigma\) be the subsidy. It is then apparent that by setting, e.g., \(\sigma\) such that \(F^I < \Delta f - \sigma < F^{II}\) the central and one niche firm will apply for the subsidy and adopt the **LF-sm** technology. Clearly, the subsidy can be large enough so as to induce a generalized adoption of the **LF-sm** technology.

**Proposition 5.** A minimum subsidy exists

\[
\sigma^* = \Delta f - \left[ \frac{5M\Delta c}{24t} \left( t - \frac{5\Delta c}{12} \right) \right],
\]

fostering the adoption of the **LF-sm** technology by all firms. It is defined as \(\sigma^* = \Delta f - F^I\) it is decreasing in \(M\), the mass of consumers, in \(t\), and in \(\Delta c\).

A consequence of the foregoing result is that policies aimed at increasing market integration and at introducing standards for digital protocols and norms (e.g. like in the European Union) act as an increase in the mass of consumers and a decrease in switching costs, favouring the adoption of digital technologies.

Combining these observations with Propositions 1 and 3 it is immediate to ascertain that any subsidy that increases the diffusion of the **LF-sm** technology benefits consumers. Likewise, it is also easy to see that a subsidy inducing one firm only to shift the **LF-sm** technology makes the profit of that firm increase, at the expenses of the rivals’ profits, which decrease because of the drop in the marginal cost of the shifting firm. Yet, the effect of a subsidy that makes more than one firm adopt the **LF-sm** technology is more complex. Indeed each firm individually benefits from adopting the **LF-sm** technology (at a lower net cost) because of the drop in own marginal and average cost, but the simultaneous adoption by the rival(s) increases the competitive pressure in the industry, thereby eroding profit margins. The overall

\(^{23}\)The treatment of variable input taxation can be found in a previous version of the present paper, available as WP. See footnote 7.
effect is ambiguous and depends on the balance between the levels of $\sigma$, $\Delta f$ and $\Delta c$. There is, however, a case where a subsidy unambiguously harms all firms in the industry, which is the one identified in Proposition 5. To see the reason let, before the subsidy is in place, $\Delta f > F^{III}$, so that the pre-intervention technological pattern is $sss$. Now by setting up at least equal to $\sigma^*$, all firms switch to the $LF-sm$ technology. It is apparent that, in this case, the profits of the firms after adoption are $\pi_{i}^{lll} + \sigma$, which fall short of the profits before adoption $\pi_{i}^{sss}$ if $\sigma < \Delta f$, namely so long as the subsidy does not make the cost associated to the $LF-sm$ technology always lower than that of the $SF-lm$ one. This is a consequence of the prisoner’s dilemma nature of the game, as pointed out in Remark 3.

Remark 4. A subsidy on the fixed input that triggers a change in technology adoption is always beneficial to consumers. If the subsidy is aimed at generating universal adoption of the $LF-sm$ technology, when this is not in use, it leads to lower profits for all firms.

Our model predicts that subsidies, while generally benefiting consumers and fostering the adoption of the $LF-sm$ technology, have ambiguous effects on the profits of firms, as they are affected by both on the number of firms adopting the $LF-sm$ technology following the subsidy and by the relative values of the fixed and marginal costs and of the subsidy itself.\footnote{That subsidies may reduce profits is also found in Rebolledo and Sandonís (2012), due to asymmetric information.}

If the $LF-sm$ technology is the "digital" one, this means that subsidies can influence the pace of digitalization. One can also show that a tax on the variable input may lead to much the same results in terms of technology adoption as a subsidy, if it leads to a welfare increasing change in the technology configuration. We omit here a full analysis of such a tax.

5.2 Subsidy to production

A subsidy to production decreases the unit cost by an amount $\sigma$.\footnote{As Adda and Ottaviani (2005) point out, subsidies for the adoption of digital technologies are usually granted to final consumers, rather than to firms. Here, for the sake of conciseness, we assume that they are given to firms. Because of the pass-through, however, the price paid by consumers net of the subsidy coincide with the price that the firms set when they are granted the transfer. As a consequence, quantities and profits are independent of the actual beneficiary of the policy.} The profit of firm $i$ adopting the $LF-sm$ technology in the presence of a subsidy to production under the $LF-sm$ is then $(p_i - c_i + \sigma)q_i(p_1, p_2, p_3)$. First, due to the inelastic aggregate demand property of the Hotelling model, there is no subsidy that leads to a switch to $lll$. Indeed the new price triplet in $lll$ with the subsidy is $p_i^{\sigma} = c_i + t/2 - \sigma$ for $i = 1, 2, 3$. With quantities as in the no subsidies configuration $lll$ and profits given by $p_i^{\sigma} = (c_i + t/2 - \sigma + \sigma)q_i^{lll} = \pi_{i}^{lll}$ there is no subsidy that will make a switch to $lll$ profitable if it is not so without a subsidy.

However a switch from $sss$ to $sls$ can be induced by a subsidy to production. The price

\[ p_i = c_i + t/2 - \sigma \]
triplet under $sls$ is computed assigning cost $c_2 = -c_l - \sigma$ to firm 2 and $c_s$ to firms 1 and 3:

$$(p_1^s, p_2^s, p_3^s) = \left(\frac{t}{2} + \frac{8c_s + 4c_l - 4\sigma}{12}, \frac{t}{2} + \frac{2c_s + 4c_l - 4\sigma}{6}, \frac{t}{2} + \frac{8c_s + 4c_l - 4\sigma}{12}\right).$$

The quantity triplet is $\left(\frac{3t-2\Delta c-2\sigma}{12t}, \frac{2(2\Delta c+3\sigma+2\delta)}{12t}, \frac{3t-2\Delta c-2\sigma}{12t}\right)$ and survival of 1 and 3 is guaranteed only if $\sigma < (3t - 2\Delta c)/2$. A subsidy then triggers a change from $sss$ to $sls$ if: $\pi_{2\sigma}^{sls} > \pi_{2\sigma}^{sss}$ where $\pi_{2\sigma}^{sls} = \frac{M(2\Delta f+Mt)}{t} - \frac{\Delta c - \frac{3}{2}t}{2}.26$

 Proposition 6. The subsidy to production needed to obtain a switch to the $LF-sm$ by the central firm is declining in the mass of consumers, $M$, and it is increasing in the level of consumer loyalty, $t$.

