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# Effects of rubber shear modulus variability on the seismic response of isolated bridges

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ABSTRACT: The mechanical behavior of bridge decks equipped with High Damping Rubber Bearings is strongly affected by the rubber shear modulus that governs their lateral stiffness. It is a random variable, which can be described by a normal distribution.

In this paper, the effects of the variability of the shear modulus are investigated in terms of displacements for both the isolators and the piers, with reference to a case-study RC bridge with continuous caisson deck over four spans. The bridge is modelled with minimal systems with a reduced number of degrees of freedom along the longitudinal and transversal directions. Linear time-history analyses have been carried out using 7 natural seismic records, by considering 10,000 samples for the shear modulus fitting the chosen distribution. A statistical analysis has been then developed on the numerical results in order to obtain percentile values. Finally, Upper and Lower Bounds are identified for design purposes.

## 1 INTRODUCTION

The dynamic and seismic behaviour of bridges is governed by the connection system ("bearings layout") between the superstructure (deck) and the substructures (piers and abutments). Among all possible traditional and innovative solutions (Tubaldi et al. 2015, Silvestri et al. 2019), nowadays the seismic isolation (Naeim & Kelly 1999, Franchin et al. 2001) of the bridge deck with respect to the substructures is widely used for both the seismic design of new bridges and the seismic retrofit of existing ones. Even though in the last years the adoption of Curved Surface Sliders is fast increasing (Furinghetti 2022), High Damping Rubber Bearings (HDRBs) still represent a viable choice (De Luca & Guidi 2019). Design of HDRB isolation system is typically conducted by practitioners assuming a deterministic value of the shear modulus of the rubber material of the HDRB devices (Furinghetti 2022). However, it is a random variable with a non-negligible coefficient of variation, that can be reasonably described by a Gaussian distribution, as highlighted by the analysis of the results of several experimental campaigns carried out for commercial orders at various laboratories (Furinghetti & Pavese 2019). The objective of this paper is to investigate the effects, in terms of displacements, of the variability of the shear modulus of the rubber material of HDRBs and to eventually provide coefficients, to be applied to the displacement response parameter values obtained using the mean value of the shear modulus, for the estimation of Upper and Lower Bound coefficients, for design purposes.

## 2 THE CASE-STUDY BRIDGE

The considered bridge is characterised by a continuous deck over four equal spans, for a total length $L$ = 180 m. The three piers are characterised by different heights, equal to 8 m, 20 m and 10 m (Figure 1). The configuration is thus symmetric with reference to the longitudinal direction, while it is slightly eccentric with reference to the transversal direction. The RC caisson deck is 11.50 m wide and 0.50 m thick (Figure 2a), whilst the RC piers have a squared cellular cross-section with side length equal to 1.75 m and thickness equal to 0.30 m (Figure 2b). On the

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top of the piers there are RC prismatic transversal elements with a rectangular cross-section of 2.00 m x 1.90 m and a length of 6.40 m, which provide the support base for the isolators. There are two isolators for each pier and for the two abutments, for a total number of ten isolators. Concrete class is C40/50. The bridge is supposed to be located in L'Aquila, Italy. The weight of the RC structural elements is 37723 kN, whilst the weight of the non structural elements (slope screed and bituminous layers, such as waterproofing, binder and wear layer) is 10624 kN. Consequently, the total dead load of the bridge is 48347 kN.

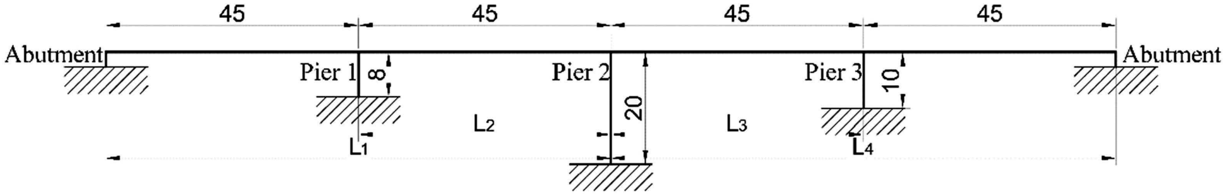

Figure 1. Schematised longitudinal view of the case-study bridge.

