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R&D investments with spillovers and endogenous horizontal differentiation

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Abstract

We investigate the implications of cost-reducing R&D activities with spillovers in a Hotelling model with endogenous product differentiation driven by quadratic transportation costs. We consider two different three-stage games: in the first, firms choose locations, then R&D efforts and finally prices; in the second, R&D strategies are decided at the first stage and locations are chosen at the second, with price competition taking place at the third. We identify the conditions whereby process innovation brings about a reduction of product differentiation in equilibrium, consequently implying lower profits. Equilibrium product differentiation and profits are increasing in R&D spillovers. Then, transforming the structure into a twofold version of a four-stage game, we allow firms to commit not to invest in R&D, to show the presence of an underlying prisoners' dilemma affecting R&D decisions. Finally, we illustrate R&D cartels' ability to eliminate wasteful sunk costs, and outline policy prescriptions.

JEL Codes: D43; L13; L41; O31

Keywords: horizontal differentiation; process innovation; endogenous locations

1 Introduction

Research and development (R&D) investments represent a crucial component of the competitive strategy of a firm. Indeed, as long as they concern a reduction of the product costs (thus increasing the production process efficiency), R&D investments might allow a firm to be more efficient and get greater profits than those rivals that invest less. That is, firms hugely invest in cost-reducing activities with the purpose of boosting profits. However, Klette and Kortum (2002) and Loof and Heshmati (2006) illustrate some stylized facts concerning the relationship between R&D efforts and firms' performance, and show that the sign of the relationship between firms' R&D investments and profits might depend on the characteristics of the industry. In other words, there is not consensus about the final effect of cost-reducing (or process) R&D activities.

Also from a theoretical perspective, there is a wide debate about the equilibrium properties of R&D investments when several firms have the possibility to adopt them. In other words, what happens if all firms invest in R&D activities? Are the equilibrium profits greater or lower with respect to a situation where no firm invests in R&D activities?

The seminal contribution by d'Aspremont and Jacquemin (1988) considers the case of two competing firms that have to decide how much to invest in cost-reducing R&D activities. In particular, the R&D activities are assumed to generate positive externalities, such that some benefits of each firm's R&D flow to other firms without any costly transaction. That is, when a firm invests in order to reduce its own productions costs, the rival's production costs are also reduced to some extent. It is shown that when firms cooperate on R&D activities (but not on the production stage), both the R&D expenditure and the quantities are increased in the equilibrium, with a positive effect on welfare, while the final effect on profits remain uncertain.

d'Aspremont and Jacquemin (1988) assume linear demand functions and quantity competition in a model where firms sell a homogenous good, thus leaving aside any interplay between R&D investments and product differentiation.[1] Here, we model the R&D technology as in d'Aspremont and Jacquemin (1988) and nest it into a three-stage Hotelling game where firms set also location and prices. This is done in two different ways. The first gives rise to a structure in line with the backbone of the debate on product life cycle, in which product design (in this case, location choice) takes place in the first stage, R&D in the second and price competition in the third. This game, which we lable as LRP (standing for location, research and price) allows one to assess the bearings of horizontal differentiation on R&D efforts for process innovation. The second architecture of the game, labelled as RLP (research, location, price), consists in assuming that R&D investments are chosen first, and are followed by the location and price choices in the ensuing stages. This amounts to saying that the second structure illustrates the impact of cost-reducing innovation on the degree of horizontal differentiation. In both cases, we will also draw the implications of the alternative sequences on profits and welfare.

Our main results are as follows. If the R&D investment decision is taken after the location decision (but before the decision about the product price), we show that product differentiation is reduced in equilibrium. In particular, the incentive for firms to locate closer is stronger when the externality effect of R&D activities is weaker. Furthermore, if the externality effect is weak enough, firms locate within the Hotelling city boundaries, thus contrasting the Maximum Differentiation Principle (d'Aspremont *et al.*, 1979). Interestingly, the equilibrium profits are shown to monotonically increase with the level of the externalities. Furthermore, when the externalities are maximal, the equilibrium amount of R&D innovation is zero. It follows that the profits are greater when no firm invests in R&D activities. At the opposite, when the R&D investment decision is taken at the beginning of the game (that is, before both the location decision and the price decision), the equilibrium product differentiation is not affected by the possibility for firms to invest in cost-reducing R&D activities.

Our setting can be interpreted as a model with both process innovation

[1] See also Henriques (1990) for a correction of d'Aspremont and Jacquemin (1988).

and product innovation along the horizontal dimension. As in Lambertini and Mantovani (2009) and Lin and Saggi (2002), product innovation is about the location in the relevant space (the horizontal dimension), whereas process innovation is about the reduction of the production costs. Therefore, the first timing refers to a situation where the firms decide first about which variety to produce (product innovation in the horizontal dimension), and then how efficiently to produce it (process innovation); the second timing refers to a situation where the firms decide first about the efficiency of the production process, and then about the variety to commercialize. As product innovation along the horizontal dimension can be seen as a restyling of existing products (rather than the introduction of new ones), the second timing is in accordance with casual observation of real world-cases.[2] Indeed, both Pine *et al.* (1993) and Adner and Levinthal (2001) document that there is no predetermined timing between product and process innovation.[3] Furthermore, Imai (1992) shows, for a large sample of Japanese firms, that both product and process innovation occur during the entire life cycle with a 60/40 ratio, thus documenting the importance of both types of innovation.

Otherwise, sticking to the spatial interpretation of the model, the second version of the model sounds more sensible, as firms selling goods whose embodied technology (i.e., marginal costs) has been chosen earlier may later decide where to locate their respective outlets inside or outside urban areas.

We also extend the game to discuss the possibility for firms to choose whether to engage or not in R&D activities in an additional stage in discrete strategies prior to the endogenous determination of R&D levels in continuous strategies, if at least one firm has indeed decided to activate an R&D project. In other words, we allow firms to commit not to invest in cost-reducing

[2] For example, "when Volkswagen-Audi introduced the new city cars Lupo and Fox, it kept in production existing models like Polo and Golf, but restyled them almost anew, while carrying out steadily its investment in process innovation" (Mantovani and Lambertini, 2009, p. 509).

[3] Note that when product innovation consists in improving the quality of an existing product (product innovation along the vertical dimension, see for example Bonanno and Haworth, 1998), the second timline is likely to be less realistic than the first.

innovation. If a firm chooses to commit not to invest, the R&D amount is zero for that firm. We show that the unique R&D investment policy equilibrium is characterized by both firms not committing, irrespective of the sequence of location and R&D choices. For given locations, the resulting equilibrium with both firms investing is the outcome of a prisoners' dilemma if the R&D cost function is sufficiently steep. In the alternative game structure in which locations are chosen between R&D efforts and price, the unique equilibrium with industry-wide R&D investment is systematically Pareto-inefficient for firms over the whole admissible parameter range.

As in d'Aspremont and Jacquemin (1988), these findings prompt for the analysis of cartel behaviour as a potential remedy to wasteful investment ultimately hindering also social welfare. Indeed, joint-profit maximisation at the R&D stage makes firms aware of the undesirable consequences of costly reductions of their marginal production costs, to such an extent that individual and aggregate R&D investments drop to zero in both games. However, the presence of cooperation at the R&D stage has different implications on product design in the two scenarios. If the first stage hosts location decision, the equilibrium degree of product differentiation is lower than in the alternative case in which the degree of differentiation is determined at the second stage. In the latter case, the subgame perfect equilibrium is exactly the same as in Lambertini (1994, 1997) and Tabuchi and Thisse (1995). Hence, a public authority would strictly prefer firms to activate a cartel so as to avoid undesirable sunk costs. Moreover, the game with locations being decided upon at the first stage, then followed by the R&D cartel can be regulated so as to induce firms to locate within the linear city boundaries (although not at the socially optimal locations), in order to reduce the burden represented by total transportation costs.

