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Published Version:
Juan-Vidal, F., Roca Martínez, A., Fantini, F., Cipriani, L. (2024). Symmetria, Mensuris and Domed Architectures in Hadrian’s Time. Cham : Springer [10.1007/978-3-031-57583-9_1].

Availability:
This version is available at: https://hdl.handle.net/11585/967656 since: 2024-04-16

Published:

DOI: http://doi.org/10.1007/978-3-031-57583-9_1

(Article begins on next page)

Symmetria, Mensuris and domed architectures in Hadrian's time

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Abstract. Recent decades have seen a renewed academic interest addressed to the study of centric-plan architectures covered with complex domes – built during the period of the High Roman Empire – thanks to new philological discoveries. In part, this trend is motivated by the dissemination of studies developed in recent years with digital documentation technologies, which have provided new information to understand the original shape, as well as the constructive and structural principles of many of these buildings. However, to understand the true novel features of these architectures, developed between the I and the II century AD, it is necessary to go beyond the simple use of new digital technologies for the documentation of architectural heritage by implementing a new method. This paper presents an interdisciplinary research approach (architecture, mathematics, archaeology, philology) that aims to understand how the three "species dispositionis", or "dispositio" (τάξις), "ordinatio", and "distributio" (οικονομία) were integrated to achieve a consistent and proper architecture with the required qualities (in particular, "symmetria") in the construction of domed buildings erected in that temporal context. To that end, in the paper we compare buildings, besides Vitruvius's one, also with the teachings by the Greek mathematician Hero of Alexandria and traces the application of his dimensional calculation formulae in the unique structures of Hadrian's Villa, recently surveyed with advanced digital technology.

Keywords: Heron of Alexandria; Hadrian's Villa; Domes; modular layout; digital survey.

1 Introduction and general context

Written sources on architecture (theory and practice) from the ancient world are quite limited. We usually refer to Vitruvius [1], or also the writings by Gromatici Veteres [2]. To decrease this philological limitation, the writings of Hero of Alexandria (10 - 70 AD) – famous author working in both the field of mathematics and mechanical engineering – should be added and integrated to those direct sources given their relevance to understanding ancient design processes. Most of Heron's writings appear to be like an operational manual, intended for architects/engineers to verify dimensions of construction elements in buildings such as domes, vaults, niches, basins, as well as other constituent elements of the vaulted spaces that it defines as "trikentron" (gores), widespread in the second century AD under Hadrian (Fig.1). Apparently, those calculations did not present the nature of a treatise [3]. Rather than a theoretical text, in fact, they survived in the form of numbered lists of calculations with simplified formulae. The same elements are calculated several times, but with different methodologies and measures.

(a) (b)

Fig. 1. Examples of Hadrianic domes from Baiae to Tivoli: (a) a quarter of a sphere from the bath complex known as the Temple of Venus; (b) a dome with wedges alternatively spherical and concave gores or "trikentron" from the Golden Court. 2023, Fantini, F.

Nowadays we can be aware of a large part of Heron's work thanks to the effort by the Danish philologist Johan Ludvig Heiberg (1854 -1928). Books IV and V of the German edition [4, 5], are of interest: Book IV contains Definitions and Geometrica, both oriented to the study of plane figures; Book V contains stereometry (Stereometrica I and Stereometrica II) and measurements (De Mensuris) (Fig. 2). It deals with a series of formulae and simplified procedures for calculating areas of plane figures, and surfaces and volumes of spherical figures (spheres, spherical caps, pendentives, quarter-spheres... with various thicknesses) and of other three-dimensional bodies (cones, pyramids...).