Obviously, subsidies must be financed and can eventually lead to levying distortionary taxes elsewhere in the economy. By contrast a tax on the variable input can induce a welfare improving distortion, as discussed above. Eventually, a combination of a tax on the variable input and of a subsidy can be studied. Of course, if all firms embrace the $LF-sm$ technology the tax on the variable input is avoided and the subsidy must still be entirely financed.

6 Regulation

6.1 Price-ceilings

In the standard analysis a price distortion induced by market power, whether under monopoly or oligopoly, reduces quantity and hence welfare. Price caps can force the market price to a lower level and increase the quantity sold, increasing welfare in the industry (as welfare is measured only as the area below market demand and above production costs minus fixed costs). Price caps are therefore welfare increasing in the standard analyses. In an address model price caps redistribute consumers through firms, affecting transportation costs. This marks a first noteworthy difference between the present framework and the one with non-localized competition.

A further difference is due to the technology dimension here introduced, which adds a second channel through which price caps affect the equilibrium. The first channel is the price-lowering effect for a given technology, the second is the possible dampening in the gains from adopting the $SF-lm$ technology. To be more systematic, consider first that a price cap can be binding or non-binding. A binding price cap can be globally binding (when the existing unregulated configuration is $sss$ or $lll$) or locally: when the configuration is $sll$ or $lls$ it binds

\[\text{It can be easily checked that this level of subsidy allows firms 1 and 3 to survive if A.1 holds.}\]
only on the niche firm with cost $c_s$, when it is $sls$ it binds on both the niche firms. The following reasoning shows that in both cases a cap can have an impact of prices.

**Case a)** In a set-up where technologies are exogenous, a price cap, by affecting prices, changes the allocation of consumers to firms and hence it affects welfare thorough this reallocation. Under $sss$ and $lll$ a price ceiling bites equally on all firms and does not change the market shares, hence it only redistributes surplus from firms to consumers without aggregate change in total surplus. Under $sls$, by contrast, the prices of the niche firms will be constrained and consumers will be reallocated from the central firm to the niche ones. Hence a price cap will affect total production costs and total transportation costs. Firms 1 and 3 are producing at a marginal cost $c_s > c_l$ so that an increase in their market share increases aggregated production costs with a negative effect on welfare, given that firm 2 produces at a lower marginal costs and $sls$ implies a fixed cost equal to $f_l$. However, the niche firms in the equilibrium $sls$ are receiving a share of consumers lower than the transportation-cost-minimizing share of 1/3, so that the reallocation of consumers in their favor will reduce transportation costs. The two contrasting effects on welfare must therefore be added up to get the direction of change. To make our point, in the following we shall assume that the price cap is "close to" the highest prices prevailing under $sls$, namely $p_1$ and $p_3$. Since prices simply redistribute surplus, total welfare under $sls$ writes as:

$$W_{sls} = M \left\{ V - \int_{0}^{x_{12}} (c_s + tx) \, dx - \int_{x_{12}}^{x_{23}} (c_l + t|1/2 - x|) \, dx - \int_{x_{23}}^{1} (c_s + t(1 - x)) \, dx \right\} - 2f_s - f_l. \quad (5)$$

A reduction in $p_1$ and $p_3$ to $\bar{p}$ following the price cap leads the central firm to modify its optimal price according to its best reply $p_2(\bar{p}) = t/4 + (c_l + \bar{p})/2$. At these prices, the locations of the indifferent consumers are $x_{12}(\bar{p}) = (2c_l + 3t - 2\bar{p})/(8t)$ and $x_{23} = 1 - x_{12}(\bar{p})$. It is a matter of direct inspection to ascertain that $\partial W_{sls}/\partial \bar{p}$ is positive if evaluated at $(\bar{p} = p_{1s}^{sls} - \epsilon)$, where it takes the value $M(4\Delta c + 3\epsilon)/12$.\(^{27}\) Hence, as long as the price cap does not trigger a change in the technological configuration, it reduces welfare. It is also easy to check that transportation costs are lowered and consumer surplus is increased by a reduction in $\bar{p}$ in a neighborhood of $p_{1s}^{sls}$. A similar computation shows that the same results hold under $sll$ with a price cap binding on firm 1 (recall that $p_1^{sll} > p_2^{sll} > p_3^{sll}$) –a complete proof is available in Appendix F.

**Proposition 7.** A price cap which has no effect on technology choices never increases total

\(^{27}\)By evaluating (5) then one obtains $W_{sls}(\bar{p}) = M \left[ V - \frac{8c_s(2c_s + 3t - 2\bar{p}) - (12\epsilon^2 - 4c_s(3t + 2\bar{p}) - 5\epsilon^2 + 4t^2 - 4\epsilon^2)}{32t} \right] - f_l - 2f_s$ and its derivative with respect to $\bar{p}$ is $(4\Delta c - 2\bar{p} + t) / 8t$. 20
welfare; it always increases consumer surplus; furthermore (a) under sss and lll a binding
price cap reduces firms’ profits, and is neutral on total welfare. (b) under all other equilibrium
configurations it decreases total welfare.

