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Applicability Evidence of Constructal Design in Structural Engineering: Case Study of Biaxial Elasto-Plastic Buckling of Square Steel Plates with Elliptical Cutout

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Abstract. The application of the Constructal Design method in Heat Transfer and Fluid Mechanics areas is an already consecrated approach to geometrically evaluate these flow engineering systems. However, this approach in Mechanics of Materials realm is not yet widely used, since one can find only few publications about it in literature. The Constructal Design is based on the Constructal Law, a physical law that explains the universal phenomenon of evolution of any finite size flow system. Therefore, the main goal here is to show that the Constructal Design can also be used in dedicated Structural Engineering problems as an effective method for geometric evaluation. The obtained results prove the Constructal Design applicability definitively in Mechanics of Materials.

Keywords: Constructal Design Method, Constructal Law, Geometric Evaluation, Biaxial Elasto-Plastic Buckling, Perforated Plate.

1. Introduction

Constructal Theory is the understanding that the design of all finite size flow systems (such as roots and branches of trees, deltas of rivers, atmospheric electrical discharges, human body veins, social dynamics, engineering and technology and so on) are generated by a physical phenomenon: the Constructal Law [1]. The Constructal Law states: "For a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that provides greater and greater access to the currents that flow through it" [2].

The animate or inanimate live systems have two universal characteristics: they flows (due to a thermodynamic disequilibrium) and they morphs (in a free way aiming to achieve geometric configurations that facilitates the flow of its currents over time) [3, 4]. It is important to highlight that the Constructal Law is not related to optimization, maximization, minimization or any other idea of superior performance. The Constructal Law is related to the evolution direction in time and with the fact that the design phenomenon is a dynamic and continuous process [4]. Therefore, in evolutionary design there is no "best"; but it is possible to define a current "better", which can turn out to be not as good in future [5, 6].

According to [5] the flow systems are inherently imperfect. Thus, the imperfections cannot be eliminated; however, they can be reduced. The systems evolution occurs to distribute its imperfections better, enabling an easier flow to its currents. This natural trend of flowing with better geometric configuration is the Principle of Optimal Distribution of Imperfections, being one of the bases of the Constructal Law.

In this sense, the Constructal Design is the philosophy of evolutionary design in any flow system, i.e., the Constructal Law can be applied in any flow system through the Constructal Design method [4, 6].

The Constructal Design is a geometric evaluation method and not an optimization method. For its application, the Constructal Design needs to define: constraints (global or local), degrees of freedom (free to vary, respecting the constraints) and performance parameters that can be maximized or minimized (which represents a view of ease of flow access). However, the Constructal Design can be employed with an optimization method (such as Heuristic Methods or Exhaustive Search). In this case, the Constructal Design is responsible for generating the search space (composed by the possible geometric configurations) in which

the optimization method identifies the system geometry that conducts to its superior performance [4, 6].

As the Constructal Law is a universal law, its application already reached several areas beyond engineering, such as: social [7-9], economics [10-12], biology [13-15], sports [16-19], transport [20-22], among others. However, until this moment, the more comprehensive scientific contribution of the Constructal Law is addressed to Heat Transfer (HT) and Fluid Mechanics (FM) engineering areas.

This fact can be explained because the Constructal Law was defined based on a study involving HT and FM [2], since then it has been broadly used in engineering systems related to these areas. There are countless publications about this subject, already being a consecrate approach to deal with these kinds of engineering problems. Some recent publications applying the Constructal Design method in HT and/or FM are presented in Table 1.

Nevertheless, the Constructal Law employment in Mechanics of Materials (MM) engineering is still quite restricted. Among the few published works, in [5, 45-47] analogies regarding HT, FM, and MM problems were used to indicate that the Constructal Design can be applied in Structural Engineering systems. In addition, the association of FM and MM can be found in the studies promoted by [48-50], in which aircraft structures were geometrically evaluated through the Constructal Design method. Finally, regarding studies exclusively dedicated to MM problems, it is possible to highlight researches that investigated the: elastic or elasto-plastic uniaxial buckling of steel plates with cutouts [51-58]; elasto-plastic buckling of uniaxial stiffened steel plates [59, 60]; and elastic bending of steel stiffened plates [61-69].

Table 1. Recent works applying Constructal Design in HT and/or FM systems.