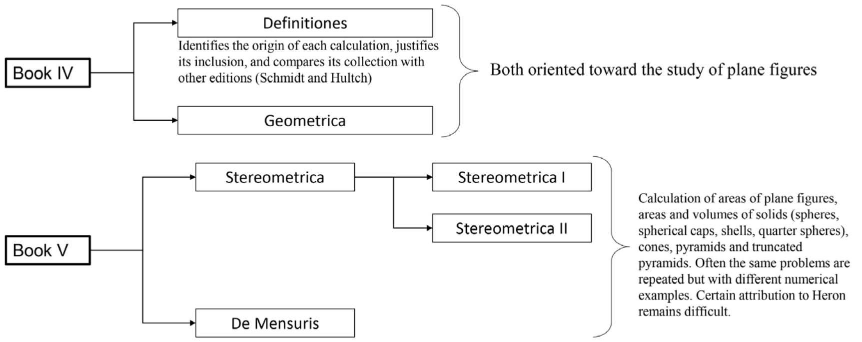

Fig. 2. Outline summarizing the contents of Heiberg's work on the texts of Heron, or attributed to him, 2023, Vidal, F., Roca, A., Fantini, F., Cipriani, L.

The aim of our research is to trace the possible use of Heron's calculation formulae in Hadrianic context, with special regards to the construction of domed buildings in *opus caementicium* [6,7], and to reveal their integration with the modular patterns and regulatory schemes used at the time for the layout of centralized buildings. Conti and Martines on Nero's Domus Aurea [8], and Svenshon [9] on Justinian buildings, have already begun a suggestive investigation based on a comparative reading of Heron's surviving texts. Today, we rely on the detailed documentation provided by the recent surveys carried out with advanced digital technology of the most singular architectures of Hadrian's Villa.

2 Discussion

The layout of classical architecture (and of the city too) was based on a general geometrical order in the form of a grid. The grid or network of squares, generated from a basic module, has been used in the past as a guide for the layout of architecture, especially for plans and made consistent with constructions based on irrational numbers generated using compass and regular polygons. It was common to start from modules of integer dimensions or simple numbers that gave rise to proportions based on simple, easily controllable arithmetical ratios. These quantities responded to the design logic of the buildings (ordinatio), but also fulfilled the ideal of simple proportionality between the parts that both Vitruvius and Archimedes referred to with the term "symmetria".
The layout of most of the domes of the Hadrianic period is related to the geometry of the octagon, which introduces dimensions based on the $\sqrt{2}$ (the diagonal of the square) and, therefore, on irrational numbers, at that time only controllable by drawing figures with ruler and compass [10]. There are countless studies that trace the use of regulatory schemes based on circles and rotated squares and successively inscribed for the centric-plan layout buildings in antiquity. The quadrature was already studied by authors such as Guglielmo De Angelis [11] for cases such as the octagonal rooms of the Baths of Caracalla, the Villa of the Gordiani or the Domus Augustana. Special reference should be made to the recent work of Demetrius Savvides on the Octagon of Thessaloniky [12] which provides archaeological evidence of the historical use of these regulatory layouts

based on circles and squares, by relating the plan of this building to the “carved” drawings on the lower part of a rectangular stone slab (61 X 134 cm) found at the site. On the other hand, in the dimensional field, the limitation to calculate surfaces and volumes characterized by circular shapes (due to obvious constraints in dealing with infinitesimal calculus) was overcome with the approximation of π to 22/7. It is not surprising, therefore, that in some of the examples he gives in his work, Hero made use of dimensions of radii or diameters that were multiples of 7 or 11. We propose to check to what extent they were present and, above all, how all these assumptions were interrelated, using as case studies singular buildings of Hadrian’s Villa recently surveyed with digital technologies: the vestibule of Golden Court; the octagonal hall of Small Baths; and the Serapeum-Canopus complex, a still ongoing study in collaboration with the Dipartimento di Ingegneria Civile e Ingegneria Inforrmatica at the Università degli Studi di Roma Tor Vergata. The entire Golden Court complex was designed on a square grid of 5.5-foot modules (11/2), evident in the rhythm of the pilasters against the cloister walls. The layout of the octagonal vestibule starts from a circle which diameter is the diagonal of a square of 6 modules in this grid (33 *pedes* on each side; 46.67 *pedes* in diameter = 33 x √2). From there the resulting outline is almost identical to the one demonstrated by Felipe Soler [13] for the early Christian chapel of San Aquilino in Milan. The average hemisphere – in the middle between intrados end extrados – has a diameter of 35 *pedes* (consistent with the approximation of π) since 33 pedes added to half of the cupola’s thickness (2 *pedes*) is 35 *pedes*. This result is also in line with the well-known solution given by Heron to obtain the volume of curved solids by multiplying the constant thickness of a geometric figure with either a two-dimensional profile or a three-dimensional surface (simple or double curvature) that are averaged with respect to the intrados and extrados [14]. The vault opens on each side of the octagon by means of concave gores (fig. 3).

