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Abstract. Structural Health Monitoring (SHM) systems are adopted to acquire timely and continuous data on the state of civil structures, aerospace vehicles, and industrial machines, which deteriorate due to slow processes, such as corrosion and fatigue, and shock events, including natural and handmade disasters. The components of SHM systems are exposed themselves to deterioration after their installation; thereby, they might provide altered information to decision-makers. To account for this, Sensor Validation Tools (SVTs) have been developed to give insight into the actual condition of the SHM systems. In the last decade, researchers have exploited the Value of Information (VoI) from Bayesian decision theory to quantify the benefit of the information provided by an SHM system, implicitly assuming that it is working correctly when interrogated. The benefit of the information provided by SVTs on the state of an SHM system has never been investigated. This paper addresses this topic and extends the VoI framework to quantify the additional benefit brought by the information on the state of the SHM system to the decision problems the SHM is meant to support. Exemplary case studies are presented to demonstrate the application of the framework.

Keywords: Value of Information, Bayesian decision theory, Structural Health Monitoring, data quality, sensor fault.

1. Introduction

Civil structures, as well as aerospace vehicles and industrial machines, deteriorate in time, e.g., due to the effect of environmental factors, slow material degradation, and sudden damaging events. Structural

Health Monitoring (SHM) systems have been applied in the last few decades to several fields to provide timely health-condition evaluation and security warnings by identifying and tracking the evolution over time of so-called "damage-sensitive features". These are parameters representative of the structural state, and their variations with respect to an initial reference condition provide an indication of damage. In general, synthetic damage indicators are used to quantify the variations of the damage sensitive features. They can be defined as functions of such features that might not have an intuitive physical interpretation, such as the T^2 metric [1] and the Modal Assurance Criterion (MAC) [2]. Depending on the approach adopted for damage identification and on the available data, damage can be identified at different levels of refinement known as damage detection, damage localization, and damage quantification [3]. Damage detection provides a binary outcome related to the presence (or not) of damage, whereas localization and quantification also inform about the position of the damage and its severity, respectively. Novelty detection approaches with a binary outcome are often used for damage detection of civil structures such as bridges [1], historical monuments [4,5], and individual structural elements [2].

Condition monitoring using SHM systems is generally based on the assumption that the data acquired from sensors contain reliable information about the health state of the structure. The prediction of response and performance of a civil structure must be referred to the entire system and not only to the structural elements. For a monitored structure, the system consists of the structure itself and the SHM system. In turn, an SHM system typically consists of three subsystems: a sensor apparatus, a data transmission module, and a health evaluation section [6]. Very frequently, the harsh environmental conditions in which civil and mechanical structures operate generate malfunctions in the sensing apparatus, which generally result in anomalies in the measured data [7]. Moreover, inner system malfunctions and interferences in the wireless transmissions employed in the most recent sensor networks can lead to data loss [8]. Such anomalies pose an important limitation for effective damage identification and, in some cases, can cause malfunctioning of the entire monitoring network [9].

As a matter of fact, inaccurate or missing data in the structural assessment and life-cycle management can lead to significant economic loss [10], as anomalous data may generate false alarms, resulting in unnecessary operational interruptions and structural maintenance, and missed detection, which may increase the probability of catastrophic accidents [6]. Promptly identifying recordings containing anomalous or incomplete data is thus an essential step in developing a successful monitoring system.

Due to the broad palette of disturbing factors, several sensor fault types exist, each of them leading to a different effect on collected data. Concerning vibration-based SHM, Kullaa [11] categorized seven recurrent sensor fault types: bias, drift, gain, precision degradation, constant recording, constant recording with noise, and bottom noise. The first four types have been targeted as "soft" sensor faults since it could still be possible to retrieve structural information, while the last three are called "hard" sensor faults, for which the data does not carry any useful information. In the scenario where collected data is directly processed without assessing its quality, both soft and hard sensor faults may lead to severe malfunctioning of the monitoring systems. However, if sensor faults are detected, isolation and data reconstruction procedures could be implemented to limit the disservice.

The importance of Sensor Validation Tools (SVTs) for the assessment of data quality was first recognized by Dunia et al. [12] in the field of chemical process monitoring. Friswell and Inman [13] studied this aspect a few years later for SHM applications. Since then, several researchers have proposed SVTs employing one-class classifiers and multivariate statistical analysis [6]. Specifically, the first category studies each sensor individually to understand whether the sensing apparatus is normal or faulty [14]. The second category is based on the correlations among the sensors of the network, thus identifying sensor faults by comparing the data collected by the different sensors that form the monitoring system [6,7,11,12,15–20]. Lately, machine learning has gained particular interest in the fault identification field [21–23]. It should be kept in mind that SVTs are inherently imperfect, as working sensors can be classified as faulty, and some data anomalies could not be recognized, leading to the aforementioned risks. Since SVTs generally have a cost and require a dedicated computational apparatus, which may dictate the selection of more expensive hardware to realize the sensor network, the real value of providing an SHM system with SVTs should be quantified.

In addition to the possible malfunctioning of the monitoring network, a remaining issue in SHM applications is to convince owners and operators of what its "added-value" is and what its social and economic benefits are. To this aim, the Value of Information (VoI) from Bayesian decision theory [24,25] has been the core of several studies oriented to evaluate the long-term economic benefit provided by an SHM system before it is adopted. The VoI is defined as the expected reduction in management costs associated with the acquisition of new information [26–28]. Application examples include emergency management following damaging events [29,30], the optimization of sensor deployment [31,32], and the definition of optimal maintenance and data collection strategies [33,34]. The interested reader is referred to Reference [35] for a recent state of the art on VoI.

Information modeling is a critical task in VoI computations [36] since the VoI is evaluated in the so-called Pre-Posterior analysis framework (as explained better in Section 2), i.e., before observing the "real" SHM outcome. Information types are typically classified as perfect information and imperfect information [24]. In the case of perfect information, which is an ideal situation, the state of the structure is known without any uncertainty. On the other hand, imperfect information reduces the uncertainty on the state of the structure but does not eliminate it. Common ways to model information in the VoI literature are by means of additive error [26,37] and likelihood functions of the damage-sensitive feature [38,39]. Imperfect information can be modeled employing either simplified [28] or sophisticated [40] simulation methods. Few authors have studied the effect of data quality on VoI. For instance, Ali et Al. [41] investigated the effect of introducing biases and dependences on the VoI. To the authors' knowledge, the VoI relevant to the conditions of the SHM system itself has never been addressed.

In this paper, the classical VoI framework is extended to include different states of the SHM system and quantify the added value of the information provided by an SVT. A key novelty of the proposed framework is an original formulation of the likelihood functions of the SHM outcome, in which the probability of observing an SHM outcome is not only conditioned on the state of the structure but also the state of the SHM system. Specifically, three "faulty" conditions of the SHM system are studied and compared to the "properly working" system. The effects of these faulty conditions on the damage indicator tracked during the monitoring process are modeled as three phenomena representative of the fault type classification identified by Kullaa [11]: missing information (representing "hard" fault types), noisy data (i.e., precision degradation), and drift (which is also representative of bias and gain in this framework). These effects are considered in a damage detection perspective, i.e., for SHM systems that only identify whether damage is present or not. The framework is general and can be applied to damage detection problems regardless of the specific damage-sensitive features and damage indicator selected for damage identification. As aforementioned, some widely used damage indicators, as well as damage-sensitive features [42], do not have an intuitive physical interpretation. For instance, the T^2 and the MAC are only meant to provide - in the most effective way - information about the variation of a damage sensitive feature. Nevertheless, damage indicators carry information about the state of the structure and are affected by the state of the SHM system, which may alter the SHM outcome and thus impact the decision process.

Uncertainty in the SVT results is also accounted for to show that, in general, the adoption of an SVT enhances the overall benefit provided by an SHM system. Different case studies are undertaken to demonstrate the extended framework and to explore the effect of different sensor fault conditions on the VoI.

The paper is structured as follows. After the Introduction, Section 2 presents the theoretical framework of the VoI for SHM systems, including both the classical Pre-Posterior analysis (Section 2.1) and its extension to quantify the benefit of SVT-related information (Section 2.2). Section 3 discusses a numerical case study to study the effect of two common sensor fault conditions (drift and noise) on the VoI. Then, in Section 4, a practical demonstration of the framework is shown using the data collected during a real experimental campaign, in which the absence of information is considered. Both in Sections 3 and 4, several sensitivity analyses that considers different application scenarios are carried out. Concluding remarks containing the most relevant findings are lastly reported.

2. Theoretical framework

The VoI is defined in the realm of the Bayesian decision theory, which deals with the rational selection of actions in an uncertain environment [24,25]. According to the available information, different types of analyses can be carried out, i.e., the Prior analysis, the Posterior analysis, and the Pre-Posterior analysis. Henceforth, it is assumed that the source of information is an SHM system: the Prior information is carried out without information from the SHM system, using the prior knowledge of the decision-maker; the Posterior analysis is carried out when the information is available; the Pre-Posterior analysis is carried out before collecting the new information. The Bayesian decision theory allows to handle decisions on actions (e.g., restrict traffic on a bridge or not) and about collecting new information (e.g., install an SHM system or not). The latter type of decision is based on the VoI, which is obtained by comparing the results of the Prior analysis with the results of the Pre-Posterior analysis. The classical Pre-Posterior analysis does not consider different states of an SHM system (e.g., working, or faulty sensors) nor the possibility that information about the state of the SHM system is collected. These issues are addressed in this section, where the framework of the VoI is first briefly presented in the classical form, and then extended to assess the benefit of collecting information about the state of the SHM system.