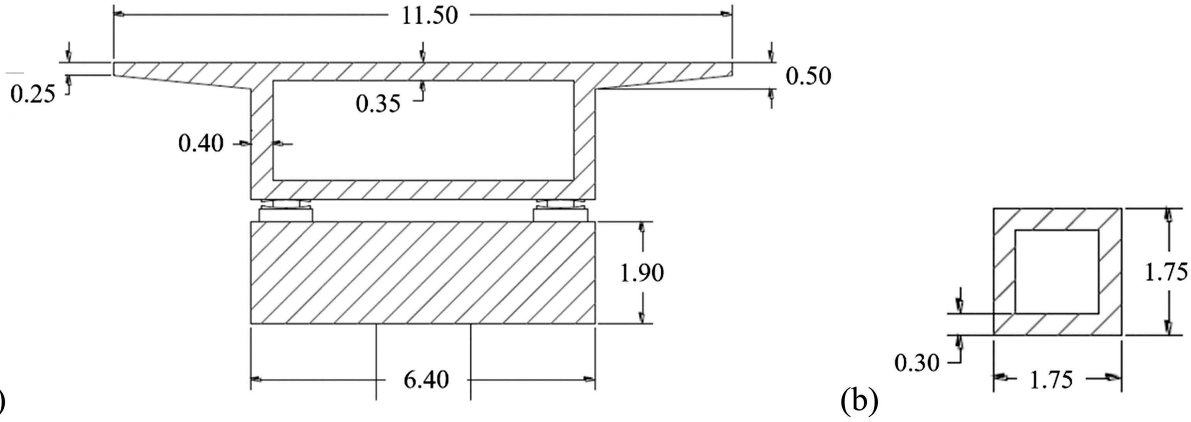

Figure 2. (a) Cross-section of the RC cassoin deck and pier top. (b) Cross-section of the three RC piers.

## 3 MODELLING OF THE ISOLATED BRIDGE

Different models are possible for the isolated bridge depending on the objective of the analysis: (i) a Single-Degree-Of-Freedom (SDOF) idealisation, representing the deck mass connected through the lateral stiffness of the isolation system to the substructures (assumed as fixed); (ii) minimal systems, that describe the behaviour of the deck and the piers by means of a reduced number of degrees of freedom (Silvestri et al. 2019); (iii) Finite Element models, that describe the bridge behaviour by means of several nodes and beam and/or shell elements.

The first one is usually sufficient for the dimensioning of the isolation system. The second one allows to perform time-history analyses with lower computational effort than the third one.

In the minimal systems, the masses that define the degrees of freedom in the equations of motion are the mass of the deck and the masses of each half pier (the remaining parts are assumed to be directly transferred to the ground base). Figure 3 shows the minimal systems used for the evaluation of the seismic response of the bridge along the two directions ($u$ referring to longitudinal displacements; $v$ referring to transversal displacements):

- along the (symmetric) longitudinal direction, the minimal system is composed of four degrees of freedom: the displacement of the deck ($u_{deck}$) and the three displacements of the top of the piers ($u_{pier1}$, $u_{pier2}$, $u_{pier3}$);
- along the (slightly eccentric) transversal direction, the minimal system is composed of five degrees of freedom: the displacement of the deck ($v_{deck}$), the three displacements of the top of the piers ($v_{pier1}$, $v_{pier2}$, $v_{pier3}$), and the in-plane rotation of the deck ($\varphi_{deck}$).

