



ALMA MATER STUDIORUM  
UNIVERSITÀ DI BOLOGNA

ARCHIVIO ISTITUZIONALE  
DELLA RICERCA

## Alma Mater Studiorum Università di Bologna Archivio istituzionale della ricerca

A Damage Detection Strategy Based on Autoregressive Parameters

This is the final peer-reviewed author's accepted manuscript (postprint) of the following publication:

*Published Version:*

Siddiqui, M.A., Zonzini, F., Quqa, S., Palermo, A., Landucci, M. (2024). A Damage Detection Strategy Based on Autoregressive Parameters. Cham : Springer [10.1007/978-3-031-61425-5\_3].

*Availability:*

This version is available at: <https://hdl.handle.net/11585/981078> since: 2024-09-04

*Published:*

DOI: [http://doi.org/10.1007/978-3-031-61425-5\\_3](http://doi.org/10.1007/978-3-031-61425-5_3)

*Terms of use:*

Some rights reserved. The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

This item was downloaded from IRIS Università di Bologna (<https://cris.unibo.it/>).  
When citing, please refer to the published version.

(Article begins on next page)

# A damage detection strategy based on autoregressive parameters<sup>\*</sup>

M. A. Siddiqui<sup>1,2</sup>[0009-0006-0499-6660], F. Zonzini<sup>2,3</sup>[0000-0002-2429-1469], S. Quqa<sup>1</sup>[0000-0001-6388-370X], A. Palermo<sup>1,2</sup>[0000-0001-9431-0461], and M. Landucci<sup>4</sup>

<sup>1</sup> Department of Civil, Chemical, Environmental, and Materials Engineering, University of Bologna, Bologna, Italy

<sup>2</sup> Advanced Research Center on Electronic Systems (ARCES), University of Bologna, Bologna, Italy

<sup>3</sup> Department of Electrical, Electronics, and Information Engineering, University of Bologna, Bologna, Italy

<sup>4</sup> SINT Technology, Italy

{mohammad.siddiqui3, federica.zonzini, said.quqa2, antonio.palermo6}@unibo.it, marco.landucci@sinttechnology.com,

**Abstract.** Structural Health Monitoring (SHM) based on Operational Modal Analysis (OMA) is pivotal in assessing the integrity of structures and infrastructures in dynamic regimes. However, the successful extraction of modal parameters and damage indexes through OMA typically relies on a dense network of sensors working synchronously. This research aims at alleviating this issue by resorting to autoregressive (AR) models computed at individual sensing locations for damage detection, paving the way to a fully decentralized monitoring approach. Such framework, in which sensors can extract AR parameters in an independent manner, is explored to alleviate the need for strict data synchronization, which is instead a typical requirement of OMA procedures. The Mahalanobis distance is then used in combination with the Receiver Operating Curve (ROC) as a damage indicator to identify potential anomalies upon aggregating the collected sets of AR features from different sensors. The methodology has been applied to a numerical model and a real steel bridge, comparing the performance of the proposed damage detection strategy with a traditional approach based on modal parameters. Results demonstrate that the proposed AR-based procedure can be very competitive over a pure natural frequency-driven alternative, reaching a classification score as high as 98% in both scenarios.

**Keywords:** Autoregressive models · Damage detection · Structural Health Monitoring · Vibration-based monitoring.

## 1 Introduction

Structural Health Monitoring (SHM) is defined as the comprehensive process of implementing strategies for the long-term and over-time health assessment of

---

<sup>\*</sup> Supported by TÜV AUSTRIA

mechanical assets via a rigorous protocol passing through damage identification, classification, and prognostics [1]. To achieve this goal, it involves the continuous monitoring of a set of damage-sensitive features, i.e., structural parameters. A considerable piece of research has been devoted to SHM in the last few decades, especially in the fields of aerospace, mechanical, and civil engineering [2]. In the latter context, the early identification of damages holds significant importance, especially in the maintenance and management of infrastructures (e.g, buildings, bridges, and other strategic infrastructures), since it is crucial to minimize repair costs, extend the service life of the infrastructure, and ensure its safety and reliability.