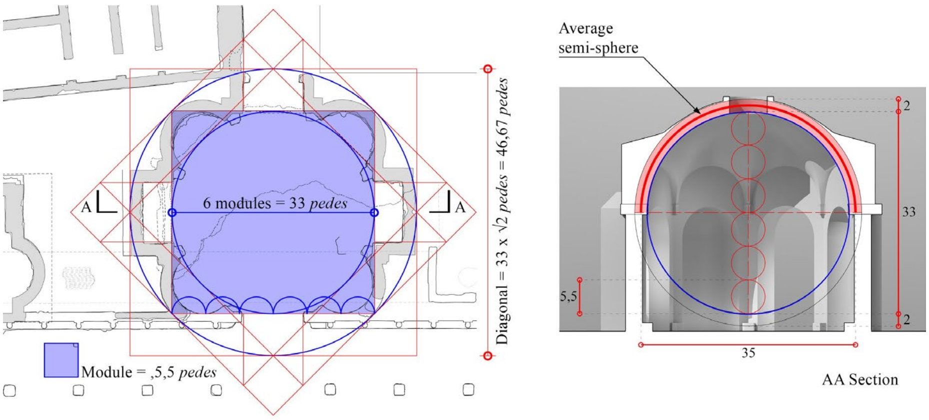

Fig. 3. Study of the layout and elevation of the octagonal vestibule of Golden Court in Hadrian's Villa (Tivoli), 2023, Vidal, F. Fantini, F.

The complex can be divided into simple constructive elements: flat walls, cylindrical walls, and spherical sectors in the vaults (caps and shells with their thicknesses), all of which can be dimensioned with the formulae provided by Heron in his writings. The lunettes could be dimensioned with the formula that the Alexandrian mathematician called trikentron: a geometric figure, like the pendentives of vaulted spaces, apparently similar to a spherical triangle whose sides are arcs of circumference which are particularly evident in better-preserved buildings such as the Serapeum at Hadrian's Villa and in the "twin" building of the Horti Sallustiani in Rome (fig. 4).

The Small Baths complex was built upon a square grid of 5 *pedes* module, as demonstrated in previous studies [15]. The 5x5 grid, in addition to generating dimensions of base 10, introduces the 7 in the layouts (the diagonal of the square of side 5 is approximately equal to 7). It is worth mentioning here the Pell series, already enunciated by Theon of Smyrna (2nd century AD) in his work "On mathematics useful for understanding Plato" [13], based on the number θ (1+√2).

The series starting from the square of unit diagonal is: 1, 3, 7, 17...; being 0'7, 2, 5, 12... the sides of the corresponding squares [16]. The layout of the octagonal space of Small Baths starts from a circle that takes as its diameter the diagonal of the square of 8 modules on each side of the mesh (40 *pedes* on each side; 56.57 *pedes* in diameter = 40 x √2). The dome is solved by following generatrices in the form of circles of 35.35 *pedes* in diameter, which pass through the keystone and are tangent to the ground (fig. 5). The abovementioned surfaces between the central spherical cap and the alternating flat and convex faces are solved with an irregular surface of complex curvature.