| Area	Reference	Year	Description
HT	[23]	2020	A flat plate solar collector has been analyzed in order to improve its performance in terms of thermal efficiency and cost considering two different situations.
HT	[24]	2020	A medium-scale solar thermal system with an evacuated flat plate solar collector structure was studied, achieving a thermal efficiency improvement of up to 59.67% under medium temperature.
HT	[25]	2020	An array of fork shaped fins operating under partially wet conditions has been evaluated. It was found that the array of fork shaped fins with two branches have a better performance than the rectangular fin array, concerning the heat transfer rate.
FM	[26]	2020	It was evaluated the influence of I and T-shaped empty channels' geometry on the resin filling time in a rectangular porous enclosed mold, simulating the main operating principle of a Liquid Resin Infusion (LRI) process.
HT	[27]	2019	In association with the Exhaustive Search (ES) technique, the geometric optimization of an Earth-Air Heat Exchanger (EAHE) was performed for the better ducts arrangements thermal performance possible.
HT	[28]	2019	Regarding entropy generation minimization, a plate with an I-shaped fin has been studied. The geometric analysis allowed reaching an optimal configuration and a dimensionless number was presented for the study of Constructal Designed plates.
HT	[29]	2019	The thermochemical energy storage in an open reactor in terms of architecture evolution had been studied aiming for a better maximum heat transfer and minimum pumping power.
FM	[30]	2019	The optimal design of a dual-pressure turbine in an ocean thermal energy conversion system was carried out, taking the total power output as the optimization objective.
FM	[31]	2019	A comb-like network (single manifold duct ramified to several branches, all subject to a pressure reservoir) is evaluated and the news geometries obtained presented better performance than those with constant spacing and diameter.
FM	[32]	2019	Two cases were considered to analyze the stability of objects in laminar fluid flow by the Finite Element Method (FEM): an elliptical cylinder and a rectangular cylinder. The results showed that there is no universal relation between drag and stability.
FM	[33]	2019	Several cases of networks T-shaped underfloor air ducts were analyzed by Computational Fluid Dynamics (CFD). The new proposed geometric configuration network improved the uniformity and efficiency of airflow distribution.
FM	[34]	2018	This work studied the influence of geometry on the performance of an Oscillating Water Column (OWC) wave energy converter, aiming to maximize its hydrodynamic power.
HT and FM	[35]	2020	A geometric investigation on a SB superheater was made, fixing the total heat exchange area, the outer tube diameter (TOD), the number of tubes per row (NTPR), and the number of the tube rows (NTR) were analyzed.
HT and FM	[36]	2020	It was proposed a new 3D finite element model with non-uniform heat generation. The model was optimized through an Exhaustive Search (ES) technique, taking account as a performance indicator, a complex function composed of hot spot temperature and pumping power.
HT and FM	[37]	2020	Study focused on an Ocean Thermal Energy Conversion System (OTECS) optimization. The optimal performance was obtained, considering the net power output as performance indicator.
HT and FM	[38]	2019	This study investigates the global performance of suspended radiant cooling panels concerning the flow channels layouts. The results proved that canopy-to-canopy (dendritic) flow channels have a superior performance in comparison with the serpentine design.
HT and FM	[39]	2019	Geometrical optimization of internal longitudinal fins on a tube was performed to assure the maximum heat transfer and thermal efficiency, being the fins inserted in the pipe perimeter and restricted to a prescribed radius.
HT and FM	[40]	2019	Different diamond-shaped pin fins are placed in a fixed volume in vertical cross flow to maximize the heat transfer density. An optimized configuration has been indicated.
HT and FM	[41]	2019	Six new geometries of a square substrate imposed to a uniform heat flux have been investigated to evaluate thermo-hydraulic and entropy generation characteristics. Highlighting that among all cases, the diamond shape configuration presented better performance.
HT and FM	[42]	2018	It was investigated geometries that minimize pressure drop and maximize the heat transfer for visco-plastic fluids in the cross flow around elliptical tubes.
HT and FM	[43]	2017	It was performed a geometrical evaluation of a triangular arrangement of circular cylinders subjected to transient, two-dimensional, incompressible, laminar, and aiding mixed convective flows; aiming to maximize Nusselt number and minimize drag coefficient.
HT and FM	[44]	2017	This work showed that pressure drop and velocity distribution in the microdevice manifold play an integral role in the flow uniformity, underscoring that its design should be tapered to reach a uniform flow rate distribution.
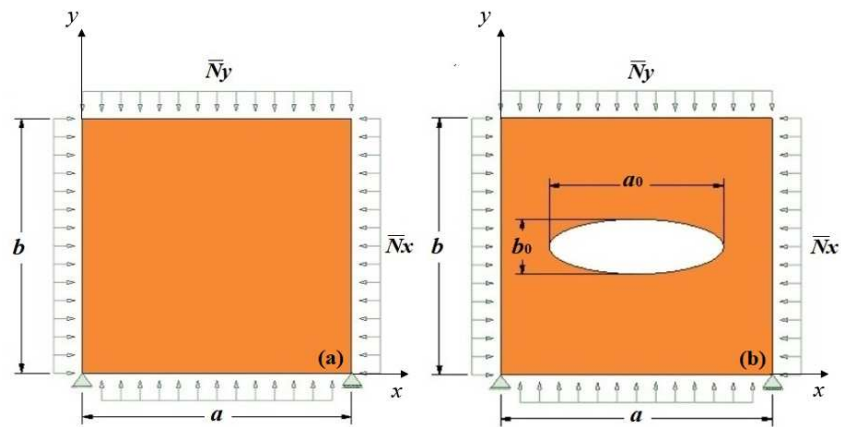
Fig. 1. Simply-supported (a) reference plate and (b) plate with cutout, under biaxial compressive loading.

Despite these already published works applying the Constructal Design in MM systems, there is still no definitive proof showing the effective applicability of the Constructal Design method in Structural Engineering. Therefore, the main goal of the present work is to prove that the Constructal Law through Constructal Design can be employed to the geometric evaluation of MM problems, as well as, if associated with an optimization method, it can also be used to the geometric optimization of these engineering structures.

A case study having symmetry of geometry, support, and load was adopted: a square steel perforated plate, simply-supported in its four edges and submitted to a biaxial compressive load. The plate has a centered elliptical cutout, which can freely vary under the imposed restrictions. The search space defined by the Constructal Design is used to promote a geometric optimization using the Exhaustive Search (ES) technique. The ultimate buckling stress and the maximum deflection of the perforated plate were chosen as performance parameters, both normalized by the correspondent values of the reference plate (with no hole). The purpose is to maximize the Normalized Ultimate Stress (NUS) and to minimize the Normalized Maximum Deflection (NMD). All proposed geometric configurations were numerically simulated with a computational model developed in ANSYS® software and based on the Finite Element Method (FEM).

2. Buckling and Post-Buckling of Plates

Plates are thin structural components where the in-plane dimensions, length (a) and width (b) (see Fig. 1), are much bigger than its out-of-plane dimension, thickness (t). These components are widely used in civil, naval, aerospace, and automotive Structural Engineering applications. However, if the plates are under compressive loadings, an instability phenomenon called buckling can occur [70]. A thin plate does not enter in collapse soon after the occurrence of the elastic buckling, but it can support loads significantly higher than its critical load (P_{cr}) without deforming excessively. The load that defines a plate's collapse, regarding elasto-plastic behavior, is called post-buckling or ultimate load (P_u) [71].

There are several studies addressed to the plate buckling phenomenon, being possible to highlight some recent works here: Dong et al. [72] performed a local buckling analysis of thin plates on tensionless elastic foundations subjected to interactive uniaxial compression and shear; Seifi et al. [73] investigated the global buckling of perforated plates reinforced with circumferential strip or short tube; Milazzo et al. [74] developed a single-domain formulation to model the buckling and post-buckling behavior of cracked multilayered composite plates; Zureick [75] studied the buckling of an isotropic rectangular plate uniformly compressed on two simply supported edges and with two free unloaded edges; Hu et al. [76] investigated the local buckling of steel plates in composite members with tie bars under axial compression; and Yuan et al. [77] proposed a set of similarity criteria aiming to predict the whole buckling process of stiffened plates subjected to compressive load.

From this, among the countless possibilities for using plates, some practical situations require the existence of cutouts on the plate (for inspection, passing pipes, or to be used as a weight reduction solution) [78], as depicted in Fig. 1b. According to [79] when the cutout is centered on the plate, the critical buckling load for uniaxial loading is higher than for other hole's positions. In addition, the post-buckling analysis for rectangular plates with centered perforation and biaxial compressive loading shows that the buckling resistance is lower than half of the case of uniaxial loading. This significant reduction in plate strength occurs because the presence of holes in the plate redistributes the membrane stresses causing change in the plate's buckling characteristics [80].

The exact analysis and design of such components are complex, particularly when the shape of holes and their arrangement are unusual. Regarding the elasto-plastic buckling analysis of plates with cutouts, an important observation is that it is not a simple task. However, computational modeling via the Finite Element Method (FEM) is an effective tool to solve these problems with good accuracy [80, 81]. Shanmugam et al. [82] used the FEM to develop design equations to determine the ultimate load carrying capacity of axially and biaxially compressed square plates with centrally located, circular or square, cutouts. They concluded that the square perforated plate's ultimate load capacity is affected significantly by the hole size and plate slenderness ratio. Also, affirm that plates with circular holes, in general, have higher ultimate load carrying capacity than plates with square holes.