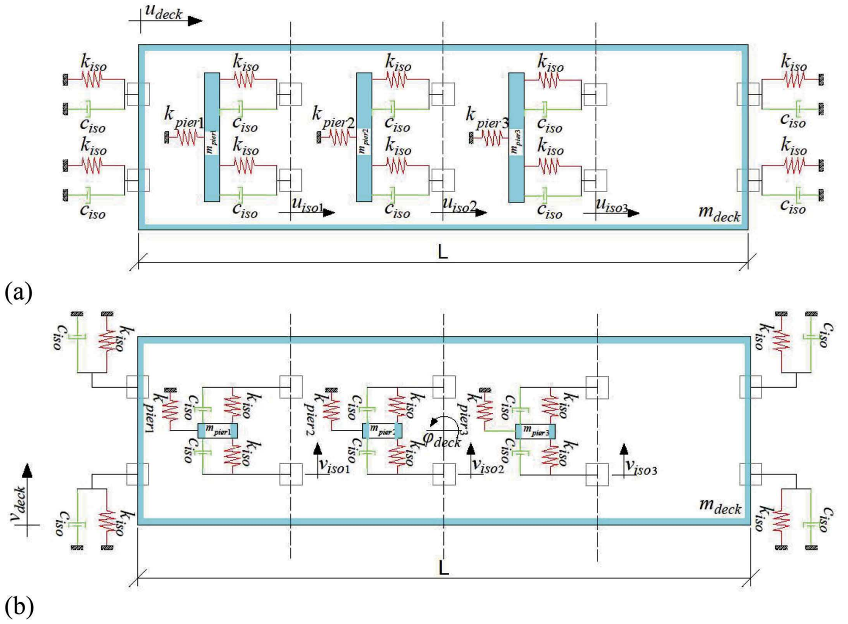

Figure 3. Minimal systems: (a) along the longitudinal direction and (b) along the transversal direction.

## 4 SEISMIC INPUT

The pseudo-acceleration elastic response spectrum of the horizontal component of the earthquake input is reported in Figure 4. Peak ground acceleration is equal to 0.459g. The maximum (plateau) spectral acceleration is equal to 1.103g. To perform time-history analyses seven natural earthquakes have been chosen in such a way as to result compatible with the 5% damped elastic spectrum.

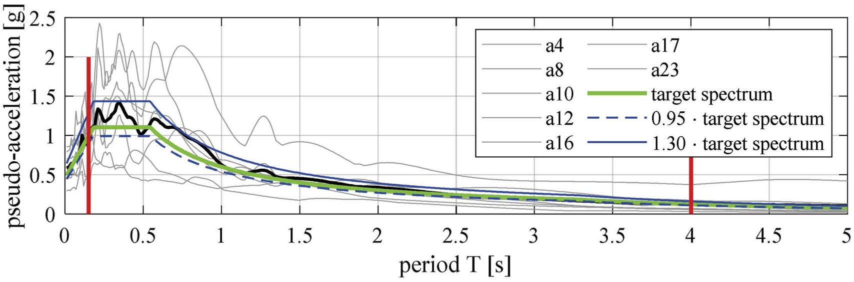

Figure 4. Horizontal elastic spectrum and spectra of the seven natural seismic records.

## 5 PROCEDURE ADOPTED FOR THE IDENTIFICATION OF THE HDRB ISOLATORS

The isolation system (to be placed between the deck and the substructures) has been pre-identified according to a simplified procedure based on the SDOF idealization, in which the degree of freedom is represented by the horizontal displacement of the deck. The SDOF model consists of the mass of the deck ($m_{deck}$) concentrated at the deck level. The stiffness ($k_{iso,tot}$) and the damping coefficient ($c_{iso,tot}$) are provided by the isolation system, which is made of ten devices. Since they work in a parallel system, the values of the stiffness and the damping coefficient of the equivalent SDOF model are given by the sum of the stiffness values and the damping coefficient values of the ten isolators, respectively.