2.1. Bayesian decision analysis

The Bayesian decision theory is rooted in the Bayesian definition of probability [43] and the utility theorem [44]. Specifically, a Bayesian probability quantifies a personal belief on a certain state of a structure, whereas the utility theorem defines the behavior of a rational decision-maker. The decision problem involves a set of states of the structure (which can be in different damage states or in a healthy conditions) s_l, $l = 1, \dots, L$, a set of actions A_n , $n = 1, \dots, N$, and the utility corresponding to different combinations of actions and states, $E[u(A_n)|s_l]$, which is a numerical value that expresses the desirability of a combination of actions and states of the structure. The decision-maker ranks the actions based on their expected utility and then selects the one associated with the maximum utility. To this purpose, the decision-maker must define the probability of the states of the structure, $P(s_l)$, according to their knowledge. Typically, the prior probabilities are retrieved by means of engineering judgments or reliability analyses if probabilistic models of capacity and demand are available, which can also include the effect of time and related degradation phenomena [26,30,37,45]. For the sake of clarity, this study focuses on a static problem, in which the effects of slow degradation phenomena are not accounted for.

The Prior analysis is carried out employing prior probabilities of the states of the structure, i.e., without information from the SHM system. The expected utility of each action is computed as follows:

$$E[u(A_n)] = \sum_{l=1}^{L} E[u(A_n)|s_l]P(s_l) \tag{1}$$

The optimal action $\hat{A}$ and the corresponding expected utility u_1 are evaluated, respectively, as

$$\hat{A} = \arg\max_n E[u(A_n)] \tag{2}$$

and

$$u_1 = E\big[u(\hat{A})\big] = \sum_{l=1}^{L} E\big[u(\hat{A})|s_l\big]P(s_l) \tag{3}$$

When an outcome o_j, with $j = 1, \dots, J$, becomes available from the SHM system, it can be used to update the prior probabilities of the states of the structure using the well-known Bayes theorem, which reads:

$$P(s_l|o_j) = \frac{P(o_j|s_l)P(s_l)}{P(o_j)} \tag{4}$$

where $P(s_l|o_j)$ is the posterior, i.e., updated, probability of the state s_l given the monitoring outcome o_j; $P(o_j|s_l)$ is the likelihood function which expresses the probability of observing the outcome o_j when the state of the structure is s_l; and $P(o_j)$ is the total probability of the outcome o_j, which is obtained as

$$P(o_j) = \sum_{l=1}^{L} P(o_j|s_l)P(s_l) \tag{5}$$

The expected utility of the action A_n, given o_j reads:

$$E[u(A_n)|o_j] = \sum_{l=1}^{L} E[u(A_n)|s_l]P(s_l|o_j) \tag{6}$$

Since the SHM outcome is available, a Posterior analysis can be performed, where the optimal action $\breve{A}_{o_j}$ and the associated expected utility $E[u(\breve{A}_{o_j})|o_j]$ are respectively computed as:

$$\breve{A}_{o_j} = \breve{A}(o_j) = \arg\max_n E[u(A_n)|o_j] \tag{7}$$

$$E\left[u\left(\breve{A}_{o_j}\right)|o_j\right] = \sum_{l=1}^{L} E\left[u\left(\breve{A}_{o_j}\right)|s_l\right]P(s_l|o_j) \tag{8}$$

The decision on installing or not an SHM system is based on the results of the Pre-Posterior analysis, which is carried out before (i.e., “Pre-”) installing the SHM system whose outcomes will be used – if the SHM is installed – to obtain updated (i.e., “Posterior”) probabilities of the states of the structure. The expected utility associated with a given SHM system is computed during the Pre-Posterior analysis in two steps (in its extensive form, see [24]). First, a Posterior analysis is carried out for each possible SHM outcome to obtain the expected utility of the optimal action, $E\left[u\left(\breve{A}_{o_j}\right)|o_j\right]$. After that, the SHM outcome is marginalized out to obtain the expected utility of the informed decision making, u_0, as follows:

$$u_0 = \sum_{j=1}^{J} E\left[u\left(\breve{A}_{o_j}\right)|o_j\right]P(o_j) \tag{9}$$

The VoI is defined as the difference between u_0 (from Eq. 9) and u_1 (from Eq. 3) and represents the increase in expected utility associated with a given SHM system:

$$\mathrm{VoI} = u_0 - u_1 \tag{10}$$

Generally, in engineering applications, the utility is expressed as a negative cost [28]. In this case, the VoI can be directly compared with the cost of the SHM system to decide if its installation is cost-effective. Specifically, if the difference between the VoI and the cost of the SHM system is lower than zero, the SHM system should not be installed.

2.2. Extension of the framework

In this section, the framework of the VoI from Bayesian analysis is extended to quantify the additional benefit provided by an SVT. For this purpose, it is necessary to introduce two additional random variables, specifically, the states of the SHM system (which can be in a faulty state or can be working properly), m_k, with $k = 1, \dots, K$, and the outcomes of the SVT, c_h, with $h = 1, \dots, H$ (normally $H = K$). The main assumptions made to extend the VoI framework are the following:

- The observations made using SHM on the state of the structure (o_j) depend on the state of both the structure (s_l) and the SHM system (m_k), i.e., $P(o_j|s_l, m_k)$;
- The state of the SHM system does not depend on the state of the structure, i.e., $P(s_l, m_k) = P(s_l)P(m_k)$, where $P(s_l, m_k)$ is the joint probability distribution of s_l and m_k. This

assumption can be justified by the fact that when damage occurs in the structural elements, sensing devices can be properly functioning (if the damage does not involve collapses).

- The observations on the state of the SHM system (c_h) depend on the state of the SHM system (m_k) and not on the state of the structure, i.e., $P(c_h|m_k)$.

Based on these assumptions, the classical Pre-Posterior analysis is thus extended to account for (i) the situation in which only the prior knowledge of the conditions of the SHM system is available and (ii) the condition in which the decision-maker is planning to collect information from both the SVT and the SHM system.

2.2.1. Pre-Posterior analysis with Prior Knowledge of the SHM system

The decision analysis addressed in this section is carried out before the installation of the SHM system, accounting for the different states of the SHM system and all the possible SHM outcomes. The introduction of multiple states of the SHM system has two main effects on the Pre-Posterior analysis. First, the state of the system now comprises both the state of the structure and the state of the SHM system. Therefore, a joint probability distribution $P(s_l, m_k)$ should be employed to represent the prior probability of occurrence of the different states. Secondly, the introduction of multiple states of the SHM system directly affects the likelihood functions of the SHM outcome, which now are not only conditional on the state of the structure, but also the state of the SHM system, i.e., $P(o_j|s_l, m_k)$. In this case, the Bayes' theorem in Eq. 4 becomes:

$$P(s_l, m_k|o_j) = \frac{P(o_j|s_l, m_k)P(s_l, m_k)}{P(o_j)} \tag{11}$$

Since the states of the structure and the SVT are considered independent events, Eq. 11 can be reformulated as follows:

$$P(s_l, m_k|o_j) = \frac{P(o_j|s_l, m_k)P(s_l)P(m_k)}{P(o_j)} \tag{12}$$

The novel formulation of the likelihood function is a key aspect of the proposed framework since it makes the result of the SVT have an impact on the choice of the optimal action. In this regard, it is important to consider that the new information has value only if it is able to modify the choice of the optimal action selected during the prior analysis, with prior knowledge [37,46].

The expected utility of an action A_n given the outcome o_j computed according to Eq. 6 is modified, accounting for the multiple states of the SHM system, as follows:

$$E[u(A_n)|o_j] = \sum_{l=1}^{L}\sum_{k=1}^{K} E[u(A_n)|s_l] \frac{P(o_j|s_l, m_k)P(s_l)P(m_k)}{P(o_j)} \tag{13}$$

For each SHM outcome, the optimal action $\breve{A}_{o_j}$ is the one associated with the maximum expected utility $E[u(\breve{A}_{o_j})|o_j]$, with

$$\breve{A}_{o_j} = \breve{A}(o_j) = \arg\max_n E[u(A_n)|o_j] \tag{14}$$

$$E\left[u\left(\breve{A}_{o_j}\right)|o_j\right] = \sum_{l=1}^{L}\sum_{k=1}^{K} E\left[u\left(\breve{A}_{o_j}\right)|s_l\right] \frac{P(o_j|s_l, m_k)P(s_l)P(m_k)}{P(o_j)} \tag{15}$$

Therefore, the expected utility of the informed decision-making considering multiple states of the SHM system reads:

$$
\begin{aligned}
u_{0,M} &= \sum_{j=1}^{J} E\left[u\left(\breve{A}_{o_j}\right)|o_j\right] P(o_j) \\
&= \sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{k=1}^{K} E\left[u\left(\breve{A}_{o_j}\right)|s_l\right] P(o_j|s_l, m_k)P(s_l)P(m_k)
\end{aligned}
\tag{16}
$$

