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A Spatial Stochastic Frontier Model Including Both Frontier and Error Based Spatial Cross-Sectional Dependence

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Abstract: Spatial dependence in SF models is usually handled by modelling the frontier function or the inefficiency error term through the introduction of some spatial components. The model proposed in this paper (SDF-CSD) combines the two different modelling approaches obtaining a full comprehensive specification that introduces four different sources of spatial cross-sectional dependence. The most appealing feature of our model is that it allows capturing global and local spatial spillover effects while controlling for spatial correlation related to firms' efficiency and to unobserved but spatially correlated variables. Moreover, it can be estimated using maximum likelihood techniques. Finally, we run some Monte Carlo simulations to test the final sample properties of our new spatial estimator and we provide an application to the Italian agricultural sector.

Keywords: Spatial stochastic frontier models; Spillover effects; Italian agricultural sector.

JEL codes: C21; C23; D24; L25.

1 Introduction

In the last two decades, scholars started to expand classical stochastic frontier models (SF) in order to include also some spatial components. Indeed, firms tend to concentrate in clusters, taking advantage of positive agglomeration externalities due to cooperation, shared ideas and emulation, resulting in increased productivity levels (Baptista, 2000). Thus, producers cannot be regarded as isolated entities and the hypothesis of cross-sectional independence underlying the basic SF model must no longer be considered valid. Until now scholars have introduced spatial dependence in SF models following two different paths: evaluating global and local spatial spillovers affecting the frontier function or considering spatial cross-sectional correlation in the inefficiency and/or in the error term.

Considering the first approach, Adetutu et al. (2015) is the first contribution introducing local spatial dependence in SF models through the spatial lag of the exogenous variables. Afterwards, Glass, Kenjegalieva, and Sickles (2016) proposed a spatial Durbin stochastic frontier specification that accounts for both local and global spatial dependence, including the spatial lag of the input variables (SLX term) and of the dependent variable (SAR term). While the former allows capturing input sharing and labour matching phenomena, the latter involves spatial dependence arising from joint practises, knowledge transmission, adoption of common technologies, creation of a collective identity, and market-oriented externalities. Further extensions of the spatial Durbin SF model introduced by Glass, Kenjegalieva, and Sickles (2016) were developed by Ramajo and Hewings (2018) and Gude, Alvarez, and Orea (2018) considering respectively a time-varying decay specification for the inefficiency error term and time-varying exogenous influences on both the degree of global and local spatial dependence for each time period. Finally, Tsukamoto (2019), following the specification of Battese and Coelli (1995) for non-spatial SF models, introduced the possibility to consider some determinants of firms' technical inefficiency in the spatial autoregressive SF model.

On the other hand, Schmidt et al. (2009) is the first contribution making distributional assumptions on both the two error terms that considers spatial dependence in the inefficiency error term. Specifically, they specified the technical inefficiency error term using a prior distribution that captures unobserved spatial features and estimated the model through a Bayesian approach. Differently, Areal, Balcombe, and Tiffin (2012) introduced an autoregressive specification of the inefficiency error term to capture spatial dependence and used a Gibbs sampler and two Metropolis-Hastings steps for estimating the unknown parameters. Moreover, Fusco and Vidoli (2013) managed to solve the autoregressive error model proposed by Areal, Balcombe, and Tiffin (2012) using a maximum likelihood estimation technique instead of a Bayesian approach. Afterwards, Tsionas and Michaelides (2016), making use of a Bayesian estimator, developed a spatial SF model splitting the inefficiency error term into an idiosyncratic term and a component capturing regional spillover effects. Finally, Orea and Alvarez (2019) considered cross-sectional dependence in both the inefficiency and the random error term and obtained a spatial SF model with closed-form for the likelihood function using a transposed version of the Wang and Ho (2010) model. Specifically, Wang and Ho (2010) took advantage of the scaling property proposed by Wang and Schmidt (2002) to remove the time-invariant effects from the firm-specific inefficiency error term. Transposing the within temporal transformation to adapt to a cross-sectional setting, Orea and Alvarez (2019) replaced the firm-specific error term with a time-varying industry-specific term in order to develop a spatial SF model considering cross-sectional correlation in the inefficiency component that can be estimated using maximum likelihood techniques.

Considering the pros and cons of the two different modelling approaches, the most interesting advantage of introducing the spatial lag of Y and of X in SF models relates to the possibility of directly measuring direct and indirect effects originating from the variables that determine firms' productivity level and that also affect neighbouring producers. However, ignoring spatial dependence in the inefficiency error term can result in biased estimates of the inefficiency distribution and in different rankings of inefficiencies across agents (Areal, Balcombe, and Tiffin, 2012; Schmidt et al., 2009). Moreover, estimating a SF model without taking cross-sectional dependence in the error term into consideration decreases the statistical efficiency of the model (Orea and Alvarez, 2019). On the other hand, considering spatial dependence in the error term is useful to take unobserved but spatially correlated variables such as environmental and climatic conditions or location-specific attributes into account. Moreover, introducing a spatial structure in the inefficiency error term allows capturing spatial correlation deriving from the specific attributes that commonly characterize all the producers belonging to the same area and that affect firms' efficiency level. Nevertheless, not introducing the spatial lag of the dependent variable in SF models can result in biased estimates due to an omitted variable bias (Glass, Kenjegalieva, and Sickles, 2016)

Therefore, in this paper, we merge together all the different spatial structures described before in order to jointly consider all the different features outlined above, obtaining a comprehensive specification that takes four different sources of spatial dependence into account. Specifically, we introduce the spatial lag of Y and of X to capture global and local spatial spillovers affecting the frontier function as introduced by Glass, Kenjegalieva, and Sickles (2016) and we also add a spatial structure to the inefficiency and to the error term to capture respectively behavioural and environmental spatial correlation as proposed by Orea and Alvarez (2019). Hence, we obtain a spatial Durbin stochastic frontier model for panel data introducing cross-sectional dependence both in the inefficiency and in the error term (SDF-CSD). The most appealing feature of our model is that it allows capturing global and local spatial spillover effects while controlling for spatial correlation related to firms' efficiency and to unobserved but spatially correlated variables. The SDF-CSD model can be estimated using maximum likelihood techniques, modifying the estimation procedure suggested by Orea and Alvarez (2019) in order to consider the endogeneity deriving from the inclusion of the spatial lag of the dependent variable. In the simulation section, we show that our spatial estimator is able to distinguish between frontier and error-based spillovers considering sparse spatial weight matrices (as binary contiguity or truncated inverse distance matrices).

In the remaining of the paper, we introduce the model specification, the estimation procedure and the derivation of the marginal effects and of the TE scores in Section 2; in Section 3 we test the finite sample properties of our estimator by performing some Monte Carlo simulations; in Section 4 we show a brief application on aggregated data from the Italian agricultural sector and in Section 5 we conclude.

2 The SDF-CSD Model

2.1 Model Specification

The spatial Durbin stochastic frontier production function for panel data allowing for spatial cross-sectional dependence both in the inefficiency and in the error term (SDF-CSD model) is specified as in Eq.(1)-(5), for $i = 1, ..., N$ and $t = 1, ..., T$.

$$Y_{it} = \rho \sum_{j=1}^{N} w_{ij} Y_{jt} + X_{it}\beta + \sum_{j=1}^{N} w_{ij} X_{jt}\theta + \widetilde{v}_{it} - \widetilde{h}_{it} u_t^{\star} \tag{1}$$

$$\widetilde{v}_t \sim MVN\left(0, \Pi\right) \tag{2}$$

$$\Pi = \sigma_v^2 M_\gamma M_\gamma^T, \quad M_\gamma = \left(I_N - \gamma W\right)^{-1} \tag{3}$$

$$\widetilde{h}_{it} = \left(I_N - \tau W\right)^{-1} h_{it}, \quad h_{it} = f\left(Z_{it}, \phi\right) \tag{4}$$

$$u_t^{\star} \sim \mathcal{N}^{+}\left(0, \sigma_u^2\right) \tag{5}$$

Specifically, Y_{it} represents the production output (usually expressed in log-form) of firm i at time t and X_{it} is a $1 \times k$ vector of production inputs used by firm i at time t with associated parameter vector β $(k \times 1)$. To take spatial dependence related to the frontier function into consideration we include in the model the spatial lag of the dependent variable and the spatial lag of the production inputs, detecting global and local spatial spillovers through ρ (1×1) and θ $(k \times 1)$, respectively. In particular, w_{ij} represents the generic element of the spatial weight matrix W, built as a row-normalized $(N \times N)$ matrix identifying the spatial structure of the cross-sectional units. Focusing on the two error terms, we follow the specification proposed by

Orea and Alvarez (2019) for the definition of both the inefficiency and the random error component assuming that they are independently distributed across the cross-sectional dimension. In particular, the vector $\widetilde{v}_t = (\widetilde{v}_{1t}, ..., \widetilde{v}_{Nt})$ represents the error term distributed as a multivariate normal random variable with variance-covariance matrix Π accounting for unobserved but spatially correlated variables thought the inclusion of M_γ, as defined in Eq.(3). Finally, assuming that the scaling property (Wang and Schmidt, 2002) holds, the inefficiency error term u_{it} is decomposed as the product of two components: a scaling function $\widetilde{h}_{it}$ and a basic distribution $u_t^\star$. Specifically, $\widetilde{h}_{it}$ depends on a positive function f of firm exogenous variables Z_{it} with associated parameter vector ϕ as specified in Eq.(4), and on the spatial lag $(I_N - \tau W)^{-1}$ capturing spatial dependence in the variables that determine technical efficiency. On the other hand, $u_t^\star$ is an industry-specific inefficiency error term following a truncated normal distribution with mean 0 and variance σ_u^2. Therefore, inefficiency depends on an industry-specific term common to all firms but varying in time, and on firm-specific exogenous variables that also affect neighbouring producers. Therefore, this model allows both the technical inefficiency error term $u_{it} = \widetilde{h}_{it} u_t^\star$ and the random noise $\widetilde{v}_{it}$ to be cross-sectionally (spatially) correlated.

The use of the scaling property in defining the inefficiency error term is fundamental to obtaining a closed-form for the likelihood function. Indeed, considering cross-sectional dependence in the inefficiency error term generally precludes using standard maximum likelihood techniques for the estimation of the unknown parameters. Therefore, defining u_{it} using the scaling property helps in overcoming this issue and allows to estimate the SDF-CSD using ML algorithms implemented in standard software.

For the model specified in Eq.(1)-(5), the following conditions must hold: (i) for a symmetric W, the spatial autoregressive parameters ρ, τ and γ must be within the range $\left(\frac{1}{\omega_{min}}, 1\right)$, where ω_{min} is the smallest eigenvalue of the spatial weight matrix W and the upper bound equals 1 if W is row-normalized; (ii) the cross-sectional correlation must converge to zero when the distance separating two spatial units goes to infinity (Kelejian and Prucha, 1998, 1999). The first condition ensures that $(I_N - \rho W)$, $(I_N - \tau W)$ and $(I_N - \gamma W)$ are non-singular matrices and the second one guarantees that the row and columns sums of W, $(I_N - \rho W)^{-1}$, $(I_N - \tau W)^{-1}$

and $(I_N - \gamma W)^{-1}$, before W is row-normalized, are uniformly bounded in absolute value as N goes to infinity. In particular, assumption (ii) is always satisfied when W is a binary contiguity matrix while for inverse distance matrices it should be introduced a cut-off point in the distance measure (Elhorst, 2010).

2.2 Estimation Procedure

The partial loglikelihood function, referring to a single time period t, is shown in Eq.(6), where μ_* and σ_*^2 are defined as in Eq.(7)-(8), $\widetilde{h}_t = (\widetilde{h}_{1t}, ..., \widetilde{h}_{Nt})$, and the generic element ε_{it} belonging to the vector ε_t is specified as in Eq.(9). Moreover, Φ represents the cumulative distribution function of the standard normal random variable and $\log |I_N - \rho W|$ is the determinant of the Jacobian of the transformation from ε_{it} to Y_{it}. Indeed, since $\frac{\partial \varepsilon}{\partial Y} = (I_N - \rho W)$, we take into account the endogeneity deriving from the inclusion of the spatial lag of the dependent variable. Starting from Eq.(6), the final loglikelihood function is equal to $\mathcal{L}(\Theta; y) = \sum_{t=1}^{T} \ell_t$, that is the sum of the partial loglikelihood functions computed for each year in the sample.

$$\begin{aligned} \ell_t = & \log |I_N - \rho W| - \frac{N}{2} \log (2\pi) - \frac{1}{2} \log |\Pi| - \log(0.5\sigma_u) - \frac{1}{2} \varepsilon_t^T \Pi^{-1} \varepsilon_t \\ & + \frac{1}{2} \left(\frac{\mu_*^2}{\sigma_*^2} \right) + \log \left[\sigma_* \Phi \left(\frac{\mu_*}{\sigma_*} \right) \right] \end{aligned} \tag{6}$$

$$\mu_* = \frac{-\varepsilon_t^T \Pi^{-1} \widetilde{h}_t}{\widetilde{h}_t^T \Pi^{-1} \widetilde{h}_t + \frac{1}{\sigma_u^2}} \tag{7}$$

$$\sigma_*^2 = \frac{1}{\widetilde{h}_t^T \Pi^{-1} \widetilde{h}_t + \frac{1}{\sigma_u^2}} \tag{8}$$

$$\varepsilon_{it} = Y_{it} - X_{it}\beta - \rho \sum_{j=1}^{N} w_{ij} Y_{jt} - \sum_{j=1}^{N} w_{ij} X_{jt} \theta \tag{9}$$

Consistent parameter estimates can be obtained by maximising the final loglikelihood function using a numerical constrained maximization algorithm (Matlab codes are available under request). According to the first assumption described above, the parameter space for an autoregressive process is $(\frac{1}{\omega_{min}}, 1)$ where ω_{min} is the smallest eigenvalue of W and the upper bound equals 1 because W is row normalized (Kelejian and Prucha, 1999). Therefore, all the autoregressive parameters (ρ, τ, γ) should be bounded to the previous interval to ensure that $(I_N - \rho W), (I_N - \tau W)$ and $(I_N - \gamma W)$ are non-singular matrices. Moreover, the two variance parameters σ_u^2 and σ_v^2 should only take positive values. A further characteristic of the SDF-CSD model is that its loglikelihood function is the same both considering a production function and a cost function. More details on the estimation procedure can be found in Appendix A.