The last task we have to accomplish consists in locating our contribution at the intersection of two debates concerning process innovation with spillovers, on one side, and price competition in spatial models with firms using (at least potentially) heterogeneous technologies, on the other. To begin with, the papers discussing the strategic choice of R&D efforts can be classified into two categories: that excluding the spatial dimension and the alternative one including it. Looking at the former, one finds several papers discuss the implications of R&D when the firms are supposed to set quantities,[4] whereas the literature of R&D and price competition is rather scant. In particular, Amir *et al.* (2003) consider a general version of the two-stage model of d'Aspremont and Jacquemin (1988) with both Cournot and Bertrand competition in the last stage of the game in order to compare the performance of cooperative and non-cooperative R&D. More recently, Buccella *et al.* (2021), building on Bacchiega *et al.* (2010),[5] focus on the conditions for a prisoners' dilemma to arise under Cournot and Bertrand behaviour in a model *à la* Singh and Vives (1984). Both product and process R&D investments are studied by Bacchiega *et al.* (2011) in a vertical differentiation model, where it is shown that process innovation fosters (hinders) product innovation for the low-quality (high-quality) firm, whereas the relationship between product and process R&D is studied by Lin and Saggi (2002) under different modes of market competition (Cournot and Bertrand competition) in a non-spatial set-up.

Considering the second subset, many authors have included the spatial dimension into the analysis of R&D. For example, Harter (1993) considers R&D investments in a Hotelling set-up, but he assumes that the R&D activity is a discovery process that ends up with the variety produced in the characteristics space by the firm. Furthermore, he does not consider spillovers from R&D activities. Analogous considerations hold for Lambertini (2002), where a stochastic innovation race endogenously determines the timing of entry and the equilibrium degree of differentiation in the long run. Other

[4]For example, Suzumura (1992) extends d'Aspremont and Jacquemin (1988) to the case of an oligopoly, Poyago-Theotoky (1999) investigates the case of endogenous spillovers with cost-reducing R&D activities in a non-spatial homogenous framework; Poyago and Teeerasuwannajak (2020) consider the case where the firms have different productivity in their R&D activity.

[5]Bacchiega *et al.* (2010) extend d'Aspremont and Jacquemin's (1988) original contribution to investigate on the incentives to collude on R&D. However, they do not consider price competition.

authors assume three-stage games similar to the ones we consider in this paper. For example, Piga and Poyago-Theotoki (2005) consider a three-stage game in a Hotelling set-up where two firms choose location, R&D and finally price, assuming that the extent of the R&D spillovers depend on the firms' location. Another difference with respect to our work is that they consider quality-enhancing R&D rather than cost-reducing R&D. They find that minimal quality differentiation always occurs in equilibrium. Long and Soubeyran (1998) assume that the extent of spillovers is related to the distance between the firms, as in Piga and Poyago-Theotoki (2005), but they consider Cournot competition and cost-reducing R&D. They find that firms agglomerate when the endogenous spillover is a convex function of the distance between firms, but they do not explicitly model the choice of R&D expenditures. Piga e Poyago-Theotoki (2004) analyse a three-stage game in which the location choice is followed by process innovation. However, they consider spatial price discrimination in the last stage of the game, rather than uniform pricing. An interesting article by Sun (2013) considers an analogous sequence of stages and evaluates alternatively quality-enhancing R&D and cost-reducing R&D. He shows that these two kinds of R&D activities yield the same equilibrium price-cost margins, R&D expenditures, and locations, provided that some symmetry conditions hold. Therefore, there exists a duality between quality-enhancing R&D and cost-reducing R&D in a Hotelling model. As in Long and Soubeyran (1998) and Piga and Poyago-Theotoki (2005), spillovers depend on the distance between the firms. Furthermore, unlike our work, firms' locations are restricted within the Hotelling segment. Matsumura *et al.* (2010) introduce licensing in the spatial game, by adding a stage between the R&D and the price stage where the firm that has a cost advantage might license the cost-reducing innovation to the rival.[6] Finally, Heywood and Zhang (2017) consider a spatial model *à la* Hotelling when the cost-reducing R&D investment decision might happen either before or after the location stage (RLP and LRP game, respectively). Differently from

[6]It should be observed, however, that in Matsumura *et al.* (2010), the R&D outcome is stochastic rather than deterministic.

our work, in the last stage the firms might spatially price discriminate. They find that in the RLP game the firms locate at the efficient locations (first and third quartile), earn less profit than without R&D, and might under-invest or over-invest depending on the degree of the spillovers. At the opposite, in the LRP game, the firms never locate at the efficient location, earn less profit than without R&D, and always under-invest in R&D.

Other papers focus exclusively on the case where R&D investment is decided before the location stage within a standard Hotelling segment. For example, Matsumura and Matsushima (2009), building on Ziss (1993), assume that the two firms have different marginal costs (which can be interpreted as the result of a first stage R&D decision), and fully characterize the mixed strategy location-price equilibria when equilibria in pure strategies do not exist. Matsumura and Matsushima (2004) and Zhang and Li (2013) consider a RLP three-stage game where a public firm competes with and a private firm. Matsumura and Matsushima (2004) show that the private firm over-invests in cost-reducing activities. Zhang and Li (2013), by introducing distance-dependent spillovers into the model by Matsumura and Matsushima (2004) and assuming that R&D improves product quality, find that the public firm might engage more aggressively in R&D expenditure than the private firm, depending on the degree of the spillovers. Quite related to our paper, Matsumura and Matsushima (2012) consider a three-stage game with R&D taking place at the first, and show that, if firms' locations are restricted (unrestricted) within the linear city, R&D investment is insufficient (excessive) as compared to the socially efficient level. However, their model does not account for technological spillovers. We'll come back to their paper in Sections 3.2 and 5.

The remainder of the paper is structured as follows. In Section 2 we introduce the model. In Section 3 we characterize the equilibrium under two alternative versions of the R&D investment game. In Section 4 we consider the possibility that only one firm might invest in cost-reduction R&D activities (asymmetric investment policies), and we characterize the outcome of the related pre-play stage in which firms non cooperatively choose whether to invest in process innovation. The beneficial effects of R&D cartels are illustrated in Section 5, while policy prescriptions are discussed in Section 6. Section 7 concludes.

2 The model

Two firms, A and B, compete along a market segment *á la* Hotelling (1929) of length one, ranging from 0 to 1. Firm A (B) is located at the left (right) of 1/2. The location of firm A is x_A and the location of firm B is x_B, with $x_A \leq 1/2 \leq x_B$. Firms are not constrained to locate themselves within the segment (as in Lambertini, 1994, 1997; Tabuchi and Thisse, 1995, and others). Consumers are uniformly distributed along the segment. We assume density one and we denote by $x \in [0, 1]$ the location of each consumer in the market. The utility function of a consumer located at location x when buying from firm $J = A, B$ is:

$$u_J = v - p_J - t\,(x - x_J)^2 \tag{1}$$

where p_J is the price set by firm J, $t > 0$ is the unit transportation cost, and v is the reservation price, which is assumed to be sufficiently high so that the market is always covered in equilibrium. For any given vector of prices and locations, the consumer indifferent between products A and B is at

$$\widetilde{x} = x_A + \frac{x_B - x_A}{2} + \frac{p_B - p_A}{2t\,(x_B - x_A)} \tag{2}$$

Consequently, any consumer at $x \in [0, \widetilde{x})$ will patronise firm A, her/his peers in $(\widetilde{x}, 1]$ will buy from firm B, and the single individual at $\widetilde{x}$ will flip a coin. The expression on the r.h.s. of (2) tells that the demand basin accruing to firm A consists of its hinterland x_A plus the balance between the effects of product differentiation and price competition, the latter clearly being itself affected by differentiation.

Following d'Aspremont and Jacquemin (1988), we suppose that the average and marginal production cost of each firm is a function of the amount of research (R&D) that it undertakes, as well as the amount of research that its rival undertakes. In particular, we define as $c_J > 0$ firm J's unit production cost, with

$$c_J = \overline{c} - k_J - \beta k_{-J} \tag{3}$$

where $\overline{c} > 0$ is the (exogenous) common marginal cost when no firm undertakes any R&D efforts,[7] and $k_J \geq 0$ is the amount of research undertaken by firm J. As in d'Aspremont and Jacquemin (1988), we allow for the existence of some positive technological externalities: in particular, the R&D investment k_{-J} carried out by the rival firm also contributes to reduce the marginal cost of firm J. Parameter $\beta \in [0, 1]$ describes the intensity of the spillover: the greater is β, the larger is the spillover effect. If $\beta = 1$, innovation is a public good freely available to firms. The profit function of firm J is

$$\pi_J = (p_J - c_J)\, q_J - bk_J^2 \tag{4}$$

where q_J, corresponding either to $\widetilde{x}$ or $1 - \widetilde{x}$, is firm J's demand. Note that the cost bk_J^2 associated with process innovation is quadratic, which amounts to assuming diminishing returns to R&D efforts, as in d'Aspremont and Jacquemin (1988) and the resulting stream of literature. In this respect, we also add that at the outset of the illustration of any given game, we will adopt appropriate assumptions concerning parameter $b > 0$, in order to ensure concavity, the reality of equilibrium magnitudes, or the viability of firms at at equilibrium (in particular, to exclude degenerate monopoly outcomes).