The VoI evaluated accounting for the state of the SHM system, $\mathrm{VoI_M}$, is computed as follows:

$$\mathrm{VoI_M} = u_{0,M} - u_1 \tag{17}$$

2.2.2. *Pre-Posterior analysis with Pre-Posterior Knowledge of the SHM system*

The decision analysis presented herein is carried out before the installation of the SHM system and the related SVT, accounting for the different states of the SHM system and all the possible outcomes of the two monitoring systems. The state of the structure is updated based on the outcomes of the SHM system, whereas the state of the SHM system is updated based on the outcomes of the SVT. Specifically, the Bayes' theorem is used to update the prior probabilities $P(m_k)$ associated with the states of the SHM system, as follows:

$$P(m_k|c_h) = \frac{P(c_h|m_k)P(m_k)}{P(c_h)} \tag{18}$$

where $P(c_h|m_k)$ is the probability that the SVT provides the outcome c_h when the true state of the SHM system is m_k, and the denominator $P(c_h)$ is defined as:

$$P(c_h) = \sum_{k=1}^{K} P(c_h|m_k)P(m_k) \tag{19}$$

Repeated checks of the state of the SHM system using an SVT could be included in the VoI analysis, if a more complex decision scenario is considered. This issue can be addressed in different ways, according to the method adopted to model the likelihood functions [26,28,37,40]. One possibility involves the formulation of the likelihood functions for sets of SVT outcomes [47]. In case of independent SVT outcomes, these likelihood functions can be obtained by multiplying the probabilities of observing single SVT outcomes for a given state of the structure. When appropriate, dependency between monitoring outcomes must be modelled explicitly [47].

The expected utility of an action A_n is obtained by substituting the prior probability $P(m_k)$ with its posterior counterpart $P(m_k|c_h)$ in Eq. 13, thus leading to

$$E\left[u(A_n)\middle|o_j, c_h\right] = \sum_{l=1}^{L}\sum_{k=1}^{K} E[u(A_n)|s_l] \frac{P(o_j|s_l, m_k)P(s_l)P(c_h|m_k)P(m_k)}{P(o_j)P(c_h)} \tag{20}$$

In this case, the optimal action $\breve{A}(o_j, c_h)$, associated with the minimum expected utility $E[u(\breve{A}_{o_j c_h})|o_j, c_h]$, is

$$\breve{A}_{o_j c_h} = \breve{A}\left(o_j, c_h\right) = \arg\max_n E\left[u(A_n)\middle|o_j, c_h\right] \tag{21}$$

$$E\left[u\left(\breve{A}_{o_j c_h}\right)|o_j, c_h\right] = \sum_{l=1}^{L}\sum_{k=1}^{K} E\left[u\left(\breve{A}_{o_j c_h}\right)|s_l\right] \frac{P\left(o_j|s_l, m_k\right)P(s_l)P(c_h|m_k)P(m_k)}{P\left(o_j\right)P(c_h)} \tag{22}$$

The expected utility c_{0,M^2} of the informed decision making is computed as follows:

$$\begin{aligned} u_{0,M^2} &= \sum_{h=1}^{H}\sum_{j=1}^{J} E\left[u\left(\breve{A}_{o_j c_h}\right)|o_j, c_h\right] P\left(o_j\right)P(c_h) \\ &= \sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{k=1}^{K} E\left[u\left(\breve{A}_{o_j c_h}\right)|s_l\right] P\left(o_j|s_l, m_k\right)P(s_l)P(c_h|m_k)P(m_k) \end{aligned} \tag{23}$$

Finally, the benefit gained from monitoring both the states of the SHM system and the structure, $\mathrm{VoI}_{\mathrm{M}^2}$, reads:

$$\mathrm{VoI}_{\mathrm{M}^2} = u_{0,M^2} - u_1 \tag{24}$$

where the term u_1 is computed according to Eq. 3 and u_{0,M^2} is computed according to Eq. 23. The additional VoI provided by the SVT, ΔVoI, is:

$$\Delta\mathrm{VoI} = \mathrm{VoI}_{\mathrm{M}^2} - \mathrm{VoI}_{\mathrm{M}} = u_{0,M^2} - u_{0,M} \tag{25}$$

The framework proposed in this section allows accounting for the situation in which the decision-maker does not know the real condition of the SHM system. This is the actual situation in practice. The framework is general and enables to model and consider simultaneously different types of sensor fault conditions (e.g., noise, drift, absence of data) which can affect the SHM outcome. The classic VoI theory described in Section 2.1 cannot be applied to account for these phenomena. Instead, this is possible through the novelties introduced in Section 2.2. In the following sections, the proposed framework is applied to demonstrate its applicability and to investigate the effects of monitoring different sensor fault conditions on the VoI. The case studies consist of a numerical case study and a real bridge. The effects of drift and noise are addressed in the numerical case study, while the absence of SHM data is considered with the real bridge application. This latter case also brings an added value related to using faulty sensor data measured during a real monitoring application.

3. Numerical case study

The proposed approach is applied in this section to a reference structure to investigate the effects of different sensor faulty conditions. For the sake of clarity, a simple decision problem is addressed, as follows: the structure can be in two states, s_1 – healthy condition – or s_2 – damaged condition – and the decision-maker has to select the optimal action between A_1 – Do nothing, i.e., keep the structure functional – and A_2 – shut it down.

Each combination of actions and states of the structure is associated with a utility, as shown in **Error! Reference source not found.**. Specifically, it is assumed that if the structure is functional and in the healthy condition, there is no loss and thus the utility is zero; if the structure is functional and in the damaged condition it might fail due to external actions, thereby, a negative utility $u_F = -1$ is associated with this situation; shutting the structure down generates only indirect losses, quantified by the negative utility $u_{SD} = -0.5$, which do not depend on the state of the structure. Figure 1 displays the results of the Prior analysis, i.e., the expected utility of the two actions (Do nothing and Shut down) computed using the prior knowledge. Results are expressed as a function of $P(s_2)$. For values of $P(s_2)$ lower than 0.5, keeping the structure operative is the optimal action, i.e., the action associated with the maximum expected utility. Instead, for values of $P(s_2)$, higher than 0.5, shutting the structure down is preferable. For $P(s_2) = 0.5$, the two actions have the same expected utility.

Table 1. *Utility of different combinations of actions and structural states.*

	$s_1 = healthy$	$s_2 = damaged$
$a_1 = Do\ nothing$	0	u_F
$a_2 = Shut\ down$	u_{SD}	u_{SD}

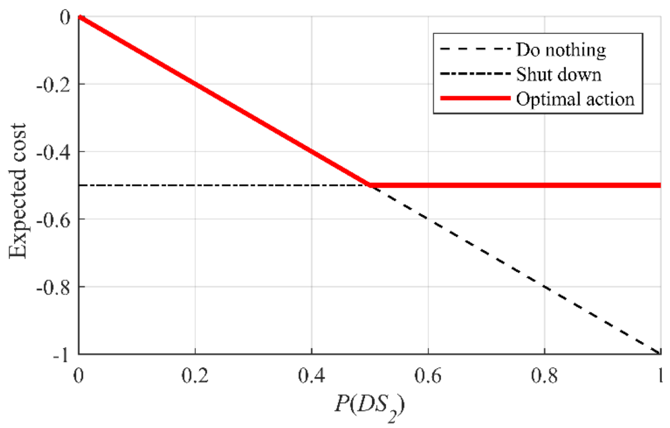

Figure 1. Results of the Prior analysis.

The SHM system (not yet installed) provides a continuous outcome, modeled through Normal distributions, $N(\mu, \sigma)$, with mean μ and standard deviation σ. The parameters of such distributions depend on both the state of the structure and the state of the SHM system. The SHM system can be in two states, namely the properly working condition m_1 and the faulty condition m_2. In this application, the following prior probabilities are assumed: $P(m_1) = P(m_2) = 0.5$. Two faulty conditions are analyzed – separately – in the following sections, namely drift and noise. When the SHM is working correctly (state m_1) the distributions of the (unitless) SHM outcomes are $N(1{,}0.1)$ and $N(0.7{,}0.1)$ for the structural states s_1 and s_2, respectively. The Probability Density Functions (PDFs) of these distributions are shown in Figure 2(a). An SVT can give insight into the actual state of the SHM system by providing two discrete outcomes c_1 and c_2 associated with the properly working and the faulty conditions, respectively.

The drift consists of a shift δ (either positive or negative) of the mean value of the distributions of the SHM outcome, as displayed in Figure 2(b). Instead, the noise is modeled as an increase ε of the standard deviation, see Figure 2(c). It is assumed that the drift and the noise affect equally the distributions of the SHM outcome in the healthy and damaged conditions of the structure.

In the following sections, the effects of drift and noise of the SHM outcomes on the VoI are investigated through sensitivity analyses.