3. Computational Model

The present work was developed by using the software ANSYS®, which is based on FEM. This engineering simulation software was preferred for the accurate results it provided in investigating the behaviour of thin plates in the presence of bucking phenomena [83, 84]. The finite element SHELL281 was adopted, since it is suitable for analyzing thin to moderately-thick plate or shell structures. The element has eight nodes with six degrees of freedom at each node: translations in the x, y, z-axes, and rotations about the x, y, z-axes [85].

An initial imperfection is considered starting from the first elastic buckling mode configuration, obeying the relation $b/2000$ as the maximum value to be assumed. Then, the plate's ultimate load can be found using as reference the load $P_y = \sigma_y t$, where σ_y represents the material yielding strength [86]. More details about the elastic and elasto-plastic buckling computational model can be found in [54] and [60].

Computational model verification was made based on [74]. To do so, a plate with $a = b = 125$ mm, $t = 6.25$ mm, circular cutout with $a_0 = b_0 = 25$ mm, boundary conditions as simply-supported and equal biaxial compressive loads on x and y-directions was considered (see Fig. 1b). The material used in the investigation was AH-36 steel with $\sigma_y = 355$ MPa, $E = 210$ GPa and $v = 0.30$. Ultimate stress obtained by the proposed equation of [82] resulted in $\sigma_u = 257.13$ MPa, while $\sigma_u = 276.42$ MPa was numerically obtained in the present work. This value represents a difference of 6.98% between numerical and analytical solutions, being acceptable according to [82].

In addition, computational model validation was made through the simply-supported square plate, with centralized circular cutout, studied by Narayanan and Chow [87]. It was considered a square plate (see Fig. 1b) with dimensions $a = b = 125$ mm and $t = 1.625$ mm, having a centered circular cutout with dimensions of $a_0 = b_0 = 25$ mm, being made of a material with $\sigma_y = 323.3$ MPa, $E = 205$ GPa and $v = 0.30$. The experimental result presented by [87] is $\sigma_u = 73.8$ MPa, while the obtained numerical solution is $\sigma_u = 77.59$ MPa, representing an error of 5.13%.

Therefore, considering the percentages of difference and error achieved, it is possible to infer that the computational model for the biaxial elasto-plastic buckling of perforated plates was, respectively, verified and validated.

It is worth to mention that for the other numerical simulations performed in this work, it was adopted a converged SHELL281 mesh of finite quadrilateral elements of 50 mm side and refined at the line around the cutout.

4. Constructal Law and Constructal Design Application

According to [88], the Constructal Law is revolutionary because it is a law of physics and governs the design and rhythm of any finite size flow system, anywhere, encompassing inanimate (rivers and lightning bolts), animate (trees and animals) and engineered (technology) phenomena. In other words, all designs arise and evolve following the same law. In agreement with [89], the Constructal Law can be understood as a unifying principle of design.

The Constructal Law has been applied in engineering systems through the Constructal Design method, allowing to understand the effect of geometric configuration over the system performance. In addition, if the Constructal Design is employed together with an optimization technique, a geometric optimization can also be developed [6].

Moreover, as earlier mentioned, the flow systems are destined to be imperfect. Hence, its imperfections cannot be vanished, but can be distributed better to enable its currents to flow easier. So, with the Constructal Design application is possible to obtain not only better configurations but also better strategies for generating the geometries which will provide the optimum distribution of imperfections [5].

The following steps are recommended to apply the Constructal Design in a flow system: i) define the system (identify exactly what constitutes the system); ii) define the flow (identify what is flowing in the system and if the system has freedom to change); iii) start simple (define one degree of freedom to vary, understanding how these changes increase the flow access of the system currents); iv) add a degree of freedom (include other degree of freedom to vary, evaluating its effect over the system); v) and another (consider another degree of freedom, analyzing its influence in the system); vi) and so on (this process has no end...) [4].

Therefore, the Constructal Design application requires the definition of constraints (global or local), at least one degree of freedom (to vary freely, having as limits the constraints) and at least one performance parameter (which must be improved, indicating the easy of the access of internal currents in the system).

In agreement with [5], the Structural Engineering systems can be understood as flow systems configured and morph to facilitate the flow of stresses. It is quite unusual to understand stress as a flow, but is effective when defining the best geometric configuration of the stressed structural component. For each failure mechanism there is a way to allow the stresses to flow to maximize the load, considering a fixed volume, and minimize the volume when the load is fixed [46]. Furthermore, in MM problems the imperfections are the concentrations of maximum stresses. Therefore, the superior structural performance is achieved if the maximum allowable stresses are uniformly distributed through the available material [5].

A case study was proposed in this context: as reference a simply-supported steel plate subjected to a biaxial compressive load with the same magnitude in both directions (see Fig. 1a). From this, a centered elliptical perforation was considered in the reference plate (see Fig. 1b). As building material for these plates the AH-36 steel was used, having the following mechanical properties: yield stress $\sigma_y = 355$ MPa, Young's modulus $E = 210$ GPa, and Poisson's ratio $v = 0.30$. The Constructal Design application was made considering as constraints the plate dimensions $a = b = 1414.214$ mm and $t = 12$ mm, which were kept constant; and the volume fraction ϕ (defined as the ratio between perforation volume and the reference plate volume), assuming five different values ($\phi = 0.025, 0.05, 0.10, 0.15,$ and 0.20) which were kept constant during the cutout's geometries changing. Besides, the following degrees of freedom were defined: the cutout inclination angle α in relation to the x-axis; and the b_0/a_0 ratio, being the ratio between the dimensions of the elliptical cutout axes (see Fig. 1b). The α angle assumed the values of 0°, 15°, 30°, and 45°; while the b_0/a_0 ratio assumed several different values in each evaluation.

After that, all geometric configurations proposed by the Constructal Design application (the search space) were numerically simulated and its results were compared with each other, characterizing a geometric optimization by means the Exhaustive Search (ES) technique.

For this, as performance parameter the Normalized Ultimate Stress (NUS) was adopted, being obtained through the ratio between the ultimate buckling stress of perforated plates (σ_u) and the ultimate buckling stress of reference plate (σ_{ur}). The maximization of the NUS factor indicates the mechanical behavior improvement of the perforated plates. Thus, only for the geometries that maximized the NUS factor, a second performance parameter was analyzed: the Normalized Maximum Deflection (NMD), defined as the ratio between the maximum deflection of perforated plates (U_z) and the maximum deflection of reference plate (U_{zr}). The NMD, in turn, needs to be minimized to conduct to the superior structural performance.