3 Game structures and equilibrium outcomes

In this section, we consider two different three-stage games. In the first, the timing is the following: in stage 1, firms choose their respective locations; then, in stage 2, they invest in process innovation; and finally, in stage 3,

[7] Parameter $\overline{c}$ is assumed to be sufficiently high to ensure that all prices are positive in equilibrium.

play in the price space. In the second game, the order of the first two stages is reversed, that is, firms choose first the amount of research and then their locations along the linear segment, while the third stage is for Bertrand competition on the market place. Information is complete, symmetric and imperfect at every stage, while it is perfect across stages, i.e., firms play Nash equilibria whose outcomes become public domain in the ensuing stages (if any). The solution concept is subgame perfection by backward induction.

3.1 Game 1: location-research-price (LRP) game

Here, we examine the first game, in which firms choose first locations, then research efforts, and finally prices. In the last stage of the game, the indifferent consumer at $\widetilde{x}$ is identified by (2), so that $q_A = \widetilde{x}$ and $q_B = 1 - \widetilde{x}$. In this subsection, we adopt the following

Assumption I To ensure real solutions at the location stage, $b \geq b_{\mathbb{R}} \equiv (1-\beta)^2 \left[3 + 2\sqrt{2}\right] / (27t)$.

Now we may proceed to the analysis of the three stages, starting with the price one. By maximizing the profits functions with respect to prices, we get the Nash equilibrium at the market stage,

$$\begin{aligned} p_A(x_{A,B}, c_{A,B}) &= \frac{2c_A + c_B + t(x_B - x_A)(2 + x_B + x_A)}{3} \\ p_B(x_{A,B}, c_{A,B}) &= \frac{2c_B + c_A + t(x_B - x_A)(4 - x_B - x_A)}{3} \end{aligned} \tag{5}$$

By substituting (3) and (5) into the profit functions, and solving the first order conditions (FOCs) at the R&D stage, we observe that the best reply functions of the firms are

$$\begin{aligned} k_A(k_B) &= \frac{(1-\beta)\left[t(x_B - x_A)(2 + x_B + x_A) - k_B(1-\beta)\right]}{18bt(x_B - x_A) - (1-\beta)^2} \\ k_B(k_A) &= \frac{(1-\beta)\left[t(x_B - x_A)(4 - x_B - x_A) - k_B(1-\beta)\right]}{18bt(x_B - x_A) - (1-\beta)^2} \end{aligned} \tag{6}$$

These reaction functions are downward-sloping, that is, $\partial k_J(k_{-J})/\partial k_{-J} < 0$, so that when the rival increases the R&D amount, firm J reduces its own investment (or, in the jargon of Bulow *et al.*, 1985, R&D investments are strategic substitutes).[8] By solving the system (6), we obtain the following equilibrium R&D efforts,

$$\begin{aligned} k_A(x_A, x_B) &= \frac{(1-\beta)\left[3bt(x_B - x_A)(2 + x_B + x_A) - (1-\beta)^2\right]}{6b\left[9bt(x_B - x_A) - (1-\beta)^2\right]} \\ k_B(x_A, x_B) &= \frac{(1-\beta)\left[3bt(x_B - x_A)(4 - x_B - x_A) - (1-\beta)^2\right]}{6b\left[9bt(x_B - x_A) - (1-\beta)^2\right]} \end{aligned} \tag{7}$$

The impact of locations on optimal investments can be evaluated through the following exercise. Take the partial derivatives of $k_J(x_J, x_L)$, $J, L = A, B$, $L \neq J$, w.r.t. x_J and x_L, respectively; then, impose the symmetry condition $x_2 = 1 - x_1$ and simplify these partial derivatives to discover that

$$\left.\frac{\partial k_J(x_J, x_L)}{\partial x_J}\right|_{x_2 = 1 - x_1} = \left.\frac{\partial k_J(x_J, x_L)}{\partial x_L}\right|_{x_2 = 1 - x_1} = \frac{t(1 - 2x_1)(1-\beta)}{2\left[9bt(1 - 2x_1) - (1-\beta)^2\right]} > 0 \tag{8}$$

for all b satisfying Assumption I. The fact that $k_J(x_J, x_L)$ reacts exactly in the same way to an increase (or decrease) in either x_J or x_L, at least for symmetric locations, can be explained by looking at the location of the indifferent consumer (or, equivalently, at firm A's demand). Taking the same partial derivatives on $\widetilde{x}(x_J, x_L)$, we have

$$\left.\frac{\partial \widetilde{x}(x_J, x_L)}{\partial x_J}\right|_{x_2 = 1 - x_1} = \left.\frac{\partial \widetilde{x}(x_J, x_L)}{\partial x_L}\right|_{x_2 = 1 - x_1} = \frac{3bt(1 - 2x_1)}{2\left[9bt(1 - 2x_1) - (1-\beta)^2\right]} > 0 \tag{9}$$

Now we may explicitly think of $J = A$ and $L = B$. In plain words, the sign of the above partial derivatives means that, given symmetric prices and locations, any increase in x_A or x_B brings about an increase in $q_A(x_J, x_L)$, and conversely. Irrespective of the specific source of such increase, its outcome is a higher incentive to invest for firm A.

[8] On this aspect, see also Henriques (1990).

The stability condition $\left(\partial^2\pi_J/\partial k_J^2\right)^2 > \left(\partial^2\pi_J/\partial k_J\partial k_{-J}\right)^2$ is satisfied if and only if $b > b_{LRP}\left(x_A, x_B\right) = \left(1-\beta\right)^2 / \left[9t\left(x_B - x_A\right)\right]$. This necessary and sufficient condition also ensures the positivity of equilibrium innovation efforts (7). This threshold must be assessed against that appearing in Assumption I, to see that $b_{\mathbb{R}} > b_{LRP}\left(x_A, x_B\right)$ for all $x_B - x_A > 3\left(3 - 2\sqrt{2}\right) \simeq 0.515$. As we shall see below, this is always true at equilibrium.

We may now move to the first stage of the game. We concentrate on symmetric equilibria to obtain analytical solutions. Therefore, by taking FOCs w.r.t. locations and imposing symmetry, it is immediate to find the coordinates of the upstream Nash equilibrium:

$$\begin{aligned} x_A^{LRP} &= 1 - x_B^{LRP} = \frac{9bt - 1 + \beta\left(2-\beta\right) - \sqrt{\Gamma}}{72bt} \\ \Gamma &\equiv 729b^2t^2 - \left(1-\beta\right)^2\left[162bt + \left(1-\beta\right)^2\right] \end{aligned} \qquad (10)$$

and $\Gamma \geq 0$ for all $b \geq b_{\mathbb{R}}$. Plugging equilibrium locations x_J^{LRP} into (7), the subgame perfect R&D efforts simplify to $k_J^{LRP} = \left(1-\beta\right)/\left(6b\right)$, which drop to zero if $\beta = 1$ or, in the limit, if b shoots up to infinity. Then, it also appears that the stability requirement at the R&D stage is also ensured by any $b \geq b_{\mathbb{R}}$.