Among the various SHM techniques, Vibration-Based Monitoring (VBM) stands out as a well-established method for the analysis of dynamic systems. Its application is grounded on the assumption that damage induces alterations in structural stiffness and/or damping. To identify such changes, VBM tracks the variations of vibration-related parameters (e.g., natural frequencies) over time. However, typical modal identification methods (such as the Frequency Domain Decomposition–FDD, or the Stochastic Subspace Identification–SSI) require strict data synchronization and involve relatively complex computations that cannot be performed at the sensor level.

Among the alternative methods to perform system identification using time series, AutoRegressive (AR) models have become an effective tool due to their simplicity in constructing a parametric model of the structure [3]. Notably, AR models can be implemented in a decentralized manner by using data collected at each sensor node. Therefore, they can alleviate the above mentioned need for centralized processing. Given these premises, in this work, we propose an approach based on the use of AR parameters to perform damage detection directly from the latter.

The paper is organized as follows. Section 2 briefly describes the proposed methodology, while Section 3 presents the results obtained for two case studies, namely, a numerical model consisting of a 4 degrees-of-freedom (DOF) shear-type model, and a steel railway bridge, which serves to test the strategy in a real-life scenario. Concluding remarks are reported in Section 4.

## 2 Methodology

System identification based on AR models aims at estimating a set of parameters, also known as model parameters, that can capture the observed structural dynamics [4]. As such, they can be a powerful tool for SHM. The proposed strategy consists of exploiting the AR parameters themselves, without the need for extracting modal parameters, as it is conventionally done in VBM. To accomplish this task, several techniques, either statistical or based on machine learning approaches, can be applied to identify anomalies from a set of representative features. In the proposed framework, the Mahalanobis distance is used in combination with the Receiver Operating Curve (ROC).

Specifically, the Mahalanobis distance is a metric that serves to quantify the distance between a point and a distribution [5]. In our SHM-oriented workflow, this metric can be applied to evaluate whether the identified AR parameters deviate from a distribution obtained in a reference condition (e.g., at the beginning of the monitoring process), since this becomes a symptom of initiated degradation process affecting the target structure. The ROC [6] is, instead, a tool used to evaluate the performance of the classification model by reporting the trade-off between sensitivity (Probability of Detection–POD), and specificity (Probability of False Alarm–PFA). In this context, the so-called Area Under the Curve (AUC) of a ROC curve is a scalar value that quantifies this trade-off: AUC = 100% indicates perfect anomaly detection, while AUC = 50% means totally random prediction. Hence, our goal becomes reaching a score of AUC = 100% with AR as input features of the Mahalanobis distance.

A summary of the proposed damage detection method, depicted in Fig. 1, is reported herein:

1. Obtain the AR parameters from the acceleration signals acquired under ambient vibrations. This process can be realized in a decentralized fashion, with each sensor returning its set of AR parameters;
2. Repeat step 1 for successive monitoring times, both in undamaged (baseline) and potentially damaged configurations;
3. Compare the AR parameters obtained in the potentially damaged time intervals with those of the baseline using Mahalanobis distance;
4. Track the Mahalanobis distance over time and check whether it is below or above a user-defined threshold that discriminates between healthy and damaged states. In this phase, assuming to know the labels of damaged intervals, the ROC curve can be used as a performance metric to quantify the anomaly detection task.

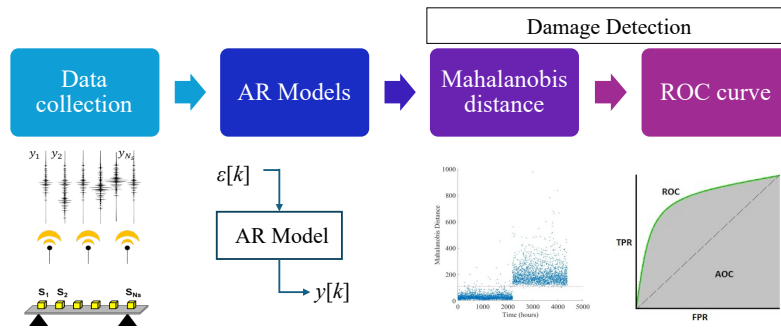


Fig. 1. Proposed approach of damage detection using AR parameters

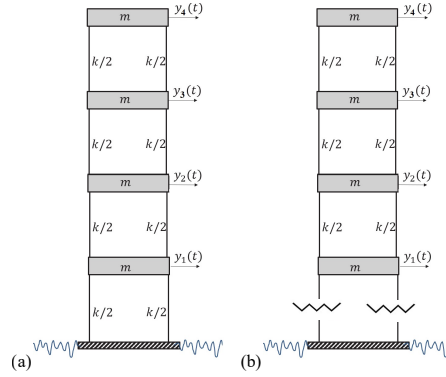
### 3 Experimental validation

This section presents the results of the proposed method for anomaly detection in two case studies. The objective is twofold:

- Analyze the performance of AR-based techniques in detecting anomalies;
- Compare AR-based approach with a more conventional VBM technique reliant on natural frequencies as damage-sensitive features.