The simplified procedure is based on the following hypotheses: (i) the deck is rigid in both horizontal (longitudinal and transversal) directions, (ii) the piers are rigid in the vertical (axial) direction, (iii) the target lateral deformation $\gamma_{iso}$ of the isolator is imposed to be equal to the height $h_{iso}$ of the isolator itself (i.e. shear deformation $\gamma_{iso} = d_{iso}/h_{iso}$ equal to 100%, where $d_{iso}$ is the displacement of the isolator), (iv) the shear modulus of the rubber material that governs the lateral stiffness of the isolator is assumed in the range 0.7-1.0 MPa (this corresponds to a normal rubber and to a damping ratio $\xi_{iso}$ of the isolation system around 10%), (v) the total damping coefficient provided by the isolation system is evaluated as: $c_{iso,tot} = 2 \cdot m_{deck} \cdot \omega_{iso} \cdot \xi_{iso}$ (where $\omega_{iso}$ is the fundamental circular frequency of the isolated bridge and $\xi_{iso}$ is the damping ratio), (vi) the isolators have radial symmetry (i.e. the procedure keeps its validity along the two horizontal directions). The procedure is summarized in the following steps:

1. Choice of the target period $T_{iso}$ for the isolated bridge, typically in the range 2.5-3.5 s.
2. Evaluation of the spectral acceleration corresponding to the target period: $S_a$ ($T_{iso}$). Note that it has to be evaluated on the 10%-damped elastic spectrum since an isolation system usually provides a damping ratio $\xi_{iso}$ of around 10%.
3. Evaluation of the spectral displacement:

$$S_d(T_{iso}) = S_a(T_{iso})/\omega_{iso}^2 = S_a(T_{iso}) \cdot (T_{iso}/2\pi)^2 \tag{1}$$

   Then, assuming that $\gamma_{iso} = d_{iso}/h_{iso} = 100\%$ leads to:

$$h_{iso} = d_{iso} = S_d(T_{iso}) \tag{2}$$

4. Choice of the diameter of the isolators in order to avoid instability phenomena. The ratio between diameter $D_{iso}$ and height $h_{iso}$ should be larger than 2.0 (Furinghetti & Pavese 2019, Furinghetti 2022). Therefore a typical relationship is:

$$D_{iso} = 2.5 \cdot h_{iso} \tag{3}$$

5. Calculation of the lateral stiffness of each single isolator $k_{iso}$, assuming that all isolators are equal to each other:

$$k_{iso} = \frac{m_{deck} \cdot \omega_{iso}^2}{n_{iso}} \tag{4}$$

   where $n_{iso}$ is the total number of isolators.
6. Check that the shear modulus $G$ falls within the assumed range (0.7-1.0 MPa):

$$G = \frac{4 \cdot k_{iso} \cdot \chi_V \cdot h_{iso}}{\pi \cdot D_{iso}^2} \tag{5}$$

   where $\chi_V$ represents the shear shape factor.
   If the hypothesis is not satisfied, then $G$ is taken equal to 1.0 MPa and the diameter of the isolator is calculated as:

$$D_{iso} = 2 \cdot \sqrt{\frac{k_{iso} \cdot \chi_V \cdot h_{iso}}{\pi \cdot G(= 1\text{MPa})}} \tag{6}$$

7. Check that the isolator shear displacement roughly respects the assumption $\gamma_{iso} = d_{iso}/h_{iso} = 100\%$ (made in Step 3).
8. Evaluation of the maximum vertical axial force on the isolators. They have to carry out the function of support during the entire life of the bridge both in static conditions, with reference to the Ultimate Limit State (ULS), and in seismic conditions, with reference to the Collapse Limit State (CLS). The axial force can be evaluated either according to tributary areas afferent to each isolator or by means of a numerical model.
9. Check that the isolators satisfy the two conditions: in terms of target lateral stiffness $k_{iso}$ (Steps 1-7) and in terms of axial force capacity (Step 8). If this latter is not satisfied, then the geometric characteristics of the isolators should be updated.