A digression on volume effects is in order here: price caps in our analysis cannot have
effects on the volume of trade as the market is of fixed size and it is entirely covered in
equilibrium. If volumes increased the result can be that a price cap increases welfare, which is
in general true under non-localized competition as in a Cournot setting (we have found only
one counter-example under Stackelberg competition in Chang, 2004). Yet, this may not be
true in an address model.28

Case b). Let us now move to the technological effects of price caps. To figure out how
the technology configuration can be affected by a price ceiling, let us start with the initial cost
configuration sss and let us assume that the Government pursues the adoption of the LF-sm
technology.29 In order to do so, the Government can impose a price cap equal to
\[ \bar{p} = p_{1}^{ssl} = p_{3}^{ssl} \]
or anywhere in between that price and the unregulated equilibrium price \( p_{i}^{sss} \), namely with
\( \bar{p} \in (p_{1}^{ssl}, p_{3}^{ssl}) \). In such a regulated industry the technology change may be profitable or not,
according to the level of the allowed price \( \bar{p} \).

To ascertain this, consider the profits of the firms under a price cap, represented by the
price \( \bar{p} \), when they all adopt the SF-lm technology, which are
\[ \pi_{2}(\bar{p}, sss) = (\bar{p} - c_{s})(M/2) - f_{s} \]
and \( \pi_{i}(\bar{p}, sss) = (\bar{p} - c_{s})(M/4) - f_{s} \), for \( i \in \{1, 3\} \). The profit of firm 2 at
ssl is instead given
\[ \pi_{2}^{ssl} = M(3t + 2\Delta c)^{2}/(36t) - f_{l} \]
Hence it is easy to check that the switch to the LF-sm
technology is profitable for firm 2 as long as
\[ \bar{p} < \tilde{p} = \frac{1}{18} \left( 30c_{s} - 12c_{l} + 9t - \frac{6\Delta f}{M} + \frac{4\Delta c^{2}}{t} \right) . \] \[ (6) \]

One can show that there exist a range for \( \Delta f \) such that the maximum price price \( \bar{p} \) is
lower than \( \tilde{p} \) but higher than the unregulated price of firms 1 and 3 after the switch from sss
to ssl, so that the firm 2’s profits are actually equal to \( \pi_{2}^{ssl} \). A change from ssl to sll or

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28To make the point, assume that at each point in [0,1] there are two types of consumers in proportion \( 1 - \alpha \)
and \( \alpha \), one which has utility as the typical Hotelling consumer and the other which is insensitive to distance
and only care about price, with a reservation price equal to \( \varepsilon \). If the price ceiling is such that \( p^{ssl} > \varepsilon > \bar{p} \), the
introduction of a ceiling leads to purchase by consumers of type 2. If \( \alpha \) is small enough the beneficial effect
on surplus and total welfare of the demand increase is small and cannot counterbalance the negative effect
computed for the case where \( \alpha = 0 \). This is never the case in Cournotian models.

29Greenstein et al. (1995) claim that price regulation, and price caps in particular, are a powerful instrument
to incentivize the adoption of digital technologies; similarly, Ai and Sappington (2002) find that price cap
regulation is one of the most effective policy instruments to foster infrastructure modernization (optic fiber
and digital switches) in the communications industry.

30The desired inequality is: \( p_{1}^{ssl} < \tilde{p} \) where \( p_{1}^{ssl} = (4c_{s} + 2c_{l} + 3t)/6 > 0 \). Then, \( \bar{p} - p_{1}^{ssl} \) is equal to
\[ \frac{1}{18M^{t}} (2M(\Delta c)^{2} - 3t\Delta f + 9Mt\Delta c) \]
In the region for sss, \( \Delta f > \frac{M\Delta c}{3t}(t + \frac{3}{2}) \) substituting this value for \( \Delta f \)
from $lls$ to $lll$ can be fostered by the same argument. As a general point: a price cap may lead to favour the LF-sm technology and to alter the equilibrium configuration. Interestingly, Liu and Chuang (2015) report that progressively lower price caps have been used to force Taiwanese TV operators to switch to the digital technology.

**Proposition 8.** Price caps, even if not binding at equilibrium, are a viable regulatory instrument to influence the technological configuration.

This last Proposition follows from the observation that any individual switch in technology generates a jump in the prices (Proposition 3), which implies that if the price cap is set between the current highest price and the lowest price that would emerge if a firm switched to the SF-lm technology, the switch is forestalled even if the cap is not actually binding.

**Proposition 9.** A price cap is welfare increasing only if it leads firms to switch to (or keep with) the LF-sm technology when at the unregulated equilibrium the SF-lm technology is over-adopted.

The two Propositions follow from the discussion of the effects on prices and from the analysis of the welfare maximizing configurations performed in Section 4. Indeed, Proposition 9 simply states that a change from e.g. $sls$ to $sss$ by firm 2 can be forestalled when it is welfare decreasing. It is clear that the effect driving the last proposition is the change in technologies.

7 Conclusion

In this paper we have analyzed the relationship between the relative positioning in a market, with a central versus two peripheral firms, and the choice of technologies. We have in particular discussed the strategic choice of adopting a technology that is more efficient at low output levels or one that is more efficient at high output levels. Our analysis fits relevant recent trends concerning the digital transformation of industries, whereby "analog" technologies are phased out and replaced by "digital" ones, which require a drastic transformation in the infrastructure, with huge capital investments. Once adopted, these technologies allow firms to save on variable costs, but oligopoly interaction may result in delays in adoption. The equilibrium configurations reveal that the central position, in particular due to the intensity of the business stealing effect, favors the adoption of the LF-sm/digital technology, which is consistent with observed patterns of digitalization. By contrast, niche (peripheral) firms are less incentivized to abandon labour intensive technologies, and therefore appear to behave as "laggards" in adopting new technologies. Furthermore, exogenous shocks to technologies,

\[
\tilde{p} - p_{1}^{sls} = \frac{1}{\Delta t} \Delta c (24t + 5\Delta c) > 0,
\]

as sought. Hence there is a range for $\Delta f > \Delta F^{III}$ such that there exist a viable $\bar{p} < \tilde{p}$.
that change the cost structure, lead to non-obvious changes in the equilibrium prices. As an example, imagine to start with a high fixed cost in the LF-sm/digital technology and with a low marginal cost in the SF-lm/analog one, so that all firms adopt this second technology. Let the fixed cost decrease: the central firm is the first one to embrace the LF-sm technology; then as the fixed cost decreases further only one of the peripheral firms adopts it, and finally all firms do. Hence, fixed costs contribute in shaping the market outcome and determine the price configuration. Similarly, an increase in variable costs that triggers firms to switch to the LF-sm technology may induce a price reduction instead of a price increase.