It can be easily checked that x_A^{LRP} is strictly decreasing in β. Therefore, it is minimized at $\beta = 1$, whereby $x_A^{LRP} = -1/4$ for all $b, t \geq 0$, as in Lambertini (1994, 1997) and Tabuchi and Thisse (1995). Indeed, when $\beta = 1$, the externality effect is very strong. This means that each firm receives the highest possible benefit by the cost-reducing research performed by the rival, thus diminishing the incentive to perform in-house research on its own. This of course is driven by the very fact that reaction functions in the innovation stage are downward sloping, and the ultimate consequence is that when technical knowledge is publicly and entirely accessible by both firms, neither of them has any incentive to invest. It follows naturally that the equilibrium locations must coincide with the ones obtained in a model without cost-reducing innovation. Similarly, x_A^{LRP} is strictly decreasing in

b: when this parameter tends to plus infinity, x_A^{LRP} tends to $-1/4$. Indeed, when the slope of the R&D cost curve becomes infinitely high, the equilibrium amount of research tends to zero once again. Therefore, the effect of an increase in b is analogous to the impact of an increase in the technological externality. These findings deserve to be stressed in the following

Proposition 1 *At the subgame perfect equilibrium of the LRP game, The degree of product differentiation* $x_B^{LRP} - x_A^{LRP}$ *is monotonically increasing in both* b *and* β, *for any* $t > 0$, *with*

$$\lim_{b\to\infty} x_A^{LRP} = x_A^{LRP}\Big|_{\beta=1} = -\frac{1}{4}\,;\, \lim_{b\to\infty} x_B^{LRP} = x_B^{LRP}\Big|_{\beta=1} = \frac{5}{4}$$

$$x_A^{LRP} = 0\,;\, x_B^{LRP} = 1 \ \text{at}\ b = \frac{2\left(1-\beta\right)^2}{9t}$$

and

$$x_A^{LRP} = \left(3\sqrt{2}-4\right)/4 \simeq 0.061\,;\, x_B^{LRP} = 1 - x_A^{LRP} \simeq 0.939$$

in correspondence of $(b = b_{\mathbb{R}}, \beta = 0)$.

The details are omitted since the contents of Proposition 1 can be easily verified on the basis of the expressions in (10). The interpretation of the Proposition can be spelled out as follows. To begin with, if the R&D cost becomes prohibitively high because b shoots up to infinity, the equilibrium R&D effort $k_J^{LRP} = (1-\beta)/(6b)$ falls down to zero in the limit, and the same happens if $\beta = 1$, i.e., technical knowledge is a public good. In either case, the game obviously replicates the outcome of the two-stage game with unconstrained locations. This ceases to hold if $b \in [b_{\mathbb{R}}, \infty)$ and spillovers are less than full with firms progressively locating closer to each other while investing resources in process R&D, thereby reducing their marginal costs and increasing mark-ups. As the mark-up becomes higher, each firm has a greater incentive to expand the demand. In other words, the traditional *demand effect* becomes stronger as process innovation efforts increase, thereby inducing firms to locate closer to each other. They reproduce the Maximum Differentiation Principle (d'Aspremont *et al.*, 1979) by locating at the city boundaries if $b = 2(1-\beta)^2 / (9t) > b_{\mathbb{R}}$, for all admissible values of β. This, of course, reveals the existence of infinitely many parameter triples in correspondence of which the equilibrium degree of differentiation is less than maximum, i.e., all those satisfying $b \in \left[b_{\mathbb{R}}, 2(1-\beta)^2 / (9t)\right)$. Then, we end up treating the last special case, i.e., $\beta = 0$, which means $b \geq b_{\mathbb{R}} \equiv \left[3 + 2\sqrt{2}\right] / (27t)$. We may impose this condition to be satisfied at the margin, to find that, in correspondence of $(b = b_{\mathbb{R}}, \beta = 0)$, $x_A^{LRP} = \left(3\sqrt{2} - 4\right) / 4 \simeq 0.061$, which is the innermost equilibrium location.

To conclude the analysis of this game, we derive the equilibrium profits:

$$\pi_J^{LRP} = \frac{27bt - (1-\beta)^2 - \sqrt{\Gamma}}{27b} \tag{11}$$

Note that the equilibrium profits are increasing both in β and in b. Therefore, the maximum of the profits is attained when $\beta = 1$ and/or $b \to \infty$. Recall that, in this case, the equilibrium research effort is nil. This amounts to saying that carrying out any positive investment in process innovation unambiguously reduces equilibrium profits. In other words, by comparing a situation where no firm engages in process R&D - which is the case in the two-stage game in locations and prices - with a situation where both firms engage in process R&D, it turns out that both firms would be better off in the case of no research at all. Indeed, strategic substitutability between firms' R&D efforts induces a privately excessive amount of R&D. We summarize this result in the next proposition:

Proposition 2 *The industry-wide adoption of process R&D is privately inefficient for all* $\beta \in [0, 1)$ *and all* $b \in [b_{\mathbb{R}}, \infty)$*.*

The intuition is the following. In presence of any positive R&D efforts, firms locate closer than they would do otherwise, tempted by the possibility of exploiting marginal cost reductions which reflect themselves into more aggressive prices at the market stage. The latter effect, at equilibrium, jeopardises revenues. In addition, the R&D costs exacerbate the negative impact of this chain of implications on the equilibrium profits. Proposition 2 has to be interpreted in the following sense: firms would strictly prefer to play the traditional two-stage game, without engaging themselves in a technological competition. This raises the question as to whether firm may design their R&D behaviour so as to reduce or eliminate altogether this inefficiency, and indeed a cartel at the second stage may serve this purpose, as shown in Section 5.

3.2 Game 2: research-location-price (RLP) game

Here we consider the RLP game, in which firms choose first the research efforts, then locations, and finally prices, the latter being the same as in (5). To begin with, we adopt

Assumption II To ensure stability at the R&D stage, $b > b_{RLP} \equiv 8(1-\beta)^2/(27t)$.

In the second stage of the game, the relevant FOCs are

$$\frac{\partial \pi_A}{\partial x_A} = \frac{[c_A - c_B + t(x_B - x_A)(2 + 3x_A - x_B)][c_A - c_B - t(x_B - x_A)(2 + x_A + x_B)]}{18t(x_B - x_A)^2} = 0 \tag{12}$$

$$\frac{\partial \pi_B}{\partial x_B} = -\frac{[c_A - c_B - t(x_B - x_A)(4 + x_A - 3x_B)][c_A - c_B + t(x_B - x_A)(4 - x_A - x_B)]}{18t(x_B - x_A)^2} = 0 \tag{13}$$

which deliver the following outcome:

Lemma 3 *In the RLP game, equilibrium locations are, respectively,* $x_A^{RLP} = [4(c_B - c_A) - 3t]/(12t)$ *and* $x_B^{RLP} = [4(c_B - c_A) + 15t]/(12t)$, *with (i)* $x_A^{RLP} < 0$ *and* $x_B^{RLP} > 1$ *if*

$$c_A = c_B$$
$$c_A > c_B \text{ and } c_A - c_B < 3t/4$$
$$c_A < c_B \text{ and } c_B - c_A < 3t/4$$

(ii) $x_A^{RLP} < 0$ *and* $x_B^{RLP} \in (1/2, 1)$ *if* $c_A > c_B$ *and* $c_A - c_B \in (3t/4, 9t/4)$*; and (iii)* $x_A^{RLP} \in (0, 1/2)$ *and* $x_B^{RLP} > 1$ *if* $c_A < c_B$ *and* $c_B - c_A \in (3t/4, 9t/4)$. *If (iv)* $|c_A - c_B| \geq 9t/4$, *the firm with the lower marginal cost locates at* $1/2$ *while the rival locates at either at* $x_A^{RLP} = -1$ *or* $x_B^{RLP} = 2$.

Proof. See Appendix A1. ■

In absence of the R&D stage, the structure of this game replicates that appearing in Ziss (1993), except for the assumption about the admissible location space, which Ziss assumes to be confined to the linear city. By doing so, he is able to identify the critical level of the cost differential which allows the efficient firm to annihilate the rival's market share, and eventually deter entry, precisely because firms have to pick locations inside the city limits (Ziss, 1993, Propositions 2-3, pp. 536-37). This cannot happen here, although of course the more efficient firm may acquire monopoly power. This is the case, say, for firm A, if $c_B - c_A \geq 9t/4$, as in such a case $x_A^{RLP} = 1/2$ and $x_B^{RLP} = 2$ and the demand functions

$$q_A = \frac{1}{2} + \frac{2\left(c_B - c_A\right)}{9t} \, ; \, q_B = \frac{1}{2} + \frac{2\left(c_A - c_B\right)}{9t} \tag{14}$$

collapse to one and zero, respectively, while firms' prices are $p_A^{RLP} = \left(4c_B - c_A\right)/3$ and $p_B^{RLP} = c_B$. It is also worth noting that, under analogous conditions on parameters, the monopoly configuration also obtains from the other solutions of (12-13) that can be found in Appendix A1, with each firm alternatively playing the monopolist's role at $x_J = 1/2$ and the rival locating itself outside the linear city.