#### 3.1 Numerical Case: Four Story Building

The first case study involves a simple numerical representation of a four-story building. This model represents a controlled environment, allowing assessment of the algorithm's effectiveness in identifying structural changes across different simulated conditions. A scheme of the numerical model is shown in Fig.



**Fig. 2.** (a) A four-story building model, (b) Depiction of location of loss of stiffness as damage to the model.

2. The model is assumed to have a mass of 1000 tonnes. The stiffness of the columns is modeled as a function of temperature to simulate the effects of seasonal changes due to temperature over one year. The temperature is varied daily from a mean temperature of  $25^{\circ}C$  to  $\pm 10^{\circ}C$ . Additionally, seasonal variations are modeled with temperatures in summers reaching  $40^{\circ}C$  and in winters around  $10^{\circ}C$ . Specifically, the following formulations have been employed:

$$T = T_d + T_s + T_m \quad (1)$$

$$T_d = 5 \sin\left(\frac{2\pi}{h}\right) \quad (2)$$

$$T_s = 10 \sin\left(\frac{2\pi}{N}\right) + 3X \quad (3)$$

where the temperature function  $T$  in Eq. (1) depends on i) the daily variation of temperature  $T_d$  in Eq. (2) with  $h$  the number of measurements per day (every 6 hours), ii) the seasonal variation of temperature  $T_s$  in Eq. (3) where  $N$  is the total number of measurements taken over one year and  $X$ , which adds a randomness to the dataset, is sampled from a uniform distribution, i.e.  $X \in [1-N]$ , and iii) the reference temperature  $T_m$  of  $25^\circ C$ . The relation between stiffness  $k$  and temperature  $T$  has been modeled as

$$k = k_o + (T - T_m)\beta k_o \quad (4)$$

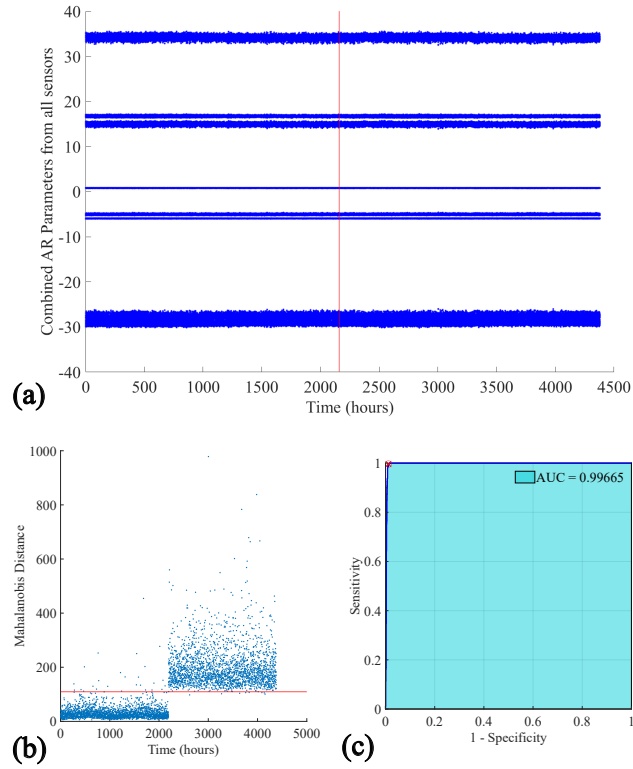
where  $k_o$  is the initial stiffness of the structure of  $10^6$  kN/m and scaling factor  $\beta$  is 0.01. In this case study, a damaged configuration is simulated through a 5% loss in stiffness in the columns below the first floor.