Following Orea and Alvarez (2019), technical efficiency scores can be found substituting the parameter estimates in $TE_{it} = \exp(-E\left(u_{it}|\hat{\varepsilon}_{it}\right))$, where the conditional expectation of u_{it} given $\hat{\varepsilon}_{it}$ is shown in Eq.(10).

$$E\left(u_{it}|\hat{\varepsilon}_{it}\right) = \widetilde{h}_{it} E\left(u_t^{\star}|\hat{\varepsilon}_{it}\right) = \widetilde{h}_{it}\left[\mu_* + \frac{\sigma_* \phi\left(\frac{\mu_*}{\sigma_*}\right)}{\phi\left(\frac{\mu_*}{\sigma_*}\right)}\right] \tag{10}$$

2.3 Marginal Effects

As usual in spatial models including the spatial lag of Y, the β estimates cannot be interpreted as marginal effects because changes in the generic regressor X_r of firm i influence the production output of firm j. To show this, the SDF-CSD model is written in the reduced form in Eq.(11), where Y is a $(NT \times 1)$ vector representing firm's output, X is a $(NT \times k)$ matrix containing the k production inputs with associated parameter vector β $(k \times 1)$, W is the block diagonal $NT \times NT$ spatial weight matrix, I_{NT} is the $NT \times NT$ identity matrix, $\widetilde{v}$ is the error term distributed as shown in Eq.(2)-(3) and u is the inefficiency error term specified following the scaling property as described in the previous paragraph. Moreover, ρ is the autoregressive parameter capturing global spatial spillovers, θ is the parameter vector $(k \times 1)$ associated with input spillovers, τ, belonging to the spatial structure included in the scaling function, is the parameter capturing behavioural correlation depending on spillovers effects related to the determinants of firms' inefficiency and γ, embedded in the structure of $\widetilde{v}$, is the spatial parameter capturing environmental spatial dependence associated to unobserved but spatially correlated variables.

$$Y = (I_{NT} - \rho W)^{-1}(X\beta + WX\theta + \widetilde{v} - u) \tag{11}$$

Deriving Eq.(11) with respect to the generic regressor X_r lead to $\frac{dY}{dX_r} = (I_{NT} - \rho W)^{-1}(I_{NT}\beta_r + W\theta_r) = S_r(W)$. Thus, it is evident that in spatial models that include that spatial lag of Y, the β estimates no longer represent the marginal effects of the X variables on Y. LeSage and Pace (2009) proposed to compute the marginal effects starting from the $S_r(W)$ matrix differentiating into (i) direct effects computed as the average of the diagonal elements of $S_r(W)$, (ii) indirect effects calculated as the average of the sum of the non-diagonal elements of $S_r(W)$ and (iii) total effects equal to the sum of the direct and the indirect effects. The related standard errors or t-statistics can be computed either following the simulation method proposed by LeSage and Pace (2009) or using the delta method.

In order to compute the marginal effect of a generic Z variable on firms' inefficiency level, we consider the unconditional definition of firms' inefficiency. Indeed, within our SDF-CSD framework, the expression $u_{it} = (I_N - \tau W)^{-1} f(Z_{it}, \phi) u_t^\star$ provides a measure of u_{it} that is conditional on the frontier feedback effects. Starting from the reduced form in Eq.(11), the unconditional definition of the inefficiency error term is given by $\widetilde{u}_{it} = (I_N - \rho W)^{-1} u_{it}$. Thus, the first derivative of $\widetilde{u}_{it}$ with respect to Z, representing the marginal effect of Z on $\widetilde{u}_{it}$, includes the spatial filter $(I_N - \rho W)^{-1}$ related to the endogenous spatial lag of Y, as shown in Eq.(12a)-(12d). However, it should be noted that both the conditional and the unconditional definition of the inefficiency error term lead to the same interpretation of ϕ.

$$\frac{\partial \log(\widetilde{u}_{it})}{\partial Z_{it}} = \frac{\partial \log((I_N - \rho W)^{-1}(I_N - \tau W)^{-1} \exp(Z_{it}\phi) u_t^\star)}{\partial Z_{it}} \tag{12a}$$

$$= \frac{\partial(\log((I_N - \rho W)^{-1}) + \log((I_N - \tau W)^{-1}) + \log(\exp(Z_{it}\phi)) + \log(u_t^\star))}{\partial Z_{it}} \tag{12b}$$

$$= \frac{\partial \log((I_N - \rho W)^{-1})}{\partial Z_{it}} + \frac{\partial \log((I_N - \tau W)^{-1})}{\partial Z_{it}} + \frac{\partial (Z_{it}\phi)}{\partial Z_{it}} + \frac{\partial \log(u_t^\star)}{\partial Z_{it}} \tag{12c}$$

$$= \phi \tag{12d}$$

In particular, we specify $f(Z_{it}, \phi) = \exp(Z_{it}\phi)$ and we apply the logarithm to $\widetilde{u}_{it}$, as suggested by Wang and Schmidt (2002) to generate a very simple expression for the effect of the generic determinant Z on firm's inefficiency level. Thus, the interpretation of ϕ is straightforward, as it represents the marginal effect of Z on $\log(\widetilde{u}_{it})$, that is the logarithm of the level of technical inefficiency of the firm.

3 Monte Carlo Simulations

To test the finite sample properties of our spatial estimator, we simulated NT data. In particular, we defined for each time period t: the input variable X and the exogenous variable Z that determines technical inefficiency as standard normal random vectors $(N \times 1)$; h as an exponential function, and W as an $(N \times N)$ row normalized spatial weight matrix. Moreover, to introduce cross-sectional correlation in the error term, we defined v as a multivariate normal random vector $(N \times 1)$ with zero mean and variance-covariance matrix equal to Π, with $\Pi = \sigma_v^2 (I_N - \gamma W)^{-1} \left((I_N - \gamma W)^{-1}\right)^T$. Finally, the inefficiency error term $u^\star$, common to all firms but varying in time, is generated as a random value drawn from a truncated normal distribution with zero mean and variance equal to σ_u^2. Therefore, following the scaling property, the complete inefficiency term u $(N \times 1)$ accounting for cross-sectional dependence, is equal to the product of $u^\star$, of the positive function of the firm exogenous variable $h = \exp(Z\phi)$, and of the spatial filter $(I_N - \tau W)^{-1}$. Then, the dependent variable Y, for each time period t, is generated as a $(N \times 1)$ vector, as defined in Eq.(13), and the whole procedure is repeated for all time periods. Lastly, the partial loglikelihood function ℓ_t is calculated for each time period and the final loglikelihood function is obtained as $\mathcal{L}(\Theta, y) = \sum_{t=1}^T \ell_t$.

$$Y = (I_N - \rho W)^{-1} (X\beta + WX\theta + v - (I_N - \tau W)^{-1} \exp(Z\phi) u^\star) \tag{13}$$

We performed three Monte Carlo simulations to test how the choice of different spatial weight matrices, of different true values of the parameters and of the sample sizes affects the bias, the standard deviation (SD) and the mean squared error (MSE) of the estimated parameters. In particular, in the first simulation, we let N and T vary across different values ($N = 100, 200, 300$ and $T = 5, 10, 15$), keeping the true values of the parameters constant, while in the second one we change the true values of the parameters keeping N and T fixed at $N = 100$ and $T = 10$. Moreover, in the third block of simulations, we consider different spatial weight matrices. In particular, in the first two blocks of simulations, W is defined as a binary contiguity spatial weight matrix, while in the third block of simulations, also inverse distance spatial weight matrices with different truncation criteria are taken into consideration, keeping the true values of the parameters and N and T constant. In the first and third blocks of simulations the true values of the parameters are fixed at $\{\beta = 0.50,\ \rho = 0.30,\ \theta = 0.30,\ \phi = 0.50,\ \tau = 0.30,\ \gamma = 0.30,\ \sigma_u^2 = 0.10,\ \sigma_v^2 = 0.20\}$. All the simulations are performed using 1000 repetitions. Table 1 shows the results of the Monte Carlo experiment for different values of N and T, Table B1 in Appendix B shows how the final sample properties of the estimated parameters are affected by changes in one value at a time, keeping N and T constant, while Table 2 takes different kinds of spatial weight matrices into consideration with T and N fixed at 5 and 300, respectively.

Insert Table 1. Monte Carlo Simulation Results for different values of N and T

The results in Table 1 show that $\beta, \theta, \gamma, \sigma_u^2$ and σ_v^2 are estimated with negligible bias even using a very small sample while the bias of the other parameters (ρ, ϕ, τ) quickly reduces increasing N or considering more time periods. In particular, ρ, ϕ and τ tend to be slightly underestimated with very small N and T but overall, the bias of all parameters approaches zero considering a sufficiently high sample size. Likewise, the SD and the MSE tend to decrease, increasing the numerosity of the sample. Moreover, Table B1 in Appendix B, presenting the results of the second block of simulations for $T = 10$ and $N = 300$, shows that the estimates are robust to different changes in the true value of one parameter at a time, keeping all the other constant.

Insert Table 2. Monte Carlo Simulation Results: Sensitivity to the choice of W

Finally, Table 2 shows the results of the simulations considering different kinds of spatial weight matrices. Differently from the previous simulations taking a binary contiguity spatial weight matrix into consideration, here, W indicates a dense inverse distance spatial weight matrix, W50 and W30 indicate two inverse distance spatial weight matrices truncated at 50 and 30 kilometres, respectively, and W250n, W100n and W50n stand for three inverse distance spatial weight matrices considering only the 250, 100, and 50 nearest neighbours, respectively. All these spatial weight matrices have been created starting from a sample of 300 tourism facilities located in the North-West of Italy belonging to the ATECO55 sector. Moreover, all these matrices are row standardized, T is fixed at 5, and the number of replications is equal to 1000. The results shown in Table 2 indicate that the bias of β, ρ, θ, ϕ and of the two variance parameters σ_u^2 and σ_v^2 is not affected by the choice of W. Conversely, τ and γ tend to be underestimated if a large number of neighbours is considered. Indeed, the bias of τ and γ is equal to -0.0661 and -0.0206 choosing a dense W, while it decreases to -0.0172 and -0.0079 truncating W at 30 kilometres. Moreover, also the standard deviations associated with these two parameters tend to be quite high considering a dense W but they tend to decrease for W50 and W30. Similarly, τ and γ tend to be underestimated when W is defined as an inverse distance spatial weight matrix considering the n nearest neighbours and the number of neighbours is high, but the bias decreases as n reduces. Indeed, using W250n the bias associated with τ and γ is equal to -0.0545 and -0.0131 respectively, but it reduces to -0.0283 and -0.0034 for W50n.

Therefore, to obtain unbiased and more efficient estimates for the unknown parameters it is better to consider sparse matrices such as a binary contiguity spatial weight matrix or an inverse distance matrix with a truncation point. This issue has already been addressed by Mizruchi and Neuman (2008) that suggested preferring sparse spatial weight matrices when using spatial models. Indeed, in accordance with our simulation results, they demonstrated that dense inverse distance matrices are likely to produce negatively biased parameter estimates and that this bias becomes more severe at higher levels of network density.

Alternatively, if a dense W is anyhow preferred, to cut down the bias detected in this simulation study, it can be assumed that the spatial weight matrix is not identical among the different spatial structures considered in the SDF-CSD model. Therefore, we implement a further simulation considering two different spatial weight matrices to evaluate if the bias detected using a dense W diminishes. The results, shown in Table B2 in Appendix B, indicate that using different spatial weight matrices to differentiate among the different kinds of spatial effects is sufficient to cut off the bias of γ, while to obtain an unbiased estimate of τ it is necessary to consider a limited number of neighbours. In conclusion, the only limitation in estimating the SDF-CSD model without bias is to necessarily assume that the spillover effects associated with the inefficiency error term result from closest neighbours. This assumption should not be considered too restrictive because spatial dependence influencing neighbouring firms' efficiency level mainly arises from local factors such as emulation, face-to-face interactions, local cooperation, and individual contact (Griliches, 1992).

4 Application to the Italian Agricultural Sector

4.1 Data, Variables, and Empirical Model

We take advantage of the SDF-CSD model to analyse the performance of the Italian agricultural sector considering four different kinds of spatial effects. This novel spatial estimator results to be particularly suitable for this sector due to the strong importance of unobserved location-specific attributes in the agricultural industry. Indeed, thanks to the spatial structure attached to the random error term it is possible to consider spatial cross-sectional dependence arising from unobserved factors common to neighbouring areas such as soil conditions or climatic, topographic and environmental characteristics (Schmidt et al., 2009). Moreover, the spatial structure related to the determinants of firms' efficiency allows capturing behavioural spatial dependence arising from emulation behaviours of agricultural producers located in nearby areas and from policies and institutions operating at the local level (Areal, Balcombe, and Tiffin, 2012). On the other hand, the spatial structures entering in the frontier function relate to productivity and input spillovers. Input spillovers may generate from a greater availability of specific products, input suppliers, assets and workers with industry-specific skills in a certain area (Marshall, 1890). Furthermore, farmers' productive performance may be related to the one of neighbours due to the transmission of knowledge and best practices between peers (Cardamone, 2020), collective behaviours such as similar financial decisions resulting from face-to-face relationships, exchange of ideas and learning from others (Skevas and Lansink, 2020), farmers' adoption of new similar technologies to face specific techno-economic problems that are common to firms operating in nearby areas (Billé, Salvioni, and Benedetti, 2018), and marketing-related externalities such as positive feedbacks deriving from 'protected designation of origin" (PDO) certifications (Vidoli et al., 2016). Indeed, the successful performance of neighbouring producers may generate economic returns also to peers because of the increased reputation of the whole area (i.e. "halo effect" (Beebe et al., 2013)).

Hence, in this section, we estimate the SDF-CSD model using data on the Italian agricultural sector at NUTS-3 level in the time period 2008-2018. The data are collected from the RICA survey, which is the Italian counterpart of the FADN survey. The FADN survey represents the only harmonized source of microeconomic data on the evolution of incomes and on the economic-structural dynamics of farms at the European level. The FADN sample does not represent the entire universe of farms surveyed in a given territory, but only those that, due to their economic dimension [1] can be considered professional and market-oriented. The methodology adopted aims to provide representative data on three dimensions: region, economic dimension, and technical-economic order. In this empirical application, we consider aggregated data at NUTS-3 level from the RICA survey because for confidentiality reasons it is not possible to know the exact location of each farm in the sample. Moreover, aggregated data at the municipal level do not ensure representative results because in most cases they do not have the minimum required sample numerosity of 5 units. Thus, the NUTS-3 aggregation results to be the finer aggregation level that also guarantees representative results.