We also find that a taste parameter related to brand loyalty, or switching costs, affects the technology choice. The mass of consumers amplifies the effects of brand loyalty/switching costs, besides leading to a higher attractiveness of the LF-sm-digital technology over the SF-lm-analog one. Hence policies that reduce switching costs and increase market integration also favor the adoption of digital technologies.

Comparison of the welfare maximizing technology configurations and the market equilibria reveal that the latter do not align with the former. The central firm tends to overadopt the LF-sm technology, while the niche firms also may overadopt it, if it is convenient enough, otherwise they may underadopt it.

The analysis of the equilibrium responses to two widely used policies, which are of particular relevance in the process of digitalization, reveals various interesting points. First, we have briefly analyzed the effects of subsidies aimed at the adoption of the LF-sm/digital technology. A subsidy on the fixed input, which may be interpreted as the "digital infrastructure", may eventually result in a "faster" adoption of the LF-sm/digital technology and also lead to higher firm profits. Yet, surprisingly, it may also actually lead to lower profits for the firms. The possibility of lowering firm profits follows from a switch to a regime with lower prices (more firms adopting the LF-sm technology). We have also analyzed the effect of a per-unit production subsidy for the adoption of the LF-sm technology. Finally, we have tackled regulation, in the form of a price ceiling. A price cap may reduce the profitability of adopting a SF-lm/analog technology, thereby favoring the digitalization process and preventing a price increase.

Caps also change total transportation costs and production costs via the reallocation of market shares to firms and, overall, they can increase welfare only if they lead to a switch in technology, otherwise they are detrimental to welfare.

We have developed our analysis by assuming that the cost function choice set available to the firms is dichotomous. One legitimate question is then about the robustness of our findings to a "smoother" formulation of the cost function set. Our analysis suggests that, if firms can fix the characteristics of their cost function $C(q) = cq + f$, with the constraint that in order to enjoy a lower marginal cost, a larger fixed cost has to be borne, and conversely, the central
firm will tend to select technologies featuring lower marginal cost and larger fixed costs than the peripheral ones.

We have left several questions for further research: one could analyze the incentives and the means by which a firm can increase the rivals’ fixed costs. Note indeed that if a firm switches from the LF-sm to the SF-lm technology it creates a positive externality to rivals due to the price complementarity: They will enjoy higher market shares and higher mark-ups after the change. If it was possible, therefore, a firm would like to forestall the adoption of the LF-sm/digital technology by the rivals, e.g. by making the LF-sm proprietary; for instance with investments in R&D larger than those of the rivals, to discover in advance the LF-sm technology or a cheaper version of it than those already available— or by raising the rivals’ fixed costs if they adopt the LF-sm technology (as in Hviid and Olczak, 2016). Finally if the central firm is a firm producing the input that is used in the SF-lm technology and selling it to the rivals then it will choose the wholesale price so as to manipulate the technology choice by rivals, namely so as to strategically avoid a change in technology from the SF-lm to the LF-sm.

A Characterization of the price equilibria.

(i) Symmetric Configuration iii.

Recall that \( p_{i}^{lll} = c_i + t/2 \) which leads to \( q_{1}^{lll} = q_{3}^{lll} = 1/4 \) and \( q_{2}^{lll} = 1/2 \), with profits

\[
\pi_{1}^{lll} = \pi_{3}^{lll} = Mt/8 - f_i, \quad \pi_{2}^{lll} = Mt/4 - f_i.
\]

(ii) Configuration sll.

In this case, \( p_{1}^{sll} = (1/2) (7c_s + 5c_l + 6t)/6, \quad p_{2}^{sll} = (c_s + 5c_l + 3t)/6, \quad p_{3}^{sll} = (1/2) (c_s + 11c_l + 6t)/6, \) which lead to quantities \( q_{1}^{sll} = (1/4) (6t - 5\Delta c)/(6t), \quad q_{2}^{sll} = (\Delta c + 3t)/(6t), \quad q_{3}^{sll} = (1/4) (\Delta c + 6t)/(6t); \) finally resulting in the following profits,

\[
\pi_{1}^{sll} = \left(M \frac{\Delta c + 6t}{36t}\right) - f_s, \quad \pi_{2}^{sll} = \left(M \frac{\Delta c + 3t}{36t}\right) - f_1, \quad \pi_{3}^{sll} = \left(M \frac{6t - 5\Delta c}{36t}\right) - f_s
\]

It is immediate to observe that, with an appropriate change of the indices, the outcomes under this configuration are equivalent to those of the configuration ll's.

(iii) Configuration sls.

Prices are \( p_{1}^{sls} = p_{3}^{sls} = (3t + 4c_s + 2c_l)/6, \quad p_{2}^{sls} = (3t + 2c_s + 4c_l)/6, \) which lead to the
quantities \( q_1^{sls} = q_3^{sls} = \frac{3t-2\Delta c}{12t} \), and \( q_2^{sls} = \frac{3t+2\Delta c}{6t} \); resulting in the following profits

\[
\pi_1^{sls} = \pi_3^{sls} = \left(\frac{M}{2}\right)\frac{(3t-2\Delta c)^2}{36t} - f_s, \quad \pi_2^{sls} = M\frac{(3t+2\Delta c)^2}{36t} - f_s.
\] (9)

(iv) Configuration lss.