The structural response is generated at all the DOFs of the structure by supposing the structure is excited at its base via a random white noise stimulus. This condition is aimed at replicating ambient vibration scenarios. Data is collected with a sampling frequency of 100 Hz and for a duration of 5 minutes every 6 hours per day. A 1-year dataset was simulated for both healthy and damaged structures for a total amount of 2190 instances, half in nominal and half in defective configurations.

As a first test, the performance of the AR-based method is compared to that obtained using the procedure outlined in Section 2, based on a set of identified natural frequencies instead of AR parameters. To this aim, natural frequencies are computed by means of FDD [7] starting from the aggregation of raw vibration data in all the tested configurations. The anomaly detection results on natural frequencies report an AUC of 93.3%, which is an effective score considering that frequency variations due to temperature are around 3% for the fourth mode, and is even superior to that due to damage.

Given this initial outcome, the AR-based framework is applied to the same dataset. To improve the reliability of the AR model, a low-pass filtering stage is used as pre-processing step necessary to filter out the high frequency components and force the process to identify patterns only in the frequency region of interest. The appropriate model order has first been determined by means of the Bayesian Information Criterion (BIC) [8], resulting in a model order of 8.

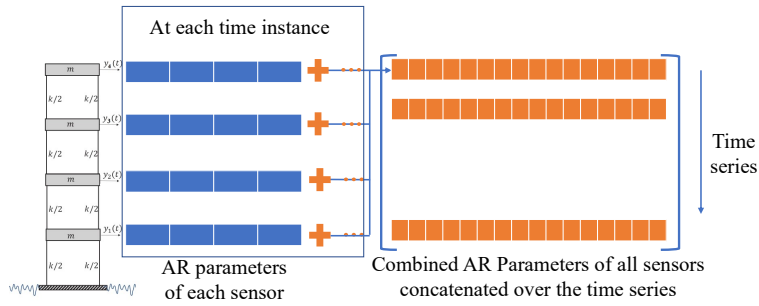
A set of AR parameters are calculated for each sensor and structural state over the entire set of data windows. Fig. 3 (c) depicts the AR parameters over time. For each case, undamaged and damaged, the AR parameters from all sensors are combined by being concatenated together before they are passed to the damage detection block. This is repeated for each time instant over the complete time series. Fig. 4 presents the scheme of steps taken to obtain the final set of combined AR parameters. The reason for preferring this all-embracing framework with respect to a sensor-driven scenario is that information from a single sensor was found to be insufficient for the distinction of the two classes. This evidence is coherent with the fact that some monitoring positions are more informative due to the higher energy of the vibration pattern at that location. Therefore, merging multiple parameter sets works as a compensation procedure.



**Fig. 3.** (a) Trend of combined AR parameters over time for both datasets, (b) Mahalanobis distance graph and (c) ROC curve using AR parameters from four sensors as damage indicator

Fig. 3 (b) shows the Mahalanobis distance of the undamaged and damaged dataset where it is observed that there is a clear distinction between the two datasets, and (c) displays the ROC curve which measures the performance of the classification shown in the mahalanobis distance. In this case, a value of  $AUC = 99.6\%$  is achieved, which is a paramount improvement in the classification performances when compared with the  $93.3\%$  of natural frequencies.

Analyses using more sophisticated representations, namely, the AutoRegressive with Moving Average (ARMA) models, are also performed. Differently from basic AR, ARMA adds a moving average term, which enables, in general, a more accurate encoding of the dynamics [9]. Applying the same methodology to ARMA parameters, with a modal order of 8, resulted in an  $AUC$  of  $98.29\%$ , which is proximal to the percentage measured for AR. This discrepancy in performance could potentially be attributed to the goodness of the model order selected in the two cases and its stability over time, which would require more accurate fine-tuning in the ARMA case.



**Fig. 4.** Schematic of how AR parameters from multiple sensors are combined

Besides AR/ARMA, which is suited for output-only scenarios, an AutoRegressive model with eXogenous input (ARX) is investigated. Specifically, the exogenous part of this model is selected as the structural response at the first floor of the structure. A model order equal to 4 is estimated as appropriate by the BIC. The superiority of ARX has been confirmed by an AUC proximal to 100%. Even though ARX has a much more appropriate response compared to AR and ARMA, it requires synchronization of at least two sensors. This is not the case for AR and ARMA models which can work in much more decentralized manner.

The results, summarized in Table 1, show that AR model and its variations, ARX and ARMA, outperformed the natural frequency-based approach and can be effectively used as damage-sensitive features for unsupervised VBM.

