Phys. Lett. B 869 (2025) 139853

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

Letter

Bounded compactness from G(E)UP

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ARTICLE INFO

Editor: Dr. A. Ringwald

Keywords:
Generalised uncertainty principle
Generalised extended uncertainty principle
Compactness
Self-gravitating systems

ABSTRACT

We analyse how different Generalised Uncertainty Principles could place bounds on the compactness of self-gravitating systems. By considering existing experimental bounds on the relevant parameters, we conclude that the compactness of large astrophysical objects is bounded above by the inverse of the GUP parameter, which would naturally be of order one. Conversely, the existence of black holes imposes stronger bounds on those parameters.

1. Uncertainty principles and quantum gravity

The nature of gravity and quantum physics seem to imply that physical measurements cannot probe arbitrarily short length scales [1,2]. Typically, a minimum scale is assumed of the order of the Planck length on dimensional grounds and considering that black hole geometries are supposed to emerge in scattering processes at Planckian energies [3]. However, semiclassical studies of collapsing compact objects (possibly of astrophysical size), generically predict a bounce at a scale that remains largely undetermined [4], and corresponds to a quantum transition from the formation of a black hole to a white hole geometry [5]. Moreover, a quantum description of the (very large number of) particles forming such objects shows that the ground state at the bounce could be a (significant) fraction of the horizon radius, hence many orders of magnitude larger than the Planck length [6]. These results suggest that quantum physics might not just imply the existence of a minimum length but also of a maximum compactness for self-gravitating objects [7].

A rather straightforward way of implementing a minimum length in quantum mechanics is to consider a Generalised Uncertainty Principle (GUP) for the position x and momentum p of a particle, which can be written as

$$\Delta x\,\Delta p \geq \hbar\left(1+\alpha\,\frac{\Delta p^2}{m_{\rm p}^2}\right), \tag{1.1}$$

where the dimensionless parameter α denotes the additional uncertainty introduced by the formalism and $m_{\rm p}$ is the Planck mass. The minimum length is then given by $\ell_{\rm min}\sim\sqrt{\alpha}\,\ell_{\rm p}$, where $\ell_{\rm p}$ is the Planck length. The literature is replete with studies devoted to the GUP, mostly as they relate to the quantum gravity regime. The GUP itself is a common feature of a variety of quantum gravity theories, including string theory [3,8–12], loop quantum gravity [13,14], ultraviolet self-complete gravity [15], non-commutative quantum mechanics [16,17], and generic minimum length models [18–20]. A full summary of GUP black hole related works would be exhaustive, but the interested reader is referred to [21–28] and references therein. A recent work [29] provided a comprehensive study of quantum and classical constraints on the GUP parameter as it relates to the "$M+1/M$" model presented in [28]. Such GUP effects are strictly relegated to the domain of Planck-scale black holes and their thermodynamics, so this work will provide a new perspective on the GUP as it relates to astrophysical scale black holes.

Compared to the GUP, the literature on the Extended Uncertainty Principle (EUP) is sparse, but has been growing steadily over the past few years. As a complement to the GUP, the EUP introduces a minimum momentum scale, or alternatively a maximum length scale, at which quantum gravity effects will manifest themselves. This is written as

$$\Delta x\,\Delta p \geq \hbar\left(1+\frac{\bar\beta}{L_*^2}\,\Delta x^2\right), \tag{1.2}$$

where L_* is some fundamental large length, possibly, but not necessarily, of the Hubble scale. In fact, the EUP implies a minimum momentum, corresponding to a maximum length scale. A number of studies have therefore considered large scale EUP effects on black holes [30–34].