The prices are \( p_1^{lss} = (1/12)(5c_s + 7c_l + 6t), \) \( p_2^{lss} = (1/6)(5c_s + c_l + 3t), \) \( p_3^{lss} = (1/12)(11c_s + c_l + 6t), \) leading to the quantities \( q_1^{lss} = (1/4)\left[(5\Delta c + 6t)/(6t)\right], \) \( q_2^{lss} = (3t - \Delta c)/(6t), \) and \( q_3^{lss} = (1/4)\left[(6t - \Delta c)/(6t)\right]; \) resulting in the following profits:

\[
\pi_1^{lss} = \left(\frac{M}{8}\right)\frac{(5\Delta c + 6t)^2}{36t} - f_l, \quad \pi_2^{lss} = M\frac{(3t - \Delta c)^2}{36t} - f_s, \quad \pi_3^{lss} = \left(\frac{M}{8}\right)\frac{(6t - \Delta c)^2}{36t} - f_s.
\] (10)

With an appropriate change of the indices, the outcomes under this configuration are equivalent to those of the configuration ssl.

(v) Configuration lsl.

Prices are \( p_1^{lsl} = p_3^{lsl} = (2c_s + 4c_l + 3t)/6, \) \( p_2^{lsl} = (4c_s + 2c_l + 3t)/6, \) the relative quantities are \( q_1^{lsl} = q_3^{lsl} = (2\Delta c + 3t)/(12t), \) \( q_2^{lsl} = (3t - 2\Delta c)/(6t), \) and the profits are

\[
\pi_1^{lsl} = \pi_3^{lsl} = \left(\frac{M}{2}\right)\frac{(2\Delta c + 3t)^2}{36t} - f_l, \quad \pi_2^{lsl} = M(3t - 2\Delta c)^2/(36t) - f_s.
\] (11)

(vi) Symmetric Configuration sss.

Prices are \( p_1^{sss} = c_s + t/2, \) and quantities are \( q_1^{sss} = q_3^{sss} = 1/4, \) and \( q_2^{sss} = 1/2. \) Profits are

\[
\pi_1^{sss} = \pi_3^{sss} = Mt/8 + f_s, \text{ and } \pi_2^{sss} = Mt/4.
\] (12)

B Proof of Proposition 1

(i) To prove the existence of configuration lll we need to check that neither one peripheral firm nor the central firm have incentives to deviate to the **SF-lm** technology.

No deviation by peripheral firm.

If firm 1 unilaterally deviates to the **SF-lm** technology it reaps a profit equal to \( \pi_1^{sll}, \) this is not profitable if

\[
\pi_1^{lll} \geq \pi_1^{sll} \Leftrightarrow f \leq 5/(24t)M\Delta c \left((12t - 5\Delta c)/12\right) \equiv F^l.
\] (13)

Clearly, this same condition guarantees that firm 3 does not want to deviate to the **SF-lm** technology too.
No deviation by central firm.

There is no profitable deviation to the \textbf{SF-lm} technology by the central firm when

\[ \pi_2^{ll} \geq \pi_1^{llst} \iff \Delta f \leq M(\Delta c/3)(1 - \Delta c/(3t)). \quad (14) \]

It is easy to prove that both conditions are fulfilled when \( \Delta f \leq F^I \) and that \( F^I < t(M/9) \forall \Delta c < 6t(M/5) \), which insures the positivity of the SPNE profits.

(ii) Existence of a SPNE with technological configuration \textit{sll} or \textit{lls} requires the following (here we focus on case \textit{sll}, which, after an appropriate permutation of the firm labels guarantees existence for configuration \textit{lls}).

No deviation by firm 1.

This requires that

\[ \pi_1^{sl} \geq \pi_1^{ll} \iff \Delta f \geq F^I. \quad (15) \]

No deviation by firm 2.

This requires that

\[ \pi_2^{sl} \geq \pi_2^{st} \iff \Delta f \leq \Delta c M/3. \quad (16) \]

No deviation by firm 3.

This requires that

\[ \pi_3^{sl} \geq \pi_3^{st} \iff \Delta f \leq \frac{5M\Delta c}{24t} \left( t - \frac{\Delta c}{4} \right) = F^{II}. \quad (17) \]

The three above conditions are simultaneously satisfied for \( F^I = \left( \frac{5\Delta c}{24} - \frac{25\Delta c^2}{288t} \right) M \leq \Delta f \leq \left( \frac{5\Delta c}{24} - \frac{25\Delta c^2}{96t} \right) M = F^{II} \). It is easy to ascertain that, in this region, the profits of the firms running the \textbf{LF-sm} technology are positive.

(iii) Existence of an equilibrium with configuration \textit{sls} requires what follows.

No deviation by peripheral firm.

For firm 1, this requires that

\[ \pi_1^{sls} \geq \pi_1^{lls} \iff \Delta f \geq F^{II}, \quad (18) \]

this same condition insures that firm 3 has no profitable deviation either.

No deviation by central firm.

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This requires that
\[
\pi_2^{ssl} \geq \pi_2^{sss} \iff \Delta f \leq (\Delta c/3) \left(1 + \Delta c/(3t)\right) M \equiv F^{III},
\]  
(19)
these conditions are simultaneously fulfilled when \(F^{II} \leq f \leq F^{III}\). As above, straightforward calculations prove that the profit of the firm adopting the LF-sm technology is positive within this parameter space.