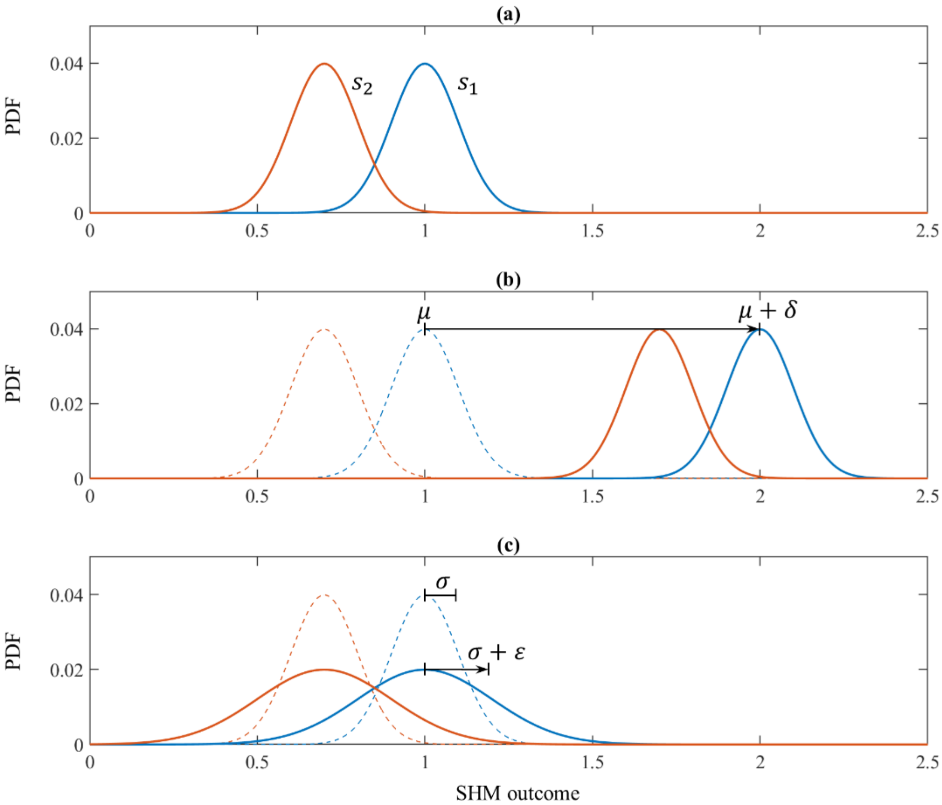

Figure 2. *Likelihood functions in the different states of the SHM system: (a) properly working system; (b) SHM outcome affected by drift; (c) SHM outcome affected by noise.*

3.1. Drift

Two situations are analyzed in this section, namely the situation in which the SVT provides imperfect information on the presence of drift in the SHM outcome and the case in which the SVT provides perfect information. In both cases, the VoI is expressed as a function of drift magnitude δ and the prior probability of the state of the structure $P(s_2)$. Both variables vary in the interval 0-1.

Error! Reference source not found. shows the probabilities of observing the SVT outcomes in different states of the SHM system (i.e., the likelihood functions of the SVT) in case of imperfect information. In the case of perfect information, the probabilities are one on the diagonal and zero otherwise.

Table 2. *Likelihood of the SVT in case of imperfect information.*

	s_1	s_2
c_1	0.8	0.2
c_2	0.2	0.8

Figure 3 shows the results of the VoI analysis in case of drift presence and imperfect SVT outcome. Specifically, Figure 3(a) displays the VoI associated with monitoring the state of the structure only, $\mathrm{VoI_M}$. For $P(s_2) = 0$ and $P(s_2) = 1$ the $\mathrm{VoI_M}$ is zero since in these situations, the new information does not modify the prior probabilities of the structural states. The $\mathrm{VoI_M}$ is maximum for $P(s_2) = 0.5$ because this is the condition of maximum uncertainty for the decision maker since the two actions have the same expected utility, see [29,46]. The $\mathrm{VoI_M}$ reduces considerably in the proximity of $\delta = 0.3$. For this drift level, the PDF of the SHM outcome s_2 for m_2 (solid red line) overlaps the PDF of the SHM outcome s_1 for m_1 (dashed blue line), see Figure 2(b). Therefore, the SHM outcome does not support the decision-maker in distinguishing the two cases. Instead, the presence of drift does not affect significantly the $\mathrm{VoI_M}$ for relatively small or high drift levels. In these situations, the distributions of the SHM outcome in m_1 and m_2 (properly working and faulty conditions of the SHM system, respectively) are well separated thereby the decision maker is able to distinguish between the healthy and the damaged conditions of the structure even when the SHM system is not working correctly. Figure 3(b) shows the VoI from monitoring both the state of the structure and the SHM system, $\mathrm{VoI_{M^2}}$, which is generally higher than the corresponding $\mathrm{VoI_M}$. This is clear by observing Figure 3(c), which shows the additional VoI provided by the SVT. Specifically, the SVT provides a higher additional benefit in the proximity of $\delta = 0.3$, that is when the decision maker is not able to distinguish between the healthy and the damaged states of the structure due to the presence of drift in the SHM outcome.

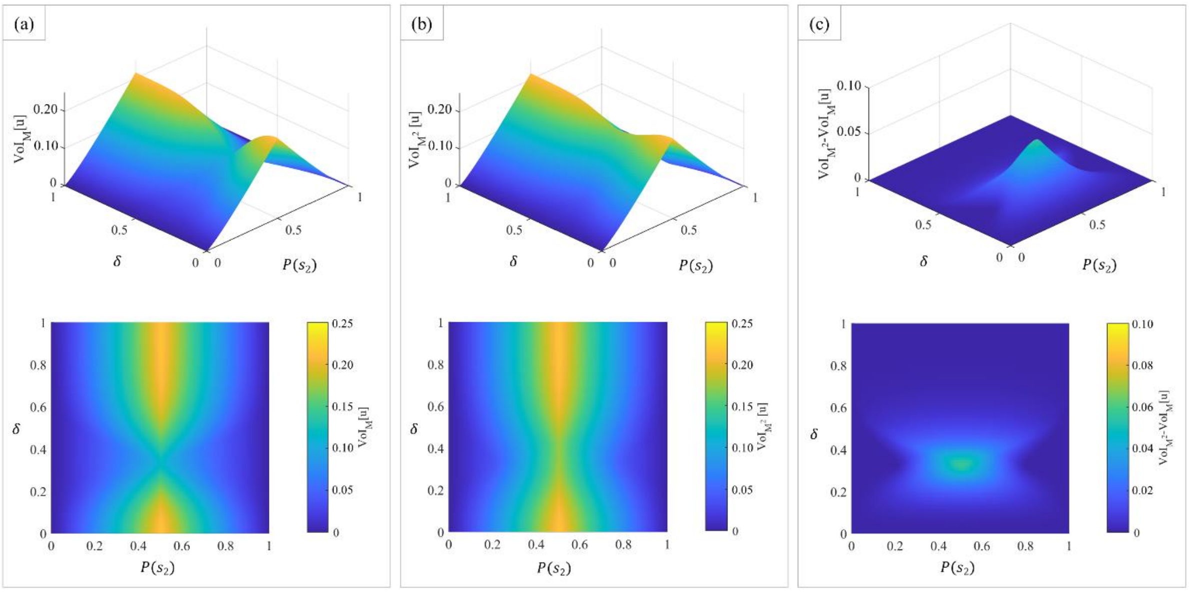

Figure 3. Results of the VoI analysis in case of drift presence and imperfect SVT outcome: (a) VoI from monitoring the state of the structure; (b) VoI from monitoring both the state of the structure and the SHM system; (c) additional VoI provided by the SVT.

Figure 4 describes the results of the VoI analysis in case of drift presence and perfect SVT outcome. Figure 4(a) is equal to Figure 3(a) since the $\mathrm{VoI_M}$ is not affected by the characteristics of the SVT. Figure 4(b) demonstrates that decision-making is not influenced by the presence of drift in case the decision-maker is informed about its presence. In fact, the drift is a systematic error, which can be eliminated when it is known. Figure 4(c) shows that the additional benefit associated with an SVT increases with the increasing quality of information.

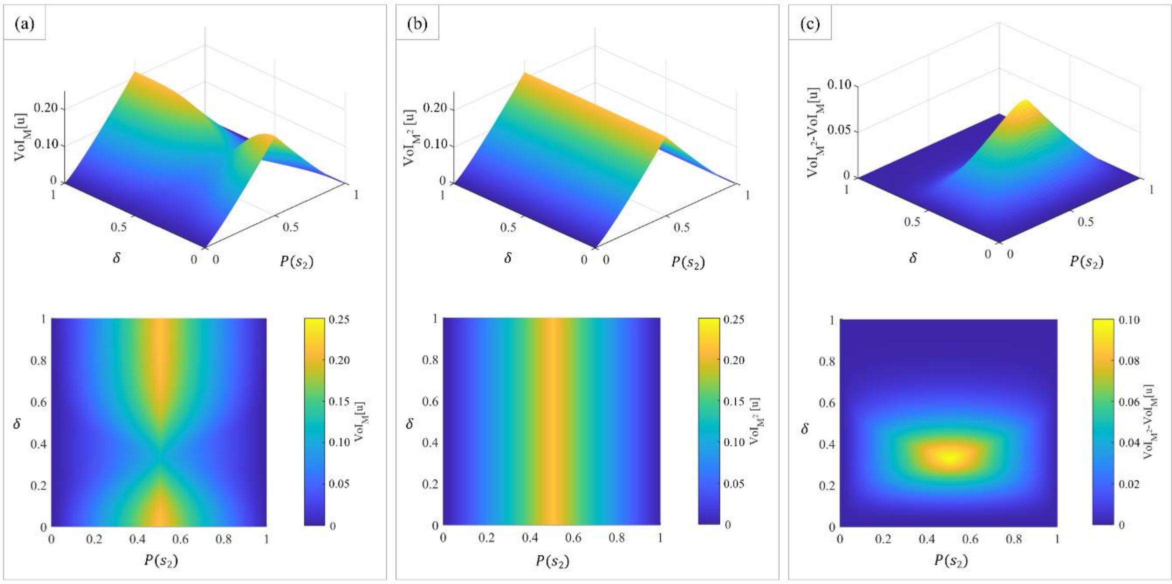

Figure 4. *Results of the VoI analysis in case of drift presence and perfect SVT outcome: (a) VoI from monitoring the state of the structure; (b) VoI from monitoring both the state of the structure and the SHM system; (c) additional VoI provided by the SVT.*

3.2. Noise

The effect of noise of the SHM outcome on the VoI is analyzed in this section. To better compare the results, the conducted VoI analysis is similar to that presented in the previous section, namely: the cases of imperfect and perfect SVT outcomes are analyzed; the VoI is expressed as a function of the noise magnitude ε, and the prior probability $P(s_2)$ which vary in the interval 0 and 1; the likelihood functions of the SVT shown in **Error! Reference source not found.** are used in the case of imperfect information.