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https://doi.org/10.1016/j.physletb.2025.139853
Received 24 July 2025; Received in revised form 25 August 2025; Accepted 27 August 2025
Available online 29 August 2025
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The weak field limit of an EUP-modified Schwarzschild black hole metric was also derived in [34], where it was suggested that the length scale L_* can fix the flattening radius of galactic rotation curves, and thus provide an alternative to dark matter. Additional related studies include the effect of the EUP on strong lensing effects in Sgr A* and M87 [35], the Shapiro time delay, Mercury's perihelion shift, and S2 star orbit precessions [36]. Ref. [37] also explored lensing effects, in addition to solar spin precession and the motion of binary pulsar systems. Lastly, the EUP's influence on cosmology has been addressed, namely through connections with the Hubble tension [38] and the cosmological constant been addressed [39]. Most recently, the authors of [40] have shown the EUP to be emergent from non-local gravity effects.

The combination of the GUP and the EUP is known as the Generalised Extended Uncertainty Principle (GEUP), and assumes the form

$$\Delta x\,\Delta p \geq \hbar\left(1+\alpha\,\Delta p^2+\frac{\bar\beta}{L_*^2}\,\Delta x^2\right), \tag{1.3}$$

The main aim of the present work is to see if the existence of a minimum length of the order of the Planck scale generically implies bounds on the compactness of self-gravitating systems. To investigate this question, we will consider the GUP [3] and the GEUP [30] as effective ways to incorporate the existence of a minimum length in quantum mechanics by modifying the Heisenberg Uncertainty Principle (HUP).

2. Compactness and quantum uncertainties

We describe a compact object by means of a quantum wavefunction $\psi=\psi(x_i)$ with Cartesian coordinates $x_i=(x,y,z)$ in its centre-of-mass reference frame, so that

$$\langle\psi\mid\hat{x}_i\mid\psi\rangle\equiv\langle\,\hat{x}_i\,\rangle=0=\langle\,\hat{p}_i\,\rangle. \tag{2.1}$$

The restriction to the centre-of-mass reference frame should remove unphysical deviations from well-established classical dynamics [41], an issue known as the "soccer ball problem" [1]. We then have the uncertainties

$$\Delta x_i^2\equiv\langle\,\hat{x}_i^2\,\rangle-\langle\,\hat{x}^i\,\rangle^2=\langle\,\hat{x}_i^2\,\rangle \tag{2.2}$$

and

$$\Delta p_i^2\equiv\langle\,\hat{p}_i^2\,\rangle-\langle\,\hat{p}_i\,\rangle^2=\langle\,\hat{p}_i^2\,\rangle. \tag{2.3}$$

Furthermore, we assume spherical symmetry, which implies

$$\Delta x_i^2=\frac{R^2}{3}\,, \tag{2.4}$$

and the distribution of momentum uncertainties is isotropic,

$$\Delta p_x^2=\Delta p_y^2=\Delta p_z^2=\frac{M^2}{3}\,, \tag{2.5}$$

where the mass of the object is given by the dispersion relation $M^2\simeq p_x^2+p_y^2+p_z^2$.

The (dimensionless) compactness of a star or black hole is defined as the ratio[1]

$$C=\frac{G_{\rm N}\,M}{R}=\frac{\ell_{\rm p}\,M}{m_{\rm p}\,R}\,, \tag{2.6}$$

where now M and R are obtained from Eqs. (2.5) and (2.4). It is important to remark that we employed Cartesian coordinates in the above argument because all of the operators $\hat{x}_i$ and $\hat{p}_i=-i\,\hbar\,\partial_i$ are Hermitian [on square integrable functions $\psi=\psi(x_i)$ of the real numbers]. Although spherical coordinates would seem a more natural choice to describe spherically symmetric systems, the operator $\hat{p}_r=-i\,\hbar\,\partial_r$ is not Hermitian, and it would not be clear how to implement uncertainty relations to bound the compactness in that frame.

[1] From now on, we mostly use units with $\hbar=\ell_{\rm p}m_{\rm p}$, and $G_{\rm N}=\ell_{\rm p}/m_{\rm p}$.