(iv) Existence of configuration sss at equilibrium requires the following.
No deviation by peripheral firm.
\[
\pi_1^{sss} \geq \pi_1^{lss} \iff \Delta f \leq \left(\frac{25\Delta c^2}{288t} + \frac{5\Delta c}{24t}\right) M,
\]  
(20)
the same condition guarantees no deviation by firm 3.
No deviation by central firm.
\[
\pi_2^{sss} \geq \pi_2^{ssl} \iff \Delta f \geq (\Delta c/3) \left(1 + \frac{5\Delta c}{3t}\right) M,
\]  
(21)
The two above conditions are simultaneously satisfied when \(\Delta f \geq F^{III}\).

To complete the proof of Proposition 1 there remains to demonstrate that no equilibrium exists under the configurations lsl, ssl and lss.

1. Equilibrium under configuration lsl requires that, simultaneously
\[
\pi_3^{lsl} = \pi_1^{lsl} \geq \pi_1^{ssl} = \pi_3^{lss} \iff \Delta f \leq \left(\frac{5\Delta c}{24} + \frac{5\Delta c^2}{96t}\right) M, \text{ and}
\pi_2^{lsl} \geq \pi_2^{lll} \iff \Delta f \geq (\Delta c/3) \left(1 - \frac{\Delta c}{3t}\right) M.
\]  
(22)
It is a matter of simple algebra to ascertain that the two conditions above cannot be simultaneously fulfilled under the assumption \(f < Mt/8\).

2. Equilibrium in configuration lss requires
\[
\pi_1^{lss} \geq \pi_1^{sss} \iff \Delta f \leq \left(\frac{5\Delta c}{24} + \frac{25\Delta c^2}{288t}\right) M, \quad \pi_2^{lss} \geq \pi_2^{lls} \iff \Delta f \geq \frac{M\Delta c}{3}, \text{ and}
\pi_3^{lss} \geq \pi_3^{lsl} \iff \Delta f \geq \left(\frac{5\Delta c}{24} + \frac{5\Delta c^2}{96t}\right) M.
\]  
(23)
As in the previous case, the tree conditions cannot be simultaneously satisfied for \(\Delta c < 6t/5\).
C Proof of Proposition 2

Consider \( \Delta f > F^{III} \) first. By Proposition 1 we know that the equilibrium configuration is \( sss \). The average cost of every firm is \( AC_{i}^{sss} = c_{s} + \frac{4f_{s}}{M} \). Now, if \( \Delta f \) exogenously decreases, the attractiveness of the LF-sm technology raises and we know that, as soon as \( \Delta f \) crosses the threshold \( F^{III} \) the first firm switching to the LF-sm technology is the central one, firm 2. In region \( F^{II} < \Delta f < F^{III} \) the equilibrium configuration is \( sls \). Here, the average production cost of firms 1 and 2 is still \( c_{s} \), but the average cost of firm 2 is now \( AC_{2}^{sls} = c_{l} + \frac{6f_{l}}{M(2\Delta c + 3t)} \). The effect of the technological switch on the average cost of firm 2 is given by

\[
\Delta AC_{2} = AC_{2}^{sls} - AC_{2}^{sss} = -\frac{6f_{l}t}{2M\Delta c + 3Mt} + \Delta c + \frac{4f_{s}}{M}.
\]

The technological switch leads to a reduction in the average cost of firm 2 if \( \Delta AC_{2} > 0 \). In order to prove that this is indeed the case, we are going to show that the inequality holds under the most unfavorable conditions, namely those where \( c_{s} > c_{l} \geq 0 \) and \( f_{l} > f_{s} = 0 \). Under these assumptions \( \Delta f = f_{l} \) and the condition for a decrease in the average cost of firm 2 reduces to

\[
\Delta AC_{2}|_{f_{s}=0} > 0 \iff f_{l} < \frac{2M\Delta c(2\Delta c + 3t)}{6t} \equiv F_{2}^{AC}.
\]

It is straightforward to observe that \( F_{2}^{AC} > F^{III} \), hence the technological switch to the LF-sm technology results in a reduction of the average cost of firm 2 even if the fixed costs associated to the SF-lm technology are nil. A positive \( f_{s} \) would increase the average cost associated to the SF-lm technology, thus making the average cost reduction following the switch larger.

By proceeding in the same manner, it can be shown that if \( \Delta f \) further reduces and crosses the \( F^{II} \) threshold, so that one of the peripheral firms switches to the LF-sm technology too, the switching firm witnesses a reduction in its average production cost. Without loss of generality, assume that firm 3 is switching: its average cost before the switch is \( AC_{3}^{sls} = \frac{2c_{s}M(c_{s} - c_{l}) - 3t(c_{s}M + 4f_{s})}{M(2c_{s} - 2c_{l} - 3t)} \), and after it is \( AC_{3}^{sl} = \frac{24f_{l}t}{M(c_{s} - c_{l} + 6t)} + c_{l} \). Under the worst conditions, namely \( f_{s} = 0 \), the switch reduces the average cost if \( f_{l} < \frac{M\Delta c(\Delta c + 6t)}{24t} \equiv F_{3}^{AC} \). It is a matter of inspection to ascertain that \( F_{3}^{AC} > F^{II} \).

Similarly if \( \Delta f \) decreases below \( F^{I} \) the last peripheral firm switches to the LF-sm technology. In this case the average cost before the switch is \( \frac{5c_{s}M\Delta c + 6t(c_{s}M + 4f_{s})}{M(5\Delta c + 6t)} \equiv F_{1}^{sl} \) and after it is \( c_{l} + 4f_{l}/M \equiv AC_{1}^{sl} \). The difference in the average costs for \( f_{s} = 0 \) is \( \Delta c - \frac{f_{l}}{M} \), which is positive if \( f_{l} < \frac{M\Delta c}{4} \equiv F_{1}^{AC} \). It is easy to ascertain that \( F_{1}^{AC} > F^{I} \).
D Extended parameter space

For the sake of simplicity, the analysis in the extended product space has been carried out under the normalization $c_l = 0$ and $f_s = 0$, which imply that $\Delta c = c_s$ and $\Delta f = f_l$.