Figure 5 reports the results of the VoI analysis for imperfect SVT outcome. The $\mathrm{VoI_M}$ in Figure 5(a) is null for $P(s_2) = 0$ and $P(s_2) = 1$ and maximum for $P(s_2) = 0.5$ due to the reasons discussed in the previous section, see discussion of Figure 4(a). In general, the $\mathrm{VoI_{M^2}}$ in Figure 5(b) is higher than the corresponding $\mathrm{VoI_M}$ in Figure 5(a). The additional VoI provided by the SVT is show in Figure 5(c). It slightly decreases for increasing ε and increases in proximity of $P(s_2) = 0.2$ and $P(s_2) = 0.8$. The decrease of the VoI for increasing ε can be explained considering that the PDF of the SHM outcomes become flatter and flatter as the noise increase. In turn, values belonging to the tails of the SHM outcome distributions, that in the properly working condition were presenting negligible probability density, increase it. Thus, the observation of such outcome suggests the decision maker that the SHM system is probability in the faulty condition thereby the benefit provided by the SVT decreases.

The increase of the VoI in proximity of $P(s_2) = 0.2$ and $P(s_2) = 0.8$ can be understand by analyzing Figure 6 that shows the absolute difference, Δ, between the posterior expected utilities of the two actions, $\Delta = \left|E\left[u(A_2)\middle|o_j\right] - E\left[u(A_1)\middle|o_j\right]\right|$, that are computed considering an SHM outcome o_j, without SVT information for a fixed level of noise ($\varepsilon = 0.2$). The two actions present very similar values of utilities (yellow color) in proximity of $P(s_2) = 0.2$ and $P(s_2) = 0.8$, that is exactly were the maximum additional VoI is located. Indeed, in this situation, the additional information from the SVT provides the maximum benefit in distinguishing the optimal action. This phenomenon is analogous to that shown in Figure 4 (a-b) and Figure 5 (a-b), where the VoI is maximum for $P(s_2) = 0.5$, that is when the expected costs of the two action is the same, also in this case

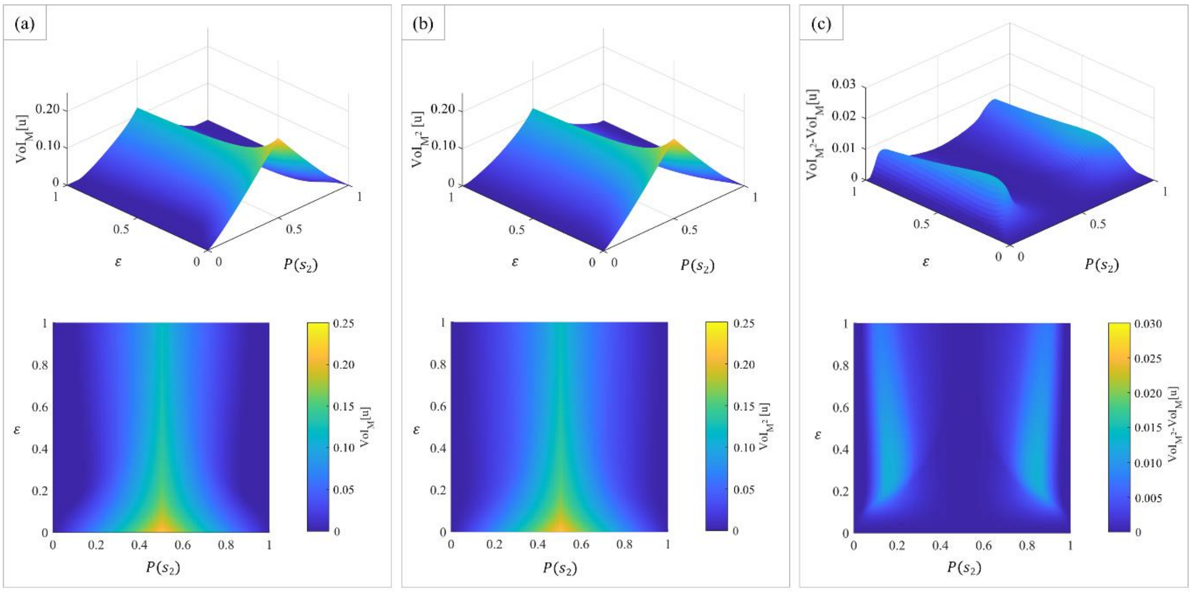

Figure 5. *Results of the VoI analysis in case of noise presence and imperfect SVT outcome: (a) VoI from monitoring the state of the structure; (b) VoI from monitoring both the state of the structure and the SHM system; (c) additional VoI provided by the SVT.*

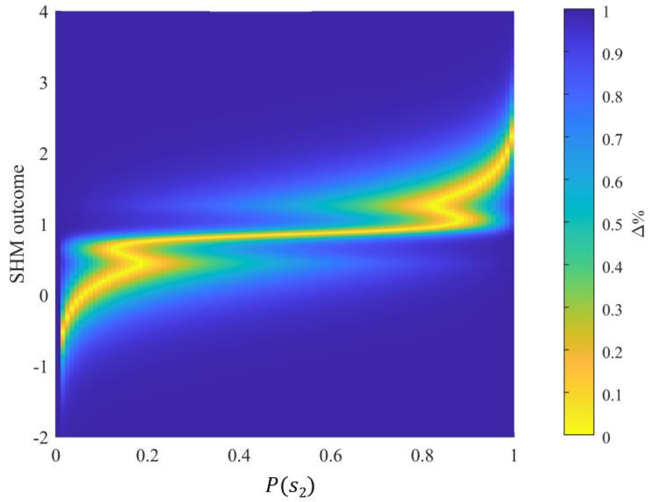

Figure 6. *Absolute difference between the posterior expected utilities of the two actions.*

Figure 7 shows the results of the VoI analysis considering perfect SVT information. The results presented in Figure 7(a) do not depend on the SVT outcome, thus they are similar to those shown in Figure 5(a). Instead, the $\mathrm{VoI}_{\mathrm{M}^2}$ in Figure 7(b) and the additional VoI in Figure 7(c) are generally higher than the values presented in Figure 5(b) and Figure 5(c), respectively due to the higher performances of the SVT. Differently from the drift, the effect of noise cannot be eliminated by the use of a perfect SVT.

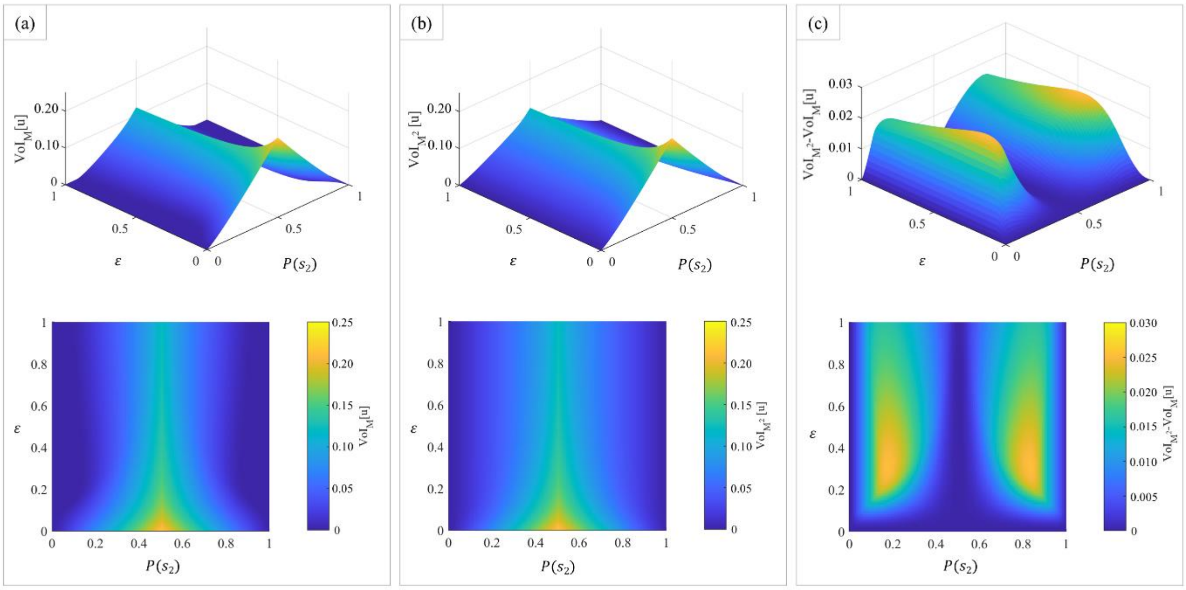

Figure 7. Results of the VoI analysis in case of noise presence and perfect SVT outcome: (a) VoI from monitoring the state of the structure; (b) VoI from monitoring both the state of the structure and the SHM system; (c) additional VoI provided by the SVT.