We use a Cobb-Douglas specification to model the production function equation, as shown in Eq.(14) for $i, j = 1, ..., N$ $(i \neq j)$ and $t = 1, ..., T$. Since we include four input variables in the analysis, we prefer a Cobb-Douglas functional form with respect to a Translog specification because it involves the estimation of fewer parameters and thus, it facilitates the interpretation of the results. Moreover, the Cobb-Douglas function is often used to estimate the production function parameters due to its ability to provide more efficient estimates, especially when dealing with small samples (Yao and Liu, 1998).

$$
\begin{aligned}
Y_{it} = {} & \beta_0 + \beta_L L_{it} + \beta_{AA} AA_{it} + \beta_M M_{it} + \beta_{WEF} WEF_{it} + \beta_t t + \rho \sum_{j=1}^{N} w_{ij} Y_{jt} \\
& + \theta_L \sum_{j=1}^{N} w_{ij} L_{jt} + \theta_{AA} \sum_{j=1}^{N} w_{ij} AA_{jt} + \theta_M \sum_{j=1}^{N} w_{ij} M_{jt} + \theta_{WEF} \sum_{j=1}^{N} w_{ij} WEF_{jt} - u_{it} + \widetilde{v}_{it}
\end{aligned}
\tag{14}
$$

Specifically, the dependent variable Y_{it} is defined as the logarithm of the total value added generated by the agricultural sector in province i at time t. Following Vidoli et al. (2016), we consider four input variables, all in log-form: total working hours (L_{it}), utilized agricultural area (AA_{it}), machinery (M_{it}), and water, energy, and fuel (WEF_{it}). Moreover, we add the time trend t to the model specification to capture the temporal dynamics, where t has a minimum value of 1 for 2008 and a maximum of 11 for 2018. To consider cross-sectional spatial dependence affecting the frontier function we include in Eq.(14) the spatial lag of the dependent variable and of the four production inputs. Specifically, following the queen contiguity criteria, we identify neighbouring observations as provinces sharing a common border or vertex. Thus, we build the spatial weight matrix W as a row standardized binary contiguity matrix considering second-order neighbours (i.e. we assign value 1 to neighbouring provinces and to neighbours of neighbours and 0 otherwise [2]). In particular, w_{ij} refers to a generic spatial weight belonging to the $(N \times N)$ sparse spatial weight matrix having all zeros on the main diagonal. Hence, we are able to detect global spatial dependence through ρ, while the parameters θ_L, θ_{AA}, θ_M, and θ_{WEF} measure local spatial dependence related to the four input variables. Finally, $\widetilde{v}_{it}$ and u_{it} represent the two error terms. The random disturbance accounts for unobserved but spatially correlated variables thought γ, capturing remaining spatial dependence resulting from unobserved but cross-sectionally correlated spatial features such as climatic conditions, soil characteristics, and local institutional and socio-economic factors. On the other hand, the inefficiency error term, defined using the scaling property, can be written as $u_{it} = \widetilde{h}_{it} u_t^{\star}$, where $\widetilde{h}_{it}$ is the scaling function defined as in Eq.(15) and $u_t^{\star}$ is the industry-specific inefficiency error term common to all firms but varying in time following a truncated normal distribution with mean 0 and variance σ_u^2.

$$\begin{aligned}\widetilde{h}_{it} = (I_N - \tau W)^{-1} \exp(&\phi_S Small_{it} + \phi_B Big_{it} + \phi_F Family_{it} + \phi_D Diversified_{it} \\ &+ \phi_H Hired_{it} + \phi_Y Youth_{it} + \phi_W Woman_{it} + \phi_{Sub} Subsidies_{it})\end{aligned} \tag{15}$$

The scaling function shown in Eq.(15) is composed by the spatial lag $(I_N - \tau W)^{-1}$ and by a positive function of the determinants of farms' inefficiency level. As usual in SF model specifying the inefficiency error term using the scaling function approach, we choose the exponential function to obtain easily interpretable estimates for the ϕ parameters. Moreover, as proposed by Orea and Alvarez (2019), we are able to capture the overall spatial dependence associated with the variables that determine cross-sectional inefficiency through τ. Indeed, farms' inefficiency level can also be related to the observable characteristics of neighbouring producers and ignoring these features can lead to heteroskedasticity issues in the inefficiency error term.
Focusing on the determinants of inefficiency, we model the scaling function introducing some variables related to the main characteristics of Italian farms at the provincial level. Specifically, we consider the average farm dimension in the province introducing two variables ($Small$ and Big) that respectively measure the proportion of small and big farms. We take the organizational type of the farms into consideration including in the model the variable $Family$ that equals the proportion of farms run by family members in the province. Moreover, we include in the scaling function equation the variable $Diversified$ measuring the proportion of diversified farms in the province and $Hired$ measuring the average percentage of hired land. Furthermore, farmers' characteristics such as age, gender, and education are often considered as a proxy for management practices and social capital. Therefore, we include in the scaling function $Youth$ and $Woman$ representing the percentage of farms run by young entrepreneurs (with less than 40 years) and by women, respectively. Finally, we investigate if subsidies positively or negatively affect farmers' inefficiency levels. Thus, we introduce the variable $Subsidies$ defined as the amount of total subsidies received by farms located in each province over total farm income

as proposed by Stetter and Sauer (2021). Further details on the definition and measurement of the variables considered in the analysis and some brief descriptive statistics can be found in Table 3.

Insert Table 3. Descriptive Statistics

4.2 Results

Table 4 shows the estimation results of the SDF-CSD model (yxuv model), of the SF models introducing only frontier and error based spillovers (yx and uv model, respectively) and of the non-spatial SF model. The complete estimates of all the nested specifications are contained in Table C1 of Appendix C. Indeed, starting from the SDF-CSD it is possible to restrict the model specification in order to capture only specific sources of spatial dependence depending on the economic phenomenon under investigation. As a consequence, the SDF-CSD model leads the way to a number of spatial specifications never introduced before but that can be very useful in empirical applications. Moreover, it is worth highlighting that we didn't have any problem simultaneously estimating the four different spatial parameters using a numerical maximisation algorithm even if our sample numerosity is not very large.

Insert Table 4. Estimation Results

Comparing the estimation results of the nested models shown in Table 4, it can be noticed that the β estimates and the ϕ estimates are quite robust to the different specifications. Nevertheless, as previously described, the β coefficients cannot be interpreted in a meaningful way when the spatial lag of Y is included in the model (a brief description of the marginal effects of the input variables is presented in sub-section 4.3). As for the ϕ estimates, our results indicate that provinces with a higher percentage of small farms as well as provinces with a lot of farms run by family members tend to be more inefficient while big farms contribute to decreasing the inefficiency level of the agricultural sector at NUTS-3 level. Moreover, we find that the degree of diversification positively affects inefficiency. Indeed, specialisation might boost technical efficiency more than diversification since it enables farmers to specialize in a few tasks, and therefore it improves management practices (Latruffe, 2010). Investigating the link between technical inefficiency and the use of external factors, we find a negative effect of the percentage of rented land in the province on technical inefficiency even if *Hired* results to be significant only in some nested specifications. Considering subsidies, our results confirm the idea that they contribute to decreasing the efficiency level of farms likely due to lowered farmers' motivation and distorted farms' production structure and factor use (Rizov, Pokrivcak, and Ciaian, 2013). Finally, provinces with a higher percentage of farms run by women or by young entrepreneurs tend to be more inefficient than others. Indeed, as Chen, Huffman, and Rozelle (2009) and Mathijs and Vranken (2001) explained, older farmers result to be more efficient thanks to increased experience and learning ability.

Concentrating on the spatial parameters, we detect positive and significant spatial dependence either at the frontier level or related to the two error terms. Thus, the estimate of ρ indicates that positive global productivity spillovers affect the Italian agricultural sector as well as positive spillover effects related to the determinants of farms' inefficiency (τ), and to unobserved environmental factors (γ). Specifically, the degree of global spatial dependence (0.14) estimated using the SDF-CSD model is considerably smaller than the level of behavioural and environmental spatial correlation associated with the two error components (both equal to 0.37) indicating that the Italian agricultural sector is more strongly affected by error-based than by frontier-based spatial dependence.

Finally, in Table C2 of Appendix C we compare the different nested models using some likelihood ratio tests and information criteria. According to both the LR test and to the AIC criteria in this case study, it is better to simplify the model specification using a Manski model that, besides considering the spatial lag of Y and of X, includes one unique spatial term referred to the whole error structure. Indeed, the degree of spatial dependence associated with the inefficiency and the random error term is the same for both the two error components using the SDF-CSD model.

4.3 Robustness Check and Marginal Effects

As a robustness check, we estimate the SDF-CSD model using different spatial weight matrices. Besides the second-order binary contiguity matrix (W2), we consider a first-order binary contiguity matrix (W) and a dense inverse distance matrix (Wid). According to the results of the Monte Carlo simulations, when we introduce the dense inverse distance matrix we associate it only to the spatial lags related to the frontier function while for the two error terms we use a sparse matrix like W or W2. The results of the robustness check are shown in Table E1 of Appendix E while Table 5 shows the corresponding marginal effects.

Insert Table 5. Marginal Effects

The direct effects of the X variables on Y do not vary depending on the choice of the spatial weight matrix. Specifically, they are all positive and significant, and labour results to be the more effective input variable (0.61), followed by water, energy, and fuel (0.20). In line with the results of Fusco and Vidoli (2013), capital inputs such as machinery (0.07) and land (0.13) contribute less to the productive performance of the Italian agricultural sector compared to human resources. Overall, the sum of the elasticities related to the four input variables results to be greater than 1 regardless of the spatial structure considered. Thus, in line with most of the European countries (Rizov, Pokrivcak, and Ciaian, 2013), the Italian agricultural sector is characterized by increasing returns to scale, indicating that increasing the inputs by 1% would produce more than a 1% increase in output. Considering the indirect effects, they tend to vary as the choice of the spatial weight matrix changes. In particular, the indirect effect associated with labour is positive and significant only when we consider a first-order contiguity matrix related to the frontier function. Conversely, the indirect effect of machinery is positive and significant only considering a dense inverse distance spatial weight matrix associated with the frontier function. Conversely, the indirect effect of utilized agricultural areas and water, energy, and fuel does not change across the different estimated models. Indeed, while the indirect effect of AA is always negative and significant, the indirect effect of WEF is never significantly different from zero. In sum, we detect positive spillover effects at the local level in terms of labour while the positive spillover effects related to machinery result to be significant only at the global level. On the contrary, provinces that dispose of larger agricultural areas tend to be located near provinces with fewer lands likely due to the specific territorial conformation and the alternation of more rural provinces with more urbanized ones.

4.4 Technical Efficiency Scores

Insert Figure 1. TE Scores: Kernel Density Plot

Figure 1 shows the kernel density plots of the technical efficiency scores computed estimating the SDF-CSD model, the non-spatial SF model, and the uv and xy models, introducing spatial dependence only in the error terms and in the frontier function, respectively. Considering the scale of the distribution, it can be noticed that the TE scores' distribution from the non-spatial model is the closest to the one obtained through the SDF-CSD. On the other hand, the xy model tends to overestimate the TE scores while the uv model tends to underestimate them. As for the shape of the distribution, the distribution obtained using the SDF-CSD model resembles the one coming from the spatial error model but is mitigated by the quasi-normal shape of the xy model. Thus, the SDF-CSD model allows to accurately combine the main features of the distributions of the TE scores resulting from the uv and xy model. In conclusion, this empirical application shows that the non-spatial TE scores distribution is the one that better approaches the SDF-CSD one while considering spatial dependence only in the frontier function or in the error terms leads to severe distortions both in the scale and in the shape of the distribution. Therefore, it is fundamental to combine frontier based and error based spatial effects both to consider different sources of spatial dependence through a comprehensive model and also to obtain consistent estimates of the TE scores.

Moreover, in Table D1 of Appendix D we present the efficiency ranking of the 107 Italian provinces where it can be observed more in detail how both the ranking and the level of the TE scores modify depending on the kind of spatial structures considered. In addition, in Table

D2 of Appendix D we show how the efficiency score ranking changes in time by comparing the 2008 and 2018 results. Overall, it can be observed that the efficiency scores reached higher values in 2008 compared to 2018, highlighting that the efficiency level of the Italian agricultural sector is decreasing over time (the average score equals 0.58 in 2008 and 0.48 in 2018). In particular, the province of Catanzaro reaches a dramatically low score in 2018 due to the structural weakness of the Calabrian agro-industrial sector depending on the small size of the farms, the fragmentation of the offer and the obsoleteness of the types of machinery and techniques used by farmers (Banca d'Italia, 2019).

Insert Figure 2. TE Scores Quantile Map, Years 2008 and 2018

Figure 2 maps the TE scores resulting from the SDF-CSD model for the 107 Italian provinces, comparing the years 2008 and 2018. Overall, provinces located in the North of Italy result to be much more efficient than those located in the South and this gap has remarkably increased in the past eleven years. Specifically, the large efficiency cluster in the Po Valley is strengthening over time while Sardinia, Calabria, the Southern Apulia, and the Northern provinces at the border with Switzerland are achieving lower and lower efficiency scores. Besides highlighting a severe North-South divide that is increasing with time, Figure 2 shows that the Italian provinces are characterized by strong spatial concentration considering the efficiency level of the agricultural sector.

Further insights on the spatial features related to technical efficiency scores are contained in Appendix D.

5 Conclusion

In this paper we propose a novel spatial stochastic frontier model that introduces four different kinds of spatial spillover effects, two related to the frontier function (i.e. productivity and input spillovers) and two related to the error structure (behavioural correlation associated with the inefficiency error term and environmental correlation referring to unobserved variables). While previous literature only considers frontier based or error based spillover effects, this is the first work that merges together the two different approaches, obtaining a very comprehensive specification never introduced before. Despite his complex spatial structure, one of the most remarkable features of the SDF-CSD model is that thanks to the modelling approach suggested by Orea and Alvarez (2019) for the two error terms, it can be estimated using standard maximum likelihood algorithms and it allows to straightforwardly interpret the effects of the inefficiency determinants.

Investigating whether our model is able to distinguish in practice between the four different sources of spatial dependence, we showed that our spatial estimator is able to differentiate between frontier and error-based spillovers considering sparse spatial weight matrices (as binary contiguity or truncated inverse distance matrices). Alternatively, unbiased estimates can be obtained using two different spatial weight matrices simultaneously and associating a sparse matrix to the spatial lag of the inefficiency error term. Therefore, the only limitation in estimating the SDF-CSD model without bias is to necessarily assume that the spillover effects associated with the inefficiency error term result from closest neighbours. This assumption should not be considered too restrictive because geographic proximity is fundamental for interaction, cooperation and for the transmission of new knowledge (Griliches, 1992).

In the empirical section of this paper, we applied the SDF-CSD model to NUTS-3 level data on the Italian agricultural sector. The main advantage resulting from estimating the SDF-CSD model in practice concerns the possibility of also evaluating a number of nested specifications never introduced before. Indeed, starting from the SDF-CSD and making some LR tests for nested models, it is possible to test whether it is better to simplify the model specification considering only specific spatial lags or if a comprehensive spatial SF model is required. Thus, it is possible to precisely assess which kind of spatial effect is more appropriate for studying the phenomenon under investigation without making a priori assumptions on the spatial structure of the data. Finally, we showed that the SDF-CSD model allows obtaining more accurate technical efficiency scores, mixing together the main distributional features resulting from spatial SF models that only include frontier based or error based spillovers.