If $\Delta f > M t / 8$, the profits of the firms under technological configuration $lll$ become negative, so that each firm has an incentive to switch to the SF-lm technology, ultimately entailing that $lll$ can no longer be part of a SPNE; to this regard, it is instructive to observe that the threshold $F^{II}$ equals $M t / 8$ for $\Delta c = 6 t / 5$. On the other hand, in the case $\Delta c > 6 t / 5$ the optimal quantity produced by a firm running the SF-lm technology equals zero under some technological configurations. In particular, when $\Delta c \in [(6 / 5) t, (3 / 2) t]$, this happens in configurations $sls$ and $lls$, and when $\Delta c > 3 t / 2$ this happens in configuration $sll$ too. A second consequence of relaxing the restrictions on the parameters is that for $\Delta c > 6 t / 5$ equilibrium multiplicity appears in a sub-region of the space $F^{II} < \Delta f < F^{III}$. In fact, within this region, $lsl$ is part of a SPNE for $(\Delta c - \Delta c^2 / 9 t) M < \Delta f < (5 \Delta c / 24 + 5 \Delta c^2 / 96 t) M$ and $\Delta c > (36 t) / 47$, together with $sls$, the green-purple region of Figure 3 (see Appendix B for the derivation of the conditions for the existence of the $lsl$ equilibrium). Notice that, in this "new" equilibrium configuration, the output of the SF-lm firm is nil for $\Delta c > (3 / 2) t$ as well. In Figure 3 the darker regions identify the parameter constellations where the output (and profit) of the firms running the SF-lm technology are zero at equilibrium. A formal characterization of the technological equilibrium partition of the unrestricted parameter space is available from the authors upon request.

E Model with five firms

Like in the previous Section, here we assume that $c_l = 0$ and $f_s = 0$, so that $\Delta c = c_s$ and $\Delta f = f_l$. Firms $\{1, \ldots, 5\}$ are located at the points $0, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}, 1$ of the unit segment respectively and charge prices $p_i$, $i = 1, \ldots, 5$. The normalized demand system in this case is

$$q_1(p_1, p_2) = 1 / 8 + (p_2 - p_1) / (2t), \quad q_5(p_4, p_5) = 1 / 8 + (p_5 - p_4) / (2t)$$

and

$$q_i(p_{i-1}, p_i, p_{i+1}) = 1 / 4 + (p_{i-1} + p_{i+1} - 2p_i) / (2t), \quad \text{for } i = 2, 3, 4.$$  

It should be noticed here that the central firm 3 and the "intermediate" firms 2 and 4 all have one firm to the left and one to the right, so that none of them has a "naturally" larger demand than the others. This can be easily ascertained because, if all prices all equal, firms 2, 3 and 4

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31 In all the cases where the best reply of one firm adopting the SSE technology dictates a negative optimal quantity, we constrain this quantity to be equal to zero, which directly entails that the profit of that firm is zero too.

29
all have a demand equal to $M/4$.

The equilibrium analysis of the 5-firm model is not conceptually different from that with three competitors, however, because of the combinatorial nature of the possible technological configurations, the analytical burden increases significantly.

Indeed, there are 32 possible technological configurations, which reduce to 17 once accounted for the "symmetrical ones" such as, e.g., $slss$ and $sssls$. Figure 4 depicts the equilibrium partition in the parameter space where all quantities, at all possible technological configurations, are non-negative. Such conditions require that $\Delta f \leq (Mt)/32$ and $\Delta c \leq (42t)/71$. For the sake of conciseness we will limit ourselves to briefly describe the conditions under which the central firm 3 adopts the LF-sm technology when all the other ones choose the SF-lm technology, which in our previous notation corresponds to configuration $sslss$.

The optimal prices in this case are $p_1^{sslss} = p_5^{sslss} = \frac{1}{12}(11c_s + c_l + 3t)$, $p_2^{sslss} = p_4^{sslss} = (10c_s + 2c_l + 3t)/12$ and $p_3^{sslss} = (5c_s + 7c_l + 3t)/12$, with corresponding normalized quantities $q_1^{sslss} = q_5^{sslss} = \frac{3}{288t} - \frac{\Delta c}{24t}$, $q_2^{sslss} = q_4^{sslss} = \frac{1}{4} - \frac{\Delta c}{6t}$ and $q_3^{sslss} = \frac{1}{4} + \frac{5\Delta c}{12t}$ and profits $\pi_1^{sslss} = \frac{M(\Delta c - 3t)^2}{288t} - f_s$, $\pi_2^{sslss} = \frac{M(2\Delta c - 3t)^2}{144t} - f_s$ and $\pi_3^{sslss} = \frac{M(5\Delta c + 3t)^2}{144t} - f_l$.

To characterize the conditions under which $sslss$ is actually part of a SPNE, one has to rule out profitable deviations. Here, because of the symmetry of the candidate equilibrium configuration, only three such deviations have to be considered, namely those of firm 1 (equiv-
alent to that of firm 5), firm 2 (equivalent to firm 4) and firm 3. If firm 1 (or 5) deviates to the LF-sm technology, the resulting configuration is fslss (cslsf). We are not going to bother the reader with a complete description of this configuration, because, to out end, suffices to notice that, in the case of deviation the profit to a niche firms $\pi_{fslss}^1 = \pi_{cslsf}^5 = M(19\Delta c + 14t)^2 - f_t$. It is immediate to observe that $\pi_{sslss}^5 = \pi_{sslss}^1 \geq \pi_{fslss}^1 = \pi_{cslsf}^5 \iff \Delta f \geq \frac{M(19\Delta c + 14t)^2}{6272} - f_t$.