5. Results and Discussion

The first numerical simulation was carried out for the plate with no hole, called reference plate (see Fig. 1a). Its ultimate buckling stress is $\sigma_{ur} = 56.80$ MPa and its maximum out-of-plane displacement is $U_{zr} = 44.73$ mm. In sequence, the geometric configurations of the search space were also numerically simulated.

Thereby, Figs. 2 to 6 presented, respectively for the ϕ values of 0.025, 0.05, 0.10, 0.15, and 0.20, the NUS factor as function of the b_0/a_0 ratio for each α value.
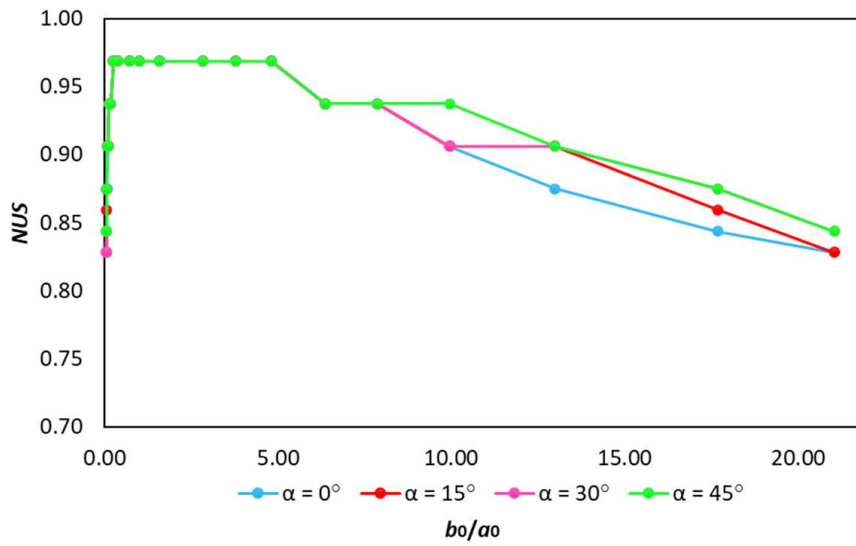
Fig. 2. Variation of NUS factor due to b_0/a_0 variation for each α and $\phi = 0.025$.
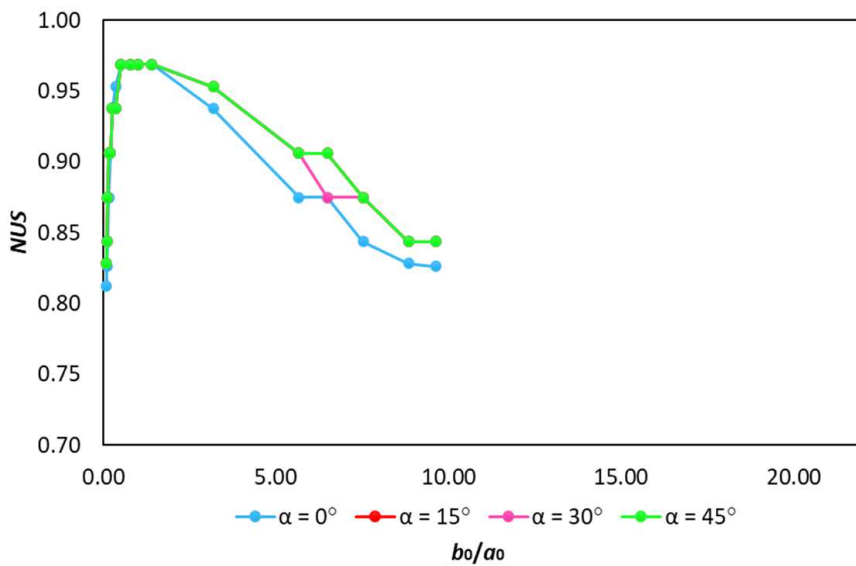

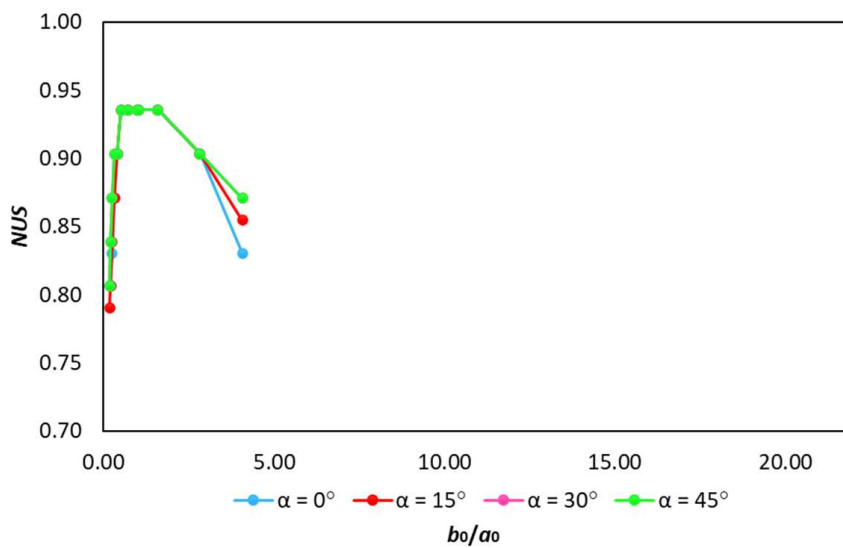**Fig. 3.** Variation of NUS factor due to b_0/a_0 variation for each α and $\phi = 0.05$.

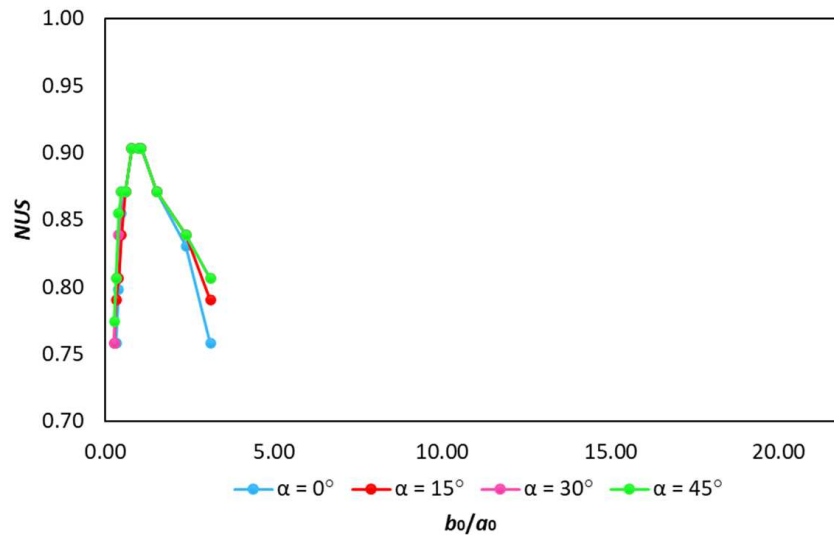**Fig. 4.** Variation of NUS factor due to b_0/a_0 variation for each α and $\phi = 0.10$.