4. Box-girder cable-stayed bridge

In this section, the proposed approach is applied to a real case study for which monitoring data are available consisting of a box-girder cable-stayed bridge located in China. The bridge presents a main span of 1088 m, two side spans of 300 m each, and two 306-m-high inverse-Y shaped towers. The girder is 41.0 m wide, including two wind fairings, and is supported by 272 cables formed of parallel steel-wire strands, deployed with intervals of 16 m at the main span, 12 m at the side spans, and 2 m at the towers.

Since its construction in 2008, an SHM system was installed on the bridge. It includes different sensor types, among which accelerometers, strain gauges, a global positioning system (GPS), and environmental monitoring devices. In this study, only acceleration data recorded continuously for two months (January and February 2012) at a sampling frequency of 20Hz, are employed. The total number of accelerometer channels used in this study is 38, including 16 two-channel accelerometers (on the two sides of the deck and on top of the towers) and 2 three-channel accelerometers (at the bottom of the towers). A scheme of the bridge is reported in Figure 8, together with relevant information about sensor deployment.

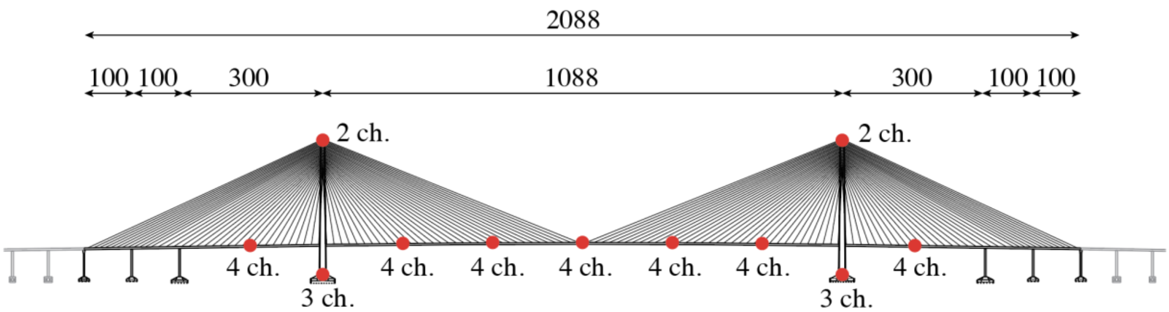

Figure 8. *Scheme of the bridge and sensor layout, indicating the number of channels (indicated as ch.) for each location; dimensions in m.*

The case study is located toward the sea, in a region with a subtropical monsoon climate, characterized by hot and humid summers alternating with cool winters. The presence of sea saltiness, together with humidity and highly variable weather, creates a challenging environment both for the structure and for the monitoring instrumentation. Weather conditions, together with hardware malfunctions and code errors, can generate abnormal datasets that would result in the identification of misleading structural parameters if processed through the usual identification algorithms.

The acceleration dataset collected on the case study was provided for a blind competition along with the 1st International Competition for Structural Health Monitoring (IPC-SHM, 2020 [48]). It should be noted that the state of the structure was healthy (i.e., no structural damage was identified) throughout the considered monitoring period. However, a considerable part of the dataset presents abnormal data. Specifically, the competition organizers labeled 1440 consecutive data segments of 1 hour collected by the 38 recording channels as "healthy" or "faulty" (relating to the state of the monitoring system), considering each channel separately. In general, the data anomalies considered in this paper do not strictly affect only accelerations but might be also found in other measurements [49].

In a reliable SHM system, the mentioned anomalies should be recognized and removed before processing the data to extract significant structural parameters for damage identification. To consider a realistic application, an SVT presented in a recent publication is considered: Martakis et al. [50] proposed an SVT based on a one-class classifier for data quality validation that classifies segments of the structural response data into "normal" or "abnormal". In the mentioned publication, the authors validate the SVT on the described case study.

The SVT used in [50] consists of a semi-supervised machine learning tool based on a nonlinear support vector machine [51] that employs a Gaussian radial basis function kernel [52]. Specifically, acceleration response segments having a user-defined length are first transformed into the feature space, i.e., they are converted into high-dimensional vectors, where each element represents a different characteristic of the signal in the time or frequency domain. A set of vectors obtained for the "normal" data configuration is employed to train the SVT, which is then able to detect anomalies (intended as outliers from the baseline set). Specifically, in this application, the outliers are defined as the points outside a hypersphere that contains 99% of the training dataset in the feature space. In other words, a hard threshold is set at the 99th percentile of the training distribution to discern between normal and abnormal data.

In the study conducted by Martakis et al., the data collected in January (31 days) from all the 38 channels are segmented into 5-minute blocks, yielding 339264 time-series samples. Of these samples, 162900 (the 48%) were labeled as "healthy" according to the information provided by the organizers of the competition and employed to train the SVT. The data from February (29 days) is employed for testing the SVT; thereby, it is considered unlabeled. Specifically, the testing data consists of 26448 data series of 1 hour each. Upon classification, 99% of the "normal" February data lies within the 99th percentile and is therefore considered "normal" in this study, while 1% is classified "abnormal", consistently with the definition of the threshold. Also, 92% of the faulty testing data is correctly classified as "abnormal" since the classifier exceeds the 99th percentile of the "normal" February data, while 8% of the "abnormal" cases are classified as "normal".

The two-month interval described above is here considered representative of a generic monitoring period. Therefore, it is used to evaluate all the parameters employed for the calculation of the VoI.

4.1. Decision problem

The decision problem tackled in this demonstration is similar to the one addressed in the previous case study and relates to the traffic management of the bridge in the aftermath of a damaging event, such as the impact of a ship. Specifically, the decision-maker has to select the optimal action between a_1 – Do nothing – and a_2 – Shut down – considering that the bridge is either in the state s_1 – healthy condition – or s_2 – damaged condition. The utilities in Table 1 are employed. While a continuous SHM outcome was considered in the numerical case study to study the effect of different sensor fault conditions, it is assumed herein that decision-maker is planning to install an SHM system for damage detection that is able to provide a binary outcome, namely, o_1 and o_2. The outcome o_1 is representative of the healthy conditions, whereas the outcome o_2 is representative of the damaged condition.

It is supposed that the SHM system can be in two states, namely, the healthy state, m_1, and a faulty state, m_2. Upon defining a threshold that discerns between the two states, the likelihood of the SHM outcome can be obtained by analyzing the distribution of the damage indicator in the undamaged and damaged states. In particular, the probability of threshold exceedance in the damage state can be interpreted as probability of detection $P(o_2|s_2)$ [53], which depends also on the state of the SHM system m_k in this framework. The likelihood of the SHM outcome for the two states of the SHM system m_1 and m_2 are displayed in Table 3 and Table 4, respectively. When the SHM system is in the healthy state, it indicates the correct state of the structure with a probability of 0.9. Instead, when the SHM is damaged, it does not provide any information on the state of the structure. This condition can be represented by assigning a probability of 0.5 to the occurrence of the SHM outcome in each state (i.e., when these conditional probabilities of the SHM outcomes are used, the prior probabilities of the bridge state are not modified when the Bayes' theorem is applied).

Table 3. Likelihood of SHM outcome for m_1.

	s_1	s_2
o_1	0.9	0.1
o_2	0.1	0.9

Table 4. Likelihood of SHM outcome for m_2.

	s_1	s_2
o_1	0.5	0.5
o_2	0.5	0.5

SVTs are typically based on binary novelty detection methods [6] since data quality can decrease due to several factors or their combinations, which are difficult to classify in a finite set of labels. As an applicative example, the performance parameters of the SVT described in Section 3.1 [50] are considered in this section to calculate the VoI through the presented general framework. Specifically, it is assumed that the SVT considered in this study is able to provide two outcomes c_1 and c_2, which indicate the properly working and the faulty conditions, respectively. Those values are taken from the so-called "confusion matrix" obtained during the testing phase of the SVT presented by Martakis et al. [54], see Table 5.

Table 5. Likelihood of the SVT

	m_1	m_2
c_1	0.99	0.08
c_2	0.01	0.92

4.2. Sensitivity analysis

The prior probabilities of the state of the structure and the SHM system, as well as the utilities of different combinations of actions and states of the structure, have not been specified so far. In this section, two sensitivity analyses are carried out to demonstrate how the VoI is affected by these parameters. Refer to Table 6 for a general overview of the two sensitivity analyses presented hereafter. In line with the aim of the paper, the variables relating to SVT are retrieved from [50], i.e., the prior probabilities of the state of the SHM system and the likelihood of the SVT (confusion matrix). The parameters considered in the sensitivity analysis (i.e., $P(s_2)$, $P(m_2)$, u_{SD}/u_F) vary in the interval 0-1. With reference to $P(s_2)$, it is remarked that, in this paper, damage refers to a generic state that deviates from a reference (e.g., undamaged) condition. This may correspond to a mild or severe damage state. Furthermore, the paper addresses a situation that occurs in the aftermath of a damaging event. While in normal conditions, the probability that a civil engineering structure is in a severe damage state is far from 1, higher values of $P(s_2)$ could be estimated in the aftermath of a damaging event. Concerning the utility ratio, it is not unusual that the indirect consequences related to loss of functionality (in this case represented by $u_{CloseSD}$) of civil structures are comparable or even higher than the associated direct costs, as documented in the relevant literature [55].