We move now to the first stage of the game. By inserting x_A^{RLP}, x_B^{RLP} and (3) into the profits functions and then maximizing them with respect to k_A and k_B, we get the symmetric Nash equilibrium in the R&D levels solving the system

$$\frac{\partial \pi_J}{\partial k_J} = \frac{2\left[k_J\left(4\left(1-\beta\right)^2 - 27bt\right) + \left(1-\beta\right)\left(9t - 4\left(1-\beta\right)k_L\right)\right]}{27t} = 0, \tag{15}$$

which is obviously independent of the common initial marginal cost $\overline{c}$ as this clears out of the profit expressions after plugging equilibium prices and locations, and therefore the equilibrium at the first stage is not only symmetric but it also depends solely on technological parameters:

$$k_A^{RLP} = k_B^{RLP} = \frac{1-\beta}{3b} \tag{16}$$

The Nash equilibrium in the R&D space (16) is analogous but not entirely equivalent to that appearing in Matsumura and Matsushima (2012, pp. 466-67), where the R&D cost function is assumed to be convex, and enough so to satisfy second-order conditions, but not specified in a quadratic form. Moreover, in Matsumura and Matsushima (2012) spillovers are assumed away.

As for the LRP game, also in the present version of the game the equilibrium effort in process innovation strictly decreases in both β and b, and either becomes nil at $\beta = 1$ or tends to zero as b becomes infinitely high. The stability condition $\left(\partial^2 \pi_J / \partial k_J^2\right)^2 > \left(\partial^2 \pi_J / \partial k_J \partial k_{-J}\right)^2$ is satisfied for all $b > b_{RLP}$.

By using (16), we can simplify the equilibrium locations, $x_A^{RLP} = -1/4$ and $x_B^{RLP} = 5/4$, the same as in Lambertini (1994, 1997) and Tabuchi and Thisse (1995). Therefore, introducing process research does not alter the equilibrium locations if the decisions about cost reducing investments are taken before setting locations. Indeed, process R&D alters the marginal production cost of the two firms. However, as the location decision is taken for a given pair of marginal production costs, equilibrium locations are unaffected by the possibility for firms to engage in research process, because of the well-known irrelevance of identical marginal production costs for the equilibrium location pattern in Hotelling-type models (see for example Ziss, 1993). Finally, equilibrium profits are:

$$\pi_J^{RLP} = \frac{27bt - 4\left(1-\beta\right)^2}{36b} > 0 \, \forall \, b > b_{RLP} \tag{17}$$

which are monotonically increasing both in β and in b as in the LRP game, following the same argument discussed for that setup in Section 3.1. Note that π_J^{RLP} corresponds to the expression of equilibrium profits emerging in absence of the R&D stage if $\beta = 1$ (see for example Lambertini, 1997, where the equilibrium profits amount to $3t/4$). Consequently, we may formulate

Proposition 4 *The presence of investments in process innovation has a negative impact on equilibrium profits irrespective of the exact position of the R&D stage, whether before or after the location stage.*

As in the LRP scenario, the presence of R&D competition is detrimental as compared to the pristine two-stage version of the model. Once again, this inefficiency can be amended by cooperating to maximise joint profits (see Section 5).

3.3 Comparative assessment

In what follows, we compare the equilibrium outcomes in the LRP game with the equilibrium outcomes in the RLP one. First, note that $b_{RLP} > b_{\mathbb{R}}$. Therefore, the parameter region wherein a comparative appraisal of the two games is admissible is identified by the condition $b > b_{RLP}$.

We can state the following proposition:

Proposition 5 *When firms choose first locations, then R&D investments, and finally prices, the equilibrium amount of innovation is lower, differentiation is lower, and equilibrium profits are larger relative to the case in which the sequence of the first two stages is reversed.*

Proof. It suffices to observe that $k_J^{LRP} \leq k_J^{RLP}$, $x_A^{LRP} \geq x_A^{RLP}$, and $\pi_J^{LRP} \geq \pi_J^{RLP}$ in the whole admissible range of the relevant parameters. ■

The intuition can be spelled out in the following terms. As noted in Proposition 1, choosing R&D efforts once locations have been set induces firms to depart from each other less, in equilibrium, in view of the gravitational attraction exerted by the median consumer and traditionally associated with the demand effect, whereas when the R&D efforts are chosen before the location stage, this fixes the level of marginal costs and therefore has no impact at all on optimal locations. Hence, $x_A^{LRP} \geq x_A^{RLP}$. This reduction in product differentiation has obvious consequences on price competition at the market stage and reduces the gross profits in the LRP game relative to the RLP game. However, in the RLP scenario, each firm invests more in R&D than in the LRP one (indeed, $k_J^{LRP} \leq k_J^{RLP}$), because of strategic substitutability and the incentive to increase one's own mark-up. As it turns out, the consequences of excess investment outweigh those brought about by too little differentiation, in such a way that net profits are higher if locations are set upfront.

4 Asymmetric R&D policies

Now we expand the picture to encompass firms' decisions as to whether to invest in order to reduce marginal production costs or not. This extension of the model delivers a four-stage game which can be envisaged in two different versions. The first is that in which the additional stage is inserted between the location stage and the one where at least one firm endogenously determines its R&D effort. The second is that where such a stage appears on top of the other three, as the endogenous amount of R&D is chosen before the location stage. In both cases, firms face the 2×2 discrete choice problem portrayed in Matrix 1,

		B: 0	B: k
A	0	$\pi_A^I(0,0);\pi_B^I(0,0)$	$\pi_A^I(0,k);\pi_B^I(k,0)$
	k	$\pi_A^I(k,0);\pi_B^I(0,k)$	$\pi_A^I(k,k);\pi_B^I(k,k)$

Matrix 1: discrete investment choices

with $I = LRP, RLP$, $J = A, B$. In correspondence of each profit $\pi_J^I(\gamma, \phi)$, with $\gamma, \phi = 0, k$, ϕ consistently refers to the opponent's decision.

Firms must solve this stage game in discrete strategies non cooperatively, under complete, symmetric and imperfect information. If a firm chooses strategy k, then in the following stage it must rationally tune its investment to maximise profits. The discrete strategies chosen at this stage become public domain before the endogenisation of R&D efforts, if any. This procedure will allow us to characterise the nature of the stage game in question, to determine, in particular, whether we must indeed expect the entire industry to invest in innovative activities or not at the subgame perfect equilibrium, in the same vein as in Bacchiega *et al.* (2010). We shall label these augmented games as LRP' and RLP', respectively.

4.1 The LRP' game

Here, the above matrix appears at the second stage, in such a way that firms have to choose whether to invest or not in cost-reducing activities *for a given location pair* (x_A, x_B). As we shall see below, this has relevant bearings on the Pareto-efficiency of R&D associated with the equilibrium outcome of this stage.

If both firms do invest in R&D activities, the relevant expressions of optimal innovation efforts are (7) and the binding condition is $b \geq b_{LRP}(x_A, x_B)$, as in Section 3.1. We need an additional one, namely

Assumption III To avoid the market configuration to collapse to monopoly, $b > b_M \equiv (1-\beta)^2 / [3t(x_B - x_A)(4 - x_A - x_B)]$.