3. Upper bound from GUP

The simple GUP is given by Eq. (1.1), which we rewrite as

$$\Delta x\,\Delta p\gtrsim\ell_{\rm p}\,m_{\rm p}\left(1+\alpha\,\frac{\Delta p^2}{m_{\rm p}^2}\right), \tag{3.1}$$

and assume that α is positive. In the extremal case, in which the equality holds, there are two solutions for Δp, to wit

$$\Delta p_\pm=\frac{m_{\rm p}\,\Delta x}{2\,\alpha\,\ell_{\rm p}}\left(1\pm\sqrt{1-4\,\alpha\,\frac{\ell_{\rm p}^2}{\Delta x^2}}\right). \tag{3.2}$$

Note that Δp_- reproduces the HUP in the limit $\alpha\to0^+$ (or for $\Delta x\gg\ell_{\rm p}$). This restriction will be taken into account to describe objects with size $R\sim\Delta x\gg\ell_{\rm p}$.

The above Eq. (3.2) for $\alpha>0$ yields complex momenta for Δx smaller than a minimum length determined by

$$\sqrt{1-4\,\alpha\,\frac{\ell_{\rm p}^2}{\Delta x^2}}=0\qquad\Rightarrow\qquad\Delta x=2\,\sqrt{\alpha}\,\ell_{\rm p}\equiv\ell_{\rm GUP}\,, \tag{3.3}$$

which is also called the remnant limit in the thermodynamic community [21,44]. This limit for the physical solution $\Delta p=\Delta p_-$ from Eq. (3.2) is shown in Fig. 1 for positive values of α. We recall that negative values of α do not imply the existence a minimum length scale. However, this case still results in objects with exotic behaviour such as an infinite lifetime [45].

The inequality in Eq. (3.1) can be solved in terms of Δx,

$$\Delta x\gtrsim\frac{\ell_{\rm p}\,m_{\rm p}}{\Delta p}\left(1+\alpha\frac{\Delta p^2}{m_{\rm p}^2}\right), \tag{3.4}$$

and replaced in the definition of the compactness from Eq. (2.6), to obtain

$$C\lesssim\frac{M^2}{3\,m_{\rm p}^2+\alpha\,M^2}\simeq\frac{1}{\alpha}\,, \tag{3.5}$$

where the approximate expression is for objects with mass $M\gg m_{\rm p}$. Assuming $\alpha\sim1$, the conclusion is that the GUP in Eq. (3.1) implies an upper bound for the compactness of order one. The unity of α is further reinforced by the results in Refs. [46–49].

It is important to recall that black holes are characterised by $C>1/2$, and Eq. (3.5) therefore allows for their formation only provided $\alpha\lesssim2$, otherwise the gravitational collapse would lead to horizonless exotic compact objects [42]. On the other hand, the limit $\alpha\to0$, for fixed M, yields

$$C\lesssim\frac{M^2}{3\,m_{\rm p}^2}\,, \tag{3.6}$$

which implies that the usual HUP does not place any significant bound on the compactness of astrophysical compact objects and formation of black holes with $M\gg m_{\rm p}$.[2]

4. Bounds from GEUP

We rewrite the GEUP in Eq. (1.3) as

$$\Delta x\,\Delta p\gtrsim\ell_{\rm p}\,m_{\rm p}\left(1+\alpha\,\frac{\Delta p^2}{m_{\rm p}^2}+\beta\,\frac{\Delta x^2}{\ell_{\rm p}^2}\right). \tag{4.1}$$

The additional term proportional to the dimensionless constant β is phenomenologically linked to a non-zero cosmological constant [30], which suggests the existence of a Hawking-Page-like phase transition. From a thermodynamical perspective, it is also noteworthy that the limit $\alpha\to0$

[2] It is somewhat intriguing that the r.h.s. of Eq. (3.6) bares a qualitative resemblance with Bekenstein's quantisation of the black hole area [43].