From a practical perspective, the empirical evidence on the existence of significant agglomeration externalities originating from various channels has important implications for policy makers dealing with the Italian agricultural sector. Indeed, strategic plans and programs aimed at improving the Italian agricultural sector's performance may take advantage of positive spillovers by strengthening the cohesiveness of the networks, providing more opportunities for farmers to learn from each other, stimulating cooperation and networking, and encouraging local farmers' associations as well as knowledge dissemination on the use of new equipment and new management practices. Exploiting existing spatial interactions by creating a collaborative environment can be an effective strategy for policy makers to overcome the technological backwardness of most Italian farmers and thus, to boost the productivity of the Italian agricultural sector.

Given the completeness of the SDF-CSD model in considering the different spatial structures, in future extensions of this work, it could be interesting to modify or relax the underlying modelling assumptions. In particular, it could be useful to introduce the possibility to account for cross-sectional variation in the inefficiency error term by implementing a Bayesian algorithm for the estimation. Moreover, rather than assuming that the inefficiency distribution is half-normal, different specifications for the two error components can be considered, such as an exponential or a gamma distribution.

Declaration of Interest:

The author reports there are no competing interests to declare.

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Notes

[1]In Italy, the minimum threshold for the inclusion in the RICA observation sample corresponds to a minimum standard gross income of 8,000 euros.

[2]A second-order matrix compared to a first-order one allows to consider more complex spatial structures and to better identify spatial clusters. Moreover, this matrix is the one that minimizes the loglikelihood function. Some robustness checks using different spatial weight matrices are provided afterwards.

Table 1: Monte Carlo Simulation Results for different values of N and T

T=5	**N=100**			**N=200**			**N=300**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	0.0009	0.025	0.0007	0.0005	0.0197	0.0004	0.0005	0.0143	0.0003
ρ	-0.0130	0.1390	0.0216	-0.0023	0.1145	0.0149	-0.0085	0.0769	0.0114
θ	0.0065	0.0747	0.0067	-0.0009	0.0604	0.0045	0.0035	0.0455	0.0034
ϕ	0.0119	0.1077	0.0127	0.0070	0.0691	0.0072	0.0127	0.0724	0.0054
τ	-0.0173	0.2059	0.0494	-0.0158	0.1443	0.0287	0.0002	0.1468	0.0199
γ	0.0047	0.1534	0.0245	-0.0037	0.1108	0.0161	0.0052	0.0813	0.0127
σ_u^2	0.0022	0.0370	0.0054	0.0044	0.0568	0.0052	-0.0001	0.0540	0.0048
σ_v^2	-0.0048	0.0128	0.0002	-0.0028	0.0094	0.0001	-0.0017	0.0075	0.0001

T=10	**N=100**			**N=200**			**N=300**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0005	0.0174	0.0004	-0.0005	0.0133	0.0002	0.0003	0.0115	0.0001
ρ	-0.0044	0.0956	0.0157	-0.0008	0.0766	0.0073	-0.0043	0.0678	0.0054
θ	0.0005	0.0527	0.0045	-0.0011	0.0425	0.0022	0.0019	0.0373	0.0017
ϕ	0.0097	0.0573	0.0048	0.0027	0.0339	0.0019	0.0026	0.0373	0.0015
τ	-0.0092	0.0915	0.0254	-0.0094	0.0877	0.0123	-0.0015	0.0882	0.0092
γ	-0.0017	0.0966	0.0170	-0.0030	0.0780	0.0081	0.0021	0.0702	0.0061
σ_u^2	0.0014	0.0442	0.0029	0.0021	0.0755	0.0024	-0.0002	0.0502	0.0023
σ_v^2	-0.0027	0.0093	0.0001	-0.0013	0.0066	4.25e-05	-0.0009	0.0053	2.88e-05

T=15	**N=100**			**N=200**			**N=300**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0004	0.0161	0.0003	-0.0002	0.0119	0.0001	2.66e-05	0.0094	0.0001
ρ	-0.0043	0.0846	0.0085	0.0060	0.0724	0.0053	-0.0004	0.0577	0.0033
θ	0.0016	0.0515	0.0027	-0.0035	0.0386	0.0016	-0.0004	0.0332	0.0010
ϕ	0.0083	0.0758	0.0026	0.0055	0.0307	0.0013	-0.0030	0.038	0.0008
τ	-0.0001	0.1422	0.0141	-0.0051	0.0883	0.0082	-0.0103	0.0874	0.0053
γ	0.0004	0.0998	0.0095	-0.0087	0.0742	0.0058	-0.0023	0.0631	0.0037
σ_u^2	-0.0020	0.0159	0.0019	-0.0011	0.0374	0.0016	-0.0004	0.0298	0.0015
σ_v^2	-0.0019	0.0078	0.0001	-0.0008	0.0055	3.06e-05	-0.0004	0.0044	1.69e-05

Table 2: Monte Carlo Simulation Results: Sensitivity to the choice of W

	W			**W50**			**W30**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0001	0.0119	0.0002	-0.0002	0.0127	0.0002	-0.0003	0.0136	0.0002
ρ	-0.0020	0.1094	0.0326	0.0007	0.1148	0.0077	0.0011	0.0650	0.0050
θ	0.0004	0.0920	0.0135	-0.0029	0.0666	0.0041	-0.0024	0.0484	0.0025
ϕ	0.0041	0.0675	0.0089	-0.0015	0.1097	0.0054	-0.0012	0.0399	0.0045
τ	-0.0661	0.2800	0.0723	-0.0255	0.1288	0.0263	-0.0172	0.0881	0.0196
γ	-0.0206	0.1686	0.0364	-0.0106	0.0951	0.0098	-0.0079	0.0675	0.0063
σ_u^2	0.0069	0.0891	0.0060	0.0027	0.0268	0.0050	0.0003	0.0831	0.0045
σ_v^2	-0.0017	0.0076	0.0001	-0.0015	0.0076	0.0001	-0.0015	0.0074	0.0001

	W250n			**W100n**			**W50n**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0005	0.0121	0.0002	0.0004	0.0122	0.0002	-0.0001	0.0125	0.0002
ρ	-0.0096	0.1615	0.0304	-0.0128	0.1059	0.0240	-0.0135	0.1336	0.0175
θ	0.0016	0.1212	0.0129	0.0036	0.0839	0.0104	0.0055	0.0818	0.0076
ϕ	0.0054	0.0934	0.0087	0.0060	0.0571	0.0079	0.0019	0.1116	0.0080
τ	-0.0545	0.3273	0.0730	-0.0372	0.2092	0.0531	-0.0283	0.1419	0.0429
γ	-0.0131	0.2033	0.0333	-0.0062	0.1452	0.0258	-0.0034	0.1201	0.0187
σ_u^2	0.0073	0.0526	0.0064	0.0081	0.1369	0.0056	0.0068	0.0270	0.0062
σ_v^2	-0.0015	0.0075	0.0001	-0.0015	0.0073	0.0001	-0.0016	0.0074	0.0001

Table 3: Variables Description

Variable	**Definition**	**Units**	**Min**	**10th Perc.**	**Mean**	**90th Perc.**	**Max**	**SD**
Y	log(Value Added)	€	10.62	13.91	15.49	18.83	17.78	1.18
L	log(Working Hours)	num.	8.40	11.26	12.58	13.69	14.41	1.04
AA	log(Utilized Agricultural Area)	km	2.46	6.04	7.63	8.93	10.07	1.16
M	log(Machinery)	€	3.69	7.82	9.29	10.55	11.49	1.1
WEF	log(Water Energy Fuel)	€	6.91	10.35	12.15	13.72	14.69	1.37
t	Time	num.	1	2	6	10	11	3.16
Small	Perc. Small Farms	%	0.00	0.05	0.24	0.47	0.89	0.16
Big	Perc. Big Farms	%	0.00	0.00	0.12	0.27	0.71	0.12
Family	Perc. Family Farm	%	0 .00	0.19	0.53	0.87	1.00	0.25
Diversified	Perc. Diversified Farms	%	0.00	0.00	0.13	0.34	0.72	0.16
Hired	Perc. Hired Land	%	0.00	0.14	0.44	0.73	0.99	0.23
Youth	Perc. Young Farmers	%	0.00	0.03	0.13	0.27	0.54	0.11
Woman	Perc. Farms Run by Woman	%	0.00	0.05	0.21	0.37	0.56	0.13
Subsidies	Tot. Subsidies/Tot. Income	%	0.01	0.07	0.28	0.59	0.92	1.06

Table 4: Estimation Results

	yxuv		**uv**		**yx**		**non-spat.**	
	Coeff.	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*
β_0	3.87***	0.53	4.25***	0.62	3.10***	0.49	3.90***	0.19
β_L	0.61***	0.02	0.63***	0.02	0.62***	0.02	0.62***	0.02
β_{AA}	0.13***	0.02	0.13***	0.02	0.14***	0.02	0.09***	0.02
β_M	0.07***	0.01	0.07***	0.01	0.07***	0.01	0.10***	0.01
β_{WEF}	0.20***	0.01	0.19***	0.01	0.20***	0.01	0.20***	0.01
β_t	-0.01	0.01	-0.01	0.02	-0.01	0.01	-0.02**	0.01
ρ	0.14*	0.08	-		0.38***	0.07	-	
θ_L	-0.06	0.08	-		-0.25***	0.07	-	
θ_{AA}	-0.25***	0.05	-		-0.27***	0.05	-	
θ_M	0.02	0.04	-		0.01	0.05	-	
θ_{WEF}	0.02	0.05	-		-0.02	0.07	-	
ϕ_S	0.70***	0.18	0.67***	0.17	0.73***	0.19	0.37***	0.11
ϕ_B	-1.72***	0.46	-1.65***	0.41	-1.71***	0.59	-1.57***	0.32
ϕ_F	0.22**	0.11	0.20**	0.10	0.22	0.14	0.13*	0.07
ϕ_D	0.37**	0.15	0.24*	0.14	0.40**	0.16	0.08	0.11
ϕ_H	-0.06	0.11	-0.07	0.10	-0.03	0.12	-0.17**	0.08
ϕ_{Sub}	0.71***	0.10	0.69***	0.09	0.72***	0.13	0.59***	0.07
ϕ_Y	0.66***	0.20	0.67***	0.19	0.69***	0.23	0.71***	0.15
ϕ_W	0.55***	0.18	0.58***	0.17	0.52**	0.19	0.63***	0.14
τ	0.37***	0.12	0.58***	0.08	-		-	
γ	0.37***	0.07	0.52***	0.05	-		-	
σ_u^2	0.05		0.06		0.04		0.17	
σ_v^2	0.09		0.09		0.09		0.11	
LL	-284.61		-297.70		-296.20		-369.50	
LR	-		26.18		23.18		169.78	
AIC	615.22		631.40		634.40		771.00	

$***: pvalue \leq 0.01; \quad **: pvalue \leq 0.05; \quad *: pvalue \leq 0.10$

Table 5: Marginal Effects

Effects		**W2**		**W**		**W2,W**		**W,W2**		**Wid,W2**		**Wid,W**	
		Coeff.	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*
L	*Direct*	0.61***	0.02	0.59***	0.02	0.61***	0.02	0.60***	0.02	0.63***	0.02	0.63***	0.02
	Indirect	0.03	0.06	0.08**	0.04	-0.01	0.07	0.11**	0.04	-0.19	0.18	-0.21	0.18
	Total	0.64***	0.06	0.67***	0.04	0.59***	0.07	0.71***	0.04	0.44**	0.18	0.42***	0.17
AA	*Direct*	0.13***	0.02	0.14***	0.02	0.13***	0.02	0.15***	0.02	0.14***	0.02	0.13***	0.02
	Indirect	-0.26***	0.05	-0.15***	0.03	-0.32***	0.05	-0.11**	0.04	-0.72***	0.21	-0.91***	0.22
	Total	-0.13**	0.06	-0.01	0.03	-0.19***	0.05	0.04	0.03	-0.59**	0.21	-0.77***	0.22
M	*Direct*	0.07***	0.01	0.09***	0.01	0.07***	0.01	0.08***	0.01	0.07***	0.01	0.07***	0.01
	Indirect	0.04	0.05	0.01	0.03	0.05	0.05	-0.01	0.03	0.41**	0.2	0.51**	0.24
	Total	0.11**	0.05	0.11***	0.03	0.12**	0.05	0.07**	0.03	0.47**	0.2	0.59***	0.24
WEF	*Direct*	0.20***	0.01	0.19***	0.01	0.20***	0.01	0.20***	0.01	0.19***	0.01	0.18***	0.01
	Indirect	0.06	0.05	0.01	0.03	0.09	0.06	0.01	0.03	-0.03	0.19	-0.09	0.21
	Total	0.26***	0.06	0.21***	0.03	0.29***	0.06	0.20***	0.03	0.16	0.2	0.08	0.22

$***$: *pvalue* $\leq$ 0.01; $**$: *pvalue* $\leq$ 0.05; $*$: *pvalue* $\leq$ 0.10

NB. W2: second order contiguity matrix associated with all the spatial terms; W: first order contiguity matrix associated with all the spatial terms; W2,W: second order contiguity matrix associated with the frontier function and first order contiguity matrix associated with the two error components; W,W2: first order contiguity matrix associated with the frontier function and second order contiguity matrix associated with the two error components; Wid,W2: dense inverse distance matrix associated with the frontier function and second order contiguity matrix associated with the two error components; Wid,W: dense inverse distance matrix associated with the frontier function and first order binary contiguity matrix associated with the two error components.