If both are satisfied, the relevant profit expressions are

$$\pi_A^{LRP}(k,k) = \frac{\left[18bt(x_B - x_A) - (1-\beta)^2\right]\left[3bt(x_A(2+x_A) - x_B(2+x_B)) + (1-\beta)^2\right]}{36b\left[9bt(x_B - x_A) - (1-\beta)^2\right]^2} \tag{18}$$

$$\pi_B^{LRP}(k,k) = \frac{\left[18bt(x_B - x_A) - (1-\beta)^2\right]\left[3bt(x_A(4-x_A) - x_B(4-x_B)) + (1-\beta)^2\right]}{36b\left[9bt(x_B - x_A) - (1-\beta)^2\right]^2} \tag{19}$$

If both refrain from investing, profit functions collapse to the well known expressions we have inherited from d'Aspremont *et al.* (1979). In particular, firm A's profits are

$$\pi_A^{LRP}(0,0) = \frac{t(x_B - x_A)(2 + x_A + x_B)^2}{18} \tag{20}$$

There remains to discuss the case in which one firm invests while the other does not. Here, we stipulate that the firm which does not invest still enjoys the benefits delivered by the spillover from the rival. This may be plausible because, in the first place, the very existence of a spillover implies an imperfect patent protection, the more so the higher the spillover is.[9]

In order to fill the cells along the secondary diagonal of Matrix 1, it suffices to investigate one of the two asymmetric situations. The analysis proceeds as in Section 3.1 after imposing $k_B = 0$. That is, suppose firm B has committed not to invest in process R&D. Of course, the last stage equilibrium prices are given by (5). Moving up to the third stage and calculating the R&D amount of firm A, we get:

$$k_{A0}^{LRP\prime}(x_A, x_B) = \frac{t(x_B - x_A)(2 + x_A + x_B)(1-\beta)}{18bt(x_B - x_A) - (1-\beta)^2} > 0 \,\forall\, b > b_{A0} \equiv \frac{(1-\beta)^2}{18t(x_B - x_A)} \tag{21}$$

where subscript $A0$ mnemonics for the asymmetric case, and the condition on b also ensures concavity and the additional fact that firm A's output and profit margin be both positive. Yet, note that $b_{LRP}(x_A, x_B) = 2b_{A0}$, so, if the symmetric equilibrium at the R&D stage is stable, then $b > b_{A0}$ for sure. However, it can be easily verified that $q_B = 0$ for all $b \in (0, b_M]$, and conversely. Moreover,

$$b_{LRP}(x_A, x_B) - b_M = \frac{(1 - x_A - x_B)(1-\beta)^2}{9t(x_B - x_A)(4 - x_A - x_B)} \gtreqless 0 \forall x_A \lesseqgtr 1 - x_B \tag{22}$$

which implies that if $x_A < 1 - x_B$ monopoly cannot emerge since the request for stability, $b > b_{LRP}(x_A, x_B)$, coupled with the fact that products are sufficiently differentiated, prevents the output of firm B to fall to zero, although firm A has a lower marginal cost. In the remainder of the analysis, we shall exclude the monopoly case by assuming $b > \max\{b_{LRP}(x_A, x_B), b_M\}$.

[9] The alternative case, where in order to learn from the rival's R&D activity one has to finance a research project, whatever small it may be, is envisaged in Kamien and Zang (2000) who label this mechanism as *absorptive capacity*.

The profits associated to the asymmetric case are

$$\pi_A^{LRP}(k,0) = \frac{bt^2 (x_B - x_A)^2 (2 + x_A + x_B)^2}{18bt(x_B - x_A) - (1-\beta)^2}$$

$$\pi_B^{LRP}(0,k) = \frac{2t(x_B - x_A)\left[3t(x_B - x_A)(4 - x_A - x_B) - (1-\beta)^2\right]^2}{\left[18bt(x_B - x_A) - (1-\beta)^2\right]^2} \tag{23}$$

We may now examine Matrix 1 to find out what follows:

Proposition 6 *In the LRP' game, for given locations, the discrete choice concerning whether to invest or not delivers (k,k) as the unique equilibrium at the intersection of dominant strategies. This is Pareto-efficient for firms for all $b \in (\max\{b_{LRP}(x_A, x_B), b_M\}, \max\{b_-, b_+\})$, while it is the outcome of a prisoners' dilemma for all $b > \max\{b_-, b_+\}$, with $b_- = b_+ = b_{LRP}(x_A, x_B)$ for all $x_A = 1 - x_B$.*

Proof. See Appendix A2. ■

Note that the above statement holds for a generic location pair, as here firms choosing whether or not to reduce their respective marginal production costs do so inheriting from the first stage a given degree of horizontal differentiation. This boils down to observing that the eventual presence of a prisoners' dilemma is independent of differentiation, being solely determined by the steepness of the R&D cost function. Additionally, in comparison with the outcome of the LRP game in Section 3.1, it may imply a reversal of the Paretian assessment of investments in technical progress. This is due to the fact that in the LRP game this evaluation was done in comparison to the subgame perfect equilibrium of the traditional two-stage game without the R&D stage, while here it concerns the Pareto-inefficiency engendered at R&D stage, evaluated in connection with an additional stage wherein firms strategically ask themselves whether to carry it out or not.

4.2 The RLP' game

The RLP' case is comparatively easier to discuss, because all admissible subgames are fully solved by backward induction and the profit expressions appearing in the relevant version of Matrix 1, this time located at the first stage, contain only the parameters of the model. If both invest in R&D, the symmetric profits $\pi^{RLP}(k,k)$ coincide with (17). If neither firm invests, profits are $\pi^{RLP}(0,0) = 3t/4$ as in Lambertini (1994, 1997) and Tabuchi and Thisse (1995). In the asymmetric subgame in which one invests in process innovation while the other does not, the relevant profit expressions are

$$\begin{aligned} \pi_A^{RLP}(k,0) &= \frac{81bt^2}{4\left[27bt - 4(1-\beta)^2\right]} \\ \pi_B^{RLP}(0,k) &= \frac{3t\left[27bt - 8(1-\beta)^2\right]^2}{4\left[27bt - 4(1-\beta)^2\right]^2} \end{aligned} \tag{24}$$

These are admissible for all $b > b_{RLP}$ ensuring stability as well as concavity and $q_A, q_B \in (0,1)$, i.e., in this version of the game an asymmetric behaviour at this stage cannot lead to monopolization. To see this, one may take a look at the output of the non-investing firm,

$$q(0,k) = \frac{27bt - 8(1-\beta)^2}{2\left[27bt - 4(1-\beta)^2\right]} \tag{25}$$

which is positive for all $b > b_{RLP}$.

At this point, it is easily established that

$$\begin{aligned} \pi^{RLP}(k,k) &> \pi^{RLP}(0,k) \\ \pi^{RLP}(k,0) &> \pi^{RLP}(0,0) \\ \pi^{RLP}(0,0) &> \pi^{RLP}(k,k) \end{aligned} \tag{26}$$

for all $b > b_{RLP}$, whereby we have proved the following

Proposition 7 *In the RLP' game, the discrete choice concerning whether to invest or not delivers (k,k) as the unique equilibrium at the intersection of dominant strategies. This is the outcome of a prisoners' dilemma for all $b > b_{RLP}$.*

This conclusion lends itself to the same qualitative considerations exposed below Proposition 6. Without replicating the whole argument, it is appropriate to recall that the inefficiency we observe here is connected with the presence of two stages not present in the standard two-stage structure of the basic game.

Now we may go back to the traditional alternative interpretations of the Hotelling model, to formulate a few considerations about the results delivered by these two versions of the four-stage game. Recall that the LRP sequence best fits the theory of product life cycle, while both RLP and LRP may describe credible sequences if location choices are interpreted in a geographical sense. So, if we look at the LRP structure as a story in which product (process) innovation comes first (second), we are bound to conclude that process R&D may or may not be affected by a prisoner's dilemma depending on the steepness of R&D costs. If instead we take both structures as different but equally plausible representations of location games in a geographical space, we see that the position of the stage for process innovation is crucial as the RLP version makes the equilibrium systematically inefficient for firms, irrespective of the steepness of the cost function (provided that the stability condition is satisfied). And all of this is systematically independent of firms' locations. Hence, one may say that firms might want to choose any location pair first and then think about decreasing marginal cost and be satisfied with it if $b \in (b_{LRP}(x_A, x_B), \max\{b_-, b_+\})$, in the sense that they will then have no afterthoughts about the Pareto-inefficiency of the R&D equilibrium. Or, they may decide to cooperate at the R&D stage precisely because this route solves any Pareto-inefficiency associated with it, as we are indeed about to see in the next section.