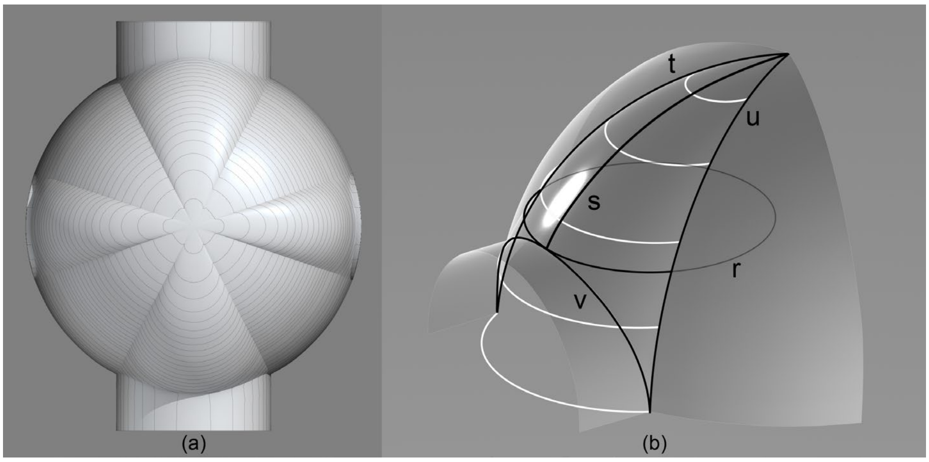

Fig. 4. (a) contour lines Horti Sallustiani dome. (b) Set of curves generating the trikentron, note the presence of a skewed curve (v) and circular horizontal sections (white), 2023, Fanini, F.

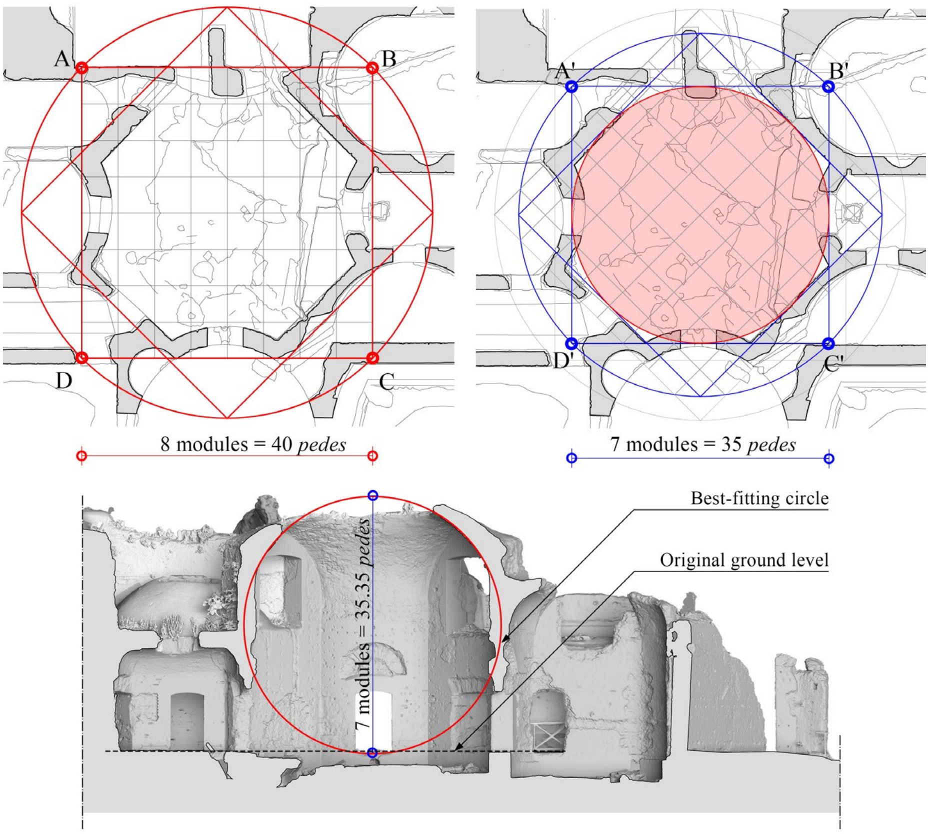

Fig. 5. Study of the layout of the octagonal room of Small Baths at Hadrian's Villa (Tivoli), 2023, Vidal, F., Fantini, F.