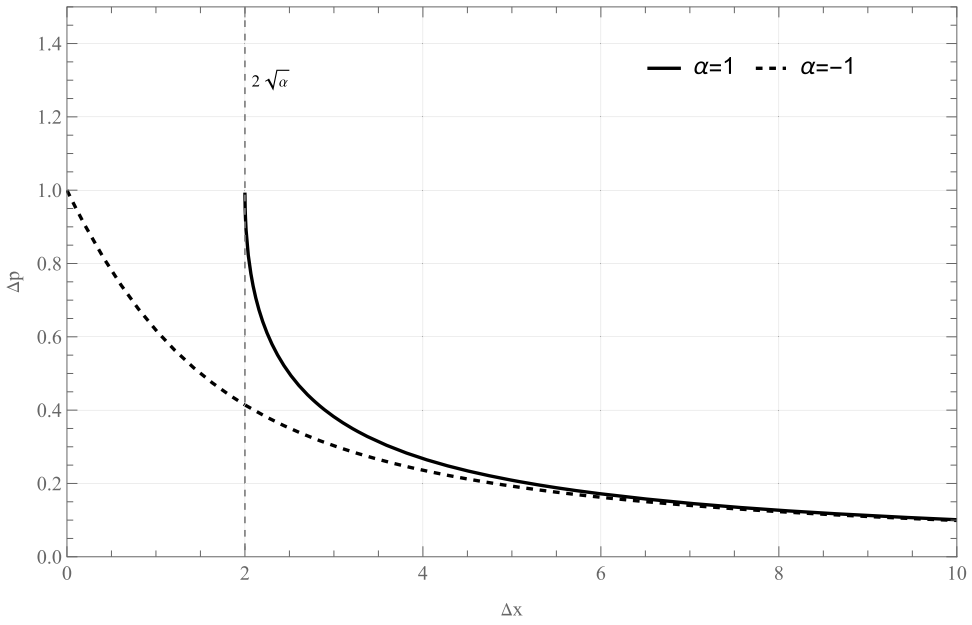

Fig. 1. $\Delta p = \Delta p_-$ from Eq. (3.2), with Δx in units of $\ell_{\rm p}$ and Δp in units of $m_{\rm p}$.

of the GEUP [that is, the EUP in Eq. (1.2)] exhibits a functional form identical to that of the Rényi entropy [50,51].

Again, in the extremal case there are two solutions

$$\Delta p_\pm = \frac{\Delta x\, m_{\rm p}}{2\,\alpha\,\ell_{\rm p}}\left(1 \pm \sqrt{1 - 4\,\alpha\,\beta - 4\,\alpha\,\frac{\ell_{\rm p}^2}{\Delta x^2}}\right), \tag{4.2}$$

where the solution Δp_- again reduces to the HUP for $\alpha \to 0^+$ and any finite value of β. Like previously, for positive α, we obtain a minimum measurable length determined by

$$\sqrt{1 - 4\,\alpha\,\beta - 4\,\alpha\,\frac{\ell_{\rm p}^2}{\Delta x^2}} = 0 \qquad \Rightarrow \qquad \Delta x = \frac{2\,\ell_{\rm p}\,\sqrt{\alpha}}{\sqrt{1 - 4\,\alpha\,\beta}} \equiv \ell_{\rm GEUP}\,, \tag{4.3}$$

which is real for $4\,\alpha\,\beta < 1$. This is the GEUP remnant limit, and the GUP case Eq. (3.3) is recovered for $\beta \to 0$. Note that the parameter space described above has already been extensively explored in the (near) quantum regime in Ref. [52].

Both solutions $\Delta p = \Delta p_\pm$ in Eq. (4.2) display a sort of *phase transition*, where the slope changes sign at

$$\Delta x_{\rm pt} = \frac{\ell_{\rm p}}{\sqrt{\beta - 4\,\alpha\,\beta^2}}\,, \tag{4.4}$$

again provided $\beta > 0$ and $4\,\alpha\,\beta < 1$. In the limit $\alpha \to 0$, we obtain $\Delta x_{\rm pt} = \ell_{\rm p}/\sqrt{\beta}$, while $\Delta x_{\rm pt}$ diverges for $\beta \to 0$, which is consistent with the GUP case of Section 3.