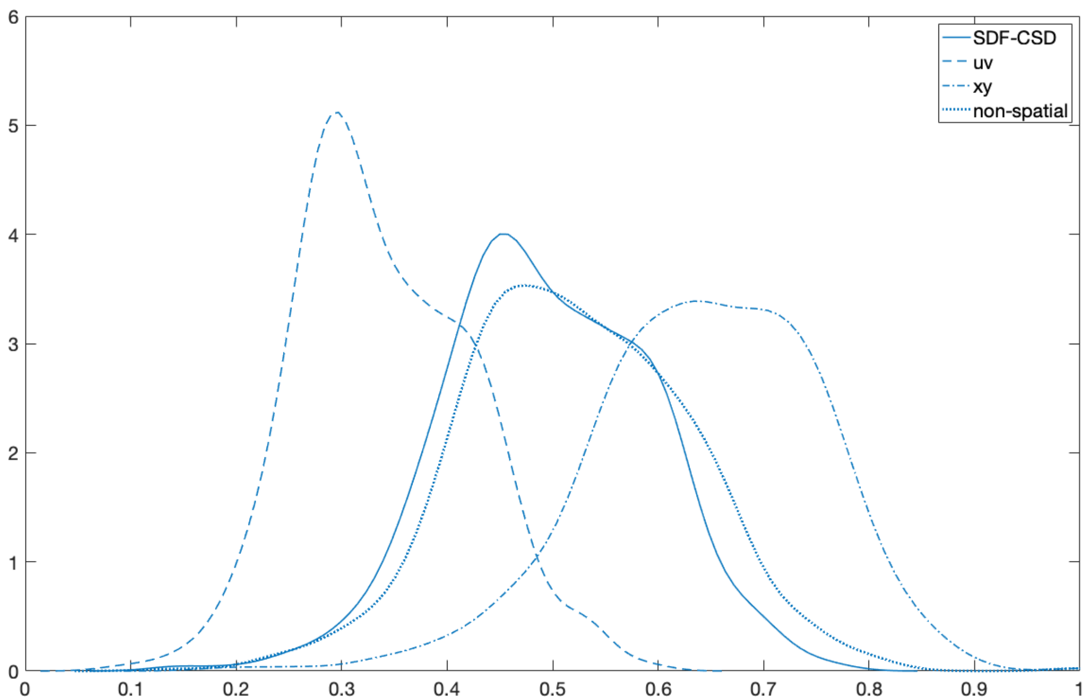

Figure 1. TE Scores: Kernel Density Plot

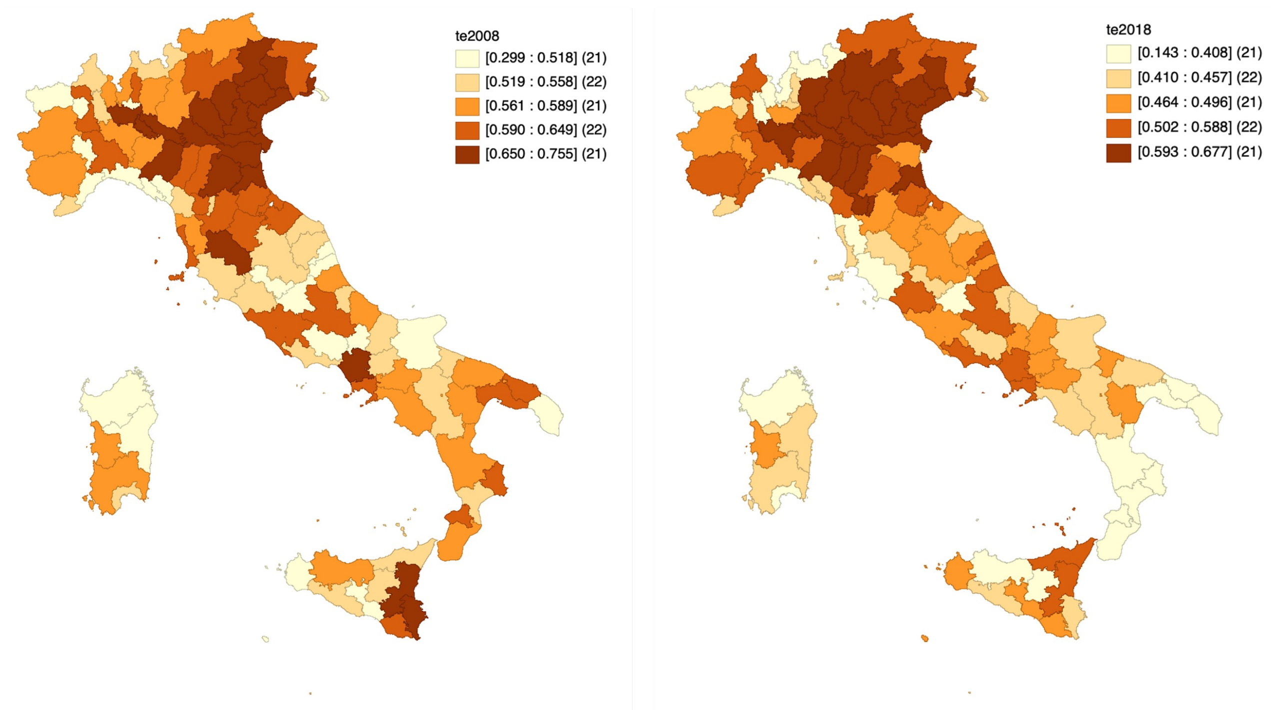

Figure 2. TE Scores Quantile Map, Years 2008 and 2018

Supplementary Materials

Appendix A: Estimation Procedure

The estimation procedure for our spatial SF model is a straightforward extension of Orea and Alvarez (2019)'s approach, which is in turn a transposed version of Wang and Ho (2010). The only additional issue concerns the computation of the log-determinant resulting from the inclusion of the spatial lag of the dependent variable in the model. As usual in spatial autoregressive models, $\log |I_N - \rho W|$ is the determinant of a large matrix and it needs to be recalculated at each iteration of the optimization procedure, leading to a computationally expensive and time-consuming routine. One potential solution to speed up the computation of the log-determinant, as suggested by Pace and Barry (1997), is to use a vector of values for the log-determinant corresponding to different values of ρ calculated before optimization belonging to the interval $(1/\omega_{min}, 1)$, where ω_{min} is the smallest eigenvalue of the spatial weight matrix W and the upper bound equals 1 for row-normalized spatial weight matrices. This involves calculating the vector of log-determinants only once during the estimation procedure and the final interpolated value for the log-determinant can be computed considering a sufficiently fine grid for ρ (for a review on these numerical approaches see LeSage and Pace (2009)).

After having adequately handled the computation of the log-determinant, the unknown parameters of the SDF-CSD model can be simultaneously estimated numerically maximising the loglikelihood function using standard software. In particular, we first derived the partial log-likelihood function ℓ_t shown in Eq.(6) referring to all cross-sectional observations in each time period t and then, the overall log-likelihood function is obtained as $\mathcal{L}(\Theta; y) = \sum_{t=1}^{T} \ell_t$. To maximize the final log-likelihood function we used the interior-point algorithm implemented in Matlab's function "fmincon". More details on this routine can be found in Byrd, Gilbert, and Nocedal (2000) and Byrd, Hribar, and Nocedal (1999). The standard errors can be recovered starting from the estimated Hessian matrix at the solution.

Appendix B: Monte Carlo Simulation Results

This appendix provides some additional insights on the Monte Carlo simulation results. Specifically, Table B1 shows how the final sample properties of the estimated parameters are affected by changes in one true value of the parameters at the time, keeping N and T constant, while Table B2 contains the simulation results using two different spatial weight matrices.

Specifically, Table B1 shows that the estimates are robust to different changes in the true value of one parameter at a time, keeping all the others constant. Moreover, we test whether the bias of τ and γ detected in the paper using inverse distance spatial weight matrices reduces considering two different kinds of spatial weight matrices simultaneously. In particular, we consider all the possible combinations of W and $W0$ associated with the four kinds of spatial effects introduced in the model: global spatial spillovers, local spatial spillovers, spatial dependence in the inefficiency term and spatial dependence in the error term, respectively captured by the parameters ρ, θ, τ, and γ. As in the simulations performed in the paper, W represents a dense inverse distance spatial weight matrix while $W0$ is a binary continuity spatial weight matrix. The results shown in Table B2 indicate that considering two different kinds of spatial weight matrices is sufficient to cut off the bias of γ, regardless of the combination of W and $W0$. Conversely, to obtain an unbiased estimate of τ it is necessary to consider a limited number of neighbours. Indeed, when a binary contiguity spatial weight matrix is used to capture the spatial dependence in the inefficiency error term, the bias of τ approaches zero, while if a dense W is taken into consideration, the bias of τ can reach a value up to -0.0313. Nevertheless, it should be noticed that -0.0313 is half of the bias detected in the simulation using a dense inverse distance spatial weight matrix for all the four spatial effects (-0.0661). Moreover, considering two different kinds of spatial weight matrices for the different spatial effects also helps in reducing the standard deviation of the estimated parameters and the MSE.

Table B1: Simulation Results for different true values of the parameters

β	0.15			0.65			0.90		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0001	0.0115	0.0001	0.0002	0.0109	0.0001	4.10e-05	0.0114	0.0001
ρ	-0.0019	0.1003	0.0081	-0.0022	0.0596	0.0039	-0.0006	0.0585	0.0033
θ	-0.0009	0.0234	0.0004	0.0009	0.0413	0.0019	0.0005	0.0554	0.0030
ϕ	0.0037	0.0287	0.0014	0.0040	0.0338	0.0018	0.0034	0.0366	0.0021
τ	-0.0062	0.1453	0.0115	1.81e-06	0.0747	0.0084	-0.0018	0.0862	0.0080
γ	-4.48e-05	0.1142	0.0086	-4.37e-05	0.0650	0.0042	-0.0019	0.0639	0.0037
σ_u^2	-0.0019	0.0739	0.0020	0.0009	0.0554	0.0024	0.0002	0.0530	0.0025
σ_v^2	-0.0015	0.0062	3.09e-05	-0.0010	0.0054	2.76e-05	-0.0010	0.0052	2.76e-05

ρ	0.15			0.65			0.90		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0006	0.0097	0.0001	0.0004	0.0106	0.0001	0.0003	0.0093	0.0001
ρ	0.0069	0.0620	0.0066	-0.0029	0.0324	0.0015	-0.0012	0.0120	0.0002
θ	-0.0044	0.0345	0.0017	0.0021	0.0269	0.0008	0.0015	0.0188	0.0004
ϕ	0.0024	0.0205	0.0016	0.0033	0.0481	0.0014	0.0034	0.0579	0.0019
τ	-0.0093	0.0626	0.0092	0.0015	0.0865	0.0067	0.0010	0.0851	0.0063
γ	-0.0085	0.0629	0.0067	0.0021	0.0474	0.0026	0.0012	0.0311	0.0011
σ_u^2	-0.0012	0.1085	0.0023	-0.0016	0.0178	0.0024	-0.0005	0.0158	0.0026
σ_v^2	-0.0009	0.0052	2.84e-05	-0.0004	0.0056	3.07e-05	-0.0003	0.0055	2.99e-05

θ	0.15			0.65			0.90		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	0.0002	0.0104	0.0001	0.0001	0.0115	0.0001	-0.0005	0.0117	0.0001
ρ	0.0017	0.1002	0.0106	0.0010	0.044	0.0018	0.0005	0.0318	0.0009
θ	-0.0007	0.0531	0.0029	-0.0001	0.0273	0.0007	-0.0007	0.0220	0.0004
ϕ	0.0040	0.0362	0.0013	0.0020	0.0364	0.0013	-0.0013	0.0462	0.0016
τ	-0.0099	0.1368	0.0136	-0.0015	0.0766	0.0046	-0.0059	0.0682	0.0044
γ	-0.0061	0.1071	0.0113	-0.0025	0.0508	0.0025	-0.0013	0.0403	0.0014
σ_u^2	0.0017	0.0385	0.0024	0.0033	0.0389	0.0025	0.0021	0.0331	0.0024
σ_v^2	-0.0015	0.0056	2.93e-05	-0.0007	0.0053	2.59e-05	-0.0005	0.0054	2.65e-05

ϕ	0.15			0.65			0.90		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0001	0.0113	0.0001	-0.0003	0.0103	0.0001	-0.0001	0.0109	0.0001
ρ	-0.0005	0.0739	0.0050	0.0026	0.0581	0.0049	-0.0004	0.0680	0.0048
θ	0.0003	0.0397	0.0016	-0.0008	0.0326	0.0014	0.0003	0.0366	0.0015
ϕ	0.0032	0.0322	0.0015	0.0016	0.0243	0.0009	0.0015	0.0122	0.0004
τ	-0.0131	0.1127	0.0231	-0.0090	0.0637	0.0066	-0.0049	0.0618	0.0055
γ	-0.0017	0.0753	0.0057	-0.0046	0.0610	0.0054	-0.0020	0.0692	0.0053
σ_u^2	0.0034	0.0833	0.0031	0.0008	0.0731	0.0021	-0.0004	0.0772	0.0021
σ_v^2	-0.0010	0.0052	2.71e-05	-0.0008	0.0053	2.83e-05	-0.0009	0.0053	2.70e-05

Table B1– continued from previous page

τ	**0.15**			**0.65**			**0.90**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	0.0006	0.0115	0.0001	0.0004	0.0113	0.0001	0.0005	0.0098	0.0001
ρ	-0.0038	0.0688	0.0054	-0.0017	0.0675	0.0046	-0.0007	0.0492	0.0028
θ	0.0008	0.0400	0.0017	-0.0002	0.0391	0.0014	0.0005	0.0292	0.0010
ϕ	0.0041	0.0327	0.0017	0.0036	0.0359	0.0017	0.0004	0.0287	0.0012
τ	0.0020	0.0934	0.0106	-0.0029	0.0534	0.0031	-0.0010	0.0117	0.0001
γ	0.0014	0.0735	0.0059	-0.0005	0.0715	0.0051	0.0002	0.0539	0.0034
σ_u^2	-0.0004	0.0721	0.0022	0.0009	0.0671	0.0024	0.0018	0.0629	0.0023
σ_v^2	-0.0012	0.0052	2.75e-05	-0.0011	0.0052	2.72e-05	-0.0010	0.0053	2.70e-05

γ	**0.15**			**0.65**			**0.90**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	0.0005	0.0119	0.0001	-0.0006	0.0109	0.0001	-0.0006	0.0107	0.0001
ρ	-0.0068	0.0765	0.0051	0.0068	0.0746	0.0050	0.0082	0.0465	0.0019
θ	0.0033	0.0467	0.0017	-0.0028	0.0355	0.0013	-0.0031	0.0258	0.0006
ϕ	0.0024	0.0317	0.0013	0.0029	0.0412	0.0020	0.0027	0.0433	0.0028
τ	0.0014	0.1065	0.0087	-0.0130	0.0886	0.0097	-0.0114	0.0922	0.0099
γ	0.0057	0.0857	0.0060	-0.0090	0.0579	0.0038	-0.0065	0.0262	0.0006
σ_u^2	-0.0001	0.0550	0.0023	3.97e-05	0.0565	0.0024	0.0016	0.0688	0.0027
σ_v^2	-0.0008	0.0057	2.94e-05	-0.0005	0.0061	3.30e-05	0.0001	0.0058	3.11e-05

σ_u^2, σ_v^2	**0.80, 0.10**			**0.10, 0.80**			**0.50,0.50**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	0.0002	0.0071	0.0001	0.0005	0.0182	0.0004	0.0005	0.0145	0.0003
ρ	-0.0034	0.0350	0.0024	-0.0094	0.0665	0.0144	-0.0093	0.0550	0.0102
θ	0.0014	0.0220	0.0007	0.0033	0.0477	0.0045	0.0036	0.0390	0.0031
ϕ	0.0011	0.0079	0.0001	0.0113	0.0593	0.0068	0.0045	0.0219	0.0008
τ	0.0019	0.0444	0.0027	-0.0072	0.1420	0.0322	0.0016	0.0823	0.0125
γ	0.0024	0.0443	0.0028	0.0043	0.0773	0.0143	0.0056	0.0652	0.0102
σ_u^2	-0.0153	0.4547	0.1213	-0.0014	0.0656	0.0031	-0.0168	0.2773	0.0502
σ_v^2	-0.0004	0.0027	7.29e-06	-0.0077	0.0240	0.0005	-0.0038	0.0144	0.0002