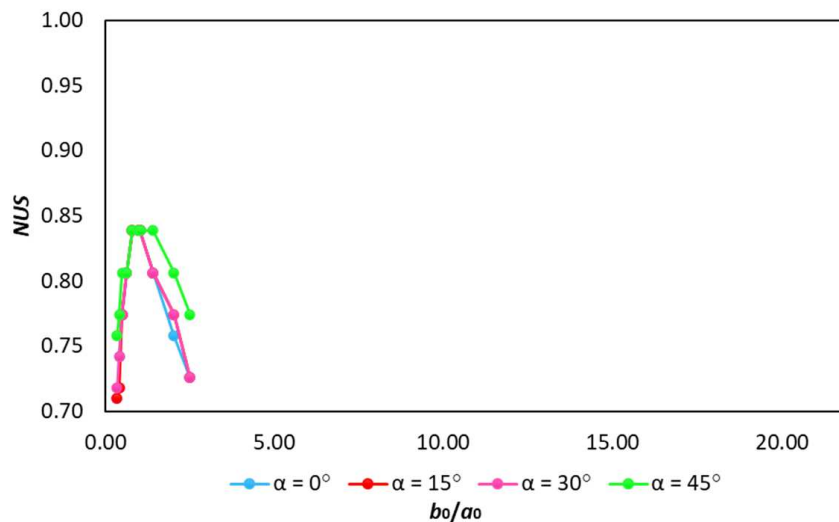**Fig. 5.** Variation of *NUS* factor due to b_0/a_0 variation for each α and $\phi = 0.15$.

Fig. 6. Variation of *NUS* factor due to b_0/a_0 variation for each α and $\phi = 0.20$.

Based on Figs. 2 to 6 one can observe that for all ϕ the mechanical behavior is similar: while b_0/a_0 increases from zero to one (0.00 to 1.00), an improvement occurs until reaching the maximized *NUS*; once reached that, the increasing of b_0/a_0 reduces the *NUS* factor magnitude. It is also possible to notice that in each ϕ for all analyzed α values the maximized *NUS* achieves the same value. In other words, it means that for the values of b_0/a_0 which maximize the *NUS* factor, the perforation inclination angle does not affect the best performances for the same ϕ. As already expected, regarding the influence of plate cutout on the maximized *NUS*, additional observations were done: the presence of hole in the plate is responsible for reducing the ultimate buckling stress when compared to the reference plate, since the maximized *NUS* values are always less than 1.00; and as the ϕ increases the maximized *NUS* values suffer a reduction in its magnitude.

Considering the structural geometric symmetry (a square plate), the boundary conditions symmetry (plate simply-supported in its four edges), and the load symmetry (plate under equal biaxial compressive loading), it is expected that the hole geometry which conducts to the superior performance is $b_0/a_0 = 1.00$. However, it was obtained the same maximized *NUS* for different b_0/a_0 ratios, as shown in Figs 2 to 6. Noting that, as ϕ increases the range of geometric configurations that reached the maximized *NUS* is smaller.

To understand how the Constructal Design defines the optimized geometries based on the *NUS* performance parameter, the von Mises stress distribution considering $\phi = 0.05$, different b_0/a_0 ratios, and $\alpha = 0°$, 15°, 30°, and 45° are, respectively, presented in Figs. 7 to 10. It is important to mention that for the other studied volume fractions ($\phi = 0.025$, 0.10, 0.15, and 0.20) the stress distributions' behavior is analogous to Figs. 7 to 10. So, for the sake of brevity, only the results for $\phi = 0.05$ were shown here.

From Figs. 7 to 10, a recurring aspect can be identified: some geometric configurations (cases c, d, and f) have more regions submitted to the maximum allowable stress of the AH-36 steel, represented in images by the red color. In other words, it is possible to state that these specific geometries can better distribute the biaxial buckling limit stress over the plate (i.e., the system imperfections). This finding was already observed in previous researches (as [54]; [57]; [59] and [60]), being in total agreement with the Constructal Principle of Optimal Distribution of Imperfections. This fact can be explained because the optimized geometric configurations can facilitate the flow of stresses in the plate, as stated by the Constructal Law.

In addition, regarding the Principle of Optimal Distribution of Imperfections and the characteristics of the proposed case study, among the geometries that maximized the *NUS* factor, those that qualitatively promote the better distribution of the imperfections are with the circular hole, i.e., with $b_0/a_0 = 1.00$, since they present a symmetric von Mises stresses distributions (see Figs 7d, 8d, 9d, and 10d). To illustrate this aspect for all ϕ values, Fig. 11 depicted the von Mises stress distribution for the reference plate together with the plates with the circular cutout ($b_0/a_0 = 1.00$).
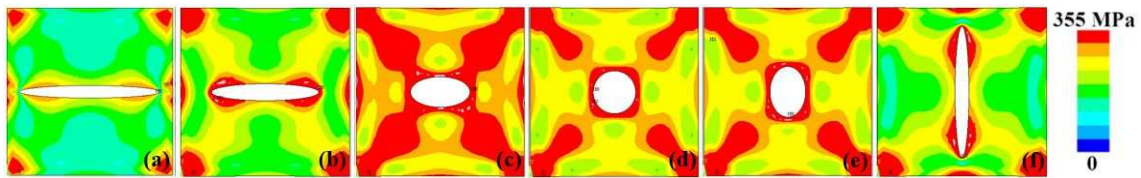
Fig. 7. Distribution of von Mises stresses for $\phi = 0.05$ and $\alpha = 0°$, being:
(a) $b_0/a_0 = 0.04$; (b) $b_0/a_0 = 0.16$; (c) $b_0/a_0 = 0.51$; (d) $b_0/a_0 = 1.00$; (e) $b_0/a_0 = 1.42$; (f) $b_0/a_0 = 8.84$.