As in the previous case, the formulae provided by the Alexandrian mathematician enable the simplified calculation of the enveloping surfaces and the mass volumes of all the constructive elements that constitute this space. For irregular surfaces Heron proposes dividing them into rectangles or tesserae (segments) as the figure allows. When the surface is not flat, but irregular, he suggests wrapping it with a canvas or paper, which can then be stretched and measured. The complexity of this vault and the indication of this method of measurement by Heron suggests the possible existence of a model or scale model, prior to its construction, to which the procedure could be applied in the design phase. In the case of the Canopus, starting from the interaxle spacing of the north colonnade of the pool (about 14 *pedes*), we can assume that the traces started from a square grid of 7 *pedes* of module. By making the 7x7 grid coincide with the central axes of the exedra of the Serapeum, the tracing starts from a circle with a radius of 8 modules. By inscribing successive squares and circles alternately (quadrature), the plan traces of most of the elements that compose its architecture are obtained. Since the geometric progression of ratio √2 generates alternate terms (2n) that double its dimension, it is verified that the successive figures of the layout are again in conformity with the grid. Thus, the third circle of the series has a radius of 4 modules (56 *pedes* in diameter) and generates the trace of the intrados of the exedra (fig. 5).

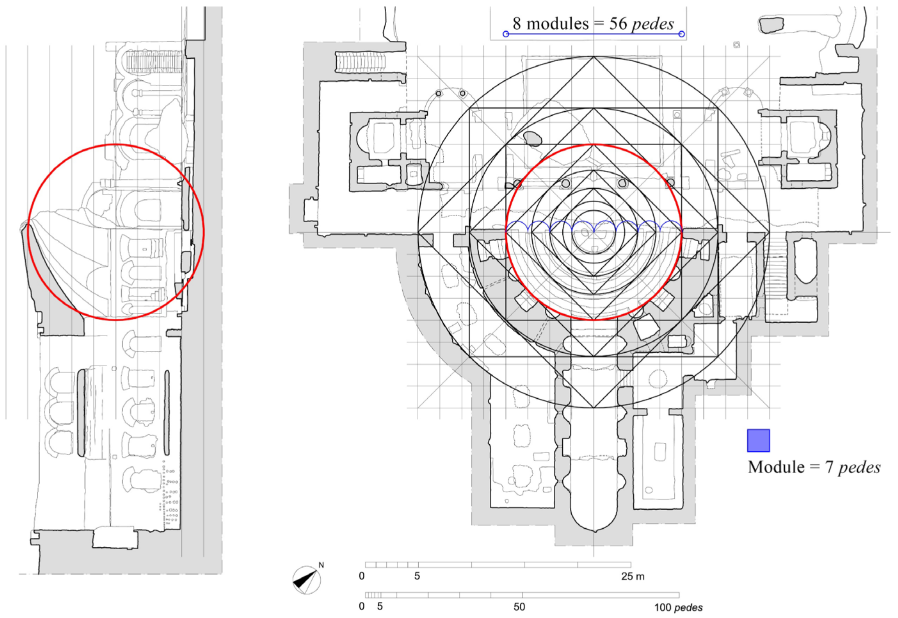

Fig. 5. Study of the layout of the Serapeum of Canopus at Hadrian's Villa, 2023, Vidal, F.

On the other hand, as in the previous cases, the simple elements that constitute this structure can be dimensioned in volume and surface area with Heron's formulae. Special mention should be made of the rounded sectors embedded in the intrados of the spherical semi-dome of the Serapeum, whose calculation was made possible by the Alexandrian mathematician with the formula of the trikentron.

3 Conclusions

It can be proved that, in the centric-plan architecture of Hadrianic age, there was a direct geometrical and dimensional relationship between the modular grid (ordination, symmetria), the regulating layout (dispositio) and the fundamental dimensions (commensus) of the building elements. In domed buildings, the module of the grid had a dimension related to the approximation of π: 22/7. In plans linked to the geometry of the octagon, the square was used as the regulating layout. The formulae prescribed by Heron of Alexandria made it possible to obtain the measurements of almost all the constructive structures that constituted these architectures. For domes with complex curvatures, the calculation procedures established by Heron suggested the existence of models or maquettes prior to the construction work, which would allow the measurements to be obtained at the design stage.

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