In Fig. 2, the minimum length values are highlighted for the solution $\Delta p = \Delta p_-$. We see that positive values of α lead to a minimum length, whereas positive values of β yield a phase transition. Negative

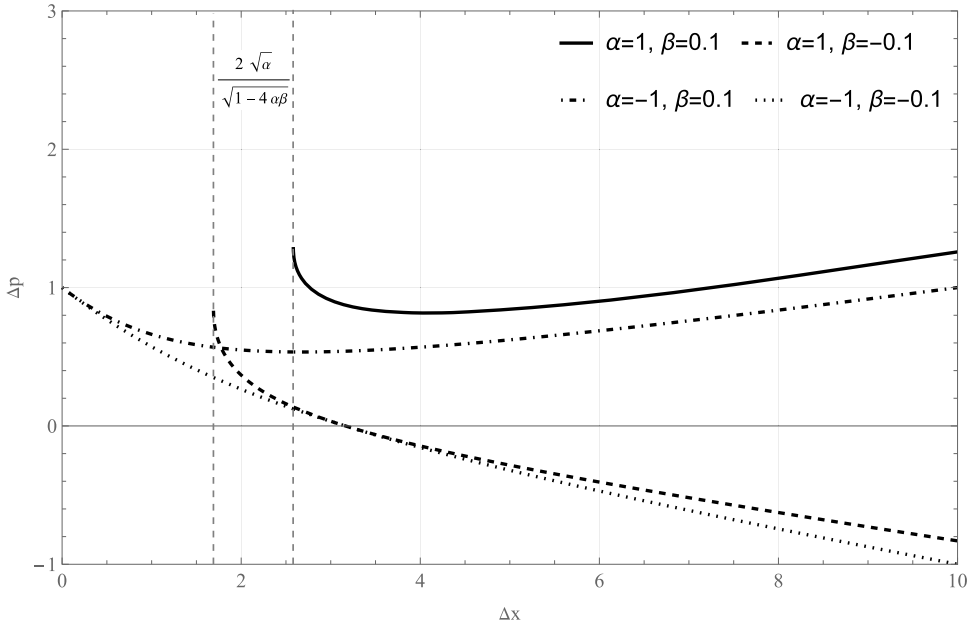

Fig. 2. $\Delta p = \Delta p_-$ from Eq. (4.2), with Δx in units of $\ell_{\rm p}$ and Δp in units of $m_{\rm p}$.