The procedure has been applied to the case-study bridge. The following data have been adopted: the elastic spectrum represented in Figure 4 reduced to account for a damping ratio of 10%, $m_{deck}$ = 4717 t, $T_{iso}$ = 3.25 s, $n_{iso}$ = 10. The following outcomes have been obtained: $h_{iso}$ = 420 mm, $D_{iso}$ = 1040 mm, $k_{iso}$ = 1.756 kN/mm, $G$ = 0.94 MPa, $c_{iso}$ = 167 kN m/s. However, the isolator obtained following Steps 1-7 does not satisfy the axial force capacity condition (Step 8). Therefore, the geometric characteristics have been slightly changed to meet this condition (the height has been reduced). The following final properties for each single isolator have been thus identified: $h_{iso}$ = 326 mm, $D_{iso}$ =

1000 mm, $k_{iso}$ = 2.99 kN/mm, $G$ = 0.8 MPa, $c_{iso}$ = 202 kN m/s, leading to $T_{iso}$ = 2.50 s for the SDOF idealisation, to $T_{iso}$ = 2.78 s for the minimal system along the longitudinal direction (roughly including the flexibility of the three piers), and to $T_{iso}$ = 2.92 s for the more refined FE model (also including the flexibility of the deck and a better representation of the piers behaviour).

## 6 ANALYSES PERFORMED

The effects of the aleatory variability in the shear modulus $G$ of the rubber material of the HDRB isolators have been investigated by means of time-history dynamic analyses carried out using the minimal systems detailed in Section 3 and the seven seismic records described in Section 4. The isolators identified in Section 5 are constituted by rubber layers with mean shear modulus $G$ equal to 0.8 MPa and equivalent viscous damping ratio of around 10%. Consequently, $G$ is assumed as a random variable characterized by a normal distribution with mean value equal to 0.8 MPa and coefficient of variation equal to 14.5% (as per the results of experimental campaigns carried out for commercial orders at various laboratories, see Furinghetti & Pavese 2019). The following assumptions have been made:

- The shear modulus is the parameter that mainly governs the mechanical behaviour of the isolator device.
- A linear relationship is assumed between the rubber shear modulus $G$ and the horizontal stiffness $k_{iso}$ of the isolator device:

$$G = \left(\frac{4 \cdot \chi_V \cdot h_{iso}}{\pi \cdot D_{iso}^2}\right) \cdot k_{iso} \tag{7}$$

  where the geometric parameters of the isolator have been considered as deterministic.
- The variability of the shear modulus has been taken into account by acting directly on the mean horizontal stiffness of the isolator device ($k_{iso,\mathrm{mean}}$ = 2.99 kN/mm corresponding to a mean shear modulus equal to 0.8 MPa) by means of an adimensional factor $g$, for which a normal distribution has been assumed:

$$k_{iso} = g \cdot k_{iso,\mathrm{mean}} \tag{8}$$

$$g = N(\mu = 1,\ \sigma = \mathrm{cov} = 0.145) \tag{9}$$

- A number of 10,000 realizations of the normal random variable corresponding to multiplication factor $g$ have been randomly generated and seven time-history simulations have been carried out (with the seven seismic records) for each realisation.
- The damping coefficient of the isolator devices has been updated for each realisation adopting the equivalent SDOF model and assuming a damping ratio equal to $\xi_{iso}$ = 10%:

$$c_{iso} = \frac{2 \cdot \xi_{iso} \cdot \omega_{iso} \cdot m_{deck}}{n_{iso}} \tag{10}$$

- The same damping coefficient $c_{iso}$ has been assumed in both longitudinal and transversal directions.

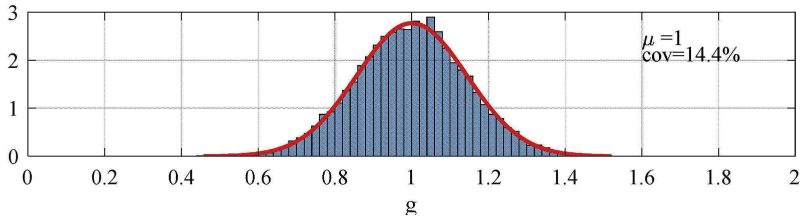

Figure 5. Histogram of the relative frequencies of the sample of the 10,000 realizations of factor $g$.