Similarly, if one of the intermediate firms deviates to the LF-sm technology, it reaps a profit equal to $\pi_{cclls}^4 = \pi_{cllsc}^2 = M/(784t)[6\Delta c + 7t]^2 - f$, which is no larger than that of the candidate equilibrium if $\pi_{fslss}^4 = \pi_{sslss}^2 \geq \pi_{cllsc}^2 = \pi_{cclls}^4 \iff \Delta f \geq 4\Delta c(2\Delta c + 21t)/(441t)$. It is a matter of simple algebra to ascertain that the most stringent non-deviation condition is the latter. Finally, if the central firm deviates to the SF-lm technology, it enjoys a profit equal to $\pi_{3sssss} = t/16 - f_s$, which is no larger than the candidate equilibrium one if $\pi_{3sssss}^3 \geq \pi_{3sssss}^3 \iff \Delta f \leq 5cM(5\Delta c + 6t)/(144t)$. By combining this condition with the preceding one we obtain that sslss is part of a SPNE for $4\Delta c(2\Delta c + 21t)/(441t) \leq \Delta f \leq 5cM(5\Delta c + 6t)/(144t)$, which corresponds to the green region in figure 4. By repeating this procedure, one can identify all the regions depicted. It is straightforward to observe that this figure is qualitatively similar to Figure 1a, with the relevant exception that, in the region where sslss is a part of a SPNE, a subregion exists (the green-purple one), where also slscc/cclsls are part of a SPNE.

Nonetheless, the figure clearly shows that there is a relevant parameter constellation where, following –say– an increase in c for given f, the central firm 3 is the first to switch to the

Figure 4: Equilibrium technology choice with 5 firms.
LF-sm technology, although it does not enjoy a larger "natural demand" than firms 2 and 4.

F Proof of Proposition 7

(a) Simply follows from the observation that under the symmetric configurations sss and lll a binding price cap does not alter the relative prices of the goods, thus does not affect the demands (and hence transport and productions costs) but only redistributes surplus from the producers to the consumers.

(b) As for total welfare, see the main text. As concerns consumer surplus, one has

\[
CS = M \left\{ V - \int_0^{x_{12}} (tx) \, dx - \int_{x_{12}}^{x_{23}} (t|1/2 - x|) \, dx - \int_{x_{23}}^1 [t(1 - x)] \, dx \right\},
\]

which, evaluated ad \( p_1 = p_3 = \bar{p} \) and \( p_2 = (\bar{p}) \) is \( CS^{sls}(\bar{p}) = MV \times \frac{M}{32} \left( 5t + 4\bar{p}^2/t - 4\bar{p} \right) \).

Its derivative with respect to \( \bar{p} \) is \( M(t - 2\bar{p})/(8t) \) which, computed at \( \bar{p} = p_1^{sls} - \epsilon \), is \(-M(2c - 3\epsilon)/(12t) < 0\) for \( \epsilon \) small enough, entailing that \( CS^{sls}(\bar{p}) \) is locally increased by a reduction on \( \bar{p} \).

(c) Under sll (lls is obtained by a suitable change in variables) total welfare is

\[
W^{sll} = M \left\{ V - \int_0^{x_{12}} (c_x + tx) \, dx - \int_{x_{12}}^{x_{23}} (c_1 + t|1/2 - x|) \, dx - \int_{x_{23}}^1 [c_1 + t(1 - x)] \, dx \right\} - 2f_1 - f_s.
\]

A price cap \( \tilde{p} \) binding on \( p_1 \) only leads to the following best replies for firms 2 and 3

\[
p_2(\tilde{p}, p_3) = (2c_l + \tilde{p} + p_3 + t)/4, \quad p_3(p_2) = (2c_l + t + 2p_2)/4,
\]

resulting in the (regulated) equilibrium price triplet \( p_1 = \tilde{p} \), \( p_2 = (1/14)(10c_l + 4\tilde{p} + 5t) \), \( p_3 = (1/7)(6c_l + \tilde{p} + 3t) \). At these prices, the marginal consumers are \( \tilde{x}_{12}(\tilde{p}) = 3/7 - (5\tilde{p} - 5c_l)/(14t) \) and \( \tilde{x}_{23}(\tilde{p}) = (1/14)(11 - (\tilde{p} - c_l)/t) \). So that welfare under a price cap biting on firm 1, in configuration sll is

\[
W^{sll}(\tilde{p}) = MV - \frac{M \left[ 14c_s(5c_l + 6t - 5z) - 44c_l^2 + 6c_l(23t + 3\tilde{p}) + 31t^2 - 26t\tilde{p} + 26\tilde{p}^2 \right]}{196t} - 2f_1 - f_s.
\]

Its derivative w.r.t. \( \tilde{p} \) is equal to \( M[35c_s - 9c_l + 13(t - 2\tilde{p})]/(98t) \) and evaluated at \( \tilde{p} = p_1^{sll} - \epsilon \) is \( M(119\Delta c + 156\epsilon)/(588t) > 0 \).

As for consumer surplus, evaluating \((28)\) at \( \tilde{x}_{12}(\tilde{p}) \) and \( \tilde{x}_{23}(\tilde{p}) \) returns

\[
CS^{sll}(\tilde{p}) = MV - \frac{M \left[ 26c_l^2 - 26\tilde{p}(2c_l + t) + 26c_l(t + 31t^2 + 26\tilde{p}^2) \right]}{196t}.
\]
Proceeding as above, its derivative w.r.t. $\bar{p}$ is $13M(2c_t + t - 2\bar{p})/(98t)$, which, evaluated at $\bar{p} = p_1^{ul} - \epsilon$ gives $-13M(7\Delta c - 12\epsilon)/(588t) < 0$ for $\epsilon$ small enough.

References