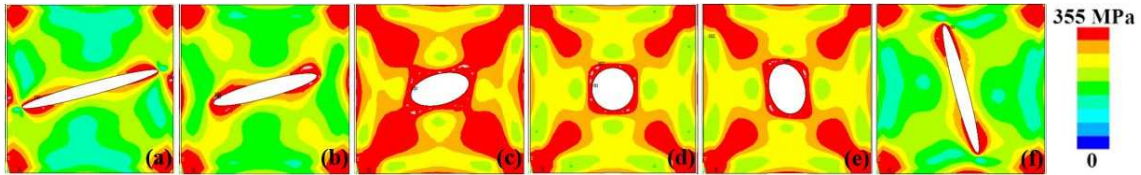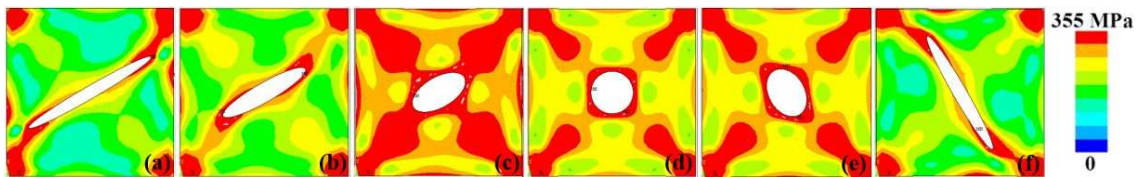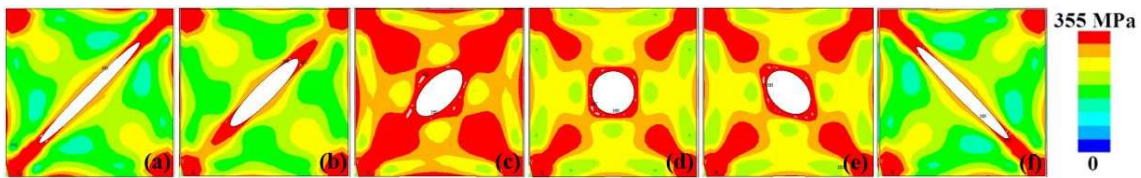**Fig. 8.** Distribution of von Mises stresses for $\phi = 0.05$ and $\alpha = 15°$, being:
(a) $b_0/a_0 = 0.04$; (b) $b_0/a_0 = 0.16$; (c) $b_0/a_0 = 0.51$; (d) $b_0/a_0 = 1.00$; (e) $b_0/a_0 = 1.42$; (f) $b_0/a_0 = 8.84$.

Fig. 9. Distribution of von Mises stresses for $\phi = 0.05$ and $\alpha = 30°$, being:
(a) $b_0/a_0 = 0.04$; (b) $b_0/a_0 = 0.16$; (c) $b_0/a_0 = 0.51$; (d) $b_0/a_0 = 1.00$; (e) $b_0/a_0 = 1.42$; (f) $b_0/a_0 = 8.84$.

Fig. 10. Distribution of von Mises stresses for $\phi = 0.05$ and $\alpha = 45°$, being:
(a) $b_0/a_0 = 0.04$; (b) $b_0/a_0 = 0.16$; (c) $b_0/a_0 = 0.51$; (d) $b_0/a_0 = 1.00$; (e) $b_0/a_0 = 1.42$; (f) $b_0/a_0 = 8.84$.
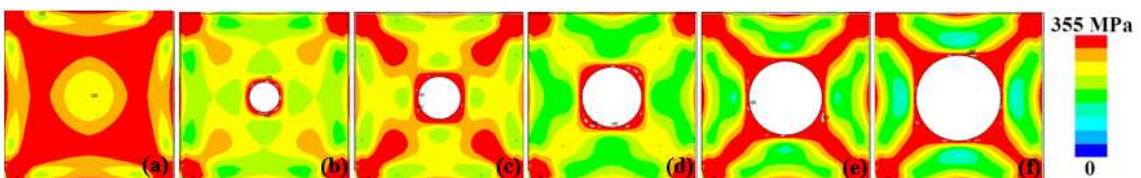

Fig. 11. Distribution of von Mises stresses for biaxial buckling of plates with no hole and with circular holes:
(a) reference plate; (b) $\phi = 0.025$; (c) $\phi = 0.05$; (d) $\phi = 0.10$; (e) $\phi = 0.15$; and (f) $\phi = 0.20$.
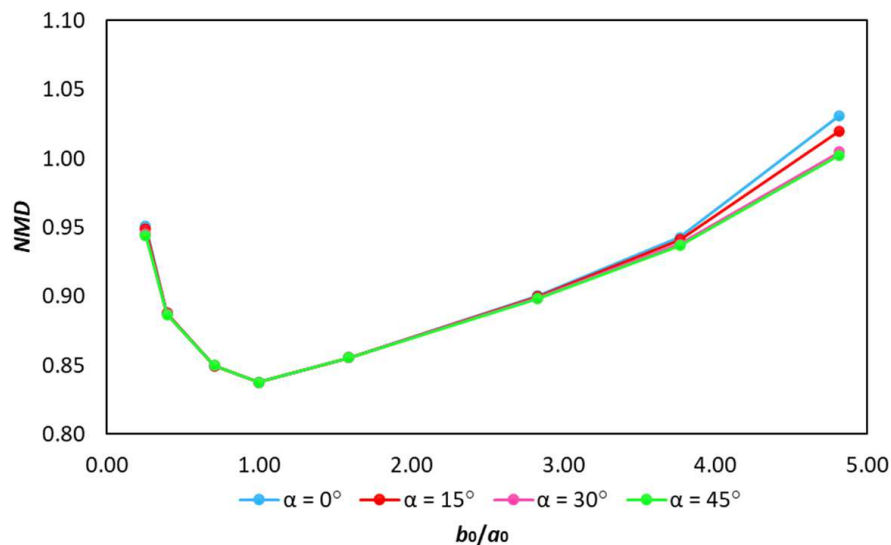

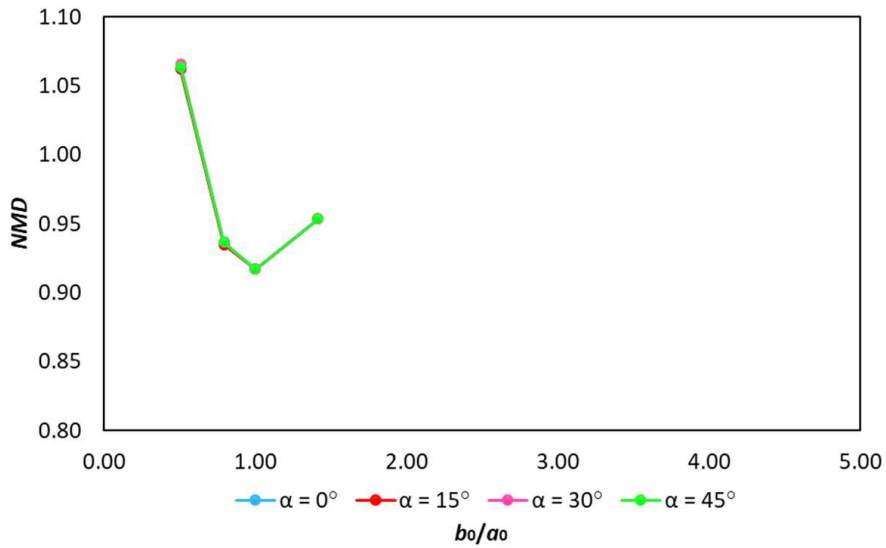**Fig. 12.** Variation of NMD factor due to b_0/a_0 variation for each α and $\phi = 0.025$.