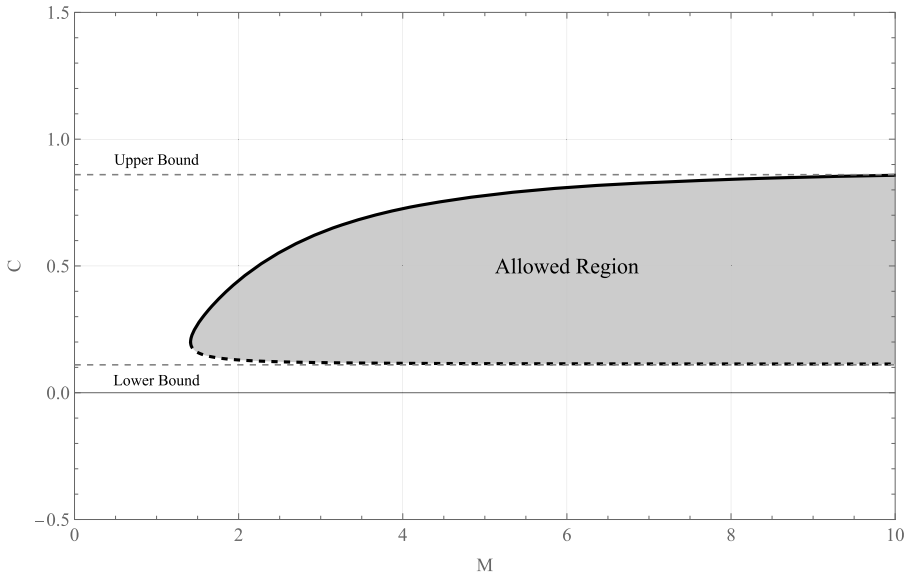

Fig. 3. Compactness as function of the mass M (in units of $m_{\rm p}$). Shaded region represents values allowed by the inequality Eq. (4.6) with $\alpha = 1$ and $\beta = 0.1$.

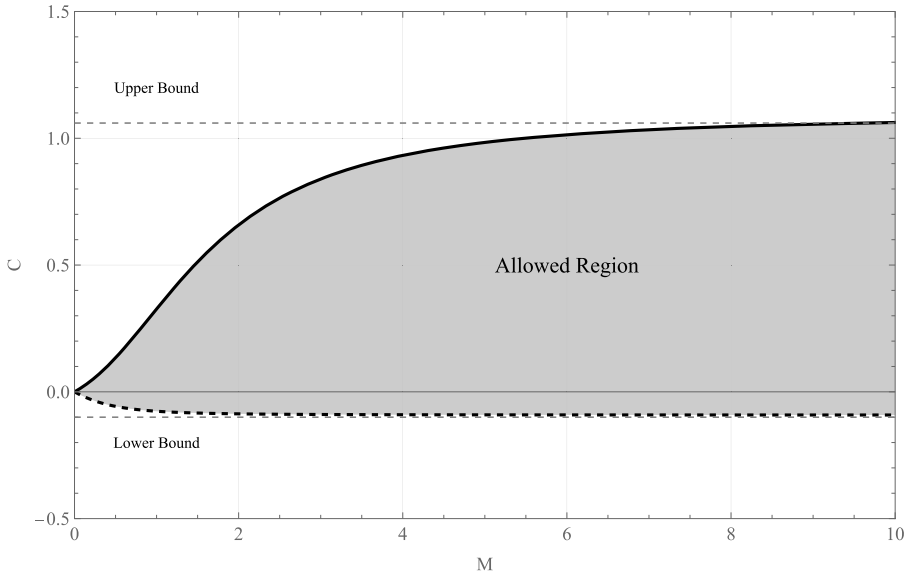

Fig. 4. Compactness as function of the mass M (in units of $m_{\rm p}$). Shaded region represents values allowed by the inequality Eq. (4.6) with $\alpha = 1$ and $\beta = -0.1$.

values of β, on the other hand, lead to unphysical negative momentum uncertainties.

Solving the inequality in Eq. (4.1) for Δx, we obtain the two constraints

$$\frac{\ell_{\rm p}\,\Delta p}{2\,\beta\,m_{\rm p}}\left(1-\sqrt{1-4\,\alpha\,\beta-4\,\beta\,\frac{m_{\rm p}^2}{\Delta p^2}}\right)\lesssim\Delta x \lesssim\frac{\ell_{\rm p}\,\Delta p}{2\,\beta\,m_{\rm p}}\left(1+\sqrt{1-4\,\alpha\,\beta-4\,\beta\,\frac{m_{\rm p}^2}{\Delta p^2}}\right), \tag{4.5}$$

with the solution Δp_- yielding the correct HUP limit this time for $\beta \to 0$. The corresponding compactness range is then given by

$$\frac{2\,\beta}{1+\sqrt{1-4\,\alpha\,\beta-12\,\beta\,m_{\rm p}^2/M^2}}\lesssim C\lesssim\frac{2\,\beta}{1-\sqrt{1-4\,\alpha\,\beta-12\,\beta\,m_{\rm p}^2/M^2}}\,, \tag{4.6}$$

which, in the large M limit, reduces to

$$\frac{1-\sqrt{1-4\,\alpha\,\beta}}{2\,\alpha}\lesssim C\lesssim\frac{1+\sqrt{1-4\,\alpha\,\beta}}{2\,\alpha}\,, \tag{4.7}$$

where we again assumed $\beta > 0$ and $0 \lesssim 4\,\alpha\,\beta < 1$. Note that the GUP bound (3.5) is recovered for $\beta \to 0$, whereas the limit $\alpha \to 0$ for finite $\beta > 0$ yields a lower bound $C \gtrsim \beta$.