Figure 5 shows the histogram of the relative frequencies of the sample of the randomly generated 10,000 realizations, which have mean value equal to 1 and coefficient of variation equal to 14.4% (compared to the 14.5% target).

To sum up, a total number of 140,000 numerical simulations have been performed:

- 70,000 analyses along the longitudinal direction, conducted on 10,000 minimal models with 4 DOFs assuming the 7 natural accelerograms as input.
- 70,000 analyses along the transversal direction, conducted on 10,000 minimal models with 5 DOFs assuming the 7 natural accelerograms as input.

## 7 RESULTS OBTAINED

The results are expressed in terms of *mean displacement* of each degree of freedom of the two minimal systems along the two directions. For each direction, it is evaluated as the mean of the seven maximum values of the displacement obtained during the seven accelerograms. With reference to the nomenclature introduced in Figure 3, the attention has been paid on the following displacements:

- $u_{isoAB} = u_{deck}$ and $v_{isoAB} = v_{deck}$ are the displacements sustained by the isolators on the abutments that coincide with the displacements of the deck with respect to the abutment;
- $u_{iso1} = u_{deck} - u_{pier1}$ and $v_{iso1} = v_{deck} - v_{pier1}$ are the longitudinal and transversal displacements sustained by the isolators on pier 1 that coincide with the relative displacements of the deck with respect to the top of pier 1;
- $u_{iso2} = u_{deck} - u_{pier2}$ and $v_{iso2} = v_{deck} - v_{pier2}$ are the displacements sustained by the isolators on pier 2;
- $u_{iso3} = u_{deck} - u_{pier3}$ and $v_{iso3} = v_{deck} - v_{pier3}$ are the displacements sustained by the isolators on pier 3.

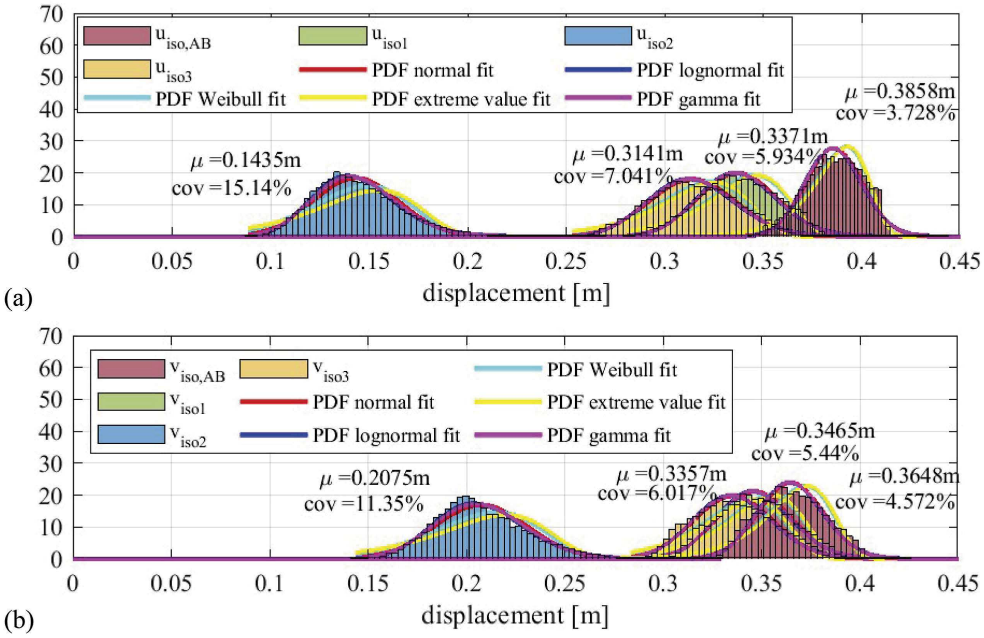

Figure 6. Results in terms of isolators displacements: (a) longitudinal direction, (b) transversal direction.