Table B2: Simulations Results: Sensitivity to the choice of W

	W W W0 W0			**W0 W0 W W**			**W0 W W0 W**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0002	0.0115	0.0001	-0.0001	0.0126	0.0002	-0.0001	0.0125	0.0002
ρ	-0.0040	0.0575	0.0038	-0.0003	0.0286	0.0009	-0.0004	0.0265	0.0007
θ	0.0027	0.0636	0.0042	-0.0002	0.0275	0.0008	0.0004	0.0586	0.0030
ϕ	0.0066	0.0471	0.0059	-0.0009	0.0781	0.0089	0.0020	0.0403	0.0036
τ	-0.0003	0.0658	0.0126	-0.0310	0.0930	0.0348	-0.0037	0.0742	0.0095
γ	0.0007	0.0332	0.0011	-0.0066	0.0753	0.0055	-0.0065	0.0756	0.0056
σ_u^2	0.0074	0.1130	0.0058	0.0101	0.0854	0.0070	0.0010	0.1419	0.0046
σ_v^2	-0.0007	0.0074	0.0001	-0.0007	0.0077	0.0001	-0.0007	0.0077	0.0001

	W W0 W W0			**W0 W W W0**			**W W0 W0 W**		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0003	0.0125	0.0002	-0.0004	0.0122	0.0001	-0.0003	0.0122	0.0001
ρ	-0.0046	0.0560	0.0032	-0.0012	0.0384	0.0016	-0.0038	0.0718	0.0065
θ	-0.0003	0.0231	0.0005	0.0002	0.0546	0.0026	-0.0005	0.0225	0.0005
ϕ	-0.0010	0.0749	0.0083	-0.0017	0.0756	0.0082	0.0026	0.0616	0.0049
τ	-0.0296	0.1112	0.0374	-0.0302	0.0918	0.0334	-0.0045	0.0733	0.0090
γ	-0.0004	0.0337	0.0012	0.0003	0.0490	0.0028	-0.0073	0.1163	0.0124
σ_u^2	0.0095	0.0879	0.0063	0.0099	0.0925	0.0064	0.0029	0.1049	0.0052
σ_v^2	-0.0007	0.0077	0.0001	-0.0008	0.0077	0.0001	-0.0008	0.0076	0.0001

	W W W W0			**W0 W0 W0 W**			**W0 W0 W W0**		
	Bias	SD	MSE	Bias	SD	MSE	Bias	SD	MSE
β	0.0001	0.0140	0.0002	-0.0004	0.0126	0.0001	-0.0001	0.0116	0.0001
ρ	-0.0030	0.0580	0.0044	0.0013	0.0286	0.0009	-0.0056	0.0685	0.0045
θ	0.0008	0.0359	0.0016	-0.0012	0.0275	0.0007	0.0056	0.0739	0.0049
ϕ	-0.0016	0.0765	0.0085	0.0022	0.0781	0.0030	-0.0050	0.1017	0.0059
τ	-0.0313	0.0923	0.0345	-0.0067	0.0930	0.0097	-0.0309	0.2600	0.0337
γ	0.0009	0.0633	0.0055	-0.0033	0.0753	0.0050	-0.0007	0.0332	0.0011
σ_u^2	0.0102	0.0934	0.0065	0.0014	0.0854	0.0050	0.0080	0.0686	0.0062
σ_v^2	-0.0011	0.0077	0.0001	-0.0011	0.0077	0.0001	-0.0011	0.0076	0.0001

Table B2– continued from previous page

	W W W0 W			W W0 W W			W0 W W0 W0		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	0.0001	0.0124	0.0002	0.0003	0.0117	0.0001	0.0000	0.0114	0.0001
ρ	-0.0126	0.0805	0.0134	-0.0004	0.0831	0.0076	0.0012	0.0423	0.0018
θ	0.0051	0.0769	0.0070	-0.0002	0.0213	0.0004	-0.0016	0.0466	0.0022
ϕ	0.0012	0.0672	0.0062	0.0033	0.0971	0.0064	0.0006	0.0296	0.0044
τ	-0.0050	0.0745	0.0093	-0.0282	0.2432	0.0388	-0.0102	0.0613	0.0137
γ	-0.0020	0.1233	0.0188	-0.0151	0.1284	0.0135	-0.0041	0.0529	0.0029
σ_u^2	0.0058	0.1004	0.0057	0.0060	0.0427	0.0056	0.0028	0.1769	0.0051
σ_v^2	-0.0009	0.0076	0.0001	-0.0012	0.0074	0.0001	-0.0010	0.0073	0.0001

	W W0 W0 W0			W0 W W W		
	Bias	*SD*	*MSE*	*Bias*	*SD*	*MSE*
β	-0.0004	0.0116	0.0001	-0.0004	0.0114	0.0001
ρ	-0.0034	0.0468	0.0023	0.0008	0.0219	0.0005
θ	-0.0006	0.0199	0.0004	-0.0006	0.0505	0.0029
ϕ	0.0013	0.0515	0.0042	0.0021	0.0375	0.0075
τ	-0.0061	0.0881	0.0092	-0.0237	0.0660	0.0285
γ	0.0015	0.0328	0.0011	-0.0053	0.0747	0.0051
σ_u^2	0.0030	0.0672	0.0054	0.0081	0.2263	0.0067
σ_v^2	-0.0011	0.0072	0.0001	-0.0010	0.0073	0.0001

NB. In the title of each simulation we report the different spatial weight matrices respectively associated with the four different spatial lags of the SDF-CSD model in the following order: spatial lag of Y, spatial lag of X, spatial lag of u, spatial lag of v. In particular, W refers to a dense inverse distance matrix while $W0$ refers to a binary contiguity matrix.

Appendix C: Nested Models Estimation Results

Table C1 contains the estimation results of the SDF-CSD model presented in subsection 4.2 of the paper and of all the nested specifications. Moreover, Table C2 shows some likelihood ratio tests and information criteria in order to choose the model specification that better fits the data.

In Tables C1 and C2, the SDF-CSD, containing the spatial lags related to the dependent variable, the input variables, the inefficiency component and the random error term, is labelled yxuv model, while the nested specifications are defined indicating the specific spatial terms that they include. In particular, the u=v model indicates the spatial error model (SEM), in which the overall error term ε follows a unique spatial process without distinguishing between the inefficiency and the noise term. Accordingly, the yxu=v model represents the general Manski model introducing the spatial lag of Y, X and ε, while the yu=v and the xu=v model are the two nested specifications also known as SARAR and SDEM model, respectively. Other than reducing to existing spatial models, the SDF-CSD model leads the way to a number of spatial specifications (such as the yuv model, the yxv model, the yxu model, etc..) never introduced before but that can be very useful in empirical applications.

Table C1: Nested Models Estimation Results

Lag	yxuv		uv		u		v		yx		y		x	
	Coeff.	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*
β_0	3.87***	0.53	4.25***	0.62	4.57***	0.28	3.69***	0.18	3.10***	0.49	3.07***	0.38	4.49***	0.41
β_L	0.61***	0.02	0.63***	0.02	0.64***	0.02	0.61***	0.02	0.62***	0.02	0.61***	0.02	0.61***	0.02
β_{AA}	0.13***	0.02	0.13***	0.02	0.10***	0.02	0.14***	0.02	0.14***	0.02	0.09***	0.02	0.13***	0.02
β_M	0.07***	0.01	0.07***	0.01	0.08***	0.01	0.07***	0.01	0.07***	0.01	0.10***	0.01	0.07***	0.02
β_{WEF}	0.20***	0.01	0.19***	0.01	0.19***	0.01	0.20***	0.01	0.20***	0.01	0.20***	0.01	0.20***	0.01
β_t	-0.01	0.01	-0.01	0.02	-0.03	0.02	-0.01	0.01	-0.01	0.01	-0.01	0.01	-0.02*	0.01
ρ	0.14*	0.08	-		-		-		0.38***	0.07	0.06***	0.02	-	
θ_L	-0.06	0.08	-		-		-		-0.25***	0.07	-		-0.03	0.06
θ_{AA}	-0.25***	0.05	-		-		-		-0.27***	0.05	-		-0.33***	0.04
θ_M	0.02	0.04	-		-		-		0.01	0.05	-		0.05	0.04
θ_{WEF}	0.02	0.05	-		-		-		-0.02	0.07	-		0.13***	0.04
ϕ_S	0.70***	0.18	0.67***	0.17	0.37***	0.11	0.86***	0.22	0.73***	0.19	0.40***	0.11	0.49***	0.14
ϕ_B	-1.72***	0.46	-1.65***	0.41	-1.47***	0.31	-1.70***	0.47	-1.71***	0.59	-1.56***	0.33	-1.54***	0.43
ϕ_F	0.22**	0.11	0.20**	0.1	0.14**	0.07	0.28**	0.13	0.22	0.14	0.13*	0.07	0.23**	0.09
ϕ_D	0.37**	0.15	0.24*	0.14	0.19*	0.11	0.35**	0.17	0.40**	0.16	0.08	0.11	0.31**	0.12
ϕ_H	-0.06	0.11	-0.07	0.1	-0.14*	0.07	-0.01	0.13	-0.03	0.12	-0.17**	0.08	-0.06	0.09
ϕ_{Sub}	0.71***	0.1	0.69***	0.09	0.55***	0.07	0.82***	0.12	0.72***	0.13	0.59***	0.07	0.64***	0.09
ϕ_Y	0.66***	0.2	0.67***	0.19	0.71***	0.15	0.55***	0.22	0.69***	0.23	0.70***	0.15	0.68***	0.17
ϕ_W	0.55***	0.18	0.58***	0.17	0.66***	0.14	0.37*	0.21	0.52**	0.19	0.65***	0.14	0.57***	0.16
τ	0.37***	0.12	0.58***	0.08	0.57***	0.05	-		-		-		-	
γ	0.37***	0.07	0.52***	0.05	-		0.63***	0.04	-		-		-	
σ_u^2	0.05		0.06		0.12		0.03		0.04		0.16		0.08	
σ_v^2	0.09		0.09		0.10		0.09		0.09		0.10		0.10	

Lag	yuv		xuv		yxu		yxv		yu		yv		xu	
	Coeff.	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*
β_0	5.09***	0.64	4.37***	0.59	3.33***	0.46	3.27***	0.49	4.97***	0.52	3.77***	0.55	4.98***	0.61
β_L	0.63***	0.02	0.61***	0.02	0.62***	0.02	0.61***	0.02	0.65***	0.02	0.61***	0.02	0.62***	0.02
β_{AA}	0.13***	0.02	0.13***	0.02	0.14***	0.02	0.14***	0.02	0.10***	0.02	0.14***	0.02	0.13***	0.02
β_M	0.07***	0.01	0.07***	0.01	0.07***	0.01	0.07***	0.01	0.08***	0.01	0.07***	0.01	0.07***	0.01
β_{WEF}	0.19***	0.01	0.20***	0.01	0.20***	0.01	0.20***	0.01	0.18***	0.01	0.20***	0.01	0.20***	0.01
β_t	-0.01	0.02	-0.01	0.02	-0.01	0.01	-0.01	0.01	-0.04**	0.02	-0.01	0.01	-0.02	0.02
ρ	-0.05	0.03	-		0.35***	0.06	0.21***	0.07	-0.02	0.03	0.00	0.03	-	
θ_L	-		0.04	0.09	-0.23***	0.06	-0.12	0.08	-		-		-0.01	0.09
θ_{AA}	-		-0.24***	0.05	-0.26***	0.04	-0.27***	0.05	-		-		-0.28***	0.04
θ_M	-		0.03	0.04	0.01	0.04	0.02	0.04	-		-		0.05	0.04
θ_{WEF}	-		0.05	0.05	-0.03	0.05	0.04	0.05	-		-		0.05	0.05
ϕ_S	0.65***	0.17	0.66***	0.17	0.71***	0.18	0.74***	0.19	0.36***	0.11	0.86***	0.22	0.49***	0.14
ϕ_B	-1.70***	0.42	-1.72***	0.44	-1.72***	0.49	-1.72***	0.46	-1.49***	0.31	-1.70***	0.47	-1.60***	0.45
ϕ_F	0.21**	0.10	0.21*	0.11	0.18	0.12	0.28**	0.13	0.14**	0.07	0.28**	0.13	0.13	0.09
ϕ_D	0.27**	0.14	0.35	0.14	0.40***	0.15	0.39***	0.16	0.20*	0.11	0.35*	0.17	0.36***	0.12
ϕ_H	-0.06	0.10	-0.07***	0.10	-0.04	0.11	-0.06	0.12	-0.14**	0.07	-0.01	0.13	-0.06	0.08
ϕ_{Sub}	0.69***	0.09	0.70***	0.09	0.71***	0.10	0.74***	0.10	0.55***	0.07	0.82***	0.12	0.64***	0.1
ϕ_Y	0.68***	0.19	0.65***	0.19	0.71***	0.22	0.63***	0.21	0.72***	0.15	0.55***	0.22	0.72***	0.18
ϕ_W	0.56***	0.17	0.56***	0.18	0.55***	0.19	0.48***	0.20	0.64***	0.14	0.37*	0.21	0.64***	0.17
τ	0.62***	0.07	0.47***	0.12	0.18	0.15	-		0.59***	0.05	-		0.44***	0.11
γ	0.52***	0.05	0.43***	0.07	-		0.35***	0.08	-		0.63***	0.04	-	
σ_u^2	0.05		0.05		0.05		0.04		0.12		0.03		0.08	
σ_v^2	0.09		0.09		0.09		0.09		0.10		0.09		0.09	