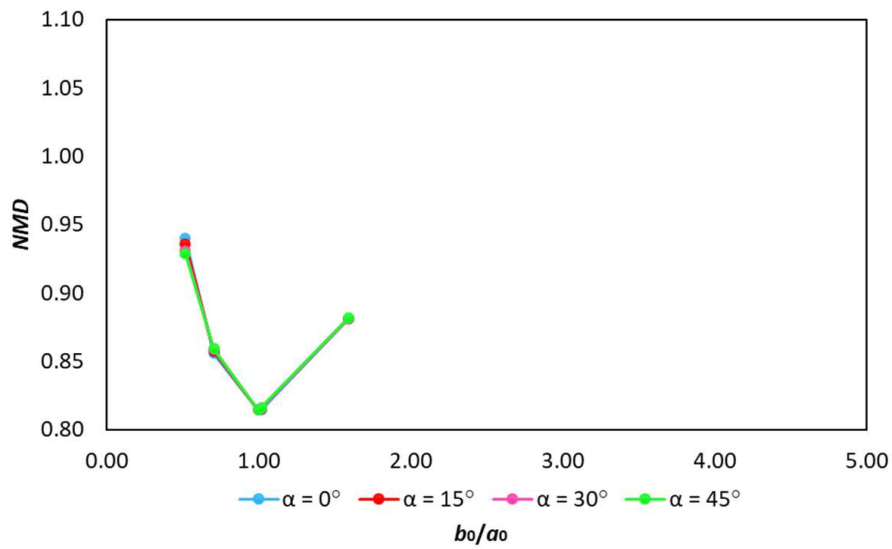**Fig. 13.** Variation of *NMD* factor due to b_0/a_0 variation for each α and $\phi = 0.05$.

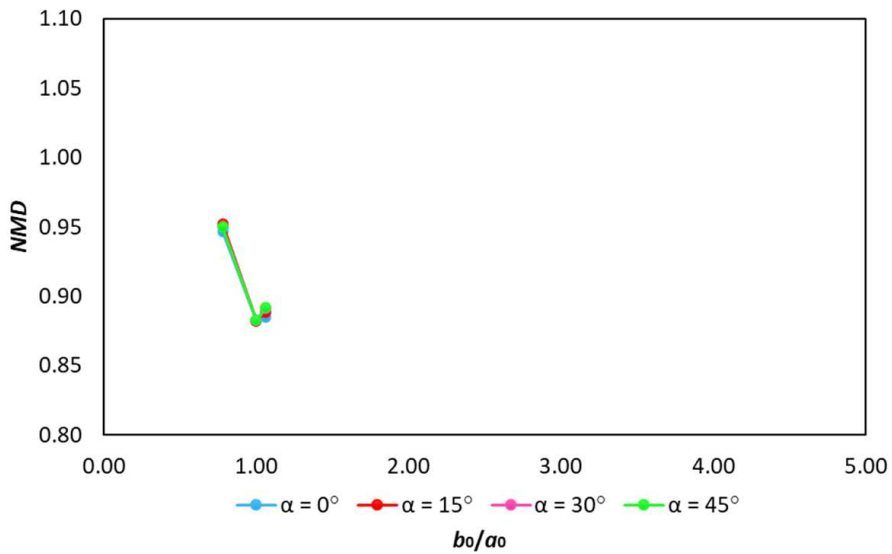**Fig. 14.** Variation of *NMD* factor due to b_0/a_0 variation for each α and $\phi = 0.10$.

Fig. 15. Variation of *NMD* factor due to b_0/a_0 variation for each α and $\phi = 0.15$.
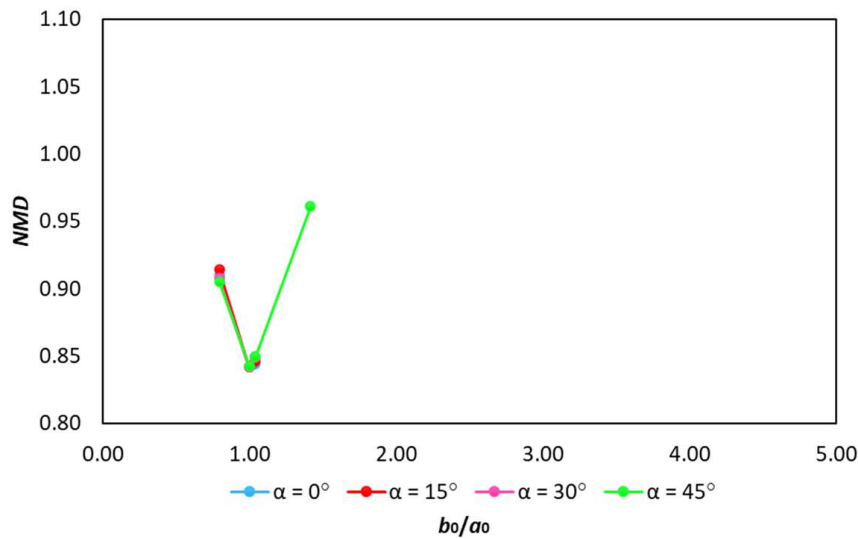
Fig. 16. Variation of NMD factor due to b_0/a_0 variation for each α and $\phi = 0.20$.
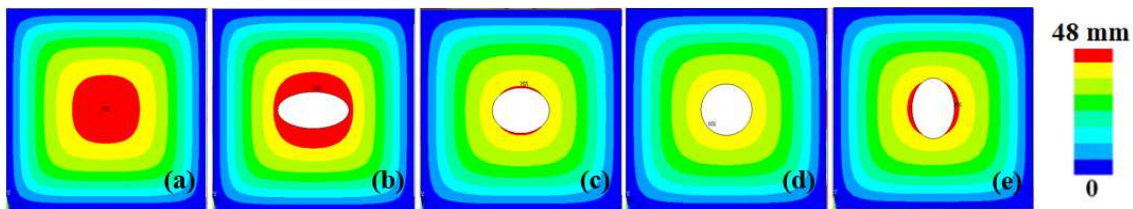

Fig. 17. Deflected configurations for plates with no hole and with elliptical holes:
(a) reference plate; (b) $b_0/a_0 = 0.51$; (c) $b_0/a_0 = 0.80$; (d) $b_0/a_0 = 1.00$; and (e) $b_0/a_0 = 1.42$.

Taking into account the earlier mentioned structural, support and loading symmetries of the case study, for all geometric configurations of Fig. 11 one can identify a symmetric stress distribution, indicating the better possible way to distribute the system imperfections in these MM problems.

The *NMD* performance parameter was adopted to support this qualitative finding, presented in Fig. 11, and give a quantitative indicator. The *NMD* factor was employed only to the geometric configurations, which maximized the *NUS* factor (see Figs. 2 to 6). In addition, it is important to remember that the *NMD* factor must be minimized towards to define the optimized geometry. Thus, the obtained *NMD* results for the five ϕ values are shown respectively, in Figs. 12 to 16, for each α value.