For each one of the above-mentioned displacements, a set of 10,000 *mean displacement* values (mean of the seven maximum values obtained for the seven accelerograms) are obtained. Each set is considered as a sample of a random variable whose distribution is unknown and has to be identified. A fitting has been then performed of the relative frequency histogram of each set of 10,000 values of the above-mentioned displacements with the Probability Density Function that better represents the set itself. The best fitting has been conducted by considering four well-known Probability Density Functions (PDFs): (i) Normal, (ii) LogNormal, (iii) Weibull, (iv) Extreme Values and (v) Gamma (Ang & Tang 2007). Figures 6a and b show an example of the results obtained in terms of isolators displacements, for the longitudinal and transversal directions, respectively. The best fit has been evaluated by means of two statistical tests: (i) the Chi-Square test and (ii) the Kolmogorov-Smirnov test (Ang & Tang 2007).

On the basis of the results of the numerical simulations (or through their best fits), also the percentile values associated with selected percentages can be obtained. Then, from the values of the percentiles, it is possible to calculate the coefficients that allow considering the variability of the rubber shear modulus on the structural response of the bridge. These coefficients can be applied to the displacements of the isolators obtained considering the mean value of the rubber shear modulus. In fact, as it can be seen from Figure 6, the coefficient of variation that characterizes the variability of the shear modulus does not keep unaltered in the variability of the response parameters, i.e. of the displacements of the deck, of the top of the piers, and of the isolator devices. Therefore, starting from the theoretical probability functions for each response parameter these coefficients can be calculated according to: $c_{x\%} = \delta_{x\%}/\delta_{Gmean}$, where $\delta_{x\%}$ indicates the percentile of the displacement response parameter characterized by a probability of non-exceedance equal to $x$%, and therefore by a probability of being exceeded equal to 1-$x$%, and $\delta_{Gmean}$ is the corresponding response parameter obtained considering the design mean value of the shear modulus $G_{mean}$.. In this respect, Table 1 reports those coefficients for all the displacements, both in the longitudinal and transversal directions.

Table 1. Coefficients for taking into account the variability of the rubber.

| | DOF | $C_{1\%}$ (Lower Bound) | $C_{5\%}$ | $C_{16\%}$ | $C_{50\%}$ | $C_{84\%}$ | $C_{95\%}$ | $C_{99\%}$ (Upper Bound) |
|---|---|---|---|---|---|---|---|---|
| Longitudinal direction | $u_{deck}$ | 0.9132 | 0.9386 | 0.9629 | 1 | 1.0371 | 1.0612 | 1.0866 |
| | $u_{pier1}$ | 0.7674 | 0.8535 | 0.9231 | 1.011 | 1.0769 | 1.1117 | 1.1447 |
| | $u_{pier2}$ | 0.9413 | 0.9652 | 0.983 | 1.0032 | 1.0174 | 1.0247 | 1.0312 |
| | $u_{pier3}$ | 0.8061 | 0.8793 | 0.9378 | 1.0098 | 1.0634 | 1.0915 | 1.1171 |
| | $u_{isoAB}$ | 0.9132 | 0.9386 | 0.9629 | 1 | 1.0371 | 1.0612 | 1.0866 |
| | $u_{iso1}$ | 0.8621 | 0.9024 | 0.941 | 1 | 1.059 | 1.0976 | 1.1379 |
| | $u_{iso2}$ | 0.699 | 0.7735 | 0.853 | 0.9889 | 1.147 | 1.2641 | 1.3993 |
| | $u_{iso3}$ | 0.8364 | 0.8841 | 0.93 | 1 | 1.07 | 1.1159 | 1.164 |
| Transversal direction | $v_{deck}$ | 0.8936 | 0.9268 | 0.9556 | 1 | 1.0463 | 1.0776 | 1.1064 |
| | $v_{pier1}$ | 0.758 | 0.8493 | 0.9224 | 1.0137 | 1.0822 | 1.1233 | 1.1553 |
| | $v_{pier2}$ | 0.9098 | 0.9358 | 0.9612 | 1 | 1.0388 | 1.0642 | 1.0909 |
| | $v_{pier3}$ | 0.7633 | 0.8521 | 0.9231 | 1.0118 | 1.0799 | 1.1154 | 1.1479 |
| | $\Delta v_{deck,max}$ | 0.717 | 0.8113 | 0.8868 | 1 | 1.1321 | 1.2264 | 1.3019 |
| | $v_{isoAB}$ | 0.8936 | 0.9268 | 0.9556 | 1 | 1.0463 | 1.0776 | 1.1064 |
| | $v_{iso1}$ | 0.8733 | 0.9105 | 0.9457 | 1 | 1.054 | 1.0837 | 1.1264 |
| | $v_{iso2}$ | 0.7658 | 0.8265 | 0.8892 | 0.9937 | 1.1108 | 1.1947 | 1.2896 |
| | $v_{iso3}$ | 0.868 | 0.9044 | 0.9404 | 0.9982 | 1.0599 | 1.1019 | 1.148 |