Lag	xv		u=v		yxu=v		yu=v		xu=v		non-spat.			
	Coeff.	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*		
β_0	3.74***	0.49	4.17***	0.29	3.86***	0.53	4.82***	0.69	4.30***	0.59	3.90***	0.19		
β_L	0.61***	0.02	0.63***	0.02	0.61***	0.02	0.62***	0.02	0.61***	0.02	0.62***	0.02		
β_{AA}	0.13***	0.02	0.13***	0.02	0.13***	0.02	0.13***	0.02	0.13***	0.02	0.09***	0.02		
β_M	0.07***	0.01	0.07**	0.01	0.07***	0.01	0.07***	0.01	0.07***	0.01	0.10***	0.01		
β_{WEF}	0.21**	0.01	0.19***	0.01	0.20***	0.01	0.19***	0.01	0.20***	0.01	0.20***	0.01		
β_t	-0.01	0.01	-0.01	0.02	-0.01	0.01	-0.01	0.02	-0.01	0.01	-0.02**	0.01		
ρ	-		-		0.14*	0.08	-0.04	0.03	-		-			
θ_L	0.04	0.09	-		-0.06	0.08	-		0.04	0.09	-			
θ_{AA}	-0.29***	0.05	-		-0.25***	0.05	-		-0.24***	0.05	-			
θ_M	0.03	0.04	-		0.02	0.04	-		0.03	0.04	-			
θ_{WEF}	0.11**	0.05	-		0.02	0.05	-		0.05	0.05	-			
ϕ_S	0.70***	0.19	0.68***	0.17	0.70***	0.18	0.67***	0.17	0.66***	0.17	0.37***	0.11		
ϕ_B	-1.74***	0.48	-1.66***	0.42	-1.72***	0.46	-1.70***	0.42	-1.72***	0.44	-1.57***	0.32		
ϕ_F	0.31**	0.13	0.20**	0.10	0.22**	0.11	0.21**	0.10	0.22**	0.11	0.13*	0.07		
ϕ_D	0.38**	0.16	0.24*	0.14	0.37***	0.15	0.26*	0.14	0.35***	0.14	0.08	0.11		
ϕ_H	-0.08	0.12	-0.06	0.10	-0.06	0.11	-0.06	0.10	-0.07	0.1	-0.17**	0.08		
ϕ_{Sub}	0.74***	0.11	0.70***	0.09	0.71***	0.10	0.71***	0.09	0.70***	0.1	0.59***	0.07		
ϕ_Y	0.61***	0.20	0.66***	0.19	0.66***	0.20	0.66***	0.19	0.65***	0.19	0.71***	0.15		
ϕ_W	0.47**	0.19	0.56***	0.17	0.55***	0.18	0.54***	0.17	0.56***	0.18	0.63***	0.14		
τ	-		0.54***	0.09	0.37***	0.12	0.55***	0.10	0.44***	0.13	-			
γ	0.48***	0.08									-			
σ_u^2	0.04		0.05		0.05		0.05		0.05		0.17			
σ_v^2	0.09		0.09		0.09		0.09		0.09		0.11			

$***: pvalue \le 0.01$; $**: pvalue \le 0.05$; $*: pvalue \le 0.10$

Table C2: Model Comparison

Model	LL	Test Statistic	Constraints	P-value	Parameters	AIC	BIC
SDF-CSD	-284.61	Base Model	-	-	23	615.22	731.85
uv	-297.70	26.18	5	0.00	18	631.40	722.67
u	-340.48	111.74	6	0.00	17	714.96	801.16
v	-305.14	41.06	6	0.00	17	644.28	730.48
yx	-296.20	23.18	2	0.00	21	634.40	740.89
y	-366.48	163.74	6	0.00	17	766.96	853.16
x	-319.66	70.10	3	0.00	20	679.32	780.73
yuv	-296.53	23.84	4	0.00	19	631.06	727.40
xuv	-285.73	2.24	1	0.09	22	615.46	727.02
yxu	-295.56	21.90	1	0.00	22	635.12	746.68
yxv	-286.70	4.18	1	0.02	22	617.40	728.96
yu	-340.02	110.82	5	0.00	18	716.04	807.31
yv	-305.13	41.04	5	0.00	18	646.26	737.53
xu	-311.35	53.48	2	0.00	21	664.70	771.19
xv	-290.09	10.96	2	0.00	21	622.18	728.67
u=v	-297.89	26.56	6	0.00	17	629.78	715.98
yxu=v	-284.61	0.00	1	6.30	22	613.22	724.78
yu=v	-297.05	24.88	5	0.00	18	630.10	721.37
xu=v	-285.77	2.32	2	0.16	21	613.54	720.03
non-spatial	-369.50	169.78	7	0.00	16	771.00	852.13

$***: pvalue \leq 0.01$; $**: pvalue \leq 0.05$; $*: pvalue \leq 0.10$

Appendix D: Efficiency Analysis

As observed by Glass, Kenjegalieva, and Sickles (2016), multiplying the TE scores for the spatial lag of the dependent variable, it is possible to obtain total technical efficiency scores $TE_{it}^{Tot} = (I_N - \rho W)^{-1} TE_{it}$, including both direct and indirect effects. Besides computing relative direct, indirect, and total efficiency scores with respect to the best performing unit in the sample as proposed by the authors, we follow the method introduced by LeSage and Pace (2009) to compute the marginal effects to disentangle the direct, indirect, and total technical efficiency scores. Specifically starting from the matrix $\epsilon = (I_N - \rho W)^{-1} (I_N \cdot TE_{it})$, direct TE scores ($TE_{it}^{Dir}$) corresponds to the elements on the main diagonal of ϵ, indirect TE scores (TE_{it}^{Ind}) are the row sums of the non-diagonal elements, and the total TE scores previously defined equals $TE_{it}^{Tot} = TE_{it}^{Dir} + TE_{it}^{Ind}$.

Figure C1 shows the distribution of direct, indirect, and total TE scores in 2018. It can be observed that the distribution of TE_{it}^{Dir} highly reflects the one of TE_{it}, while the distribution of TE_{it}^{Ind} takes smaller values and is much more concentrated around the mean value. Specifically, the average direct TE score (ADTE) equals 0.50, the average indirect TE score (AITE) equals 0.08, and the average total TE score (ATTE) equals 0.58. Thus, efficiency mostly depends on internal and controllable factors and on average, the 13.79% of provinces' overall efficiency level comes from positive spillover effects resulting from neighbours.

Table C1 presents the efficiency ranking of the 107 Italian provinces where it can be observed more in detail how both the ranking and the level of the TE scores modify depending on the kind of spatial structures considered. Table C2 shows how the efficiency score ranking changes in time comparing the 2008 and 2018 results. The province of Catanzaro reaches a dramatically low score in 2018 due to the structural weakness of the Calabrian agro-industrial sector depending on the small size of the farms, the fragmentation of the offer and the obsoleteness of the types of machinery and techniques used by farmers.

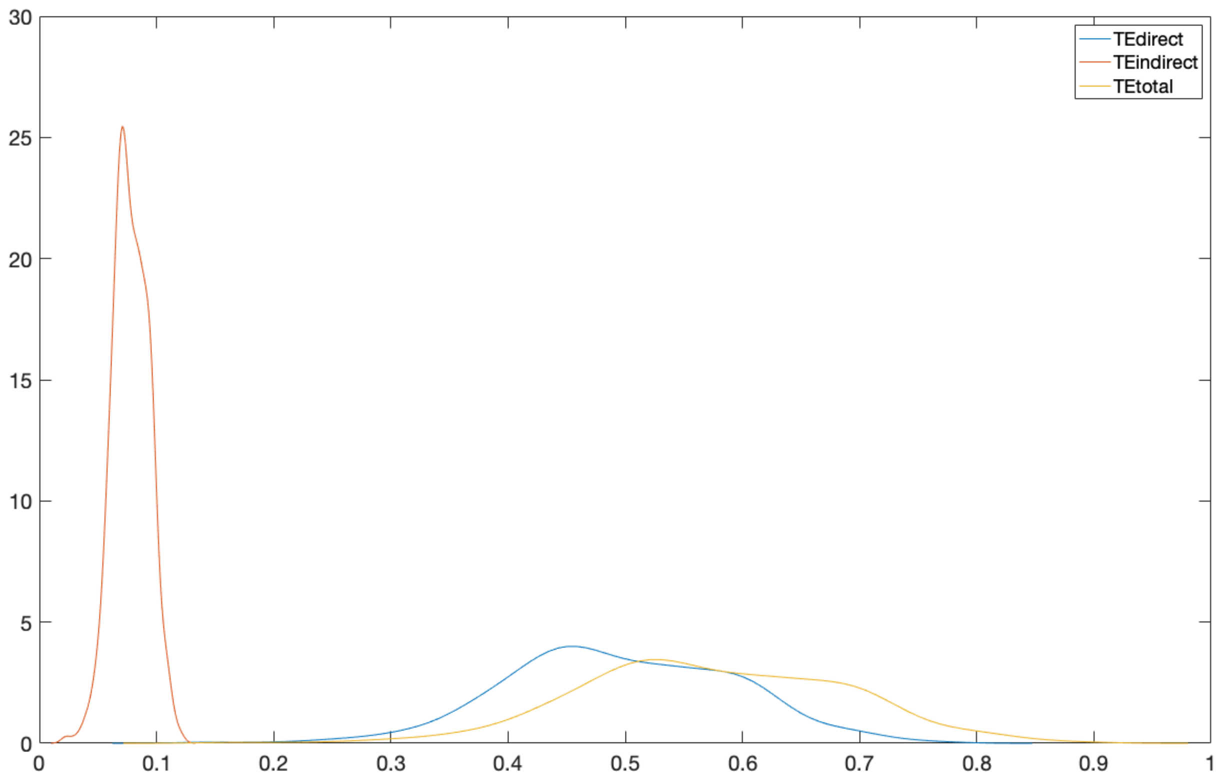

Figure D1. Direct, Indirect, and Total TE Scores: Kernel Density Plot

Table D1: Model Comparison: Efficiency Scores Ranking, Year 2018

	SDF-CSD		uv-model		yx-model		non-spatial model	
Ranking	**Provinces**	**TE**	**Provinces**	**TE**	**Provinces**	**TE**	**Provinces**	**TE**
1	Cremona	0.68	Rovigo	0.49	Cremona	0.87	Cremona	0.79
2	Rovigo	0.66	Cremona	0.48	Pavia	0.82	Lodi	0.73
3	Brescia	0.65	Pordenone	0.48	Brescia	0.81	Pavia	0.72
4	Parma	0.64	Brescia	0.47	Lodi	0.81	Brescia	0.71
5	Verona	0.63	Verona	0.46	Parma	0.8	Messina	0.69
6	Pordenone	0.63	Venezia	0.46	Rovigo	0.8	Pistoia	0.69
7	Lodi	0.63	Treviso	0.46	Pistoia	0.79	Parma	0.68
8	Pavia	0.62	Padova	0.46	Messina	0.79	Verbano-Cusio-Ossola	0.68
9	Pistoia	0.62	Parma	0.46	Reggio Emilia	0.78	Bergamo	0.68
10	Venezia	0.62	Lodi	0.45	Bergamo	0.78	Rovigo	0.68
11	Reggio Emilia	0.62	Gorizia	0.45	Verona	0.78	Reggio Emilia	0.67
12	Mantova	0.62	Trento	0.45	Prato	0.77	Pordenone	0.67
13	Treviso	0.62	Vicenza	0.45	Mantova	0.77	Ravenna	0.66
14	Padova	0.61	Mantova	0.44	Pordenone	0.77	Verona	0.66
15	Bergamo	0.61	Reggio Emilia	0.44	L'Aquila	0.77	Mantova	0.66
16	Trento	0.61	Pistoia	0.44	Ravenna	0.76	Gorizia	0.65
17	Ravenna	0.6	Bergamo	0.44	Venezia	0.76	Forlì-Cesena	0.64
18	Vicenza	0.6	Pavia	0.43	Trento	0.76	L'Aquila	0.64
19	Prato	0.6	Ravenna	0.43	Forlì-Cesena	0.76	Trento	0.64
20	Gorizia	0.6	Modena	0.43	Treviso	0.75	Padova	0.64
21	Modena	0.59	Udine	0.42	Piacenza	0.75	Venezia	0.63
22	Forlì-Cesena	0.59	Piacenza	0.41	Teramo	0.75	Piacenza	0.63
23	Piacenza	0.59	Forlì-Cesena	0.41	Padova	0.74	Vicenza	0.63
24	Messina	0.59	Prato	0.41	Gorizia	0.74	Treviso	0.62
25	L'Aquila	0.58	Bologna	0.39	Verbano-Cusio-Ossola	0.74	Cuneo	0.62
26	Teramo	0.56	Bolzano	0.39	Vicenza	0.74	Teramo	0.62
27	Verbano-Cusio-Ossola	0.55	Belluno	0.39	Modena	0.73	Modena	0.61
28	Latina	0.55	L'Aquila	0.39	Latina	0.73	Rimini	0.6
29	Udine	0.55	Verbano-Cusio-Ossola	0.38	Catania	0.72	Vercelli	0.58
30	Bologna	0.55	Messina	0.38	Cuneo	0.72	Caserta	0.58
31	Rimini	0.55	Rimini	0.38	Caserta	0.72	Viterbo	0.58
32	Caserta	0.55	Teramo	0.37	Rimini	0.72	Bologna	0.57
33	Bolzano	0.54	Lucca	0.37	Matera	0.71	Latina	0.57
34	Catania	0.53	Latina	0.37	Viterbo	0.71	Udine	0.57
35	Vercelli	0.53	Vercelli	0.36	Vercelli	0.7	Catania	0.57
36	Viterbo	0.53	Alessandria	0.36	Bologna	0.69	Alessandria	0.56
37	Cuneo	0.53	Caserta	0.36	Napoli	0.69	Fermo	0.55
38	Alessandria	0.53	Cuneo	0.36	Alessandria	0.69	Matera	0.55
39	Savona	0.52	Savona	0.36	Savona	0.68	Lucca	0.55
40	Lucca	0.52	Viterbo	0.35	Bolzano	0.68	Milano	0.55
41	Napoli	0.52	Trieste	0.35	Udine	0.67	Savona	0.55
42	Belluno	0.52	Catania	0.34	Fermo	0.67	Torino	0.55
43	Fermo	0.5	Milano	0.34	Barletta-Andria-Trani	0.67	Napoli	0.54
44	Torino	0.5	Ferrara	0.34	Lucca	0.66	Bolzano	0.54
45	Caltanissetta	0.49	Torino	0.34	Caltanissetta	0.66	Macerata	0.53
46	Roma	0.49	Napoli	0.34	Torino	0.66	Belluno	0.52
47	Matera	0.49	Fermo	0.33	Trapani	0.65	Prato	0.52
48	Barletta-Andria-Trani	0.49	Firenze	0.33	Oristano	0.65	Avellino	0.52
49	Campobasso	0.48	Asti	0.33	Roma	0.65	Arezzo	0.51
50	Benevento	0.48	Arezzo	0.33	Avellino	0.64	Caltanissetta	0.51
51	Asti	0.48	Roma	0.32	Campobasso	0.64	Barletta-Andria-Trani	0.51
52	Milano	0.48	Caltanissetta	0.32	Asti	0.64	Campobasso	0.51
53	Trapani	0.48	Macerata	0.32	Macerata	0.64	Benevento	0.51
54	Macerata	0.48	Massa-Carrara	0.32	Benevento	0.64	Asti	0.51
55	Avellino	0.48	Campobasso	0.32	Ragusa	0.64	Oristano	0.5
56	Arezzo	0.48	Perugia	0.31	Belluno	0.63	Roma	0.5
57	Isernia	0.47	Monza e della Brianza	0.31	Bari	0.63	Perugia	0.49
58	Ragusa	0.47	Lecco	0.31	Ascoli Piceno	0.63	Nuoro	0.49
59	Ascoli Piceno	0.47	Isernia	0.31	Arezzo	0.62	Firenze	0.49
60	Oristano	0.47	Pesaro Urbino	0.31	Isernia	0.62	Pesaro Urbino	0.49
61	Perugia	0.47	Ascoli Piceno	0.31	Pesaro Urbino	0.62	Ragusa	0.49
62	Ferrara	0.47	Benevento	0.31	Reggio Calabria	0.62	Trapani	0.49
63	Firenze	0.47	Siena	0.31	Perugia	0.62	Isernia	0.49
64	Pesaro Urbino	0.46	Trapani	0.31	Sud Sardegna	0.61	Ascoli Piceno	0.48
65	Trieste	0.46	La Spezia	0.31	Nuoro	0.61	Bari	0.48
66	Chieti	0.45	Barletta-Andria-Trani	0.31	Milano	0.61	Massa-Carrara	0.47
67	Salerno	0.45	Ragusa	0.3	Salerno	0.6	Salerno	0.47
68	Bari	0.45	Avellino	0.3	Firenze	0.6	Siena	0.47
69	Nuoro	0.45	Chieti	0.3	Chieti	0.6	Biella	0.47
70	Siena	0.44	Livorno	0.3	Agrigento	0.59	Monza e della Brianza	0.47
71	Massa-Carrara	0.44	Imperia	0.3	Imperia	0.59	Chieti	0.47
72	Agrigento	0.44	Terni	0.29	Terni	0.59	Trieste	0.47
73	Sud Sardegna	0.44	Oristano	0.29	Potenza	0.58	Livorno	0.46
74	Imperia	0.44	Biella	0.29	Siena	0.58	Imperia	0.46
75	Lecco	0.44	Agrigento	0.29	Siracusa	0.58	Reggio Calabria	0.46
76	Terni	0.44	Ancona	0.29	Livorno	0.58	Ancona	0.46
77	La Spezia	0.44	Matera	0.29	Monza e della Brianza	0.57	Terni	0.46
78	Monza e della Brianza	0.44	Pisa	0.29	Ferrara	0.57	Como	0.45
79	Siracusa	0.44	Salerno	0.29	Foggia	0.57	Ferrara	0.45
80	Livorno	0.43	Como	0.29	Massa-Carrara	0.57	Agrigento	0.45
81	Ancona	0.42	Siracusa	0.28	Trieste	0.56	Siracusa	0.44
82	Biella	0.42	Varese	0.28	Ancona	0.56	Sud Sardegna	0.44
83	Pescara	0.42	Nuoro	0.28	Lecco	0.56	Potenza	0.44
84	Foggia	0.42	Pescara	0.28	Pescara	0.56	Cagliari	0.43
85	Frosinone	0.41	Bari	0.28	La Spezia	0.56	Lecco	0.43
86	Potenza	0.41	Sud Sardegna	0.28	Biella	0.55	Pescara	0.43
87	Pisa	0.41	Frosinone	0.27	Sassari	0.55	Foggia	0.43
88	Sassari	0.4	Foggia	0.27	Cagliari	0.53	La Spezia	0.43
89	Cagliari	0.39	Novara	0.26	Frosinone	0.53	Pisa	0.43
90	Palermo	0.39	Grosseto	0.26	Pisa	0.52	Varese	0.4
91	Varese	0.38	Sassari	0.26	Taranto	0.52	Frosinone	0.4
92	Como	0.38	Palermo	0.26	Palermo	0.52	Sassari	0.4
93	Grosseto	0.38	Cagliari	0.26	Lecce	0.51	Palermo	0.39
94	Taranto	0.37	Potenza	0.25	Brindisi	0.51	Grosseto	0.39
95	Brindisi	0.37	Sondrio	0.25	Grosseto	0.51	Taranto	0.39
96	Novara	0.37	Rieti	0.24	Cosenza	0.51	Novara	0.39
97	Lecce	0.37	Aosta	0.24	Como	0.49	Sondrio	0.37
98	Rieti	0.36	Taranto	0.23	Rieti	0.48	Lecce	0.36
99	Reggio Calabria	0.36	Lecce	0.22	Varese	0.48	Cosenza	0.36
100	Cosenza	0.35	Brindisi	0.22	Novara	0.47	Aosta	0.35
101	Aosta	0.34	Enna	0.22	Enna	0.44	Rieti	0.35
102	Sondrio	0.34	Cosenza	0.2	Aosta	0.43	Brindisi	0.32
103	Enna	0.33	Genova	0.2	Sondrio	0.42	Enna	0.32
104	Genova	0.27	Reggio Calabria	0.18		0.4	Genova	0.29
105	Crotone	0.26	Crotone	0.14	Vibo Valentia	0.39	Vibo Valentia	0.27
106	Vibo Valentia	0.24	Vibo Valentia	0.12	Genova	0.34	Crotone	0.25
107	Catanzaro	0.14	Catanzaro	0.08	Catanzaro	0.21	Catanzaro	0.13