5 R&D cartels

Here we delve into the details of what happens if firms solve the R&D stage in a fully cooperative way, i.e., choosing k_A and k_B to maximise joint profits, all else equal. As we shall show in both games, this solution drives firms not to invest at all in process innovation, the intuition being the following: given that the unit profit margin is independent of marginal cost, unlike what happens under non cooperative behaviour, firms are aware that any effort to become more efficient translates itself in a pure sunk cost with no advantage at all, and therefore at the subgame perfect equilibrium no R&D investments are observed. However, as we are about to see, the stage sequence has a bearing on the equilibrium levels of differentiation and prices, and therefore also on profits, consumer surplus and welfare.

5.1 The R&D cartel in the LRP game

In the last stage equilibrium prices are those appearing in (5). At the second stage, the optimal efforts under R&D cooperation solve the system

$$\frac{\partial\left(\pi_J+\pi_L\right)}{\partial k_J}=\frac{2k_J\left[(1-\beta)^2-9bt\left(x_B-x_A\right)\right]}{9t\left(x_B-x_A\right)}-\frac{2\left(1-\beta\right)\left[t\left(x_B-x_A\right)\left(x_A+x_B-1\right)-k_L\left(1-\beta\right)\right]}{9t\left(x_B-x_A\right)}=0 \tag{27}$$

with $J, L = A, B$, $L \neq J$, which yields the following pair:

$$\begin{aligned} k_A &= \frac{t\left(1-\beta\right)\left(x_B-x_A\right)\left(x_A+x_B-1\right)}{9bt\left(x_B-x_A\right)-2\left(1-\beta\right)^2} \\ k_B &= \frac{t\left(1-\beta\right)\left(x_B-x_A\right)\left(1-x_A-x_B\right)}{9bt\left(x_B-x_A\right)-2\left(1-\beta\right)^2} \end{aligned} \tag{28}$$

It can be easily checked that (i) $k_B = -k_A$ for all location pairs, so that $k_A + k_B = 0$ everywhere, and (ii) under symmetric locations (that is, $x_B = 1 - x_A$) both individual R&D efforts are nil. Moreover, taking into account the concavity condition $b > b_{LRP}(x_A, x_B)$, we have that, if either $b > 2b_{LRP}(x_A, x_B)$ and $x_A > 1 - x_B$ or $b \in (b_{LRP}(x_A, x_B), 2b_{LRP}(x_A, x_B))$ and $x_A < 1 - x_B$, then $k_A > 0 > k_B - k_A$. Otherwise, if either $b > 2b_{LRP}(x_A, x_B)$ and $x_A < 1 - x_B$ or $b \in (b_{LRP}(x_A, x_B), 2b_{LRP}(x_A, x_B))$ and $x_A > 1 - x_B$, then $k_A < 0 < k_B - k_A$.

Of course, whenever one of the expressions in (28) is negative, we should pose the corresponding $k_J = 0$. However, there remains a single case not accounted for in I and II, namely the scenario in which firms locate symmetrically, with $x_A = 1 - x_B$. If so, $k_A = k_B = 0$. We focus on this case to show that it is indeed the equilibrium solution.

Plugging expressions (28) into the individual profit functions, now being defined only in terms of locations and exogenous parameters, one obtains a system of FOCs which are quartic equations in x_A and x_B, the only symmetric pair satisfying the concavity requirement being

$$x_A^{LRPc} = \frac{3bt - \sqrt{3bt\,(27bt - 16)}}{24bt}\,;\ x_B^{LRPc} = 1 - x_A \tag{29}$$

where superscript $LRPc$ indicates the presence of the R&D cartel; x_A^{LRPc}, $x_A^{LRPc} \in \mathbb{R}$ for all $b \geq 16/\,(27t)$, $\beta = 0$ since neither firms invests in R&D, and[10]

$$\begin{array}{c} \lim_{b\to\infty} x_A^{LRPc} = -\dfrac{1}{4}\,;\ \lim_{b\to\infty} x_B^{LRPc} = \dfrac{5}{4} \\ x_A^{LRPc}\Big|_{b=\frac{16}{27t}} = \dfrac{1}{8}\,;\ x_B^{LRPc}\Big|_{b=\frac{16}{27t}} = \dfrac{7}{8} \end{array} \tag{30}$$

The associated optimal R&D efforts in (28) obviously drop to zero. Comparing x_A^{LRPc} with x_A^{LRP}, one finds that $x_A^{LRPc} > x_A^{LRP}$ everywhere, except at $\beta = 1$ and $b \to \infty$, as in these cases $x_A^{LRPc} = x_A^{LRP}$. This implies that differentiation is lower when firms activate the R&D cartel, which also implies that the price is lower then at the subgame perfect equilibrium of the fully non cooperative game illustrated in subsection 3.1. In absence of investments, the marginal cost remains unaltered, and therefore the unit margin and the resulting per firm profit are lower than in the initial version of this game. Overall, this effect is strong enough to more than offset the positive R&D cost affecting firm in the non cooperative LRP game, making cartel behaviour at the second stage altogether undesirable from the firms' standpoint (although socially convenient, as it reduces total transportation costs).

[10] Including spillovers, the expression of firm A's location would be

$$x_A = \frac{3bt - \sqrt{3bt\left[27bt - 16\,(1-\beta)^2\right]}}{24bt}$$

and the concavity requiment would be $b \geq 16\,(1-\beta)^2 / \,(27t)$, while the values in (30) would remain the same.

5.2 The R&D cartel in the RLP game

The analysis in the second and third stage proceeds as in subsection 3.2. In particular, it is worth recalling from Lemma 4 that equilibrium locations are, respectively, $x_A^{RLP} = [4(c_B - c_A) - 3t] / (12t)$ and $x_B^{RLP} = [4(c_B - c_A) + 15t] / (12t)$. These, in correspondence of any symmetric pair of R&D efforts entailing $c_A = c_B$, collaps to $-1/4$ and $5/4$, respectively. And indeed, at the first stage of the game, firms decide the amount of R&D that maximizes joint profits by solving the system

$$\frac{\partial(\pi_J + \pi_L)}{\partial k_J} = -\frac{2\left[k_J\left(27bt - 8(1-\beta)^2\right) + 8k_L(1-\beta)^2\right]}{27t} = 0, \tag{31}$$

with $J, L = A, B$, $L \neq J$, which yields $k_A = k_B = 0$. Bluntly speaking, since firms know that any symmetric equilibrium makes the marginal cost differential totally immaterial, they also know that R&D is a pure sunk cost, and rationally choose not to invest at all. Since we have shown (Propositions 4 and 7) that the profits when both firms invest on R&D are lower than when no one invests, R&D cooperation allows firms to avoid the profits reduction induced by symmetric R&D, and of course this also responds to the analogous early warning inherited from Matsumura and Matsushima (2012).

Once again, as in the LRP, firms cooperatively decide not to invest in R&D. Yet, here equilibrium locations, prices, consumer surplus, profits and welfare are exactly the same as in the unconstrained two-stage Hotelling model with no R&D investment (Lambertini, 1994, 1997, and Tabuchi and Thisse, 1995), so that an uninformed external observer evaluating the equilibrium magnitudes of this market would be unable to detect whether firms have never taken cost-reducing R&D into account or have just chosen to cooperate to discover that the optimal decision was not to invest. This is not true in the LRP case, as there the very fact that a cartel exists, at least nominally, affects the equilibrium configuration.

6 Welfare assessment and policy recommendations

Here we summarise the welfare properties of games 1 and 2 against the possibility for firms to activate a cartel, with a view to deliver a simple policy prescription for a public authority aiming at increasing social welfare through an intervention tailored upon the essential features of the R&D stage.

To begin with, at the social optimum, a planner in full control of firms' strategies would locate them at $x_A = 1/4$ and $x_B = 3/4$, in order to minimise total transportation costs, and would avoid investing any amount of resources in process innovation as this has no effect on total surplus under full market coverage. This of course is true also for the price level, which may not necessarily coincide with marginal cost, if the marginal willingness to pay v is high enough to ensure full coverage across any configuration of the game, as assumed upfront.

Now, without going through analytical details, we may formulate an intuitive appraisal of the welfare properties of the two game structures, LRP and RLP, and their policy implications.

Take the LRP first. From (30), we know that game LRP with the R&D cartel in Section 5.1 delivers $k_A = k_B = 0$ for any symmetric location pair, with $x_A^{LRPc} = 1/8$ and $x_B^{LRPc} = 7/8$ if $b = 16/(27t)$ (these are the locations which are closest to the first best ones). Therefore, if $b > 16/(27t)$, the policy maker may design an appropriate subsidy to reduce the marginal R&D cost to $b = 16/(27t)$. If $b < 16/(27t)$, the policy maker should tax R&D to reach the same targeted value. Be that as it may, firms won't be receiving or paying anything at equilibrium as R&D efforts will be nought.