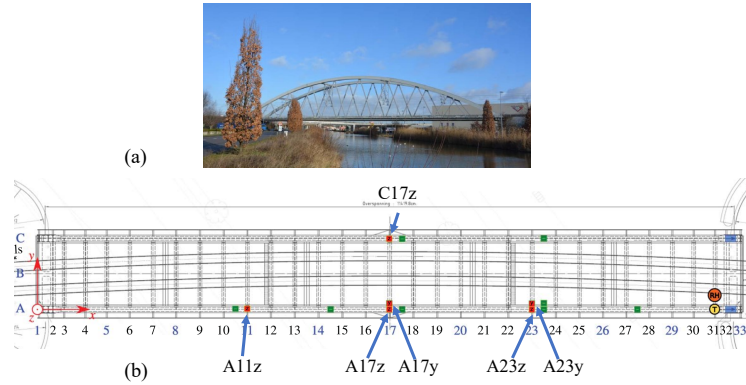
**Table 1.** Summary of AUC using different damage indicators

Damage-sensitive feature AUC (%)	
Natural Frequencies	93.39
AR parameters	99.66
ARX parameters	99.92
ARMA parameters	98.29

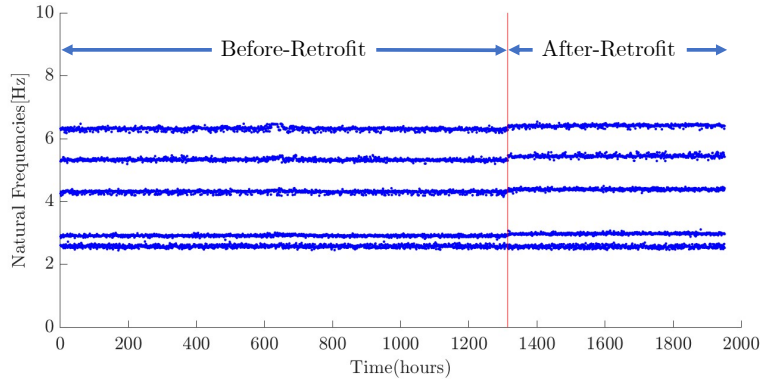
### 3.2 KW51 Bridge

The second case study shifts from theory to practice by considering a real steel bridge. Here, the focus is on assessing the capability of the method to scale and generalize to real use cases.

KW51 is a single-span steel arch railway bridge located close to the city of Leuven, Belgium. It is 115 m long and is part of 100 km Line 36 railway line



**Fig. 5.** (a) Kw51 Bridge, (b) Accelerometers installed on the bridge deck, indicated in the top view



**Fig. 6.** Natural frequencies over time for the before retrofit period and after-retrofit periods as indicated

that runs from Brussels to Liege. From May 15th to September 27th, 2019, the bridge was retrofitted to resolve a construction error. In the process, it was monitored extensively before, during, and after the retrofitting works. Throughout this period, a monitoring network has been installed on the structure, with the deployment plan shown in Fig. 5 where the selected sensors are accelerometers providing information on acceleration in vertical axis.

Fig. 6 depicts the estimated natural frequencies over time for both before-retrofit and after-retrofit periods, where the shift before and after retrofitting can be seen. To implement our AR-based methodology, the after-retrofit period dataset has been considered as the undamaged dataset, while the before-retrofit period dataset was representative of the damaged one since it relates to a defective structural configuration. A low-pass filtering has also been applied to raw vibration data.

**Results for ARX model** For the case where ARX parameters are used as damage indicators, a modal order of 16 is selected. Since ARX model requires data from two sensors simultaneously, different combinations of sensor pairs are tested and the one obtaining the highest fitness percentage is selected as final candidate for the analyses. The best fit is obtained by combining sensor A11z as input and sensor A17z as output. This may be justified by the fact that sensor A17z is at the center of the bridge and experiences the maximum deflection with respect to other sensing units placed in unfavorable locations. This is also seen in [10] where the authors pointed out that the sensor at the central point of the bridge is more informative about the global behavior of the structure. AUC of the ROC curve using ARX parameters as damage index scores to 100%.