Based on Figs. 12 to 16, it is possible to infer that for all volume fractions (ϕ) and inclination angles (α), always the minimized *NMD* is obtained for the ratio $b_0/a_0 = 1.00$. In other words, when the elliptical cutout assumes the circular shape the deflection caused by the biaxial buckling phenomenon is the smallest, for the same *NUS* (calculated through the obtained biaxial buckling ultimate stress). To exemplify, Fig. 17 shows the deflected configurations for the reference plate as well as for the perforated plates attending to $\phi = 0.05$ and $\alpha = 0°$ from Fig. 13. It is important to mention that the same mechanical behavior trend indicated in Fig. 17 occurred for all studied ϕ and α values. For this reason, only Fig. 17 was included in the present work.

Therefore, the qualitative finding regarding the definition of the optimized geometric configuration by means the symmetric von Mises stress distribution was ratified through the quantitative results given by the minimization of the *NMD* factor. Through the distribution of deflections presented in Fig. 17, one can infer for the perforated plates that the deflected configuration is affected by the cutout shape. In Fig. 17b, for example, it is noted a large area in red color representing the maximum deflection due to buckling, having a magnitude slightly greater than the reference plate. In this specific case the region in red color is the most representative among all distributions of Fig. 17, showing that this is the most affected geometry by the ultimate biaxial buckling load, considering its out-of-plane displacements. Figures 17c and 17e also presented some plate regions submitted to the largest deflections, since it is possible to identify some regions in red color. However, when analyzed the circular hole (Fig. 17d), where $b_0/a_0 = 1.00$, there are no areas represented in red color what means that the plate's displacement did not reach the maximum value of the range of deflections.

6. Conclusions

The main purpose of the present work was to prove that the Constructal Law can also be applied to the geometric evaluation of Mechanics of Materials (MM) engineering systems, as it already is widely employed in Heat Transfer (HT) and Fluid Mechanics (FM) areas. The Constructal Design method associated with the Exhaustive Search technique was the approach adopted to the case study analysis, allowing, in addition to the geometric evaluation, the realization of a geometric optimization. The case study consisted of a symmetric structural engineering system: simply-supported square steel plates, having a centered elliptical perforation, subjected to equal biaxial compressive loads. From this, an elasto-plastic computational model based on the Finite Element Method (FEM) was used to obtain the ultimate buckling stress of these components. In addition, a plate with no hole was considered as reference. The volume fraction ϕ (ratio between the cutout volume and the total volume of the reference plate) assumed five different values. The degrees of freedom α (orientation of the elliptical hole from x-direction) and b_0/a_0 (ratio between the axes dimensions of the elliptical hole) were varied respecting the problem constraints, generating the different geometric configurations which defined the search space. In sequence, geometric optimization was performed by maximizing the *NUS* factor (ratio between the ultimate buckling stresses of the perforated plates and the reference plate).

However it is important to inform that the optimized geometric configuration, for all ϕ values, was already known before obtaining the results of the numerical simulations: it is the circular hole with $b_0/a_0 = 1.00$, independently of α value. This conclusion can be achieved based on the structural, support, and loading symmetries of the investigated problem. Highlighting that this case study was chosen precisely for that reason, i.e., the certainty of which geometry would lead to the superior performance. After that and employing the Constructal Law's precepts, the optimized geometric configuration, with $b_0/a_0 = 1.00$, was also defined. First, it was identified that some geometric configurations could maximize the *NUS* performance parameter, since these geometries make easier the flow of stresses. However, for each ϕ and each α, some other geometries with different values of b_0/a_0 also reached the maximized *NUS* factor; but only the plate with $b_0/a_0 = 1.00$ promotes the better distribution of imperfections, i.e., a symmetric distribution of the von Mises stress caused by the biaxial buckling.

Finally, to avoid the definition of the optimized perforated plate geometry be only based on this qualitative finding, another performance parameter was also considered: the *NMD* factor (ratio between the maximum displacement of the perforated plates and the reference plate, due to the biaxial buckling). Thus, only the geometric configurations which in previous analysis maximized the *NUS* factor were evaluated. Since the minimization of the *NMD* is required to indicate the optimum geometry, it was also quantitatively proved that the perforated square plates' optimized geometric configurations are those with $b_0/a_0 = 1.00$, i.e., with the circular cutout.

Given the above, the Constructal Law, applied by means the Constructal Design allied to the Exhaustive Search, adequately determined the geometric configuration that conducts the Structural Engineering system to superior performance, obtaining the expected optimized geometry for the symmetric proposed problem. Therefore, in this work it was proven the effective applicability of the Constructal Law to evaluate the geometric configuration influence over the mechanical behavior in Structural Engineering problems, aiming to help spread the use of the Constructal Design approach in this scientific area in future research.

Author Contributions

Conceptualization: T. da Silveira and L. A. Isoldi; Methodology: T. da Silveira, A. Pavlovic and L. A. Isoldi; Computational Model (development, verification and validation): T. da Silveira, V. T. Pinto, J. P. S. Neufeld and A. Pavlovic; Formal Analysis: E. D. dos Santos, L. A. O. Rocha and L. A. Isoldi. Investigation: T. da Silveira, V. T. Pinto, A. Pavlovic and J. P. S. Neufeld; Resources: L. A. O. Rocha, E. D. dos Santos and L. A. Isoldi. Data Curation: T. da Silveira and L. A. Isoldi. Writing (original draft preparation): T. da Silveira and V. T. Pinto. Writing (final version): E. D. dos Santos, L. A. O. Rocha and L. A. Isoldi; Supervision: L. A. O. Rocha and E. D. dos Santos; Project Administration: L. A. Isoldi. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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Nomenclature

a	Length of plate [m]	P_y	Yield load [N]
a_0	Horizontal axis dimension of elliptical cutout [m]	t	Thickness of plate [m]
b	Width of plate [m]	U_z	Maximum deflection of perforated plate [m]
b_0	Vertical axis dimension of elliptical cutout [m]	U_{zr}	Maximum deflection of reference plate [m]
E	Young's modulus [Pa]	x, y, z	Directions of coordinate system [-]
FEM	Finite Element Method	α	Inclination angle of cutout in relation to the *x*-axis [°]
FM	Fluid Mechanics	v	Poisson's ratio [-]
HT	Heat Transfer	σ_u	Ultimate buckling stress of perforated plate [Pa]
MM	Mechanics of Materials	σ_{ur}	Ultimate buckling stress of reference plate [Pa]
NMD	Normalized Maximum Deflection [-]	σ_y	Yield stress [Pa]
NUS	Normalized Ultimate Stress [-]	ϕ	Cutout volume fraction [-]

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