Table 6. Sensitivity analyses

Sensitivity analysis n.	Prior probability $P(s_2)$	Prior probability $P(m_2)$	Utilities
1	Varying in the range 0-1	Fixed as 0.52	Ratio u_{SD}/u_F varying in the range 0-1
2	Varying in the range 0-1	Varying in the range 0-1	Fixed as: $u_F = -1$ $u_{SD} = -0.5$

Sensitivity analysis to prior probabilities of structural states and utility ratio

The first sensitivity analysis (see Table 6) is carried out to verify the impact on the VoI of the prior probabilities of the structural state $P(s_2)$, and of the utility ratio u_{Close}/u_f. The VoI results are normalized to the direct losses $|u_F|$.

Figure 9 shows the results of the Prior analysis, i.e., the optimal action between Do nothing and Shut down, for different values of $P(s_2)$ and u_{Close}/u_f. The boundary between the two regions is the line bisecting the first quadrant. For a given ratio u_{Close}/u_f, the action Shut down is preferable for relatively high values of the probability that the bridge is damaged. Instead, for a given $P(s_2)$, the optimal action is Do nothing when the indirect losses are relatively high with respect to the losses associated with failure.

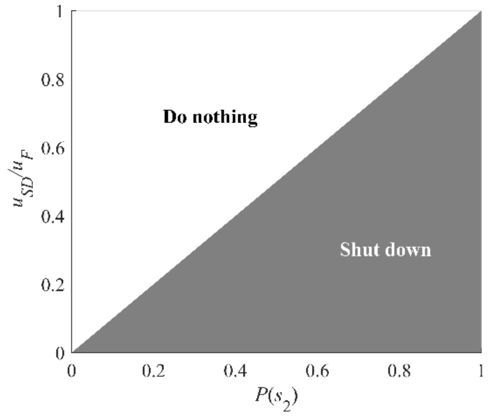

Figure 9. Optimal action from Prior analysis for varying $P(s_2)$ and u_{SD}/u_F.

The VoI computed considering the SHM information only is shown in Figure 10(a). The highest values of the ratio $\mathrm{VoI_M}/|u_F|$ are reached in correspondence of the bisector of the first quadrant, which corresponds to the boundaries between the optimal actions in Prior analysis, as shown in Figure 9. The absolute maximum of the ratio $\mathrm{VoI_M}/|u_F|$ is attained for $P(s_1) = P(s_2) = 0.50$, that is when non informative prior probabilities are used. Instead, in the blue areas, the information from the SHM system does not provide any benefit.

Figure 10(b) reports the VoI computed considering both the information from SHM system and the SVT. The ratio $\mathrm{VoI_{M^2}}/|u_F|$ is maximum in correspondence of the bisector of the first quadrant, reaching the same maxima of the ratio $\mathrm{VoI_M}/|u_F|$ displayed in Figure 10(a). Instead, the blue area corresponding to null benefit is smaller. This means that, when the information on the state of the SHM system from the SVT is available, the information from the SHM system has a stronger impact on the decision-making process. Figure 10(c) displays the normalized additional benefit provided by the SVT. The additional VoI presents an “eye” shape with zero value assumed in the proximity of the bisector of the first quadrant.

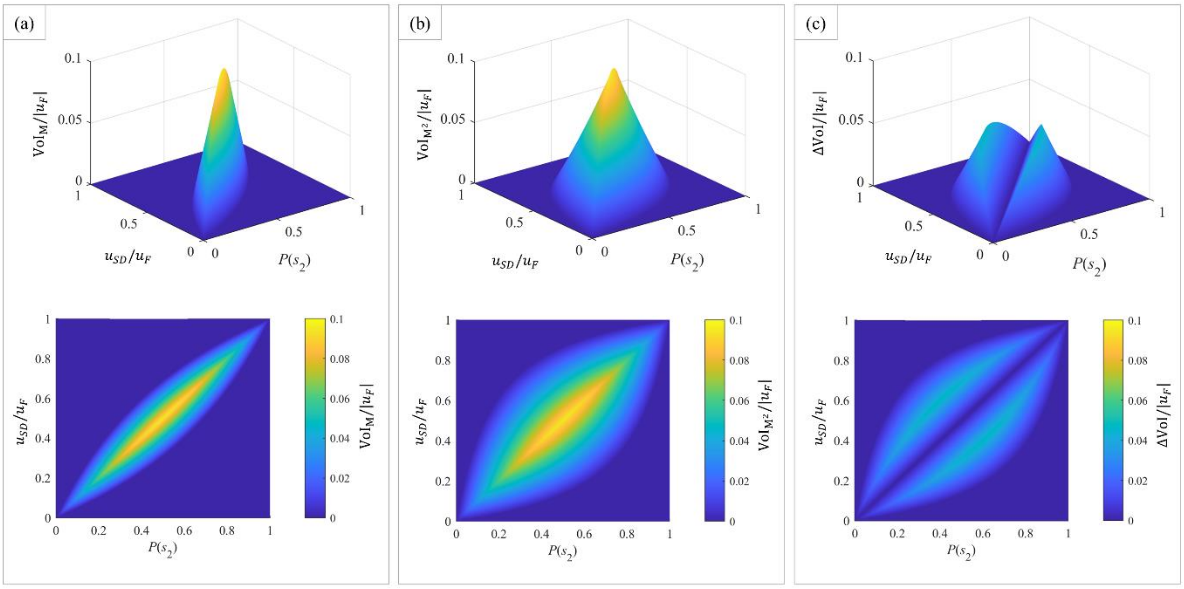

Figure 10. *Results of the VoI analysis for varying $P(s_2)$ and ratio u_{Close}/u_f: (a) Ratio $VoI_M/|u_F|$ (b) Ratio $VoI_{M^2}/|u_F|$; (c) additional VoI provided by the SVT, $\Delta VoI/|u_F|$.*

analysis

To explain the local maxima of additional VoI, the expected utilities of the two management actions computed according to Eq. 13, that is considering the prior information on the state of the SHM system $E[u(A_n)|o_j]$, are examined in Figure 11**Error! Reference source not found.**, fixing the following values: $u_{SD}/u_F = 0.5$, $u_F = -1$ and $u_{SD} = -0.5$. In particular, Figure 11**Error! Reference source not found.**(a) and Figure 11**Error! Reference source not found.**(b) show the expected utilities of the two management actions given the outcome o_1 and o_2, respectively. In both plots, the expected utility of the action Shut down is constant and equal to -0.5, as in the Prior analysis (since it does not depend on the state of the structure). Instead, the expected utilities associated with the action Do nothing decrease for the increasing probability that the structure is in the damaged state s_2. The maxima of the term ΔVoI are reached when the expected utilities of the two actions are the same for a given SHM outcome, that is approximately for $P(s_2) = 0.3$ and $P(s_2) = 0.7$. In this situation, the information from the SVT on the state of the SHM system provides the highest benefit since the expected utilities of the two actions evaluated without the information of the SVT, are similar. In other words, the benefit provided by the new information (either on the state of the structure or the SHM system) is maximum when the expected utilities of the traffic management actions computed using prior knowledge (either on the state of the structure or the SHM system) are the same. The same applies to the other values of the ratio u_{SD}/u_F. Similar considerations were made in Section 3.2 to justify the position of the local maxima of the additional VoI in Figure 5.

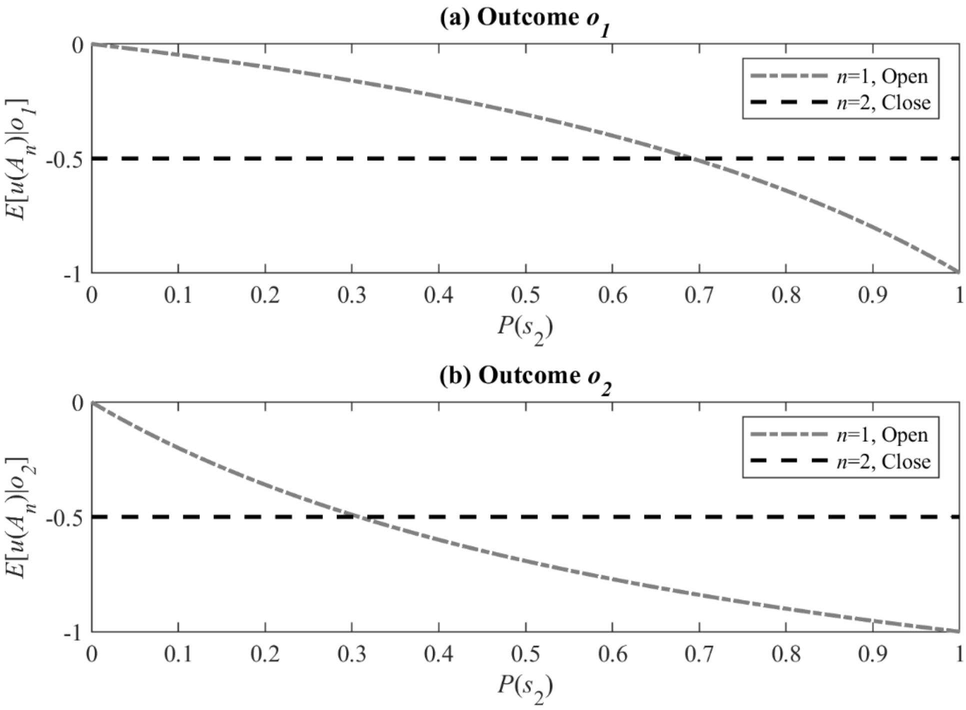

Figure 11. Contribution to the VoI_M of the two SHM outcomes.