**Results for AR model** A model order of 16 is also found compliant with AR modeling. It is observed that AUC of the ROC curve when AR parameters from all six accelerometers are exploited is 99.44%. Moreover, the possibility of exploiting only a subset of sensors and signals has been explored, verifying that parameters from a single sensor but on different axes, such as sensors at A17y and A17z, can also lead to interesting performance, as indicated by an AUC of 98.17%. The same, however, does not apply to AR parameters from the sensor at A23y and A23z which result in AUC of 80.16%, given the less favourable position of this sensor which is farther from the mid-span.

These results pave the way to an alternative scenario compatible with fully decentralized monitoring, in which it is not necessary to collect data from multiple sensors before moving to the diagnostic phase. Indeed, it opens a new perspective in which detection is computed by single sensors in a standalone manner by combining information from different directions, under the reasonable assumption that multi-axis accelerometers are installed on the sensing unit. This is a very light requirement, since most of the state-of-the-art monitoring systems are based on inertial measurement units, which measure at least tri-axis accelerations.

Lastly, a comparison with natural frequencies has also been performed, with results summarized in Table 2, reporting an AUC of 98.9% in front of an AUC of 99.44 % for the best AR score and 100% AUC in the case of ARX. The utilization of natural frequencies as damage-sensitive features demonstrates commendable performance in this case study due to the fact that, compared with the previous dataset, the shifts due to damage is significantly more pronounced for this structure rather than in the numerical case where the effect of temperature exceeds the one due to deterioration. Noteworthy, this outcome proves that the AR-based approach is potentially more robust to environmental variations, and this is a very desirable functionality for in-service applications.

## 4 Conclusion

We have proposed a damage detection procedure based on the variation of AR parameters computed from ambient vibration time-series recorded at the sensor

**Table 2.** Summary of AUC using different damage indicators

Damage-sensitive feature	AUC (%)
ARX parameters from A11z & A17z	100
AR parameters from all sensors	99.44
AR parameters from A17z & A17y	98.17
AR parameters from A23z & A23y	80.16
Natural Frequencies	98.98

nodes. The procedure exploits the Mahalanobis distance computed on the AR parameters to formulate a damage index for anomaly detection. The approach has been tested on numerical and experimental dataset showing performances comparable or even superior to the ones of VBM based on natural frequencies. Since AR models can be executed directly on sensor nodes, enabling a significant reduction in data transmission by only sending a few AR parameters at regular intervals, they introduce large benefits for the realization of more efficient monitoring systems without affecting the quality of the SHM process.

## References

1. Aktan, A.E., Catbas, F.N., Grimmelsman, K.A. and Tsikos, C.J.: Issues in infrastructure health monitoring for management. *Journal of Engineering Mechanics* **126**(7), 711–724 (2000)
2. Farrar, C. R., Worden, K.: An Introduction to Structural Health Monitoring. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **365**(1851), 303–315 (2007)
3. Entezami A, Shariatmadar H.: An unsupervised learning approach by novel damage indices in structural health monitoring for damage localization and quantification. *Structural Health Monitoring* **17**(2), 325–345 (2018)
4. Meixedo, A., Santos, J., Ribeiro, D., Calçada, R. and Todd, M.: Damage detection in railway bridges using traffic-induced dynamic responses. *Engineering Structures* **238**, 112189 (2021)
5. De Maesschalck, R., Jouan-Rimbaud, D., and Massart, D. L.: The Mahalanobis distance. *Chemometrics and intelligent laboratory systems* **50**(1), 1–18 (2000)
6. Fawcett, T.: An introduction to ROC analysis. *Pattern recognition letters* **27**(8), 861–874 (2006)
7. Brincker, R., Ventura, C. E.: *Introduction to Operational Modal Analysis*. John Wiley & Sons, 2015
8. Neath, A.A., Cavanaugh, J.E.,: *The Bayesian information criterion: background, derivation, and applications*. Wiley Interdisciplinary Reviews. *Computational Statistics* **4**(2), 199–203 (2012)
9. Zhang, C., Mousavi, A.A., Masri, S.F., Gholipour, G., Yan, K. and Li, X.: Vibration feature extraction using signal processing techniques for structural health monitoring: A review. *Mechanical Systems and Signal Processing* **177**, 109175 (2022)
10. Meixedo, A., Santos, J., Ribeiro, D., Calçada, R., Todd, M. D.: Online unsupervised detection of structural changes using train-induced dynamic responses. *Mechanical Systems and Signal Processing* (165), 108268 (2022)