Table D2: Efficiency Scores Ranking: Time Comparison

Ranking	2008 Provinces	TE	2018 Provinces	TE
1	Belluno	0.76	Cremona	0.68
2	Venezia	0.74	Rovigo	0.66
3	Milano	0.73	Brescia	0.65
4	Rovigo	0.72	Parma	0.64
5	Ravenna	0.71	Verona	0.63
6	Lodi	0.71	Pordenone	0.63
7	Catania	0.70	Lodi	0.63
8	Verona	0.70	Pavia	0.62
9	Padova	0.69	Pistoia	0.62
10	Siracusa	0.69	Venezia	0.62
11	Vicenza	0.69	Reggio Emilia	0.62
12	Mantova	0.69	Mantova	0.62
13	Gorizia	0.69	Treviso	0.62
14	Cremona	0.69	Padova	0.61
15	Ferrara	0.68	Bergamo	0.61
16	Treviso	0.68	Trento	0.61
17	Bologna	0.67	Ravenna	0.60
18	Parma	0.67	Vicenza	0.60
19	Pordenone	0.66	Prato	0.60
20	Caserta	0.65	Gorizia	0.60
21	Siena	0.65	Modena	0.59
22	Modena	0.65	Forlì-Cesena	0.59
23	Forlì-Cesena	0.65	Piacenza	0.59
24	Brindisi	0.65	Messina	0.59
25	Ragusa	0.64	L'Aquila	0.58
26	Pistoia	0.64	Teramo	0.56
27	Lecco	0.63	Verbano-Cusio-Ossola	0.55
28	Alessandria	0.62	Latina	0.55
29	Vercelli	0.62	Udine	0.55
30	Taranto	0.62	Bologna	0.55
31	Reggio Emilia	0.62	Rimini	0.55
32	Firenze	0.61	Caserta	0.55
33	Crotone	0.61	Bolzano	0.54
34	Napoli	0.61	Catania	0.53
35	Arezzo	0.60	Vercelli	0.53
36	Pesaro Urbino	0.60	Viterbo	0.53
37	Roma	0.60	Cuneo	0.53
38	Udine	0.60	Alessandria	0.53
39	Rimini	0.60	Savona	0.52
40	L'Aquila	0.60	Lucca	0.52
41	Livorno	0.59	Napoli	0.52
42	Trento	0.59	Belluno	0.52
43	Vibo Valentia	0.59	Fermo	0.50
44	Bergamo	0.59	Torino	0.50
45	Varese	0.59	Caltanissetta	0.49
46	Palermo	0.59	Roma	0.49
47	Bolzano	0.59	Matera	0.49
48	Cuneo	0.58	Barletta-Andria-Trani	0.49
49	Avellino	0.58	Campobasso	0.48
50	Piacenza	0.58	Benevento	0.48
51	Pisa	0.58	Asti	0.48
52	Brescia	0.58	Milano	0.48
53	Teramo	0.58	Trapani	0.48
54	Cosenza	0.58	Macerata	0.48
55	Oristano	0.57	Avellino	0.48
56	Salerno	0.57	Arezzo	0.48
57	Sud Sardegna	0.57	Isernia	0.47
58	Como	0.57	Ragusa	0.47
59	Reggio Calabria	0.57	Ascoli Piceno	0.47
60	Bari	0.56	Oristano	0.47
61	Torino	0.56	Perugia	0.47
62	Pavia	0.56	Ferrara	0.47
63	Matera	0.56	Firenze	0.47
64	Chieti	0.56	Pesaro Urbino	0.46
65	Novara	0.56	Trieste	0.46
66	Pescara	0.56	Chieti	0.45
67	Macerata	0.56	Salerno	0.45
68	Lucca	0.56	Bari	0.45
69	Potenza	0.55	Nuoro	0.45
70	Catanzaro	0.55	Siena	0.44
71	Latina	0.55	Massa-Carrara	0.44
72	Viterbo	0.55	Agrigento	0.44
73	Agrigento	0.55	Sud Sardegna	0.44
74	Messina	0.54	Imperia	0.44
75	Enna	0.54	Lecco	0.44
76	Cagliari	0.54	Terni	0.44
77	Verbano-Cusio-Ossola	0.54	La Spezia	0.44
78	Sondrio	0.53	Monza e della Brianza	0.44
79	Benevento	0.53	Siracusa	0.44
80	Perugia	0.53	Livorno	0.43
81	Prato	0.53	Ancona	0.42
82	Ancona	0.53	Biella	0.42
83	Grosseto	0.53	Pescara	0.42
84	Campobasso	0.52	Foggia	0.42
85	Barletta-Andria-Trani	0.52	Frosinone	0.41
86	Imperia	0.52	Potenza	0.41
87	Sassari	0.52	Pisa	0.41
88	Monza e della Brianza	0.52	Sassari	0.40
89	Trapani	0.51	Cagliari	0.39
90	Frosinone	0.51	Palermo	0.39
91	Asti	0.51	Varese	0.38
92	Terni	0.50	Como	0.38
93	Massa-Carrara	0.50	Grosseto	0.38
94	Biella	0.50	Taranto	0.37
95	Foggia	0.50	Brindisi	0.37
96	Caltanissetta	0.50	Novara	0.37
97	Ascoli Piceno	0.49	Lecce	0.37
98	Rieti	0.48	Rieti	0.36
99	Nuoro	0.48	Reggio Calabria	0.36
100	La Spezia	0.47	Cosenza	0.35
101	Isernia	0.46	Aosta	0.34
102	Savona	0.45	Sondrio	0.34
103	Trieste	0.44	Enna	0.33
104	Genova	0.43	Genova	0.27
105	Lecce	0.43	Crotone	0.26
106	Fermo	0.43	Vibo Valentia	0.24
107	Aosta	0.30	Catanzaro	0.14

Appendix E: Robustness Check

Appendix E contains the estimation results of the robustness check, testing the sensitiveness of our baseline estimates to different kinds of spatial weight matrices. The related marginal effects are presented in Table 5 of the paper.

Table E1: Robustness Check

SDF-CSD	W2		W		W2,W		W,W2		Wid,W2		Wid,W	
	Coeff.	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*	*Coeff.*	*SD*
β_0	3.87***	0.53	4.29***	0.65	3.47***	0.46	3.83***	0.42	6.20***	1.27	6.74***	1.26
β_L	0.61***	0.02	0.59***	0.02	0.61***	0.02	0.60***	0.02	0.63***	0.02	0.63***	0.02
β_{AA}	0.13***	0.02	0.14***	0.02	0.14***	0.02	0.15***	0.02	0.14***	0.02	0.14***	0.02
β_M	0.07***	0.01	0.09***	0.01	0.07***	0.01	0.08***	0.01	0.07***	0.01	0.07***	0.01
β_{WEF}	0.20***	0.01	0.19***	0.01	0.20***	0.01	0.20***	0.01	0.19***	0.01	0.18***	0.01
β_t	-0.01	0.01	-0.02	0.02	-0.01	0.01	-0.01	0.02	-0.02	0.02	-0.03	0.02
ρ	0.14*	0.08	-0.06	0.12	0.29***	0.06	0.01	0.05	0.08	0.18	0.12	0.18
θ_L	-0.06	0.08	0.12	0.09	-0.19***	0.06	0.10**	0.05	-0.23	0.20	-0.26	0.19
θ_{AA}	-0.25***	0.05	-0.15***	0.04	-0.27***	0.04	-0.11***	0.03	-0.67***	0.18	-0.81***	0.16
θ_M	0.02	0.04	0.02	0.03	0.01	0.04	-0.01	0.03	0.37**	0.16	0.44***	0.17
θ_{WEF}	0.02	0.05	0.03	0.03	0.01	0.04	0.01	0.03	-0.04	0.21	-0.11	0.22
ϕ_S	0.70***	0.18	0.51***	0.15	0.73***	0.18	0.65***	0.16	0.75***	0.19	0.73***	0.19
ϕ_B	-1.72***	0.46	-1.57***	0.37	-1.64***	0.46	-1.70***	0.41	-1.64***	0.51	-1.57***	0.54
ϕ_F	0.22**	0.11	0.12	0.08	0.22**	0.11	0.15	0.10	0.29**	0.12	0.30**	0.12
ϕ_D	0.37**	0.15	0.24*	0.13	0.39**	0.15	0.26*	0.14	0.50***	0.14	0.57***	0.14
ϕ_H	-0.06	0.11	-0.11	0.08	-0.03	0.11	-0.11	0.10	0.06	0.10	0.06	0.09
ϕ_{Sub}	0.71***	0.10	0.63***	0.09	0.71***	0.10	0.68***	0.09	0.72***	0.11	0.70***	0.11
ϕ_Y	0.66***	0.20	0.65***	0.17	0.68***	0.20	0.60***	0.19	0.61***	0.20	0.68***	0.20
ϕ_W	0.55***	0.18	0.70***	0.16	0.54***	0.18	0.64***	0.17	0.46***	0.18	0.45***	0.18
τ	0.37***	0.12	0.47***	0.13	0.15	0.12	0.55***	0.08	0.48***	0.10	0.43***	0.09
γ	0.37***	0.07	0.27***	0.09	0.12**	0.05	0.48***	0.06	0.38***	0.06	0.15***	0.05
σ_u^2	0.05	-	0.08	-	0.04	-	0.06	-	0.04	-	0.04	-
σ_v^2	0.09	-	0.09	-	0.09	-	0.09	-	0.09	-	0.09	-
LL	-284.61	-	-314.25	-	-292.08	-	-289.72	-	-275.89	-	-285.31	-

$***: pvalue \leq 0.01$; $**: pvalue \leq 0.05$; $*: pvalue \leq 0.10$

NB. W2: second order contiguity matrix associated with all the spatial terms; W: first order contiguity matrix associated with all the spatial terms; W2,W: second order contiguity matrix associated with the frontier function and first order contiguity matrix associated with the two error components; W,W2: first order contiguity matrix associated with the frontier function and second order contiguity matrix associated with the two error components; Wid,W2: dense inverse distance matrix associated with the frontier function and second order contiguity matrix associated with the two error components; Wid,W: dense inverse distance matrix associated with the frontier function and first order binary contiguity matrix associated with the two error components.

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