Starting from the values of these percentiles, for each structural element and for each isolator, the multiplicative coefficients have been calculated which, once applied to the result ($\delta_{Gmean}$) obtained assuming the mean design value of the rubber shear modulus, allow to immediately obtain the Upper Bound (here selected as the 99% percentile) and Lower Bound (1% percentile) values which take into account the variability of the rubber shear modulus: $\delta_{UpperBound} = c_{UB} \cdot \delta_{Gmean} = c_{99\%} \cdot \delta_{Gmean}$ and $\delta_{LowerBound} = c_{LB} \cdot \delta_{Gmean} = c_{1\%} \cdot \delta_{Gmean}$. Regarding the isolators displacements: (i) the Upper Bound coefficient increases as the flexibility of the underlying structural element (pile/abutment) increases; (ii) the Lower Bound coefficient decreases (and therefore deviates more and more from 1) as the flexibility of the structural element (pile/abutment) increases. Regarding the top piers displacements: (i) the Upper Bound coefficient decreases as the flexibility of the pile increases, with a trend qualitatively opposite to the coefficient for the displacement of the corresponding isolator; (ii) the Lower Bound coefficient increases (and therefore gets closer and closer to 1) as the flexibility of the pile increases, with a trend qualitatively opposite to the coefficient for the displacement of the corresponding isolator. It has been verified that, by applying a typical variability (cov = 14.5%) to the mechanical properties of the elastomeric isolators (specifically: to the shear modulus of the rubber material), with linear dynamic analyses, a variability is obtained in the main displacement response parameters (underestimates/overestimations with respect to the results obtainable assuming the design mean values) no higher than 20% with reference to the 84th-percentile and no higher than 30% with reference to the 99th-percentile. This result is substantially in line with what is reported in the last sentence of paragraph 7.10.5.1 of the Italian code (NTC 2018).

## 8 CONCLUSIONS

A simplified procedure for the identification of HDRB isolators has been presented and applied to a RC bridge. The effects of the variability in the mechanical properties of the isolators (rubber shear modulus) on the structural behaviour of the isolated bridge emerged from several time-history analyses carried out on minimal models. They are characterized by few degrees of freedom, four in the longitudinal direction and five in the transversal one. The rubber shear modulus $G$ is assumed as a random variable characterized by a normal distribution with mean value equal to 0.8 MPa and coefficient of variation equal to 14.5%. From 10,000 sets of numerical simulations (each one carried out with seven natural accelerograms) which are representative of the realizations of the random variable $G$, the best fitting of the probability functions of the mean values of the displacements (for both isolators and piers) have been performed. The analyses of the associated percentiles allow the definition of coefficients for taking into account the variability of the rubber, i.e. Upper and Lower bounds values.

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