In Fig. 3, we see that the compactness is asymptotically bounded from below and above by the values corresponding to inequality Eq. (3.5). Negative values of β also yield a valid upper bound displayed in Fig. 4 (of course, the lower bound is not physically significant for negative C).

5. Discussion and conclusions

In this work we have obtained bounds on the compactness of self-gravitating objects depending on the GUP and GEUP parameters α and β. Such parameters are in turn bounded by their effects on experimental

observations. For the GUP, one the strongest constraint $\alpha \lesssim 10^7$ comes from the study of harmonic oscillators [53].

For the EUP, bounds are typically placed on the associated length scale L_*. To our knowledge, the strongest of such bounds is obtained from the precession of S2's orbit and is given by $L_* \gtrsim 4\sqrt{\beta}\,10^{10}$ m [36], which results in

$$\beta \lesssim 10^{-90}. \tag{5.1}$$

For such small values of β, the upper bound from the GEUP in Eq. (4.7) essentially reduces to the GUP bound Eq. (3.5).

However, the Event Horizon Telescope collaboration observed the shadow resulting from the phenomena taking place near the event horizon of the supermassive black hole candidates in the centre of M87 [54] and in the centre of our Galaxy, Sgr A* [55]. Requiring that black holes do indeed form, one finds that $C \sim 1/2$ must be allowed, which then implies that values of $\alpha \sim 2$ should also be acceptable according to the GUP bound Eq. (3.5). Such cases correspond to a minimum length $\ell_{\rm GUP} \sim 3\,\ell_{\rm p}$, which is still in qualitative agreement with independent analyses, like those stemming from string theory [46–49].[3] Moreover, using the same argument, we can place independent bounds on the GEUP paramater β. Starting from the inequality on the right of Eq. (4.7) one can constrain β in terms of the value of α. When both parameters are positive and all the factors contained in the GEUP are real, the constraint which ensures that the maximum compactness is large enough to allow for black hole (horizon) formation yields $0 \lesssim \beta \lesssim 1/4\,\alpha$ for $0 \lesssim \alpha \lesssim 1$, or $0 \lesssim \beta \lesssim (2-\alpha)/4$ for $1 \lesssim \alpha \lesssim 2$, with values of α outside this range being excluded.[4]

Data availability

No data was used for the research described in the article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

R.C. is partially supported by the INFN grantnum GS501100004007 FLAG. The work of R.C. has also been carried out in the framework of activities of the National Group of Mathematical Physics (GNFM, INdAM) and COST action CA23115 (RQI). I.C. is supported by the Polish National Science Centre grant No. DEC-2020/39/O/ST2/02323 and Polish National Agency for Academic Exchange grant No. BPN/PRE/2022/-1/00068. O.M. is supported by the Romanian Ministry of Research, Innovation and Digitalization under the Romanian National Core Program LAPLAS VII-contract n. Romanian Ministry of Research 30N/2023. I.C. and J.M. thank the Department of Physics and Astronomy at the University of Bologna for hospitality during the initial stages of this project.

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[3] Smaller values of α are also possible, for instance according to the quantum model of Ref. [6] and in the context of bootstrapped Newtonian gravity where objects with stable cores (of larger compactness) can exist hidden beyond the horizon [56].

[4] Constraints using the minimum compactness can be imposed in the GEUP case, but we found those not to be relevant since they depend on an arbitrarily low defined value of this parameter.