Sensitivity analysis to prior probabilities of the states of the structure and SHM system

The second sensitivity analysis is performed to investigate the joint impact of the prior probabilities of the state of the structure and the SHM system. The utilities u_{SD} and u_F are kept constant according to Table 6.

The selection of the optimal action from Prior analysis (shown in Figure 12) does not depend on the state of the SHM system $P(m_2)$ since the prior expected benefit only depends on the prior probabilities of the structural state $P(s_2)$. The boundary between the optimal actions Do nothing and Shut down is found for $P(s_2) = 0.50$, that is non informative prior probabilities.

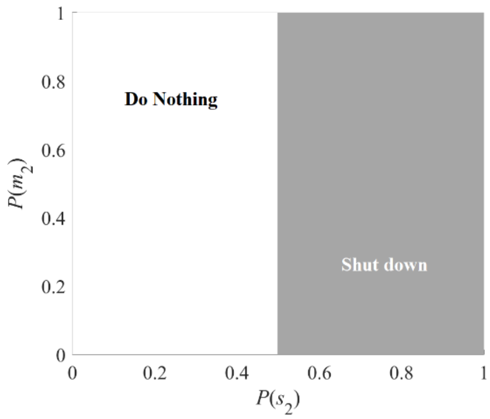

Figure 12. Optimal action from Prior analysis for varying $P(s_2)$ and $P(m_2)$.

The $\mathrm{VoI_M}$ for varying $P(s_2)$ and $P(m_2)$ is reported in Figure 13(a). Again, the $\mathrm{VoI_M}$ is maximum at the boundary between the optimal actions Do nothing and Shut down. For a given prior probability of structural state s_2, the $\mathrm{VoI_M}$ decreases for increasing prior probability that the SHM system is in the faulty state m_2. The absolute maximum (see Figure 13) is observed for $P(s_2) = 0.50$ and $P(m_2) = 0$. Indeed, when the SHM system is in m_2, it cannot provide any insight on the actual structural conditions, see Section 3.2. Thus, the benefit provided by the SHM information decreases for increasing probability $P(m_2)$.

Figure 13(b)**Error! Reference source not found.** shows the $\mathrm{VoI}_{\mathrm{M}^2}$ for varying $P(s_2)$ and $P(m_2)$. The benefit related to the combined use of the SHM system and the SVT is maximum for $P(s_2) = 0.50$, where $\mathrm{VoI}_{\mathrm{M}^2} = \mathrm{VoI}_{\mathrm{M}}$. In the other regions, the $\mathrm{VoI}_{\mathrm{M}^2}$ is higher than the $\mathrm{VoI}_{\mathrm{M}}$ for the same values of $P(s_2)$ and $P(m_2)$ (excluding the blue regions in **Error! Reference source not found.**where $\mathrm{VoI}_{\mathrm{M}^2} = \mathrm{VoI}_{\mathrm{M}} = 0$). In general, the $\mathrm{VoI}_{\mathrm{M}^2}$ decreases for increasing $P(m_2)$ due to the reduction in the quality of the information provided by the SHM system.

Finally, the additional VoI provided by the SVT, ΔVoI, for varying $P(s_2)$ and $P(m_2)$ is reported in Figure 13(c)**Error! Reference source not found.Error! Reference source not found.**. The ΔVoI surface is characterized by a "lung" shape, being equal to zero for $P(s_2) = 0.50$ and presenting the maximum values in two distinct areas on the left and on the right of this probability value. The absolute maxima of the term ΔVoI are reached for $P(m_1) = P(m_2) = 0.50$ thereby for non-informative prior probabilities on the state of the SHM system.

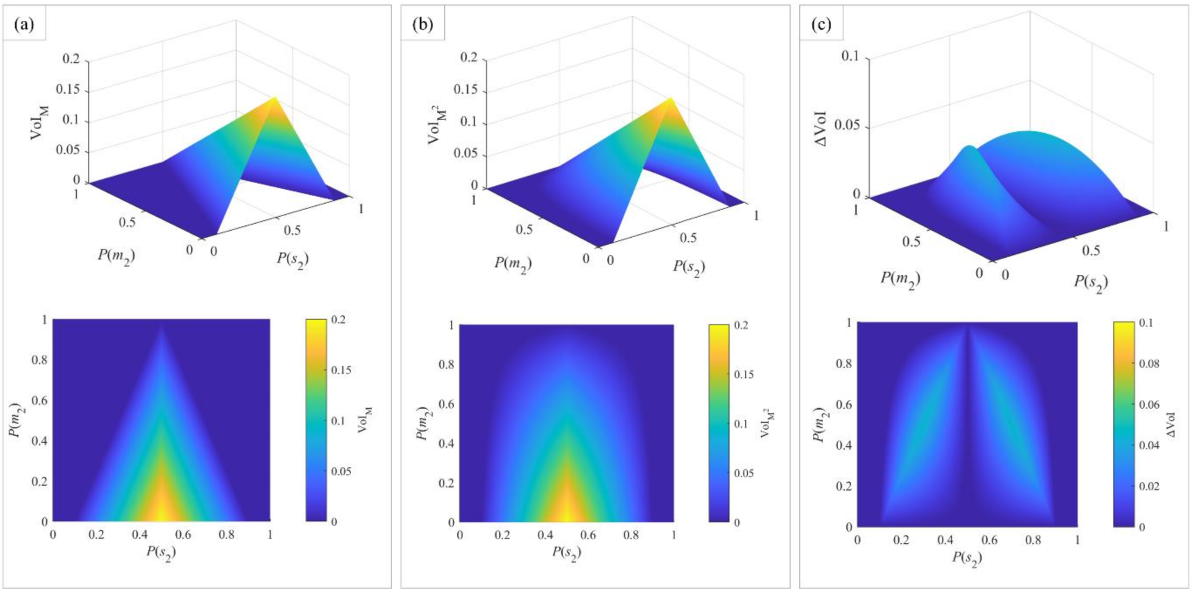

Figure 13. *Results of the VoI analysis for varying $P(s_2)$ and $P(m_2)$: (a) VoI computed without the information from the SVT, VoI_M; (b) VoI computed with the information from the SVT, VoI_{M^2}; (c) Additional VoI provided by the SVT, ΔVoI.*

5. Conclusion

SHM systems very often provide altered data to decision makers, nevertheless, very few published works deal with the modelling of information quality in VoI analysis. This paper intends to contribute to fill this gap by proposing an extension of the classical Bayesian decision theory, that enables decision-makers to quantify the overall benefit – in terms of VoI – of collecting information on both the state of a structure by means of SHM and the state of the SHM system itself through an SVT.

The main novelties of the proposed framework relate to: (i) the modelling of the different states of the SHM system, associated with different sensor fault conditions, leading to drift, noise, and absence of information in the SHM outcome; (ii) the modelling of data quality through the likelihood functions of the SHM outcome; and (iii) the modelling of the SVT outcome, which might be affected by uncertainty. The main assumptions made to develop the extended framework are the following: (1) The observations on the state of the structure, made using SHM, depend on the state of both the structure and the SHM system; (2) The state of the SHM system does not depend on the state of the structure; (3) The observations made using the SVT on the state of the SHM system depend only on the state of the SHM system.

To demonstrate the applicability of the framework, two case studies are proposed with different purposes. The first case study is focused on the effect of two common sensor fault conditions, namely drift and noise, on the VoI. The second case study relates to an emergency management scenario for a box-girder cable-stayed bridge in China for which real data on SHM outcome quality are available. The SVT considered in this case study is based on a neural network trained in the reference condition of the bridge and the SHM system. The neural network is characterized by a confusion matrix whose components coincide with the likelihood of the SVT outcomes in the different states of the SHM system. The sensor faut condition considered in this second application relates to the absence of SHM data. For both case studies, several sensitivity analyses are carried out to demonstrate how the VoI is affected by the parameters involved in its computations. The main findings of the VoI analysis are the following:

1) The benefit provided by the SHM system, coupled with the SVT, is generally higher than the benefit provided by the SHM system alone.
2) The additional benefit provided by the SVT is maximum when the expected utilities of management actions (without the support of the SVT) are the same for a given SHM outcome.
3) The use of a perfect SVT eliminates the effect of the drift on the SHM outcome, while it can only mitigate the effect of noise.
4) The prior knowledge on the state of the system, i.e., structure and SHM system, represented by the prior probabilities of these quantities, the strongly influences the VoI results.

Future works relate to several aspects of the proposed framework, such as modelling additional sensor fault conditions through likelihood functions of the SHM outcome; study of the effect on the VoI of considering different sensor fault types simultaneously; release the assumption of independency between the states of the structure and the SHM system; study of reliability models to quantify the probability of the different states of the SHM system; application of the extended framework to more complex/realistic case study addressing, for instance, repeated SVT observations and the effect of slow deterioration.

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