However, it should also be noted that $\pi^{LRPc}\big|_{b=16/(27t)} = 3t/8 < \pi^{LRP}\big|_{b=16/(27t)}$, where

$$\pi^{LRP}\big|_{b=16/(27t)} = \frac{3t\left[15 + \beta(2-\beta) + \sqrt{(7+\beta(10-\beta))(23-\beta(6+\beta))}\right]}{128} \tag{32}$$

The above inequality tells that, from the firms' standpoint, activating a cartel under these conditions is not desirable in the LRP game. Hence, in order to create the appropriate incentive for firms to cooperate at the R&D stage, the R&D policy must be accompanied by a lump-sum transfer equal to $\pi^{LRP}\big|_{b=16/(27t)} - \pi^{LRPc}\big|_{b=16/(27t)}$.

The second case is straightforward, as the cartel altogether eliminates firms' R&D expenditure while yielding the same equilibrium of the fully non-cooperative game at the location stage. Accordingly, in the RLP scenario, the regulator would prefer firms to activate a cartel *all else equal*, and firms are happy to adopt this configuration of the first stage as doing so their equilibrium profits increase.

7 Concluding remarks

While R&D investments have received great attention by the economists given their importance for the strategy mix of the firms, from a theoretical standpoint it is not entirely clear what are the implications of cost-reducing R&D investments (process innovation) for the locational decisions (product innovation along the horizontal dimension) and the equilibrium profits of the firms when R&D investments are characterized by spillovers.

This paper has tackled this issue, by investigating the implications of cost-reducing R&D activities with spillovers in a horizontally differentiation model *á la* Hotelling with endogenous product differentiation. We have shown that when the R&D amount is chosen after firms' location choices, the equilibrium degree of product differentiation is lower as compared to the traditional case without R&D investments. In particular, when the intensity of the spillover is sufficiently weak, the two firms locate within the endpoints of the Hotelling segment, in contrast with the Maximum Differentiation Principle (d'Aspremont *et al.*, 1979). If instead the R&D amount is chosen before the location choice of the firms, then firms' R&D investments do not affect product differentiation in equilibrium. When considering the equilibrium profits, we have proven that the possibility for both firms to invest in cost-reducing activities induces lower equilibrium profits. However, no firm would like to commit not to invest. Hence, the two firms may find themselves trapped into a prisoners' dilemma.

A plausible remedy to this souerce of inefficiency, already accounted for in d'Aspremont and Jacquemin (1988), would consist in activating an R&D cartel, by means of which the equilibrium R&D efforts would become nil. In the game where locations are chosen at the first stage, this design of R&D activities could then be accompanied by an appropriate R&D policy (either a subsidy or a tax) driving firm to diminish total transportation costs.

To complete the picture, it should be mentioned that Cremer and Thisse (1991) established that a horizontal model with quadratic transportation costs coincides with a vertical differentiation model with quadratic variable costs for quality improvement. However, as discussed in footnote 3, the second time structure of the game we have considered in this paper is unlikely to describe real-world situations of process and product innovation by the firms when product innovation is related to the quality of the product rather than to the variety of the good.

The present paper could be extended in several directions. For example, we have assumed that firms set uniform (mill) prices. However, it is well-known that when firms can price discriminate, the equilibrium locations are different (Lederer and Hurter, 1986). Therefore, exploring the implications of cost-reduction R&D investments in the case of price discrimination looks like a promising line of research (see for example the recent paper by Pinopoulos, 2020). Second, we have posed that the number of competing firms is exogenous and equal to two. Yet, it would be interesting to consider the impact of R&D investments decisions as a pre-emption strategy in the case of an endogenous number of firms. Finally, our paper focuses exclusively on process R&D and product innovation in the horizontal dimension. An interesting avenue for future research would be considering the implications of process R&D when also product innovation along the vertical dimension is taken into account.

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Appendices

A1 Proof of Lemma 3

The system (12-13) yields the following set of solutions:

$$x_A = \frac{4(c_B - c_A) - 3t}{12t}\,;\; x_B = \frac{4(c_B - c_A) + 15t}{12t}$$

$$x_A = \frac{1}{2}\,;\; x_B = 2 \pm \frac{\sqrt{t\left[9t - 4(c_B - c_A)\right]}}{2t} \tag{a1}$$

$$x_A = -1 \pm \frac{\sqrt{t\left[9t + 4(c_B - c_A)\right]}}{2t}\,;\; x_B = \frac{1}{2}$$

The first pair is the only one satisfying the second order conditions and collapsing to $x_A = -1/4\,;\, x_B = 5/4$ if marginal costs coincide. The first result appearing in the Lemma can be obtained by solving $x_A^{RLP} = 0$ and $x_B^{RLP} = 1$. Results (ii-iii) emerge from combining (i) with the solutions of $x_J^{RLP} = 1/2,\ J = A, B$.

Finally, the last claim obtains by observing that, should the difference between marginal costs correspond to $9t/4$, the efficient firm would find itself at $1/2$, and would not gain from relocating beyond the median consumer as this would grant the rival the chance to leapfrog.

A2 Proof of Proposition 6

To prove the claim, we may take firm A's standpoint. If the rival does invest, A does so if and only if $\pi_A^{LRP}(k,k) > \pi_A^{LRP}(0,k)$, which has the same sign as

$$324b^2t^2(x_B - x_A)^2 - (1-\beta)^4 - 18bt(x_B - x_A)(1-\beta)^2 \tag{a2}$$

This expression is positive and increasing in b in correspondence of any $b \geq \max\{b_{LRP}(x_A, x_B), b_M\}$. Therefore, $\pi_A^{LRP}(k,k) > \pi_A^{LRP}(0,k)$ over the entire admissible parameter range.

If the rival does not invest, A instead does so, since

$$\pi_A^{LRP}(k,0) - \pi_A^{LRP}(0,0) = \frac{t(x_B - x_A)(2 + x_A + x_B)^2 (1-\beta)^2}{18\left[18bt(x_B - x_A) - (1-\beta)^2\right]} \tag{a3}$$

is always positive.

Then, $\pi_A^{LRP}(k,k) - \pi_A^{LRP}(0,0)$ has the sign of

$$\begin{aligned} &-27b^2t^2(x_B - x_A)^2\left[4 - (x_B - x_A)^2\right] - (1-\beta)^4 \\ &+2bt(x_B - x_A)\left[11 - (x_B - x_A)^2 - x_A - x_B\right](1-\beta)^2 \end{aligned} \tag{a4}$$

which is concave in b and becomes nil at $b_\pm = \left(-\Lambda \pm \sqrt{\Lambda^2 - 4\Theta\Upsilon}\right)/(2\Theta)$, with

$$\begin{aligned} \Theta &\equiv -27t^2(x_B - x_A)^2\left[4 - (x_B - x_A)^2\right](1-\beta)^2 \\ \Lambda &\equiv -27t^2(x_B - x_A)^2\left[4 - (x_B - x_A)^2\right] \\ \Upsilon &\equiv -(1-\beta)^4 \end{aligned} \tag{a5}$$

Consequently, simplifying $\Lambda^2 - 4\Theta\Upsilon$ one is left with

$$\left[x_A(1-x_A) - x_B(1-x_B)\right]^2\left[13 + (x_A + x_B)(4 + x_A + x_B)\right] \tag{a6}$$

under the square root. This implies that (i) $b_- = b_+ = b_{LRP}(x_A, x_B)$ for all $x_A = 1 - x_B$ and of course $x_A = x_B$, and in this cases $\pi_A^{LRP}(k,k) \leq \pi_A^{LRP}(0,0)$ for all $b \geq \max\{b_{LRP}(x_A, x_B), b_M\}$; and (ii) the correct extraction of $\left[x_A(1-x_A) - x_B(1-x_B)\right]^2$ from the square root reveals that $\max\{b_{LRP}(x_A, x_B), b_M\} \in (\min\{b_-, b_+\}, \max\{b_-, b_+\})$ always.

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